A Two-Step Approach for Reliability Assessment of a Tunnel in Soft Soil

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Abstract

We assess the reliability of a tunnel in Keuper marl with uncertain mechanical properties. The tunnel is constructed by the conventional tunneling method. The limit state function is expressed in terms of a two-dimensional finite element model of the tunnel. Plain strain finite elements are used to represent the soil and the yield surface is modeled with a hardening plasticity soil model. The three-dimensional arching effect is approximated by application of the stress reduction method. In a first step, the reliability analysis is performed by application of the first order reliability method (FORM) and the results are verified by importance sampling. The FORM provides information on the sensitivity of the reliability in terms of the uncertain variables. This information is used in a second step to account for the inherent spatial variability of the ground parameters with the largest influence through a random field modeling. The discretization of the random field leads to a large number of random variables. Therefore, we apply the subset simulation method, which is an adaptive Monte Carlo method known to be especially efficient for such high dimensional problems. The analysis is performed using a reliability tool that is integrated into the SOFiSTiK finite element software package.

Keywords: reliability analysis, Keuper marl, FORM, sensitivities, subset simulation

1 INTRODUCTION

In tunnel construction, a typical design requirement is the restriction of surface settlements to acceptable values with sufficient reliability. This serviceability condition is particularly important in urban environments, where tunnel induced settlements may have an impact on existing structures. Predictions of the ground displacement can be made with the help of non-linear finite element models. However, there is significant uncertainty involved in the choice of the model parameters. Moreover, the mechanical properties of the soil exhibit an inherent spatial variability. These issues need to be addressed in a proper assessment of the adequacy of the design.

In this paper, we account for the uncertainties in the model parameters and evaluate the probability that the tunnel induced settlements exceed a predefined threshold. In addition, we investigate the influence of the spatial variability of soil parameters on the analysis results.

2 MODEL DESCRIPTION

2.1 Mechanical Model

A conventional driven tunnel with a horse-shoe shaped profile is considered in this study (see figure 1). The problem is modeled in the SOFiSTiK finite element (FE) software package, using plain strain finite elements. The numerical model has a width of 80m and a total height of 26m. In this study, we are interested in surface settlements over the tunnel center line (point A in figure 1). The excavation process is modeled by application of the stress reduction method, which approximately accounts for the three-dimensional arching effect of the stress-distribution.

Three different ground layers are incorporated in the model; the layers are illustrated in figure 1. The cover layer is a man-made fill and has a depth of 5.4m. Heavily weathered soft rock known as Keuper marl forms the second layer. The thickness of this layer is assumed to be 16.8m. We adopt a hardening plasticity soil model [1] to describe the material behavior of the first two layers. This material model allows for a realistic description of the stiffness and hardening behavior of soft soil in settlement analysis. The material properties of the cover layer are as follows: elastic modulus for unloading-reloading: 30MPa, Poisson's ratio: 0.2, specific weight: $20\text{kN}/\text{m}^3$, friction angle: 25°, cohesion: 10kPa, oedometric stiffness modulus: 10MPa, stiffness modulus for primary loading: 10MPa. The exponent in the hardening law is selected as 0.5 for the first and the second layer. The angle of dilatancy is assumed as zero, corresponding to an non-associated flow rule. The soil parameters of the Keuper marl layer are assumed to be random and their probabilistic description is given in section 2.2. Strong limestone constitutes the bottom layer. The Mohr-Coulomb law is applied for this layer. The material properties are: Young's modulus: 575MPa, Poisson's ratio: 0.2, specific weight: $23\mathrm{kN}/\mathrm{m}^3$, friction angle: 35°, cohesion: 200kPa. Due to the much larger stiffness of the limestone compared to the stiffness of the overlaying materials, only 3.8m of this layer are modeled.

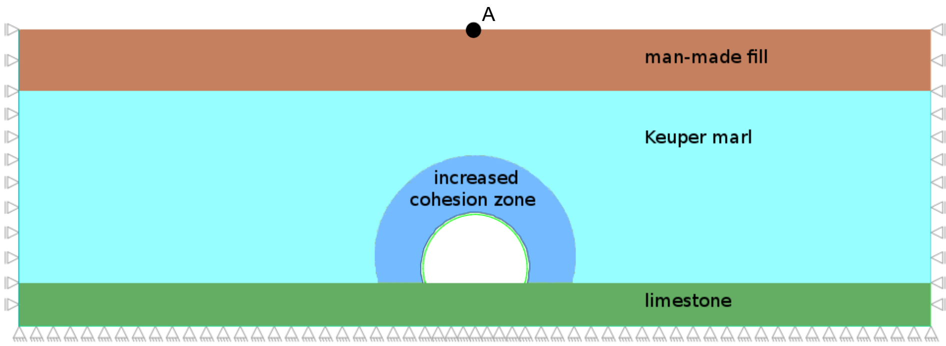

Figure 1: Ground layers considered in the model

The height of the tunnel above the limestone layer is 6.2m. Consequently, the tunnel is located in a depth of 16m below the ground surface. At the intersection of the second and the third layer, the tunnel has a width of 9.16m. In the vicinity of the tunnel the Keuper marl is reinforced with nails. This is modeled by increasing the cohesion in the affected region (see figure 1) by 25kPa. Moreover, it is assumed that the tunnel is located above the groundwater level. The shotcrete lining is modeled using linear beam elements with a normal stiffness of 10.5GN and a flexural rigidity of $26.78\mathrm{MNm}^2$.

2.2 Stochastic Model

The cover layer and the limestone layer are considered as deterministic in the analysis. Since the cover layer is a man-made fill, we assume that its soil properties are well-known, and the associated uncertainties are small compared to the uncertainties in the material description of the Keuper marl layer and can be neglected. The limestone layer is also modeled as deterministic because - due to its large stiffness - the contribution of this layer to the surface settlements is negligible. The probability dis-

tribution describing the uncertainties in the material parameters of the Keuper marl layer are listed in table 1. We assume that the stiffness modulus for primary loading E_{50}^{ref} equals the oedometric stiffness modulus $E_{\mathrm{oed}}^{\mathrm{ref}}$. We also consider a correlation of 0.7 between the parameters $E_{\mathrm{oed}}^{\mathrm{ref}}$ and E_{ur}. The friction angle and the cohesion are assumed to have a negative correlation of -0.5.
In conventionally driven tunnels, there is usually a large uncertainty in the choice of the relaxation factor $\beta \in [0,1]$ of the stress reduction method [2]. In this study β is modeled as a beta-distributed random variable (see table 1).

Table 1: Uncertain parameters of the Keuper marl layer

Parameter	Distribution	Mean	C.o.V.
Relaxation factor β	Beta(0.0,1.0)	0.5	10%
Elastic modulus for un-/reloading E_{ur} [MPa]	Lognormal	80.0	32%
Oedometric stiffness modulus $E_{\mathrm{oed}}^{\mathrm{ref}}$ [MPa]	Lognormal	30.0	32%
Poisson's ratio ν	Beta(0.0,0.5)	0.2	15%
Friction angle φ [°]	Beta(0.0,45.0)	20.0	15%
Cohesion c [kPa]	Lognormal	25.0	30%
Specific weight γ [kN/m^3]	Lognormal	24.0	5%

In the second part of this study, the inherent spatial variablity of the parameters $E_{\mathrm{oed}}^{\mathrm{ref}}$ and E_{ur} is taken into account. This is achieved by modeling the two parameters as cross-correlated homogeneous random fields. It is assumed that the spatial variability depends only on the separation in horizontal and vertical direction between two locations, denoted by Δx and Δy, respectively. The following exponential autocorrelation coefficient function is chosen for both random fields:

$$\rho(\Delta x, \Delta y) = \exp\left(-\frac{\Delta x}{l_x} - \frac{\Delta y}{l_y}\right) \tag{1}$$

where l_x and l_y denote the correlation lengths in horizontal and vertical direction, respectively. The cross-correlation coefficient function is:

$$\rho_{\mathrm{cross}}(\Delta x, \Delta y) = \rho_{\mathrm{c}} \cdot \rho(\Delta x, \Delta y) \tag{2}$$

where ρ_{c} denotes the correlation of $E_{\mathrm{oed}}^{\mathrm{ref}}$ and E_{ur} at the same location.
The midpoint method [3] is used for the discretization of the random fields. In this study, the stochastic finite element (SFE) mesh is a coarser variant of the deterministic finite element mesh. The SFE mesh consists of 142 deterministic finite element

patches and is illustrated in figure 2, where the patches in the second layer are indicated by areas of different color. In the midpoint method, the random field is assumed to be constant in each SFE and represented by its value at the midpoint of the SFE.

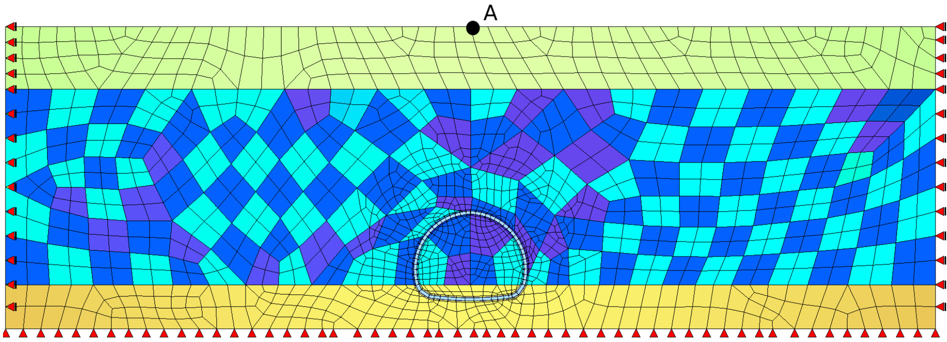

Figure 2: Stochastic and deterministic finite element mesh

3 RELIABILITY ANALYSIS

3.1 Introduction

In reliability analysis, we compute the probability of failure of a system as:

$$P_f = \Pr\{g(\mathbf{X}) \leq 0\} = \int_{g(\mathbf{X})\leq 0} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x} \tag{3}$$

where g is called limit-state function and $\mathbf{X}$ is the K-dimensional vector of random variables with joint probability density function $f_{\mathbf{X}}$. Failure of the system occurs if $g(\mathbf{x}) \leq 0$. In this study the limit state function is defined as:

$$g(\mathbf{x}) = u_{\text{threshold}} - u_{\text{A}}(\mathbf{x}) \tag{4}$$

where u_{A} denotes the surface settlement in point A (see figure 2) as computed with the FEM, and $u_{\text{threshold}}$ is the maximum allowed settlement.
For most reliability methods it is convenient to transform the original space of random variables $\mathbf{X}$ to a space of independent standard normal random variables $\mathbf{U}$. The limit-state function defined in the transformed space is denoted by $G : \mathbf{U} \to \mathbb{R}$. Consequently, equation 3 can be rewritten as:

$$P_f = \Pr\{G(\mathbf{U}) \leq 0\} = \int_{G(\mathbf{U})\leq 0} \varphi_{\mathbf{U}}(\mathbf{u})\,\mathrm{d}\mathbf{u} \tag{5}$$

where $\varphi_{\mathbf{U}}$ is the K-dimensional standard normal joint probability density function.

3.2 FORM

The first order reliability method (FORM) solves the reliability problem formulated in equation 5 approximately by a linearization of the limit-state function $G(\mathbf{u})$ at the design point $\mathbf{u}^*$. The design point is defined as the most probable point of failure. It is obtained by a minimization of $\sqrt{\mathbf{u}^{\mathrm{T}}\mathbf{u}}$ subjected to $G(\mathbf{u}) = 0$. The quality of the approximation depends on how well $G(\mathbf{u})$ can be approximated by a linear function. The probability of failure defined in equation 5 is approximated as:

$$P_f \approx \Phi(-\beta) \tag{6}$$

where Φ denotes the standard normal cumulative distribution function, and β is the FORM reliability index and is defined as $\beta = \sqrt{\mathbf{u}^{*\mathrm{T}}\mathbf{u}^*}$. In this work, the standard HL-RF method [4, 5] is applied to solve the optimization problem.
A by-product of the FORM are sensitivities at the design point. In this work the sensitivity is expressed in terms of the influence coefficients α_i. The coefficients α_i represent the direction cosines along the coordinate axes U_i; they are helpful for estimating the most important uncertain parameters in terms of their influence on the structural reliability [6].

3.3 Importance Sampling

In the standard Monte Carlo simulation, we draw N samples from $\mathbf{U}$ and count how often the failure event $G(\mathbf{U}) \leq 0$ is observed. An estimate of the probability of failure is obtained by dividing the number of failures by N. This procedure becomes inefficient if the problem to investigate has a small failure probability.
The idea of importance sampling is to draw the samples not from $\mathbf{U}$, but from a distribution that produces more samples in the failure domain. This distribution is called importance sampling function. In this work, the importance sampling function is chosen as the K-dimensional normal distribution with unit variance centered around the design point $\mathbf{u}^*$.

3.4 Subset Simulation

The subset simulation method [7] is an adaptive Monte Carlo method that is efficient for high dimensional problems. In this method, the probability of failure is expressed as a product of larger conditional probabilities.

Let us introduce M intermediate failure events F_i, with $1 \leq i \leq M$ and $F_1 \supset F_2 \supset \ldots \supset F_M = F$. The failure events F_i are defined as $F_i = \{G(\mathbf{u}) \leq c_i\}$, where $c_i \in \mathbb{R}$ with $c_1 > \ldots > c_M = 0$. The probability of failure $P_f = \Pr(F_M)$ can be expressed as:

$$P_f = \prod_{i=1}^{M} \Pr(F_i|F_{i-1}) \tag{7}$$

where F_0 denotes the certain event, and $\Pr(F_i|F_{i-1})$ is the probability of the event F_i conditioned on the occurrence of the event F_{i-1}. The values c_i can be chosen adaptively such that the conditional probabilities $\Pr(F_i|F_{i-1}),\ i < M$ correspond to a given value p_0.
Standard Monte Carlo simulation is applied to compute $\Pr(F_1)$. The conditional probabilities $\Pr(F_i|F_{i-1})$ for $2 \leq i \leq M$ are approximated by means of Markov Chain Monte Carlo (MCMC) techniques. In this work, the component-wise Metropolis-Hastings algorithm is used [7]. Moreover, p_0 is fixed to 10%.

3.5 Two-Step Procedure

To solve the reliability problem we adopt the following two-step procedure. In the first step, the spatial variability of the material parameters is neglected. The stochastic model is described by the seven random variables listed in table 1. The probability of failure is approximated by FORM. Importance sampling is used to verify the results obtained with FORM and to investigate the non-linearity of the limit-state function.
In the second step, the spatial variability of the parameters with the largest influence on the failure probability is considered. This is modeled by a cross-correlated random field. The random field model results in a large number of additional random variables, and, FORM cannot be applied efficiently. Therefore, the subset simulation method is used to perform the reliability analysis.

4 RESULTS AND DISCUSSION

4.1 Step 1: Spatial Variability is Neglected

The results of the analysis neglecting the spatial variability of the soil are listed in table 2. β_{FORM} denotes the reliability index obtained with FORM for different threshold values $u_{\text{threshold}}$. $P_{f,\text{FORM}}$ is the associated probability of failure according to equation 6. The number of steps required for convergence of the HL-RF algorithm is given in the column $N_{\text{step,FORM}}$. Importance sampling with 1000 limit-state function evaluations was used the verify the FORM results. The estimate is listed in column $P_{f,\mathrm{IS}}$, the associated coefficient of variation is given in $\mathrm{CV}_{\mathrm{IS}}$. Comparing the results obtained by FORM with the results from importance sampling, we observe that for all investigated $u_{\mathrm{threshold}}$, FORM gives a good approximation of the probability of failure.

Table 2: FORM and importance sampling results

$u_{\mathrm{threshold}}$ [cm]	β_{FORM}	$P_{f,\mathrm{FORM}}$	$N_{\mathrm{step,FORM}}$	$P_{f,\mathrm{IS}}$	$\mathrm{CV}_{\mathrm{IS}}$
1	0.12	$4.5 \cdot 10^{-1}$	2	-	-
2	2.0	$2.1 \cdot 10^{-2}$	5	$2.0 \cdot 10^{-2}$	5.0%
3	3.3	$5.6 \cdot 10^{-4}$	7	$5.6 \cdot 10^{-4}$	6.2%
4	4.1	$1.9 \cdot 10^{-5}$	9	$2.0 \cdot 10^{-5}$	7.0%
5	4.8	$8.8 \cdot 10^{-7}$	10	$9.8 \cdot 10^{-7}$	8.4%

Figure 3 depicts the squared influence coefficients α^2 in a pie graph. It is observed that the variable with the largest influence is the oedometric stiffness modulus $E_{\mathrm{oed}}^{\mathrm{ref}}$. In the next step, we account for the spatial variability of this parameter by a random field modeling. Since $E_{\mathrm{oed}}^{\mathrm{ref}}$ is strongly correlated with the elastic Young's modulus E_{ur}, the later parameter is also modeled as a random field.

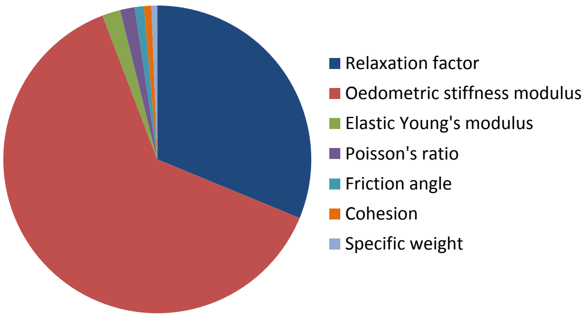

Figure 3: Squared influence coefficients

4.2 Step 2: Spatial Variability is Considered

In this section only the case $u_{\mathrm{threshold}} = 3\mathrm{cm}$ is investigated. As correlation length we choose $l_x = 20\mathrm{m}$ and $l_y = 5\mathrm{m}$. The parameters $E_{\mathrm{oed}}^{\mathrm{ref}}$ and E_{ur} are modeled as cross-correlated random fields, compare section 2.2. With subset simulation using 500 samples per conditioning step we computed a probability of failure of $6.8 \cdot 10^{-5}$. In a second run with 3500 samples per conditioning step we obtained a probability of failure of $6.1 \cdot 10^{-5}$. Comparing this to the FORM estimate of $5.6 \cdot 10^{-4}$ from table 2, we observe that neglecting spatial variability results in a significantly conservative estimate. This is due to the fact that in the model with spatial variability a local loss of strength becomes possible and a global loss of strength less likely. Among all possible local losses, only a small fraction will lead to failure.

5 SUMMARY

In this paper we accounted for the risk that tunnel induced settlements in soft soil exceed a specified threshold. We used FORM to evaluate the reliability of the problem for the case where spatial variability is neglected. The subset simulation method was used to perform reliability analysis for the case where spatial variability of the most influential ground parameters is taken into account. It was shown that the spatial variability has a significant influence on the computed reliability.

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