Control of Networked Systems Using the Scattering Transformation

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Abstract

In a networked control system (NCS) the plant and the controller are spatially separated and the control loop is closed through a communication network. Communication time delay in a NCS degrades the performance and may lead to instability. In this article the scattering transformation is applied to NCS, for the first time, in order to guarantee stability in the presence of unknown constant time delay. The scattering transformation approach in its original version relies on the assumption that all subsystems are passive. This article extends the approach to non-passive, static-output-feedback-stabilizable plants. We consider linear time invariant (LTI) systems here. It is furthermore shown that no knowledge of the time delay value is necessary for the analysis and design of the closed loop system. Lastly, experimental validation shows that the proposed approach is superior to a delay-dependent controller and the Smith predictor, as far as stability, performance, and sensitivity to time delay are concerned.

Index Terms

Networked control systems, delay-independent stability, linear systems.

I. Introduction

THE use of digital communication networks for signal transmission in control systems offers significant advantages over the traditional point-to-point connections, in terms of reduced wiring and cost, increased modularity, high flexibility, and reconfigurability. Furthermore, the use of the Internet as a communication medium offers the possibility of remote control of the systems without investing in the network infrastructure. Therefore, traditional control systems are increasingly replaced by systems in which the plant and the controller are connected through a network, see Fig. 1. Such NCS have been already adopted to numerous applications, see e.g. [1]–[3]. However, the signal transmission over the communication network can not be regarded as ideal. Time delay, packet loss and the limited communication resources constitute major challenges. These network induced effects depend principally on the policy the network adopts for sharing the common communication resources, the number of active nodes, the network configuration, or even the number of intermediate nodes in case of the Internet. Hence, these parameters are not exactly known during the controller design stage.

Specifically designed networks have been mainly used in NCS, known as control networks. In several cases control networks, e.g. DeviceNet, can guarantee constant time delay as long as the network is not saturated [4], [5]. In any case control networks can at least guarantee bounded time-varying delay. On the other hand in general purpose networks, e.g. Ethernet, time delay and packet loss behave unpredictably and no guarantees can currently be given for maximum values. With the increasing deployment of networked real time applications though, quality of service (QoS) functions are likely to be implemented in the near future. Such QoS functions guarantee a certain communication quality, e.g. in terms of bounded or even constant time delay. Time-varying delay with an upper bound can be reformulated to constant with the addition of buffers on the communication network [6], [7], a commonly used technique in praxis.

In this article the unknown constant time delay challenge is addressed. For a general overview on constant time delay methodologies see [8], [9]. Time delay in the control loop deteriorates the performance and can lead to instability. Constant time delay methodologies are distinguished between delay-dependent and delay-independent, according to whether a bound on the time delay value is necessary for stability guarantees or not. Some commonly in praxis used delay-dependent techniques include specifically tuned PID controllers [10] [11], and the Smith predictor [12]. The Smith predictor gives good performance, nevertheless it requires full plant and time delay knowledge, and is sensitive to modelling errors.

Delay-independent control methodologies on the other hand, are usually based on the small gain theorem, which requires the gain of the open loop transfer function at all frequencies to be smaller than one. The small gain theorem is rather conservative for real control applications. For instance free integrators in the open loop transfer function are not allowed, leading to unavoidable significant steady state error. Another delay-independent methodology, commonly used in force feedback telepresence applications, is the scattering transformation. The scattering transformation was initially derived from classical

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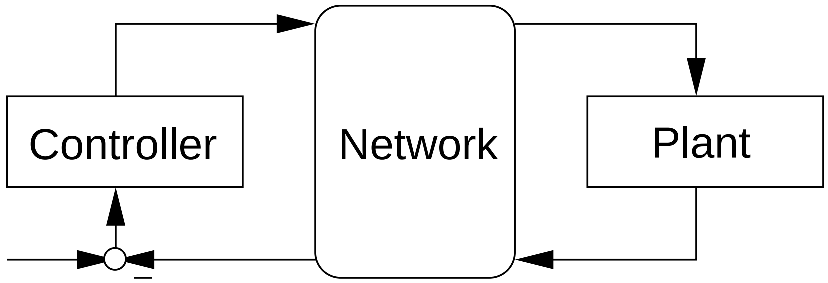

Fig. 1. Networked control system architecture.

network theory [13] in order to describe the energy flow through the network[1] and was used in a control context for the first time in [14]. It was firstly defined in continuous time and constant time delay [15]. Extensions have appeared for time-varying delay [16], discrete time [17] and packet loss [18]. However the scattering transformation is based on the passivity assumption of the plant and the controller, i.e. plant and controller must not generate energy. Passivity is a restrictive requirement in general control applications. This can be clearly seen in the single-input-single-output (SISO) LTI systems, where only transfer functions with relative degree one or zero are allowed. A simple second order system is in consequence non-passive.

In this article, for the first time, the scattering transformation is applied to non-passive LTI NCS with unknown constant time delay, and a new stability condition is derived. The approach is extended to static-output-feedback-stabilizable plants. Contrary to the small gain theorem, the scattering transformation can simultaneously guarantee delay-independent stability and steady state error zero. Performance issues are furthermore considered. Unique, compared to other time delay approaches, is the consideration of sensitivity issues with respect to time delay. Sensitivity and performance criteria are defined independently of the time delay value. No time delay knowledge is necessary during the controller design. A comparison is performed with a delay-dependent controller and a Smith predictor in which all the controllers are designed with numerical optimization. The proposed approach shows significantly lower sensitivity to time delay. In simulation and experiment the proposed approach shows better performance in a wide range of time delay values compared to the other two approaches.

The remainder of this article is organized as follows: Section II introduces the necessary background. A novel stability condition is derived in Section III, followed by performance aspects in Section IV. A case study is presented in Section V and conclusions are given in Section VI.

II. Background

Let $\mathbb{R}_+$ be the set of non-negative real numbers and $\mathbb{R}^m$ the Euclidean space of dimension m. Consider a LTI causal system $h : u \rightarrow y$ with input $u(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}^m$, output $y(\cdot) : \mathbb{R}_+ \rightarrow \mathbb{R}^m$ and $y(t) = 0,\ t \leq 0$. With $G(s)$ the $m \times m$ transfer matrix of h is denoted where $s = \sigma + j\omega$ the Laplace variable and with $s_r = \sigma_r + j\omega_r$ the roots of all elements of $G(s)$. With $|G|^\infty$ the H_∞ norm of the transfer matrix $G(s)$ is denoted. For convenience of notation, where non-ambiguous, the Laplace variable s is dropped.

A. Positive real transfer functions

Positive realness is an equivalent notion to passivity for LTI systems in the frequency domain, which will be used in the remainder of this article.

Definition 1: A transfer matrix $G(s)$ is positive real (PR) if :

$$G(s) + G^*(s) \geq 0, \quad \text{for every } \sigma > 0,$$

The positive realness of a transfer matrix can be concluded by its stability and its values on the imaginary axis.

Proposition 1: [19] A transfer matrix $G(s)$ is PR if and only if :

a) $\sigma_r \leq 0,$

b) $G(j\omega) + G^*(j\omega) \geq 0$ for every $\omega \geq 0,$

c) if $\sigma_r = 0,\ s_r$ is simple pole and $\lim_{s \rightarrow j\omega_r} (s - j\omega_r)G(s) \geq 0.$

Proposition 1 states that the transfer matrix should be stable, positive semi-definite on the imaginary axis, the poles on the imaginary axis should be simple, and their associate residues non-negative. In case of SISO systems condition b) implies that the Nyquist plot lies in the right half plane and can be easily checked.

[1]In classical network theory a network is defined as a system composed of a finite number of interconnected elements, e.g. resistors, capacitors, coils, and should not be confused with the communication network in NCS.

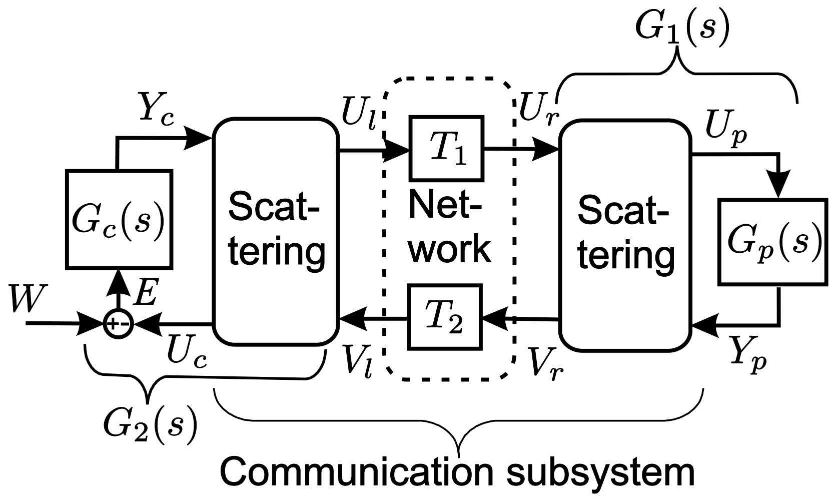

Fig. 2. NCS with scattering transformation.

Another notion necessary in the following is strictly positive realness.

Definition 2: A transfer matrix $G(s)$ is strictly positive real (SPR) if $G(s-\epsilon)$ is PR for some $\epsilon > 0$.

Necessary conditions for a SPR system is that it is strictly stable, i.e. $\sigma_r < 0$ and strictly positive definite on the $j\omega$-axis, i.e.

$$G(j\omega) + G^*(j\omega) > 0, \quad \text{for every } \omega \geq 0.$$

The major difference between PR and SPR systems is that the first can tolerate poles on the imaginary axis as long as they are simple and their associated residues are real and non-negative.

B. Scattering transformation

In the rest of this article for convenience of notation the SISO case will be considered, i.e. $m = 1$. The scattering transformation acts on the input and output U, Y of a system $G(s) = \frac{Y(s)}{U(s)}$ with the equations

$$U_s = \frac{\sqrt{2}}{2}\left(\sqrt{b}U + \frac{1}{\sqrt{b}}Y\right), \quad V_s = \frac{\sqrt{2}}{2}\left(-\sqrt{b}U + \frac{1}{\sqrt{b}}Y\right), \tag{1}$$

where $U_s(s), V_s(s)$ are the input and output of the system extended by the scattering transformation, and $b > 0$ constant, resulting to the system

$$G_s(s) = \frac{V_s(s)}{U_s(s)} = \frac{G(s) - b}{G(s) + b}. \tag{2}$$

In the case of $b = 1$ the scattering operator is obtained, originally defined in classical network theory [13]. The scattering operator provides an equivalence between positive realness and the H_∞ norm of a transfer function, i.e. the next proposition holds.

Proposition 2: [14] A transfer function G is PR if and only if the H_∞ norm of its scattering operator is less than or equal to one.

III. Main result

A. System description

We consider a system consisting of a SISO LTI plant and controller with transfer functions $G_p(s) = \frac{Y_p(s)}{U_p(s)}$ and $G_c(s) = \frac{Y_c(s)}{U_c(s)}$ respectively, where $U_p(s),\ Y_p(s)$ and $U_c(s),\ Y_c(s)$ are the Laplace transforms of the input and output of the plant and the controller. The plant is connected to the controller through the communication network. However, between the communication network, the plant and the controller the scattering transformation is inserted, see Fig. 2. Based on (2), the subsystem $G_1(s)$, including the plant $G_p(s)$ and the right hand scattering transformation, is given by $G_1(s) = \frac{V_r(s)}{U_r(s)} = \frac{G_p(s)-b}{G_p(s)+b}$, where $U_r(s), V_r(s)$ are the right hand values that are communicated through the network, see Fig. 2. Accordingly for the subsystem $G_2(s) = \frac{U_l(s)}{V_l(s)}$ where again $U_l(s), V_l(s)$ describe the left hand communicated values. The network is modelled as a time delaying two-port with forward and backward time delays T_1 and T_2 respectively. The time delays $T_1,\ T_2$ are assumed to be constant but unknown. Without loss of generality, we assume further, that no energy is stored initially in the communication network, i.e.

$$u_l(\theta) = 0, \quad \text{for every } \theta \in [-T_1, 0],$$
$$v_r(\theta) = 0, \quad \text{for every } \theta \in [-T_2, 0].$$

The relations between the left and right hand transmitted variables are given accordingly by $U_r(s) = U_l(s)e^{-j\omega T_1}$ and $V_l(s) = V_r(s)e^{-j\omega T_2}$.

B. Stability analysis

In the next it will be shown that the stability of the above system can be concluded from the positive realness of an auxiliary transfer function. Contrary to the typical case of the scattering transformation in telepresence systems, plant and controller do not have necessarily to be PR. The next theorem holds.

Theorem 1: If there exists a $b > 0$ so that

$$K(s) = \frac{1}{b}\frac{b^2 G_c + G_p}{1 + G_c G_p}, \tag{3}$$

is PR, the closed loop system is stable independently of the time delay. If $K(s)$ is SPR, then the closed loop system is asymptotically stable.

Proof: The open loop transfer function G_e from the controller input E to the output of the left hand scattering transformation U_c, see Fig. 2, is computed my means of the scattering transformation equations (1),

$$G_e(s) = bG_c(s)\frac{1 + e^{-sT}G_1(s)}{1 - e^{-sT}G_1(s)}, \tag{4}$$

with $T = T_1 + T_2$ the roundtrip time delay in the communication network and

$$G_1(s) = \frac{G_p(s) - b}{G_p(s) + b}. \tag{5}$$

The poles s_r of the closed loop system are placed where $G_e(s_r) = -1$ holds, thus, substituting (5) in (4) we get

$$-\frac{1 + e^{-s_r T}}{1 - e^{-s_r T}} = \frac{1}{b}\frac{b^2 G_c(s_r) + G_p(s_r)}{1 + G_c(s_r)G_p(s_r)} = K(s_r). \tag{6}$$

Considering the real part of (6) it follows

$$\mathrm{Re}\{K(s_r)\} = -\frac{1 - e^{-2\sigma_r T}}{(1 - e^{-\sigma_r T}\cos\omega_r T)^2 + (e^{-\sigma_r T}\sin\omega_r T)^2}. \tag{7}$$

Lets assume $K(s)$ to be PR. Then there exists no solution s_r of (7) in the open right half plane, i.e. $\sigma_r > 0$, as the real part of $K(s_r)$ is always non-negative, while the right part of (7) is negative. Furthermore, if $K(s)$ is SPR, then (7) has no solution s_r on the imaginary axis as well, because the real part of $K(s_r)$ on the imaginary axis is positive, while the right part of (7) is zero. Thus, the system is asymptotically stable. ■

Condition (a) of Proposition 1 implies the stability of $K(s)$. $K(s)$ has the same poles with the closed loop system without time delay and scattering transformation. Hence, the set of the stabilizing controllers is a subset of the controllers which stabilize the system without time delay and scattering transformation. Condition (b) of Proposition 1 restricts further the set of admissible controllers, imposing restrictions on the amplitude and argument of the controller G_c. Taking the real part of $K(j\omega)$ it follows

$$\begin{aligned}&\mathrm{Re}\{K\} = \\ &\mathrm{Re}\{G_c\}\frac{b^2 + \|G_p\|^2}{b\|1 + G_c G_p\|^2} + \mathrm{Re}\{G_p\}\frac{1 + b^2\|G_c\|^2}{b\|1 + G_c G_p\|^2} \geq 0,\end{aligned} \tag{8}$$

where the dependence on $j\omega$ is suppressed for convenience of notation. At a fixed frequency ω_0, (8) defines admissible areas on the complex plane for $G_c(j\omega_0)$. If $\mathrm{Re}\{G_p(j\omega_0)\} < 0$, (8) defines a disk of the right half complex plane with the center on the real axis in position R and radius r given by,

$$R = -\frac{b^2 + \|G_p\|^2}{2b^2\mathrm{Re}\{G_p\}}, \qquad r^2 = \frac{(b^2 + \|G_p\|^2)^2}{4\mathrm{Re}\{G_p\}^2 b^4} - \frac{1}{b^2}, \tag{9}$$

in which $G_c(j\omega_0)$ must lie in. In case $\mathrm{Re}\{G_p(j\omega_0)\} \geq 0$ a disk on the left half complex plane is defined with center and radius given again by (9), which $G_c(j\omega_0)$ must lie out from, see also Fig. 3. These restrictions on the amplitude and argument of the controller $G_c(j\omega_0)$ can be accordingly expressed

$$\begin{aligned}&|\arg\{G_c\}| \leq \tan^{-1}\left(\frac{\sqrt{r^2 - (\mathrm{Re}\{G_c\} - R)^2}}{\mathrm{Re}\{G_c\}}\right) \text{ and} \\ &R - r \leq \|G_c\| \leq R + r, \quad \text{if} \quad \mathrm{Re}\{G_p\} < 0, \\ &|\arg\{G_c\}| \leq \pi + \tan^{-1}\left(\frac{\sqrt{r^2 - (\mathrm{Re}\{G_c\} + R)^2}}{\mathrm{Re}\{G_c\}}\right) \text{ when} \\ &-R - r \leq \|G_c\| \leq -R + r, \quad \text{if} \quad \mathrm{Re}\{G_p\} \geq 0.\end{aligned}$$

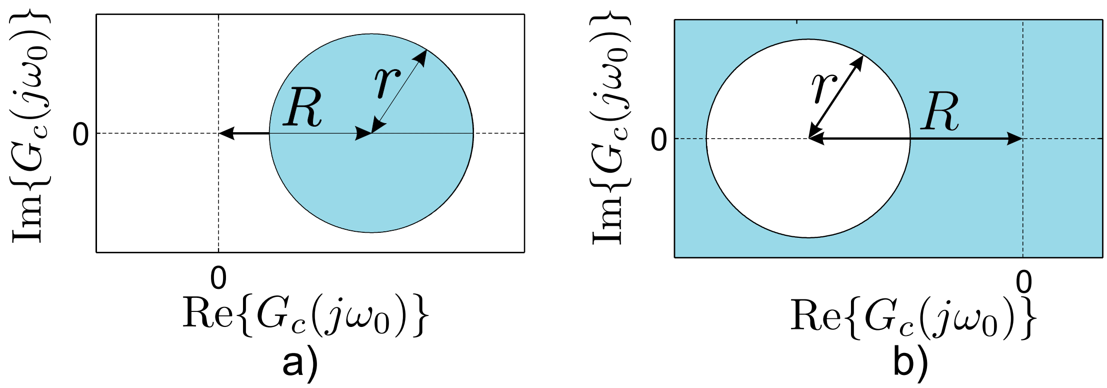

Fig. 3. a) Admissible (dark) area of $G_c(j\omega_0)$ for negative Re$\{G_p(j\omega_0)\}$. b) Admissible (dark) area of $G_c(j\omega_0)$ for positive Re$\{G_p(j\omega_0)\}$.

Based on (8) it is clear that a negative real part of G_p in some range of frequencies can be compensated by a positive real part of G_c in the same range and vice versa, thus plant and controller do not have to be PR. In case of PR plant and controller stability, independently of b, can be guaranteed as it can be directly seen from (8) for non-negative Re$\{G_p\}$ and Re$\{G_c\}$.

An additional useful interpretation can be given in terms of the small gain theorem. Using Proposition 2 the condition of Theorem 1 can be rewritten

$$|G_{OL}|^\infty = |G_1 G_2|^\infty = \left| \frac{G_p - b}{G_p + b} \frac{1 - bG_c}{1 + bG_c} \right|^\infty < 1, \tag{10}$$

where G_1, G_2 represent the transformed plant and controller respectively. Thus, Theorem 1 is equivalent to the small gain condition in the loop of the extended by the scattering transformation plant $G_1(s)$, and controller $G_2(s)$, see Fig. 2. Since the controller can be arbitrarily chosen, (10) can be satisfied as long as G_1 is stable. The roots of G_1 are those of the plant with static output feedback $\frac{1}{b}$. Since b can be freely chosen the approach is applicable to all static-output-feedback-stabilizable plants. The stability in the typical case of scattering transformation in telepresence systems can be accordingly expressed $|G_1|^\infty |G_2|^\infty < 1$. Thus, the reduction of the conservatism through Theorem 1, comes from the fact that in general $|G_1 G_2|^\infty \leq |G_1|^\infty |G_2|^\infty$. Although Theorem 1 is valid only for constant time delay, by falling back into the more conservative stability result $|G_1|^\infty |G_2|^\infty < 1$, all the extensions of the scattering transformation for time-varying delay, e.g. [16], and packet loss, e.g. [18], are straightforward to apply here.

IV. Performance aspects

The transfer function of the closed loop system from the reference input $W(s)$ to the output of the plant $Y_p(s)$, see Fig. 2, is computed from the transformation equations (1) to be

$$G(s) = \frac{Y_p(s)}{W(s)} = G_0(s) G_{sc}(s) e^{-sT_1}, \tag{11}$$

where

$$G_{sc}(s) = \frac{2}{K(s)(1 - e^{-sT}) + (1 + e^{-sT})}, \tag{12}$$

with $K(s)$ defined according to (3) and

$$G_0(s) = \frac{G_c(s) G_p(s)}{1 + G_c(s) G_p(s)},$$

the typical closed loop transfer function without communication network and time delay. The system can be interpreted as a series connection of the standard closed loop system $G_0(s)$, the subsystem $G_{sc}(s)$ which incorporates the effect of the time delay and the scattering transformation on the closed loop behavior, and the forward time delay shift e^{-sT_1}.

A. Steady state error

The steady state behavior with scattering transformation and time delay is equivalent to that without scattering transformation and without time delay, as easily derivable by setting $s = 0$ in (11) (12), resulting in $G(0) = G_0(0)$. Thus, concerning the steady state error, the controller can be designed without considering the network. In terms of steady state error the proposed approach clearly outperforms the standard small gain approach, which requires $|G_c(j\omega) G_p(j\omega)| < 1, \omega \geq 0$, i.e. free integrators in the open loop are not allowed. This leads to a rather large steady state error, e.g. $|y(t)|_{t\to\infty} < \frac{1}{2}|w|$ for a reference step input w. In the proposed approach free integrators in plant or controller do not necessarily violate the positive realness of $K(s)$, thus delay-independent stability and steady state error zero can be simultaneously guaranteed. This can be demonstrated by examples. Consider a non-PR plant with integrator $G_p(s) = \frac{k_p}{s(s+a)}$, $a > 0$, and a proportional controller $G_c(s) = k_c$. $K(s)$ (3) is stable since arg$\{G_p(s) G_c(s)\} < 180^o$. Furthermore, for the real part of $K(j\omega)$ we reach Re$\{K(j\omega)\} = \omega^4 k_c b^2 + \omega^2 [k_c b^2 (a^2 - k_p k_c) - k_p] + k_p^2 k_c$, which for sufficiently small k_c and sufficient large b, is positive for every ω.

B. Sensitivity to time delay

The sensitivity function with respect to the time delay T can be written

$$S_T^{G^*} = \frac{T}{G^*}\frac{dG^*}{dT} = -\frac{sTe^{-sT}(K-1)}{K(1-e^{-sT})+(1+e^{-sT})}, \tag{13}$$

where $G^* = G_0(s)G_{sc}(s)$ represents the closed loop transfer function (11) without the purely time delay shifting part e^{-sT_1}. As long as $K(s) \approx 1$ the sensitivity function with respect to time delay $S_T^{G^*} \approx 0$, i.e. the system shows low sensitivity to time delay and a good performance is guaranteed in a wide range of time delay values. The ideal case of $S_T^{G^*} = 0 \Rightarrow K(s) = 1$ can be achieved with a proportional controller $G_c(s) = 1/b$, which leads to $G_{sc} = 1 \Rightarrow G(s) = G_0(s)e^{-sT_1}$, i.e. the time delay has no effect on the transient response. Thus, the scattering transformation acts as the Smith predictor, moving the time delay "out of the loop", however, without requiring knowledge of the plant or the time delay value. Most of the times though, a proportional controller does not meet the performance requirements and compromise has to be made between performance and sensitivity to time delay.

An additional interpretation can be given in terms of the gain of the loop where the time delay is. Therefore, (13) is reformulated as

$$S_T^{G^*} = \frac{T}{G^*}\frac{dG^*}{dT} = sT\frac{G_{OL}e^{-sT}}{1-G_{OL}e^{-sT}}.$$

where $G_{OL} = G_1G_2$. The sensitivity to time delay becomes smaller for smaller $\|G_{OL}\|$. Note that based on Proposition 2 this does not contradict Theorem 1, i.e. $|G_{OL}|^\infty < 1$. Thus, during the controller design the stability requirement $|G_{OL}|^\infty < 1$ can be substituted with $|G_{OL}|^\infty < a < 1$, which ensures also lower sensitivity to time delay.

C. Zero time delay case

As the time delay reduces to zero, i.e. $T_1 = T_2 = T = 0$, the system reduces to that without scattering transformation, i.e. $G(s) = G_0(s)$ as straightforward computable from (11) (12). This is interesting, compared to the standard small gain approach, as the controller can be more aggressively designed for the zero time delay case. For zero time delay "nominal" performance is recovered. Together with low sensitivity to time delay, good performance is guaranteed for a wide range of time delay values.

Concluding the above, the overall design goal is to find a controller and a value for b such that the closed loop system *without* the time delay and the scattering transformation has a satisfying response, while $K(s)$ is PR and approximately one for a broad range of frequencies. Stability and sensitivity to time delay requirements can be conjointly expressed as $|G_{OL}(j\omega)|^\infty < a < 1$. Lower sensitivity to time delay is achieved for lower values of a. The significant advantage of the above is that no knowledge of the time delay value is required during the controller design.

V. Performance Evaluation

In order to show the efficacy of the proposed approach a comparison is performed with a delay-dependent controller and the Smith predictor. The comparison is in some sense "unfair" since delay-dependent methods lead to less conservative results at least for the nominal time delay. The small gain theorem, however, which is the only delay-independent alternative approach, cannot be applied because there is an integrator in the system under consideration. The system under consideration is an one-degree-of-freedom (1DOF) robotic system used later for experimental validation. The robotic system is approximated by the transfer function

$$G_p(s) = \frac{73}{s^2 + 10.15s}.$$

from the input voltage (V) (which is assumed to be proportional to the torque, see Section V-C) to the output angle (rad). The transfer function is obtained by standard least square identification of the response to square pulse input. Note that the plant is not PR as arg$\{G_p(j\omega)\} \geq 90^o$, $\omega \geq 0$, i.e. the Nyquist plot lies in the left half plane.

A. Controller design

In order to achieve a fair comparison the controllers for all approaches are designed by numerical optimization using the ITAE (Integral Time Absolute Error) performance index as cost function

$$J = \int_0^{t_f} \tau|e(\tau)|\mathrm{d}\tau.$$

The numerical optimization is performed for a nominal time delay of $T = 300$ms over a horizon of $t_f = 5$s using `fmincon` of the Matlab optimization toolbox. As basic structure for the controller a lead-lag element

$$G_c(s) = k\frac{s+a}{s+c},$$

is considered where $k, a, c > 0$ are parameters to be determined by numerical optimization. The exact design procedure for all three cases is explained in the next.

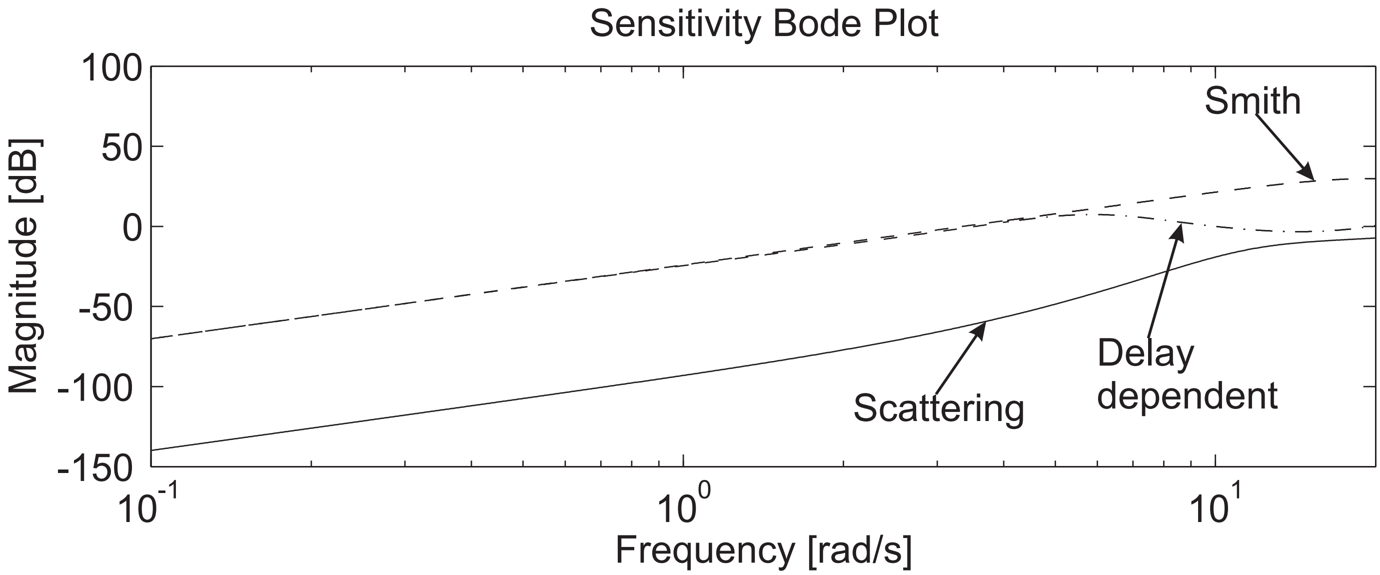

Fig. 4. Amplitude of the sensitivity function with respect to time delay, of the scattering approach, delay-dependent controller and Smith predictor.

1) Scattering transformation: The constrained optimization problem can be formulated as $\min_{k,a,c,b} J$ subject to the constraints $k, a, c, b > 0$. The numerical optimization gives the controller for the input-output transformation

$$G_{tr}(s) = \frac{1.3981(s + 9.9114)}{s + 9.1558},$$

and $b = 0.6203$. The optimal cost function value is $J = 0.1976$. For the above controller it is straightforward to compute that $K(j\omega)$ is PR and $\|K(j\omega) - 1\| < 0.1702$.

2) Delay-dependent controller: For the delay-dependent controller the lead-lag element is used directly to control the plant. Similarly to the previous case the optimization problem is represented by $\min_{k,a,c} J$ subject to the constraint $k, a, c > 0$ resulting in the controller

$$G_{dd} = \frac{3.2649(s + 4.7877)}{(s + 57.7271)},$$

with the optimal cost function value $J = 0.4797$.

3) Smith predictor: For the Smith predictor the controller G_c is designed initially without considering the time delay. The optimization problem is given by $\min_{k,a,c} J$ subject to the constraint $k, a, c > 0$ and $k < 5$ as otherwise an indefinitely large gain is given by the optimization, resulting in the controller

$$G_{c,sp} = \frac{5(s + 8.4642)}{s + 22.5354}$$

with the optimization error $J = 0.0871$. The finally used controller is given by

$$G_{sp}(s) = \frac{G_{c,sp}(s)}{1 + G_{c,sp}(s)G_p(s)[1 - e^{-sT}]},$$

where $T = 300\,\text{ms}$ represents the nominal time delay. Note, that full knowledge of the plant is required for the design. In the case of exact plant and time delay knowledge, the response of the time delayed system with Smith predictor equals the response of the system without time delay shifted in the time axis by the forward delay T_1.

TABLE I
SIMULATION RESULTS: ±5% SETTLING TIME

Time Delay (ms)	50	200	300	400
Scattering	0.58s	0.64s	0.60s	0.84s
Delay-dependent	1.64s	1.24s	0.58s	1.92s
Smith	unstable	unstable	0.30s	unstable

B. Simulations

The response to a step input with a final value of 0.2 rad is considered in the simulations. Additionally the disturbance rejection in the input of the plant is studied. A zero reference input is used, in order for the robotic system to remain to the initial position, and a step disturbance of 0.2V to its input is applied. Four different values for the round trip time delay are tested, the nominal time delay of $T = 300\,\text{ms}$, and $T = 50,\ 200,\ 400\,\text{ms}$, with equal forward and backward time delays, $T_1 = T_2 = \frac{T}{2}$.

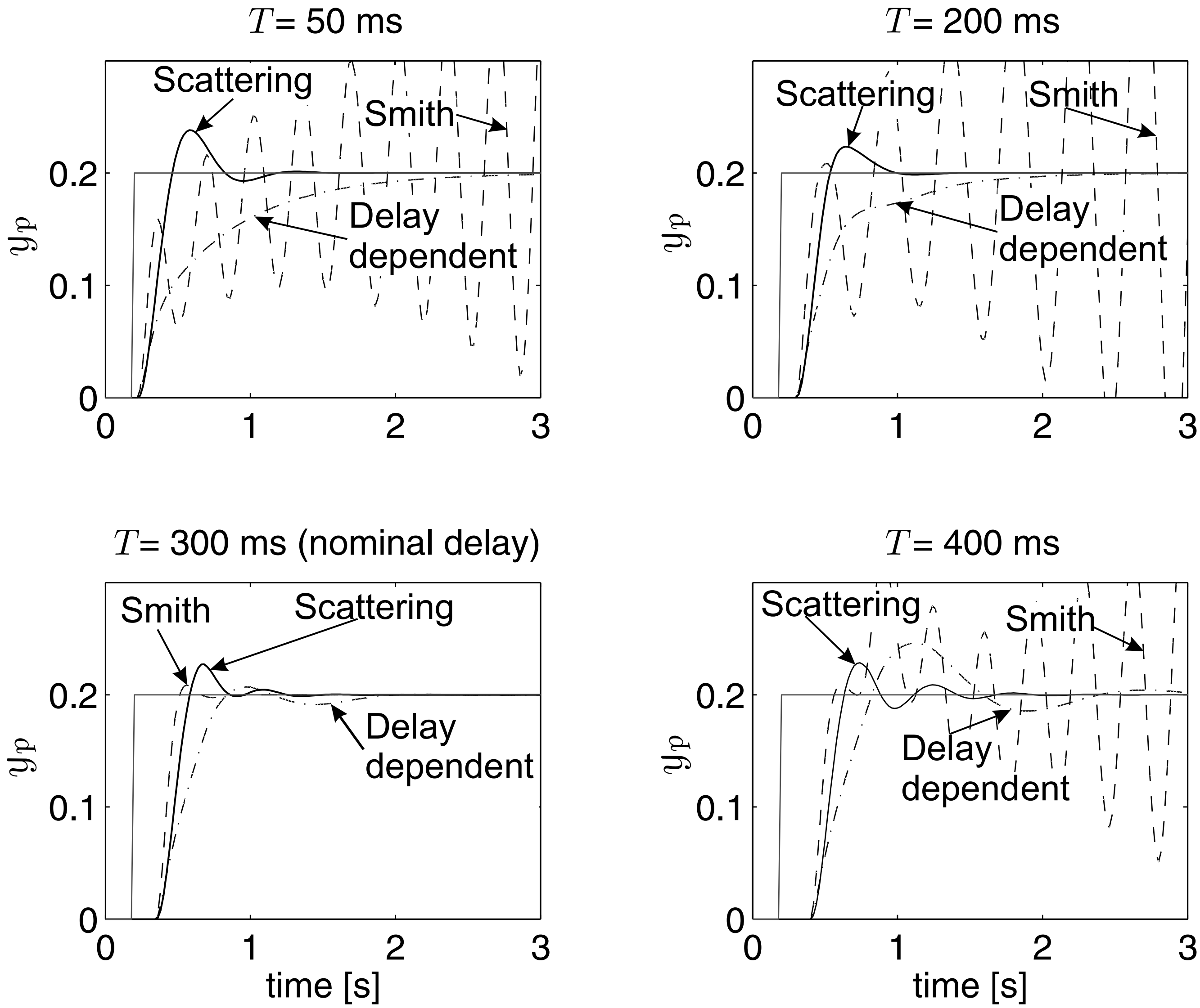

Fig. 5. Simulation step response of the system with scattering transformation, delay-dependent controller and Smith predictor for different values of time delay.

TABLE II
SIMULATION RESULTS: OVERSHOOT %

Time Delay (ms)	50	200	300	400
Scattering	19.11	11.78	13.59	14.24
Delay-dependent	none	none	3.52	23.04
Smith	unstable	unstable	4.31	unstable

Of specific interest is the stability, the performance and the sensitivity to time delay. The amplitude of the sensitivity function with respect to time delay for the various controllers in the nominal time delay $T = 300$ms, is shown in Fig. 4. The amplitude is plotted until the maximum cut-off frequency for the different systems, i.e. for the Smith predictor, $\omega = 21$ rad/sec. The proposed approach shows indeed significantly lower sensitivity to time delay. This is also verified in the simulation results that follow.

The simulation results for the step input are shown in Fig. 5, and Tables I and II. The scattering transformation approach performs well in all cases. Furthermore the response is slightly affected by the time delay value. The delay-dependent controller performs well in the nominal time delay; the settling time is slightly smaller than the scattering approach and the overshoot also small, but the performance becomes rather poor for different time delay values. The delay-dependent controller becomes unstable for $T \approx 850$ms. The Smith predictor gives indeed the best performance for the nominal time delay. However, in all the other time delay values the system becomes unstable.

The simulation results for the step disturbance in the plant input are shown in Fig. 6 and Table III. The scattering transformation approach shows the smallest remaining disturbance and the lowest sensitivity to time delay. The delay-dependent

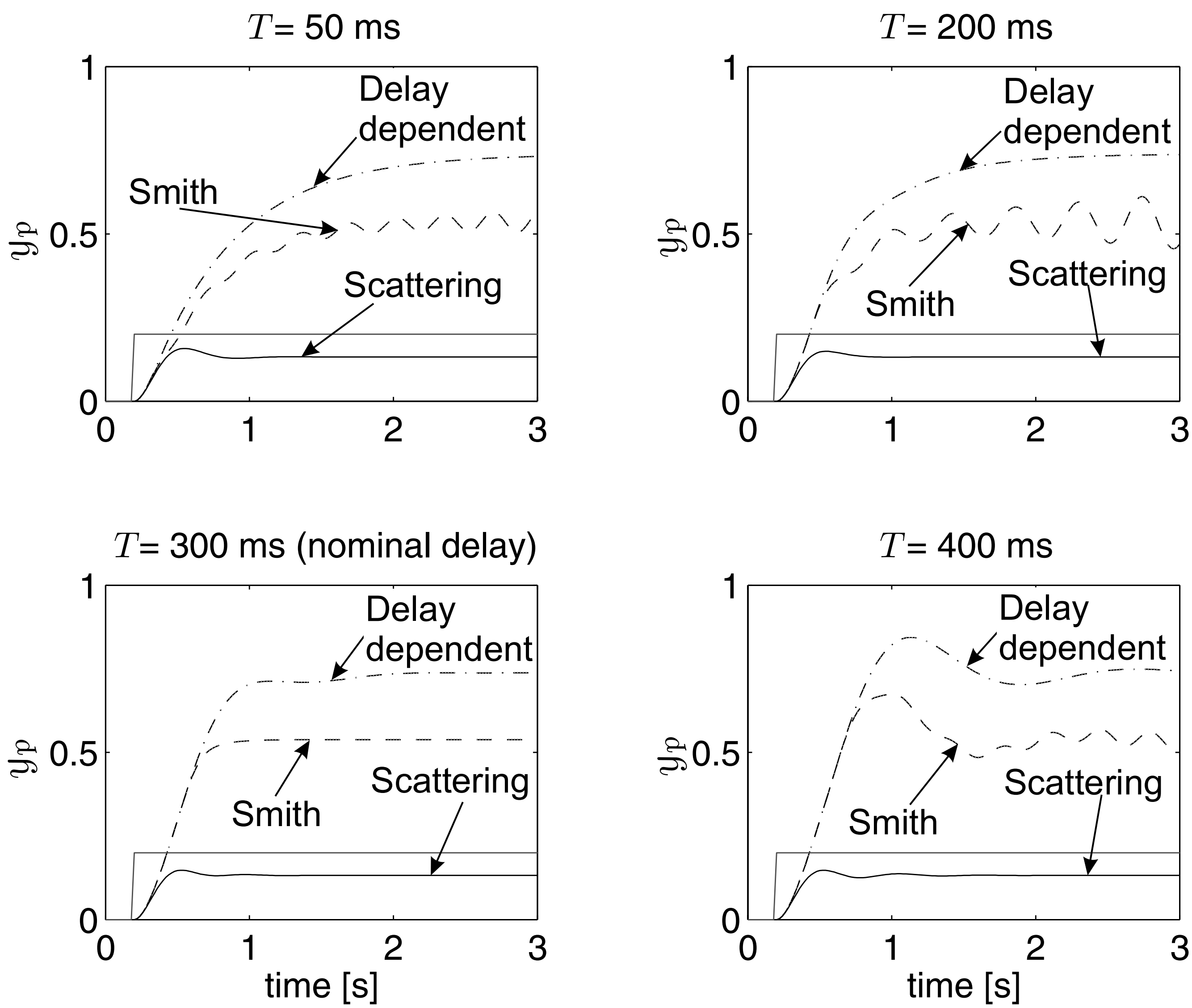

Fig. 6. Simulation response to step disturbance of the system with scattering transformation, delay-dependent controller and Smith predictor for different values of time delay.

TABLE III
SIMULATION RESULTS: REMAINING DISTURBANCE IN %

Time Delay (ms)	50	200	300	400
Scattering	66.1	66.1	66.1	66.1
Delay-dependent	369.2	369.3	369.3	369.2
Smith	unstable	unstable	269.0	unstable

controller and especially the Smith predictor are sensitive to time delay.

C. Experimental setup

The experimental testbed consists of the 1DOF robotic system shown in Fig. 7 connected to a PC running under Real-Time Linux. The original design can be found in [20]. The DC-motor torque is controlled over a PWM amplifier operated under current control with the reference signal given by a voltage from the D/A converter output of the I/O board. Thus, the input voltage can be considered to be proportional to the torque. The position of the lever is measured by an optic pulse incremental encoder and processed by a counter on the I/O board. The control loop including the controller and the communication network with constant time delay and the scattering transformation is implemented as MATLAB/SIMULINK blocksets. Standalone realtime code is generated directly from Matlab. The sampling time interval is $T_A = 1$ ms. In real NCS applications the right hand scattering transformation can be implemented as a local controller in the plant CPU which implements the network protocols. The right hand scattering transformation can be interpreted as a static-output-feedback-input-feedforward control. Since no dynamic is involved very limited computational power is necessary, so computational power is not an issue. The left hand scattering transformation can be implemented as part of the controller.

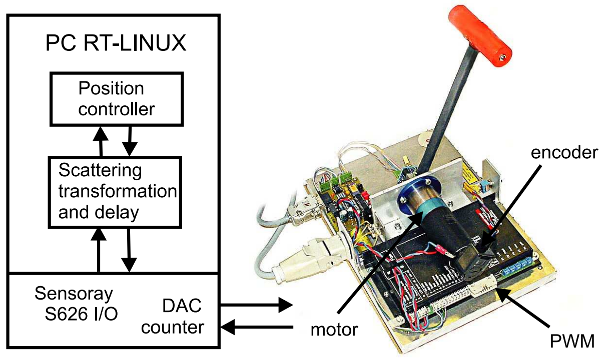

Fig. 7. Experimental testbed

TABLE IV
EXPERIMENTAL RESULTS: ±5% SETTLING TIME

Time Delay (ms)	**50**	**200**	**300**	**400**
Scattering	0.32s	0.38s	0.43s	0.48s
Delay-dependent	> 5	3.31s	3.28s	3.28s
Smith	unstable	unstable	0.48s	unstable

The experimental results for the step input are presented in Fig. 8, and Tables IV and V. The delay-dependent controller has a smaller steady state error but its response is very slow, e.g. in case of $T = 50$ms the settling time is more than 5s. The steady state error in all the cases is most likely due to unmodelled nonlinearities of the plant, e.g. backlash.

TABLE V
EXPERIMENTAL RESULTS: STEADY STATE ERROR IN %

Time Delay (ms)	**50**	**200**	**300**	**400**
Scattering	24.14	17.59	22.2	18.74
Delay-dependent	313.07	9.66	12.43	4.54
Smith	unstable	unstable	41.1	unstable

The experimental results for the step disturbance are presented in Fig. 9 and Table VI. The scattering transformation approach gives by far the smallest remaining disturbance in all cases. The Smith predictor becomes unstable only in the case with $T = 400$ms, however, the remaining disturbance is by far larger than the scattering transformation approach. In short, the proposed approach guarantees delay-independent stability and shows low sensitivity to time delay, while even delay-dependent approaches are outperformed.

VI. CONCLUSION

In this article a delay-independent control methodology is proposed for NCS based on the scattering transformation. A novel contribution is an extension of the known scattering transformation to non-PR, static-output-feedback-stabilizable LTI plants. Sensitivity to time delay and performance issues are furthermore examined. All the design goals can be defined without assuming knowledge of the time delay value. In a simulation example the proposed approach outperforms the Smith predictor and a delay-dependent controller as far as stability, performance and sensitivity to time delay are concerned. A hardware-in-the-loop experiment with a 1DOF robotic system verifies the efficacy of the proposed approach. Future work is to approach time-varying delay, packet loss, and non-linear systems.

TABLE VI
EXPERIMENTAL RESULTS: REMAINING DISTURBANCE IN %

Time Delay (ms)	**50**	**200**	**300**	**400**
Scattering	67.9	68.1	68.8	69.1
Delay-dependent	265.8	445.1	552.8	670.9
Smith	330.9	346.9	377.1	unstable

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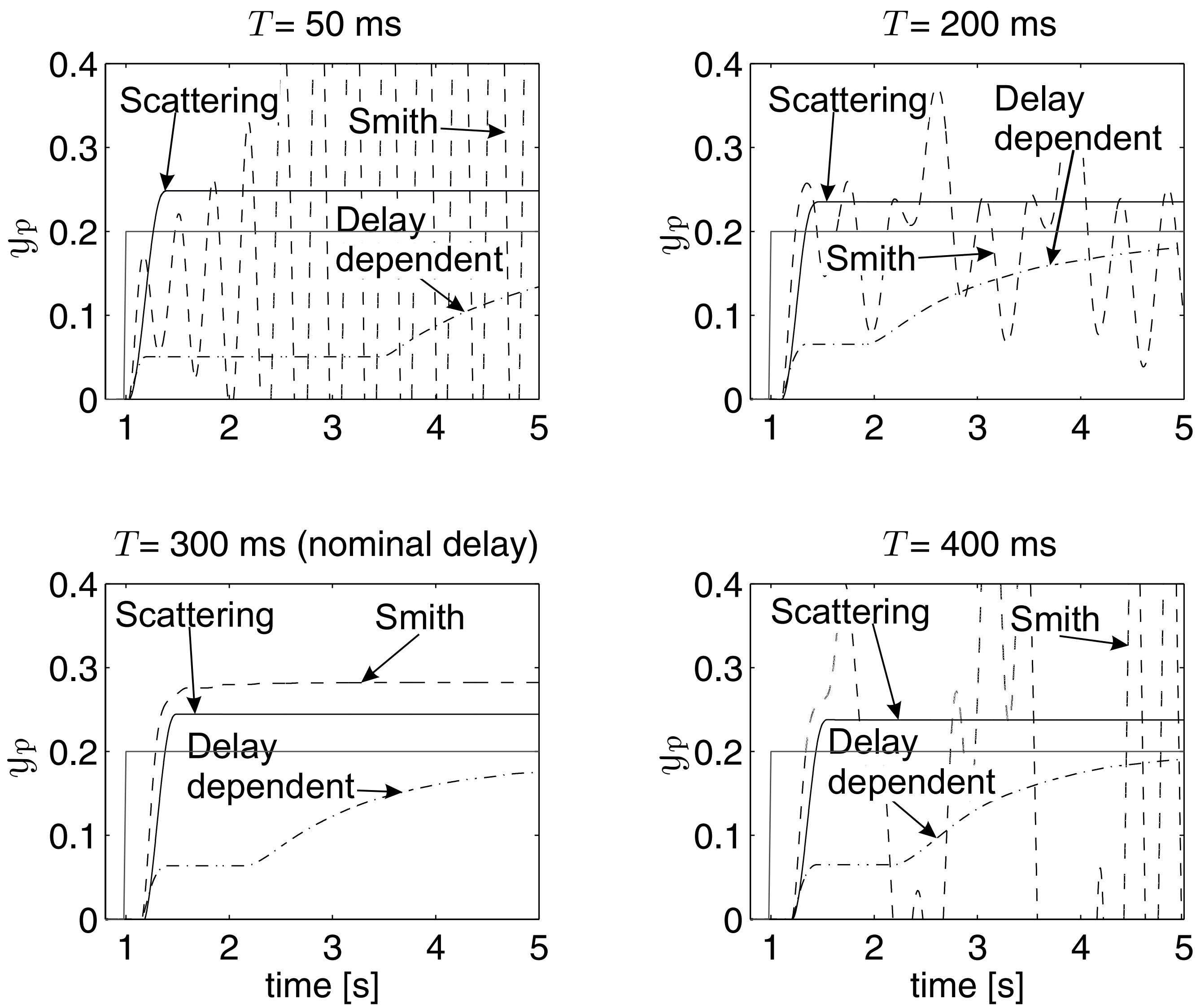

Fig. 8. Experimental step response of the system with scattering approach, delay-dependent controller and Smith predictor for different values of time delay.

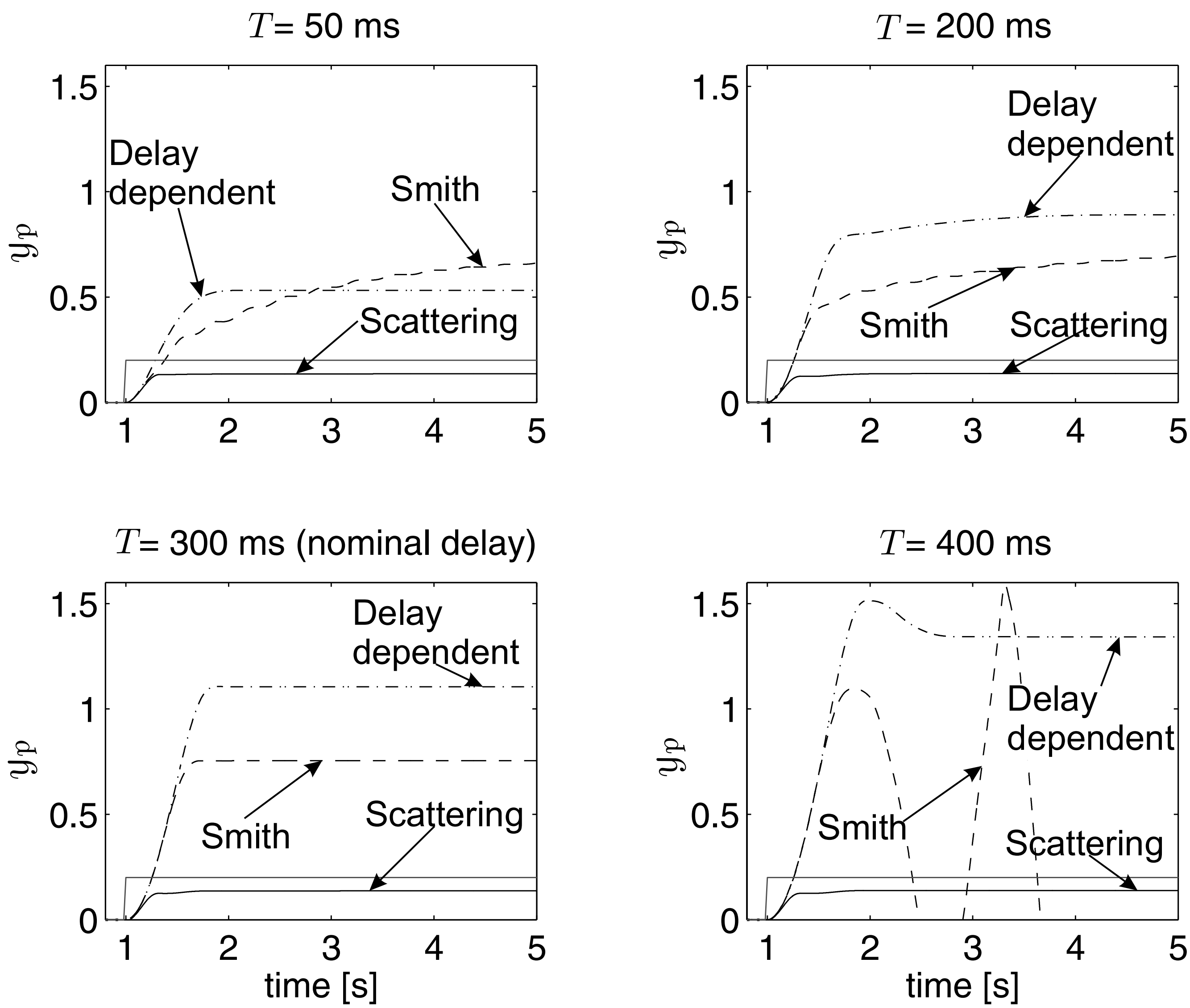

Fig. 9. Experimental response to step disturbance of the system with scattering transformation, delay-dependent controller and Smith predictor for different values of time delay.