

# Single-Stage FDE Supported by Training-Aided Channel Estimation for Coherent Optical Receivers

Fabio Pittalà<sup>1,3</sup>, Fabian N. Hauske<sup>2</sup>, Josef A. Nossek<sup>3</sup>

<sup>1</sup>: European Research Center, Huawei Technologies Duesseldorf GmbH, Riesstrasse 25, D-80992 Munich, Germany,

<sup>2</sup>: On leave from <sup>1</sup>,

<sup>3</sup>: Institute for Circuit Theory and Signal Processing, Technische Universität München, Arcisstr. 21, D-80290 Munich, Germany  
E-mail: fabio.pittalà@huawei.com

**Abstract:** Low-complexity single-stage frequency domain equalization is demonstrated in a 112-Gbps PDM-QPSK optical communication system. The bulk of CD is blindly estimated while residual CD, PMD and SOP tracking is supported by training-aided channel estimation.  
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## 1. Introduction

In coherent optical transmission system, chromatic dispersion (CD), polarization-mode dispersion (PMD) and other channel impairments accumulate during fiber transmission and cause severe inter-symbol-interference (ISI) degrading the system performance. In long-haul optical communications, the length of the equalizer of a transponder is mainly dependent on the ISI induced by CD [1]. Time-varying effects with short memory length like state of polarization (SOP) rotations and PMD require a shorter length of the filter. However, a regular update of the equalizer properties is required to maintain continuous optimum equalization [2].

Dual-stage equalization is widely considered, where a first static equalization stage compensates for the bulk of CD and a second adaptive  $2 \times 2$  multi-input-multi-output (MIMO) equalization stage compensates for residual CD and PMD effects. The coefficients of the  $2 \times 2$  MIMO equalizer may be adapted by non-training-aided (NTA) methods (typically in time domain (TD)) or based on a training sequence (TS), which refers to training-aided (TA) channel estimation (typically in frequency domain (FD)). TA  $2 \times 2$  MIMO FD channel estimation and equalization among other benefits like faster convergence speed, robustness with respect to polarization depended loss (PDL) effects, proves the lowest implementation complexity [3].

However, the implementation complexity can be further reduced by using a single-stage FD equalizer (FDE) which requires only the  $2 \times 2$  MIMO structure [1]. The length of the FD filter still follows the requirement of CD compensation (i.e. 1024 taps) and if the channel parameters change (i.e. SOP rotation) all FD coefficients need to be updated. Due the high degree of freedom of such a long filter, a stable and fast NTA filter acquisition is practically impossible for implementation in real systems. A classical TA channel estimation would be more practical in this case but as the length of the TS follows the total channel impulse length, the overhead would be tremendous.

In this paper we present a solution which combines the benefits of both dual-stage FDE and single-stage FDE: 1) use a short training length with low overhead for adaptive channel estimation and filter update; adaptively only track time varying channel impairments with low memory after initial blind CD estimation and 2) use a single-stage  $2 \times 2$  MIMO FDE that compensates for residual CD and PMD in a single filter operation.

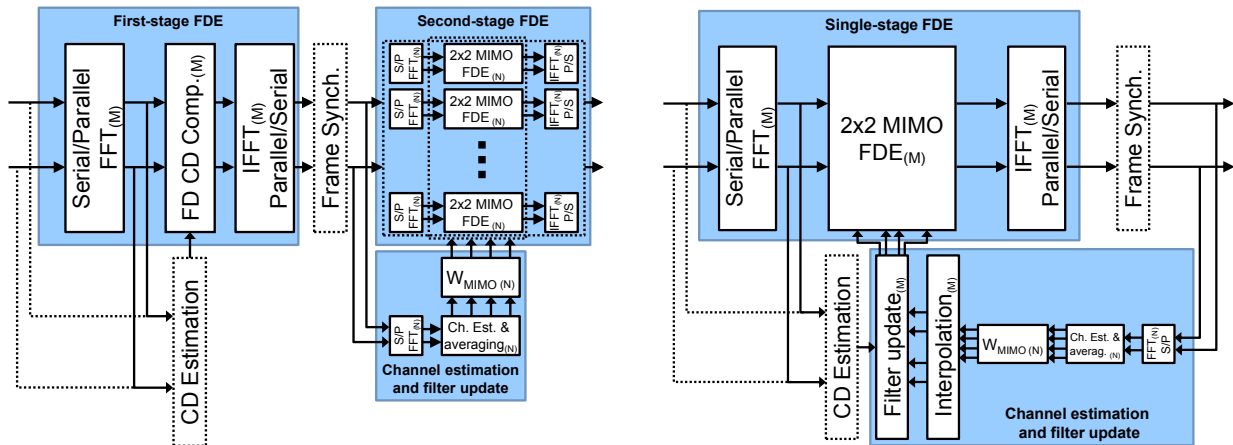


Fig. 1. Dual-stage FDE (left) and single-stage FDE (right) for coherent optical receivers.

## 2. Single-Stage Frequency Domain Equalizer Structure

The structure of the proposed single-stage FDE is presented in Fig.1-right (as reference Fig.1-left illustrates the architecture of a dual-stage FDE). The total complexity of the equalization process is reduced by incorporating in a single stage equalization the FD CD compensation and the TA  $2 \times 2$  MIMO FD equalizer and keeping the total overhead low for TA channel estimation. A detailed arithmetical complexity analysis is illustrated in Tab. 1.

The single-stage FDE has length M equal to that of a first stage FD CD compensator of a dual-stage FDE, which is initially adapted by blind CD estimation [4-6]:

$$W_{XX}[k] = H_{CD}^{-1}[k]; \quad W_{XY}[k] = 0; \quad W_{YX}[k] = 0; \quad W_{YY}[k] = H_{CD}^{-1}[k]; \quad \text{with } k = 1, 2, \dots, M \quad (1)$$

We include short TS between the payload data with sufficient repetition rate to track time-varying distortions. The length of the TS only covers residual CD after FD CD compensation (estimation error of blind CD estimation), PMD and all other impairments with memory (i.e. amplitude filtering or electrical receiver bandwidth limitation). After FD CD compensation, based on the received known TS we estimate the  $2 \times 2$  MIMO channel which has resolution (N), dependent on the effective length of TS used for channel estimation [7], and following we calculate the zero-forcing (ZF) filter, which for a 2 samples per symbol system reads as:

$$W_{2 \times 2 \text{ MIMO}}[k] = H[k]^H (H[k]H[k]^H + H[k']H[k']^H);$$

$$\text{with } k = 1, 2, \dots, N, \text{ and } k' = \begin{cases} N & \text{if } k = N/2 \\ \text{mod}(k + \frac{N}{2}, N) & \text{otherwise} \end{cases} \quad (2)$$

We linearly interpolate the estimated channel elements of length N to the length M of the FD CD compensation stage. The M-size transfer function of the interpolated TA channel estimation and the CD compensation is combined in a single-stage FD equalization function:

$$W_{XX}[k] = W_{XX_{2 \times 2 \text{ MIMO}}}[k]H_{CD}^{-1}[k]; \quad W_{XY}[k] = W_{XY_{2 \times 2 \text{ MIMO}}}[k]H_{CD}^{-1}[k];$$

$$W_{YX}[k] = W_{YX_{2 \times 2 \text{ MIMO}}}[k]H_{CD}^{-1}[k]; \quad W_{YY}[k] = W_{YY_{2 \times 2 \text{ MIMO}}}[k]H_{CD}^{-1}[k] \quad \text{with } k = 1, 2, \dots, M \quad (3)$$

For continuous update we “de-equalize” the extracted TS after the FDE:

$$R_{TS_X}[k] = Z_{TS_X}[k]H_{XX}[k] + Z_{TS_Y}[k]H_{XY}[k];$$

$$R_{TS_Y}[k] = Z_{TS_X}[k]H_{YX}[k] + Z_{TS_Y}[k]H_{YY}[k]; \quad \text{with } k = 1, 2, \dots, N \quad (4)$$

Based on  $R_{TS_X}[k]$  and  $R_{TS_Y}[k]$  we calculate the  $2 \times 2$  MIMO channel [7] of length N, perform linear interpolation to get channel elements of length M and then proceedings with the steps described by (2), (3) and (4).

TABLE 1. Arithmetic complexity analysis for frequency domain equalization (Split-Radix FFT/IFFT complexity described in [8]).

		Dual-stage FDE (50% overlap-save processing)			Single-stage FDE (50% overlap-save processing)		
		Operation	Real multiplication	Real addition	Operation	Real multiplication	Real addition
2x2 MIMO channel estim. and filter update		S/P and FFT	$2 \cdot \{N \cdot [\log_2(N) - 3] + 4\}$	$2 \cdot \{3 \cdot N \cdot [\log_2(N) - 1] + 4\}$	S/P and FFT	$2 \cdot \{N \cdot [\log_2(N) - 3] + 4\}$	$2 \cdot \{3 \cdot N \cdot [\log_2(N) - 1] + 4\}$
		Channel estimation and linear averaging	$8 \cdot N + 6 \cdot \{N \cdot [\log_2(N) - 3] + 4\} + 8 \cdot N \cdot L$	$4 \cdot N + 6 \cdot \{3 \cdot N \cdot [\log_2(N) - 1] + 4\} + 8 \cdot N \cdot L$	Channel estimation and linear averaging	$8 \cdot N + 6 \cdot \{N \cdot [\log_2(N) - 3] + 4\} + 20 \cdot N + 8 \cdot N \cdot L$	$4 \cdot N + 6 \cdot \{3 \cdot N \cdot [\log_2(N) - 1] + 4\} + 11 \cdot N + 8 \cdot N \cdot L$
		Filter implementation $W_{\text{MIMO}}$	ZF: $122 \cdot N$ MMSE: $122 \cdot N$ (SNR est.)	ZF: $94 \cdot N$ MMSE: $102 \cdot N$ (SNR est.)	Filter implementation $W_{\text{MIMO}}$	ZF: $122 \cdot N$ MMSE: $122 \cdot N$ (SNR est.)	ZF: $94 \cdot N$ MMSE: $94 \cdot N$ (SNR est.)
		Linear interpolation (optional)	$2 \cdot N$	$2 \cdot N$	Linear interpolation	$2 \cdot M$	$2 \cdot M$
					Filter update	$16 \cdot M$	$8 \cdot M$
Equalization	First-stage FDE	S/P and FFT	$4 \cdot \{M \cdot [\log_2(M) - 3] + 4\}$	$4 \cdot \{3 \cdot M \cdot [\log_2(M) - 1] + 4\}$			
		CD compensation	$16 \cdot M$	$8 \cdot M$			
		IFFT and P/S	$4 \cdot \{M \cdot [\log_2(M) - 3] + 4\}$	$4 \cdot \{3 \cdot M \cdot [\log_2(M) - 1] + 4\}$			
	Second-stage FDE	S/P and FFT	$4 \cdot M \cdot N \cdot \{N \cdot [\log_2(N) - 3] + 4\}$	$4 \cdot M \cdot N \cdot \{3 \cdot N \cdot [\log_2(N) - 1] + 4\}$	S/P and FFT	$4 \cdot \{M \cdot [\log_2(M) - 3] + 4\}$	$4 \cdot \{3 \cdot M \cdot [\log_2(M) - 1] + 4\}$
		2x2 MIMO FDE	$32 \cdot M$	$24 \cdot M$	2x2 MIMO FDE	$32 \cdot M$	$24 \cdot M$
		IFFT and P/S	$4 \cdot M \cdot N \cdot \{N \cdot [\log_2(N) - 3] + 4\}$	$4 \cdot M \cdot N \cdot \{3 \cdot N \cdot [\log_2(N) - 1] + 4\}$	IFFT and P/S	$4 \cdot \{M \cdot [\log_2(M) - 3] + 4\}$	$4 \cdot \{3 \cdot M \cdot [\log_2(M) - 1] + 4\}$

As illustrated in Tab. 1 the 2x2 MIMO channel estimation and filter update of the single-stage FDE differs from the dual-stage FDE only from the “de-equalization” of the TS and from the interpolation of the channel elements. However, the significant reduction in complexity comes from the elimination of the  $M/N$  fast Fourier transform (FFT) and inverse FFT (IFFT) required for the  $M/N$   $2 \times 2$  MIMO FDEs. As practical example, the dual-stage architecture with  $M=1024$  and  $N=32$  requires 128640 real multiplication and 359552 real additions, which can be reduced by 11.4% and 27.0%, respectively, by using a single-stage FDE architecture with  $M=1024$ .

### 3. Performance Evaluation

The performance investigation is based on a simulated 28-GBaud polarization-division multiplexed (PDM) system with quaternary phase-shift keying (QPSK) leading to a transmission rate of 112-Gbps. Simulation of the linear channel include CD, all-order PMD, initial polarization rotation angle  $\alpha$  and phase  $\phi$ , PDL and SOP rotation rate.

At the receiver, white Gaussian noise is loaded onto the signal, followed by an optical Gaussian band-pass filter (2<sup>nd</sup>-order, double-sided 35 GHz), the polarization-diverse 90°-hybrid and an electrical Bessel filter (5<sup>th</sup>-order, 19 GHz). Finally, an analog-to-digital (ADC) stage digitalizes the received signal at 2 samples per symbol. A 24-symbol (including guard intervals) perfect-square minimum-phase (MP-PS) constant amplitude zero-autocorrelation (CAZAC) sequence, transmitted every 1000 data symbols, is employed for channel estimation and synchronization.

In Fig. 2-left, bit-error-rate (BER) performance for back-to-back (B2B) and for transmission under linear static channels are shown with parameters randomly chosen from Tab. 1. For each OSNR value 10 channel trials have been performed. Before the zero-forcing (ZF) filter update, averaging over 30 channel estimations is performed. As expected, since the channel estimation is converged, no penalty is introduced by incorporating the two FDE stages into a single FDE stage and no degradation due PDL is experienced in the filter update.

In Fig. 2-right, we show the required optical-signal-to-noise-ratio (OSNR) at BER equal to  $10^{-3}$  for transmission under fast time-varying channels. The fastest state of polarization (SOP) change rate is 136.75 kHz. In accordance to [2], the number of averages over channel estimations is reduced to 5 to optimize the tradeoff between noise averaging and tracking penalty. Results show a negligible penalty between the two equalizer structures. The 0.1 dB difference comes from the channel estimation error which affects the TS “de-equalization” step of the SS-FDE. A larger number of averages over channel estimations would reduce the performance difference between the two structures but would increase the tracking penalty due to SOP rotation.

TABLE 1  
Impairments for Channel Simulations

Impairment	Distribution	Value range
PMD	Maxwellian	Mean 25 ps
CD	constant	20000 ps/nm
$\alpha$	uniform	$[0; 2\pi]$ rad
$\phi$	uniform	$[0; 2\pi]$ rad
PDL	constant	5 dB
SOP rot. rate	constant	$[0; 136.75]$ kHz

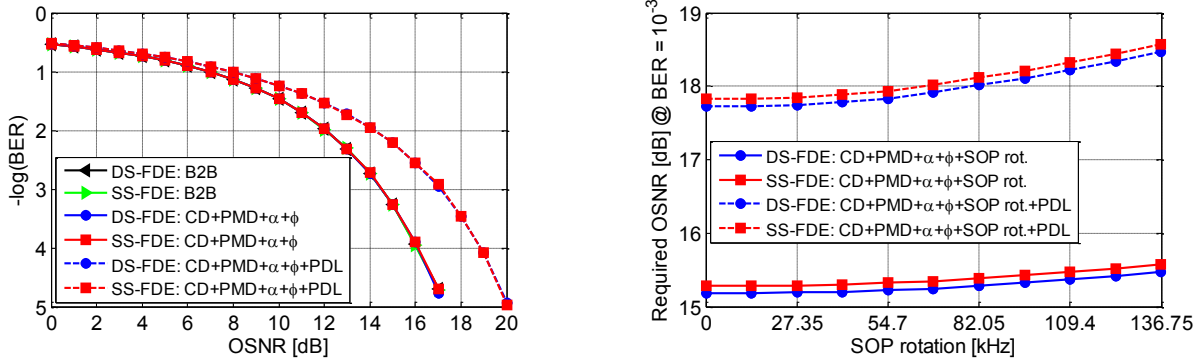


Fig. 2. Single-stage (SS)-FDE versus dual-stage (DS)-FDE: BER performance for B2B and transmission over static linear channels (left) and required OSNR at  $\text{BER}=10^{-3}$  for transmissions over fast time-varying linear channels (right).

### 4. Conclusions

We have successfully demonstrated a single-stage FDE for coherent optical communication systems with a reduction of total complexity of  $\sim 20\%$  respect to a dual-stage FDE architecture and 2.4% required overhead for channel estimation. The two equalizer structures prove similar performance in static and fast time-varying linear channels.

### 5. References

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