TECHNISCHE UNIVERSITÄT MÜNCHEN

Transient problems with the Finite Cell Method

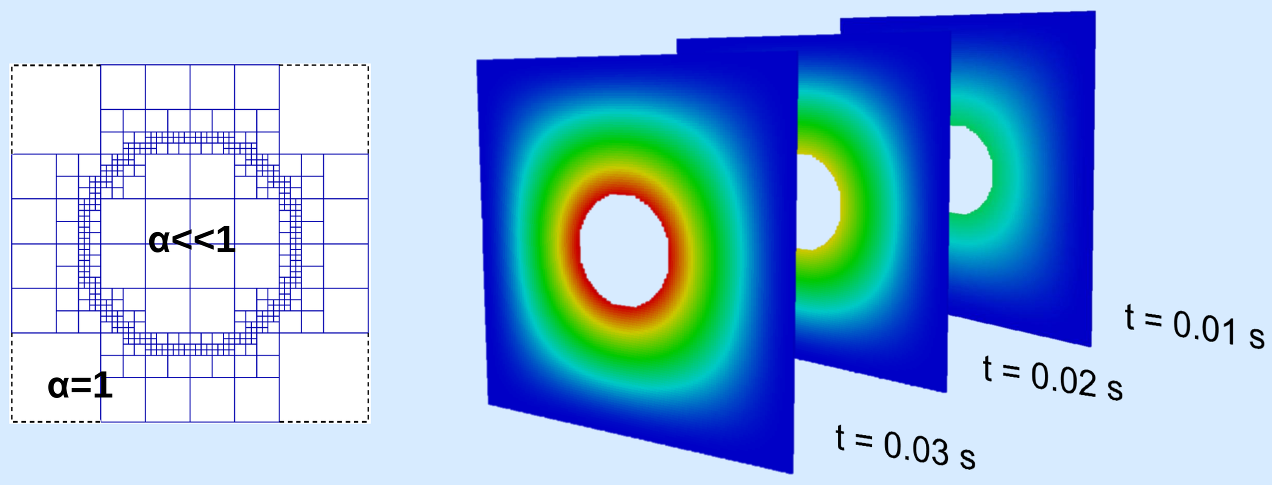
Software Lab Project 2012 :

Students: Reza Najian Asl, Koushyar Komeilizadeh, Shahed Rezaei

Supervisors: Martin Ruess, Vasco Varduhn

Abstract

In contrast to the standard FEM, fictitious domain methods do not require a boundary-fitted mesh. Instead, they embed structures of arbitrarily complex geometry in a domain of simple shape. The Finite Cell Method (FCM) [1,2] is a high-order approximation scheme that follows the fictitious domain idea.



The Finite Cell Method has been successfully applied to various problems from engineering and science in linear and non-linear analyses. Other fields like transient problems haven't yet gained much attention. This software lab project focus on the implementation of different time integration schemes including the Newmark algorithm [3] as an implicit approach and the Central Difference scheme as an explicit approach.

$$[M]\frac{\partial^2 U}{\partial t^2} + [C]\frac{dU}{dt} + [K]\,U = f$$

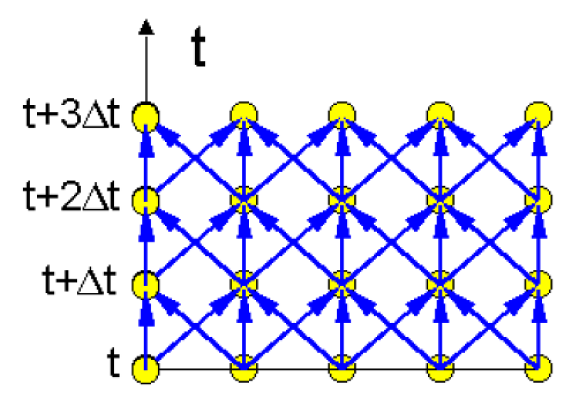
↑ Mass Matrix ↑ Damping Matrix ↑ Stiffness matrix

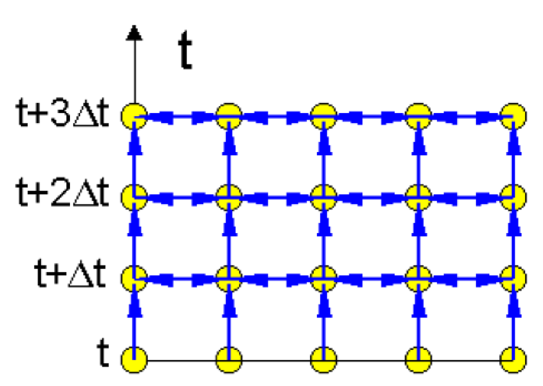
The implementation has been verified for benchmark problems in elastodynamics. The effect of different parameters in time integration schemes and the effect of the FCM specific penalty value for fictitious domain has been studied, to obtain optimal results for the time response of a structure.

Different schemes in structural dynamics

Time integration methods

Explicit	Implicit
$D_{t+\Delta t} = f(D_t, \dot{D}_t, \ldots)$	$D_{t+\Delta t} = f(D_{t+\Delta t}, \dot{D}_{t+\Delta t}, \ldots)$
Central difference	Newmark
Lumped Mass Matrix	Consistent Mass Matrix





Explicit:
- ✓ Solution is trivial for lumped mass
- ✓ No need to invert and assemble the global stiffness matrix.

Implicit:
- ✓ Solving the coupled system
- ✓ For nonlinear systems the stiffness matrix has to be solved at every time step.

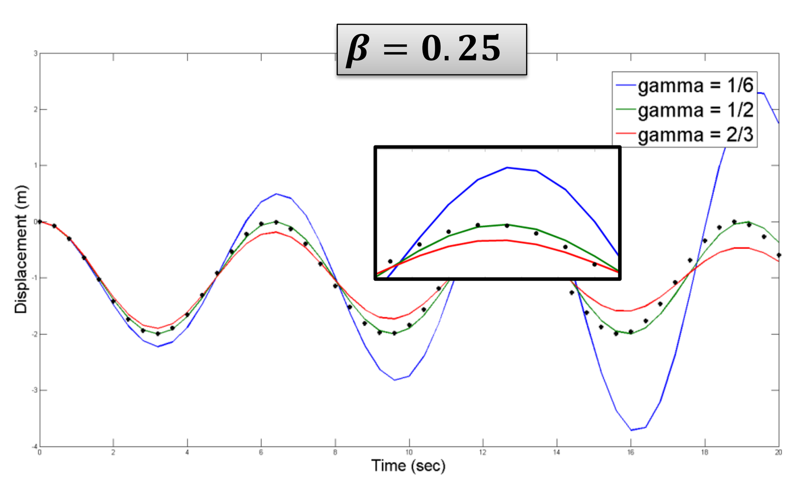
Insight into the implicit and explicit schemes

$$\dot{\boldsymbol{u}}_{t+\Delta t} = \dot{\boldsymbol{u}}_t + \Delta t\,\ddot{\boldsymbol{u}}_\gamma \quad (1)$$

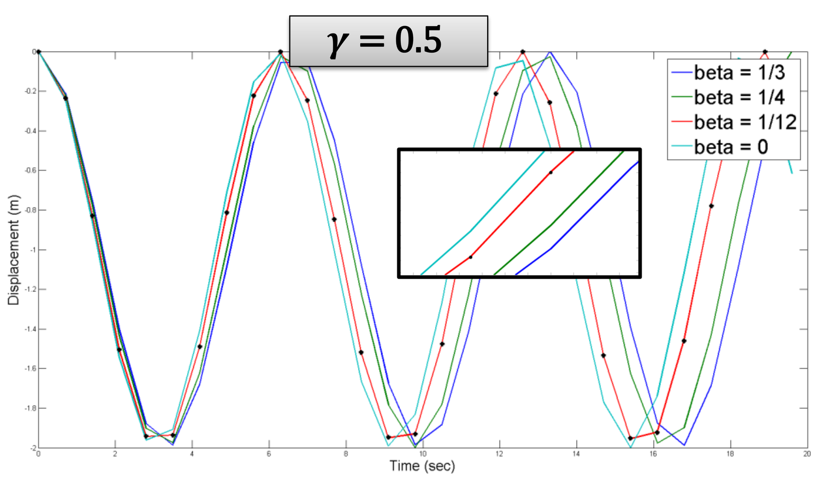
$$\ddot{\boldsymbol{u}}_\gamma = (1-\gamma)\ddot{\boldsymbol{u}}_t + \gamma\ddot{\boldsymbol{u}}_{t+\Delta t}, \; 0 \le \gamma \le 1$$

$$\boldsymbol{u}_{t+\Delta t} = \boldsymbol{u}_t + \Delta t\,\dot{\boldsymbol{u}}_t + \Delta t^2\ddot{\boldsymbol{u}}_\beta \quad (2)$$

$$\ddot{\boldsymbol{u}}_\beta = \left(\frac{1}{2} - \beta\right)\ddot{\boldsymbol{u}}_t + \beta\ddot{\boldsymbol{u}}_{t+\Delta t}, \; 0 \le \beta \le 1$$

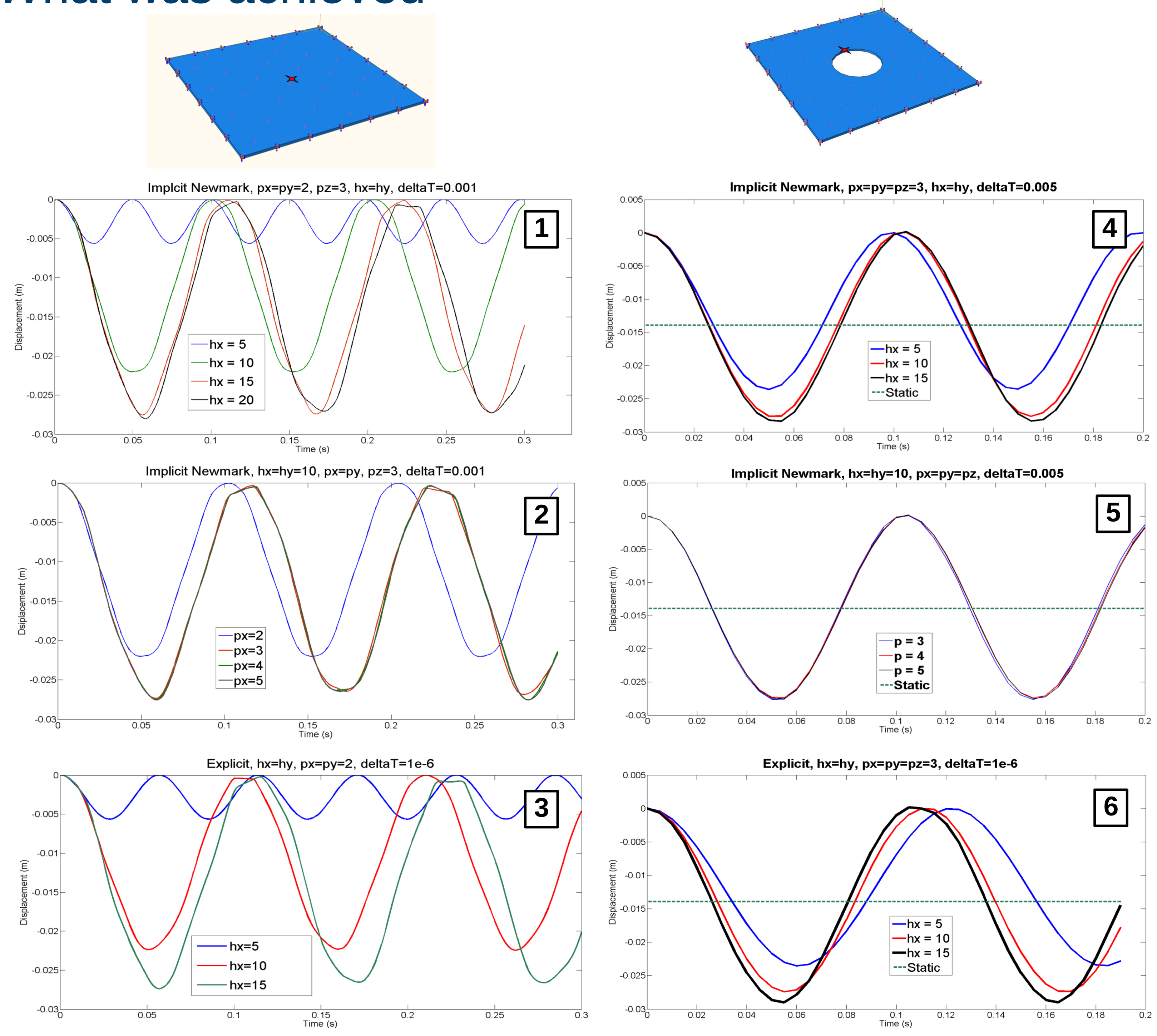


- Numerical damping for $\gamma > 0.5$
- Instability for $\gamma < 0.5$

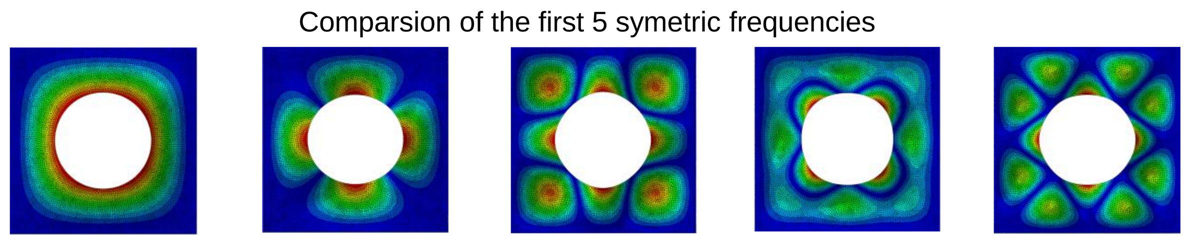


- Phase shift
- Conditionally stable $\beta < 0.25$

What was achieved



Comparsion of the first 5 symetric frequencies



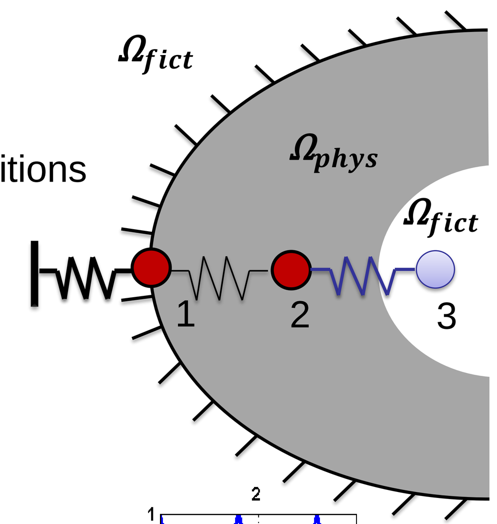
ABAQUS:	13.74 Hz	29.07 Hz	46.75 Hz	68.61 Hz	82.78 Hz
FCM FFT:	13.28 Hz	31.25 Hz	46.88 Hz	65.63 Hz	83.34 Hz

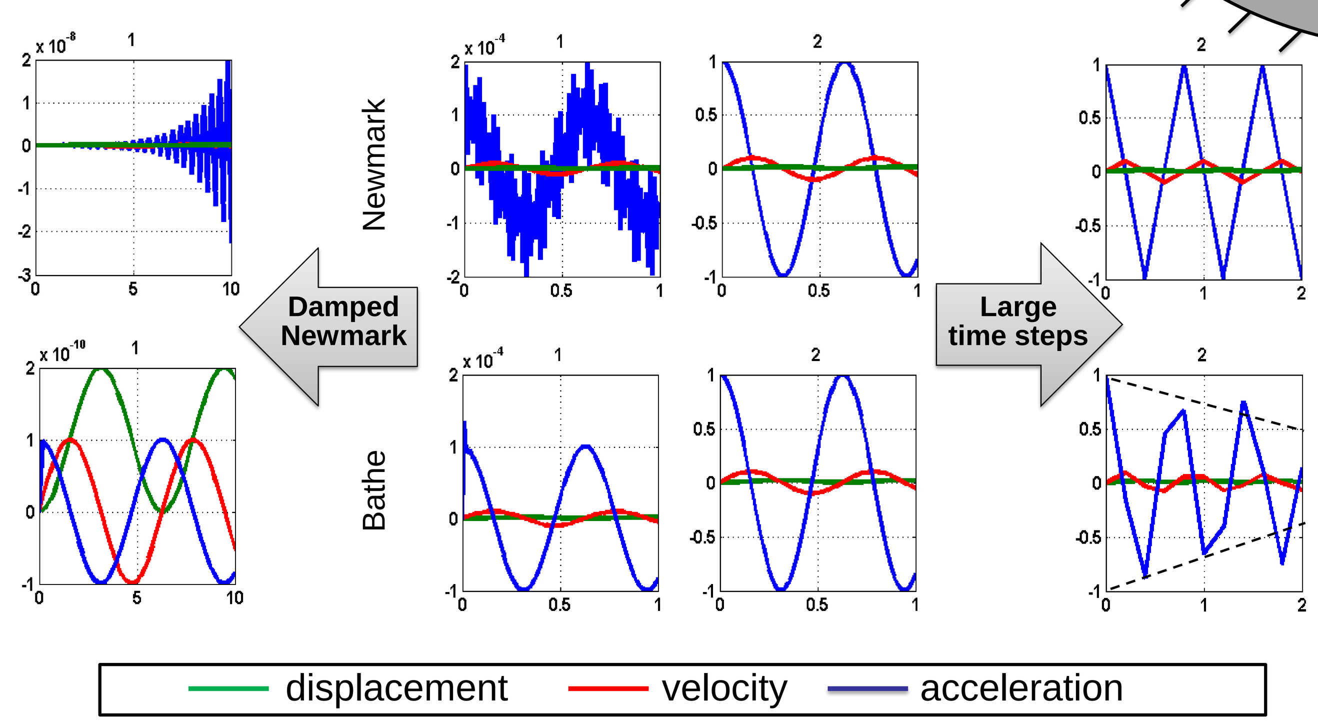
- We show faster convergence for p-refinement than h-refinement as expected from comparison between fig(1) and fig(2) or fig(4) and fig(5).
- In fig(3) it is observed that by h-refinement the frequency of the system decrease till it converged, (compare with fig(2)) while in fig(6) we observed increase in frequencies with h-refinement.
- h-refinement with moderate p-degrees is the preferred configuration.

Future works with transient FCM!

Problem: The penalty approach to enforce essential boundary conditions significantly destroys the conditioning of the problem

Solution: Bathe [4] introduced new method, considering additional midpoint and Euler backward to solve the problem at the supports





References

[1] D. Schillinger, M. Ruess, N. Zander, Y. Bazilevs, A. Düster, E. Rank, Small and large deformation analysis with the p- and B-spline versions of the Finite Cell Method, submitted to Computational Mechanics, 2011

[2] M. Ruess, D. Tal, N. Trabelsi, Z. Yosibash, E. Rank, The finite cell method for bone simulations: Verification and validation. DOI: 10.1007/s10237-011-0322-2 Biomechanics and Modeling in Mechanobiology, 2011

[3] K.-J. Bathe. Finite Element Procedures, Prentice-Hall, Inc., Upper Saddle River 1996

[4] K-J Bathe, Gunwoo Noh, Insight into an implicit time integration scheme for structural dynamics, Computers and Structures, Volumes 98–99, Pages 1-6, May 2012