Delayed Single-Tap Frequency-Domain Chromatic-Dispersion Compensation

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***Abstract*— A long-haul transmission of 100 Gb/s without optical chromatic-dispersion (CD) compensation provides a range of benefits regarding cost effectiveness, power budget, and nonlinearity tolerance. The channel memory is largely dominated by CD in this case with an intersymbol-interference spread of more than 100 symbol durations. An efficient implementation of digital CD compensation is feasible by frequency-domain (FD) filtering. Still the large size of the Fourier transform requires a high gate-count and a large chip size. We propose a new FD filtering on the basis of a nonmaximally decimated discrete Fourier transform filter bank with a trivial prototype filter and a delayed single-tap equalizer per sub-band. This method, which can be regarded as an extension to the popular overlap-save method, allows us to increase the CD tolerance drastically. At the same time, the implementation complexity is not altered apart from adding simple memory elements realizing sub-band delays. With this technique, the uncompensated trans-Pacific transmission becomes feasible with the digital CD compensation.**

***Index Terms*— Chromatic dispersion, coherent detection, digital signal processing, equalization, frequency domain.**

I. Introduction

THE combination of polarization-diverse coherent detection with digital signal processing (DSP) for equalization and synchronization enables long-haul transmission of 100 Gbit/s data rates and above. Applying optical CD compensation by the use of dispersion compensating fiber (DCF) induces additional attenuation degrading the power budget of the link and reducing the nonlinearity threshold apart from cost issues [1]. Therefore, non-dispersion managed transmission without the use of DCF is preferred, which requires digital CD compensation at the transmitter or receiver implemented by FD digital filtering to cope with the large ISI spread [2]. In state-of-the-art FD CD compensation design, the size of the fast Fourier transform (FFT) to realize fast linear convolution is governed by the specification of the maximum channel memory length with two-fold oversampling and 50% block overlap [3]. Nevertheless, the overlap-save method can be regarded not just as fast convolution method but also as a FB structure with trivial prototype filters and the equalization is done per sub-band, opening the way for more sophisticated sub-band processing [4]. Based on this observation, we investigate the possibility of using a low complexity delayed single-tap equalizer in the sub-bands exploiting the nature of the CD channel.

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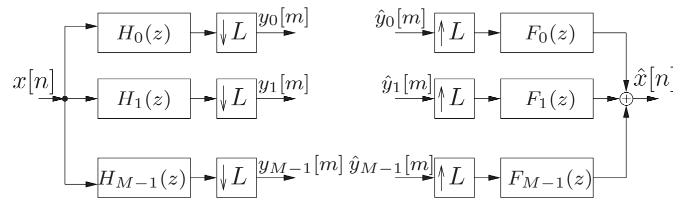

Fig. 1. Basic structure of FB. In the kth sub-band, the analysis filter is $H_k(z)$, the synthesis filter is $F_k(z)$, and the rate changing factor is L.

It turns out that this approach provides much better equalization ability with no extra complexity. In fact, by simply adding delay elements to the filter design, more than $240,000$ ps/nm CD tolerance with only 0.5 dB OSNR filtering penalty can be achieved using a 1024 FFT in a 28 GBaud transmission with two-fold oversampling.

After a short introduction of the basic theory of FB, we present a FD CD compensation design based on DFT FB with delayed single-tap equalizer per sub-band. This allows to scale the FFT size significantly below the channel memory length.

II. Efficient Non-maximally Decimated FB

Fig. 1 illustrates the general framework of a filter bank. We use the definition $z = e^{sT}$, where $s = \sigma + j\omega$ is the complex frequency variable. The analysis and the synthesis filters of the k-th sub-band are $H_k(z)$ and $F_k(z)$, respectively. The number of sub-bands is M and the rate changing factor is denoted as L. In general $L \leq M$. Throughout this letter, we will consider the case of non-maximally decimated FB, i.e. $L < M$, more specifically, we choose $L = M/2$ even though our approach can be generalized for other values of L.

For a uniform complex modulated FB, the transfer functions of $H_k(z)$ and $F_k(z)$ are obtained by complex modulating two low-pass linear phase prototype filters $H(z)$ and $F(z)$ of length P, respectively, i.e. $H_k(z) = z^{(-(P-1)/2)} H(ze^{j\frac{2\pi k}{M}})$, and $F_k(z) = z^{(-(P-1)/2)} F(ze^{-j\frac{2\pi k}{M}})$ $\forall k = 0, \ldots, M-1$. The constant delay of $(P-1)/2$ samples is translated to the factor $d^k = e^{-j\frac{\pi}{M}(P-1)k}$ in the k-th sub-band so that the phase of the modulated filters is strictly linear and not affine.

Both analysis and synthesis FBs (AFBs and SFBs) can be efficiently implemented by first applying polyphase decomposition [5] of type 1 to $H(z) = \sum_{m=0}^{M-1} z^{-m} G_m(z^M)$ and of

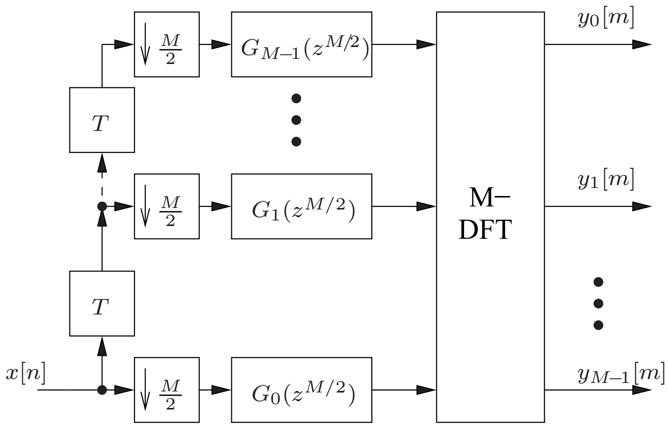

Fig. 2. Efficient structure of AFB of non-maximally decimated DFT FB.

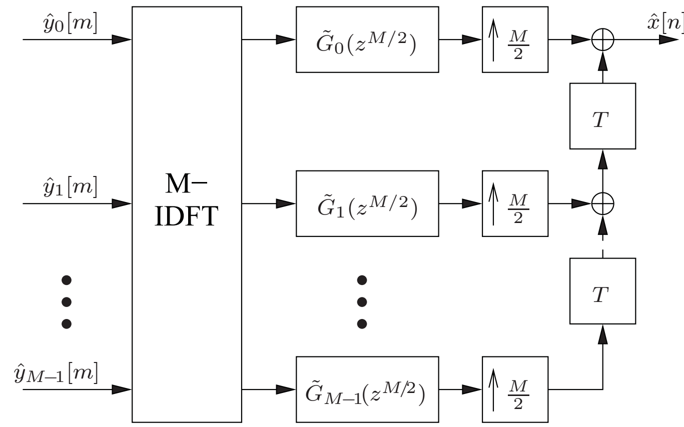

Fig. 3. Efficient structure of SFB of non-maximally decimated DFT FB.

type 3 to $F(z) = \sum_{m=0}^{M-1} z^m \tilde{G}_m(z^M)$. Then, some important identities for multirate processing are applied and finally the complex modulation by means of DFT and inverse DFT (IDFT) of size M is applied. The polyphase representation is a neat tool which enables the rearrangement of computations of the filtering operations, so as to minimize the computational load per time unit. The polyphase components $G_m(z)$ and $\tilde{G}_m(z), m = 0, \ldots, M-1$ of $H(z)$ and $F(z)$, respectively, are static filters with small number of taps K. The multiplication by the factor d^k in the analysis and synthesis FBs, given that P is even, can be transferred after the IDFT operation by exploiting the effect of a frequency shift on the IDFT. This leads to the efficient analysis and synthesis FBs structures shown in Figs. 2 and 3, respectively. To operate the FB structure as FD equalizer, short-length FIR filters are placed between the analysis and the synthesis FB as to be discussed in the next section.

III. Trivial Prototype Filter FB Based CD Compensation

The overlap-save FFT method for FD CD compensation with 50% overlap as benchmark [3] can be implemented as a non-maximally decimated DFT FB. In order to realize this structure based on Figs. 2 and 3, trivial filters (i.e. rectangular impulse response) of length M are chosen for $H(z)$ and $F(z)$. However, $M/2$ coefficients of $F(z)$ are zero for the overlap-save method. The polyphase components $G_m(z)$ and $\tilde{G}_m(z)$ are of length $K = 1$. CD equalization is done per sub-band through the FIR filter $E_k(z)$ of length N_t.

Fig. 4 shows the resulting structure implemented as an efficient non-maximally decimated DFT FB. Until now the case of a single tap $N_t = 1$ per sub-band has been considered which will be referenced as our benchmark. As we are making use of all M degrees of freedom to design the overall equalizer, it is no longer strictly the overlap-save method that implements linear convolution with the aid of FFT and IFFT, but it is a FB based CD equalization with trivial prototype filters. Throughout the previously published literature, this scheme has been widely considered without correct awareness of this issue [3].

Our method for CD compensation is based on this non-maximally decimated DFT FB with trivial prototype filters but with delayed single-tap equalizer per sub-band i.e. $N_t > 1$. In the following section, we explain the idea behind and how to design a delayed single-tap equalizer.

IV. Delayed Single-Tap Equalizer Design

The idea for the delayed single-tap equalizer is to take into account in the design the group delay (in samples) due to the nature of the inverse of CD channel which is given by:

$$H_{\mathrm{CD}}^{-1}(f) = \exp\{+j\alpha f^2\}, \tag{1}$$

where $\alpha = \frac{\pi\lambda^2 \mathrm{D}}{c}$. The value of chromatic dispersion D is expressed in ps/nm, the wavelength λ is in nm and the speed of light in vacuum c in m/s. The group delay τ_k obtained by differentiating the inverse channel phase response of Eq.(1) with respect to angular frequency has a linear behaviour given as:

$$\tau_k = \frac{-\alpha}{\pi} f_k,$$

$f_k = \frac{2k}{TM}$ for $k \leq M/2$ and $f_k = -\frac{2(M-k)}{TM}$ for $k > M/2$. Now, the single tap equalizers E_k of the overlap-save method are extended to delayed single-tap equalizer $E_k(z)$ with maximum number of taps of N_t. In Fig. 5, the idea of delayed signle-tap equalizer is illustrated. To avoid any extra complexity, only one tap $e_{k,l}$ is assumed to be active performing single-tap phase equalization where the delay elements realize a sub-band delay.

Due to the $M/2$-fold downsampling in the sub-band region, only a coarse (quantized) approximation of the sub-band group delay τ_k can be achieved. Mathematically, each equalizer reads as:

$$E_k(z) = \mathrm{e}^{j\pi Q(\tau_k)k} H_{\mathrm{CD}}^{-1}(f_k) z^{-\frac{M}{2}\left(Q(\tau_k)+\frac{N_t-1}{2}\right)} \tag{2}$$

where the group delay quantization function $Q(\tau_k)$ is given by, depending on whether N_t is even (mid rise quantizer):

$$Q(\tau_k) = \mathrm{sign}(\tau_k)\min\left(\left|\mathrm{round}\left(\frac{4\tau_k}{TM}+0.5\right)-0.5\right|, \frac{N_t-1}{2}\right) \tag{3}$$

or N_t is odd (mid thread quantizer):

$$Q(\tau_k) = \mathrm{sign}(\tau_k)\min\left(\left|\mathrm{round}\left(\frac{4\tau_k}{TM}\right)\right|, \frac{N_t-1}{2}\right). \tag{4}$$

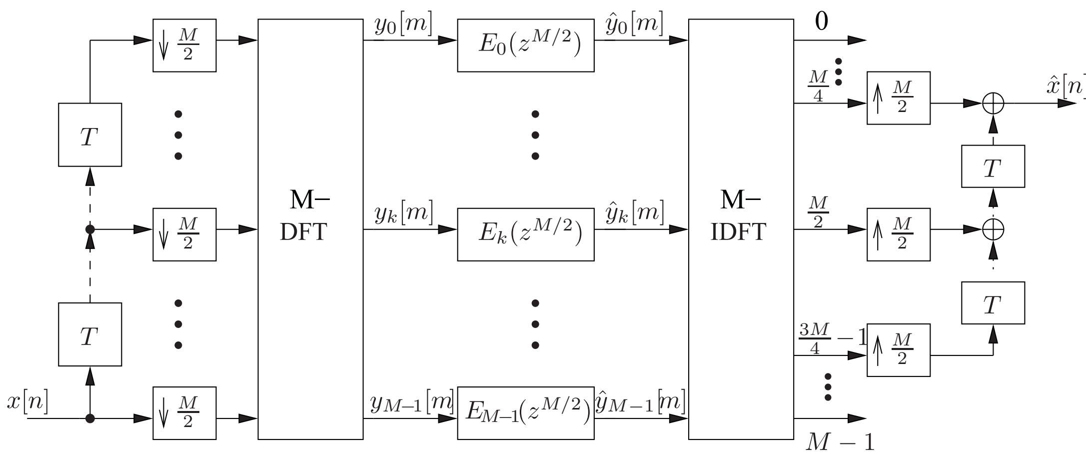

Fig. 4. Method of overlap and discard with 50% overlap realized as a nonmaximally decimated with trivial prototype filters DFT FB with sub-band equalizer $E_k(z^{M/2})$ for CD compensation.

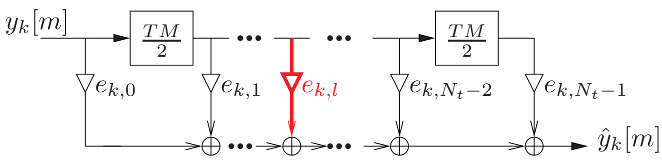

Fig. 5. Idea of a delayed single-tap equalizer. Only $e_{k,l}$ is active for the kth sub-band with $l = Q(\tau_k) + (N_t + 2)/2$. All other taps ($e_{k,p}$ for $p \neq l$) are zero.

In the special case of a single tap equalizer (N_t =1), $E_k = H_{\mathrm{CD}}^{-1}(f_k)$. The additional term $e^{j\pi Q(\tau_k)k}$ in Eq(2) is needed to fulfill:

$$E_k(z = e^{j2\pi T f_k}) = H_{\mathrm{CD}}^{-1}(f_k)e^{-j2\pi \tau_{\mathrm{cst}} f_k},$$

where τ_{cst} is a frequency independent delay induced by the sub-band processing and given by $\tau_{\mathrm{cst}} = \frac{N_t-1}{2}TM$.

In brief, this equalizer can be interpreted as a sub-band group delay filter with linear all-pass filtering of the CD in each sub-band.

V. Results

A 28 GBaud polarization division multiplexed (PDM) return to zero (RZ) QPSK transmission with digital coherent receiver applying two-fold oversampling with 56 GS/s is used to verify our technique for CD equalization. Based on linear simulations, the performance of the system with delayed single-tap equalizer and the benchmark i.e. with a single-tap equalizer have been evaluated for different N_t, FFT sizes (i.e. different M) and for compensating different CD values. System parameters are $B = 1/T = 28$ GHz and λ =1550 ps/nm. For performance analysis, the required optical signal to noise ratio (OSNR) to tolerate different CD values (accordingly different fiber lengths) at a bit error ratio (BER) of 10^{-3} is chosen as the figure of merit. The DFT and the IDFT are efficiently implemented with FFT and IFFT, respectively, by choosing M as power of 2. With this linear simulation model we want to demonstrate the capability of the linear equalizer. It should be noted that on top of the demonstrated filtering penalty, OSNR penalties resulting from the fiber attenuations and the according optical amplification need to be added

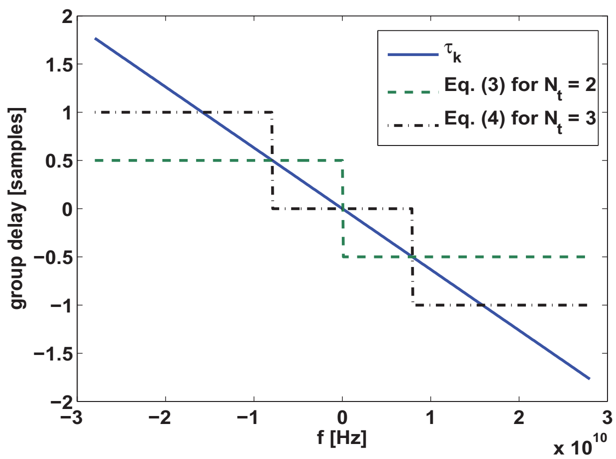

Fig. 6. Quantization of the group delay with an even and odd N_t.

as well as penalties from non-linearity and implementation constraints.

In Fig. 6, the quantization of the group delay $Q(\tau_k)$ for even and odd number of taps N_t is plotted for the case of $M = 512$ and dispersion of 36,000 ps/nm. The difference between the true group delay and the quantized group delay causes a low value of residual CD in each sub-band, which is compensated by a single-tap all-pass filtering function realizing the according parabolic phase transfer function.

In a first set of simulations, ideal synchronization of timing and carrier is assumed and only the delayed single-tap FD CD compensation is applied. In Fig. 7, the required OSNR has been plotted for different number of taps N_t for the equalizer per sub-band. The advantage of a delayed single-tap equalizer can be clearly seen where significantly higher CD values are compensated for the same FFT size when increasing N_t as long as $N_t \ll M$. The equalizer memory can be significantly increased by the combination of the group delay elements and the phase rotation. This is a clear benefit compared to a single tap equalizer with only phase filtering. For a given maximum N_t, a lower number of active taps is automatically applied at low CD values. In other words, as CD increases, the number of required taps for equalization increases linearly until the maximum N_t is reached.

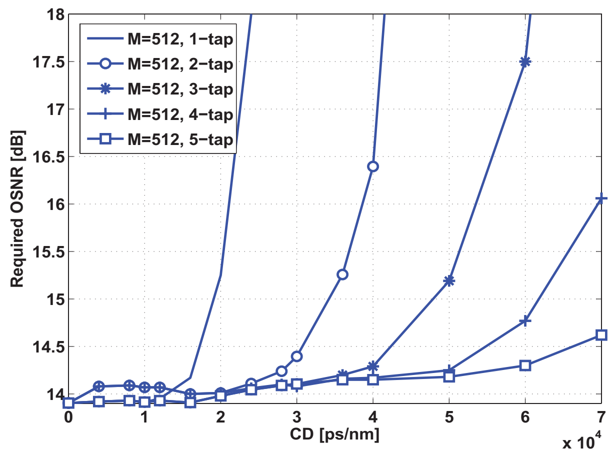

Fig. 7. Required OSNR for different CD values with a multitap per sub-band CD equalizer. Advantage of a delayed-single tap equalizer.

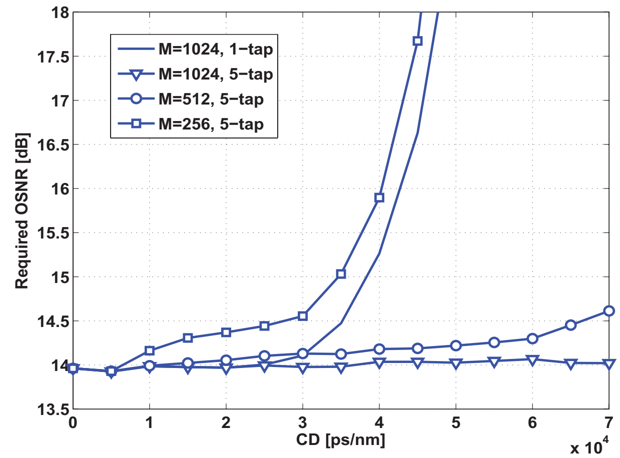

Fig. 8. Required OSNR for different CD values. Higher CD values are compensated for $N_t > 1$ at the cost of small penalty and additional memory elements.

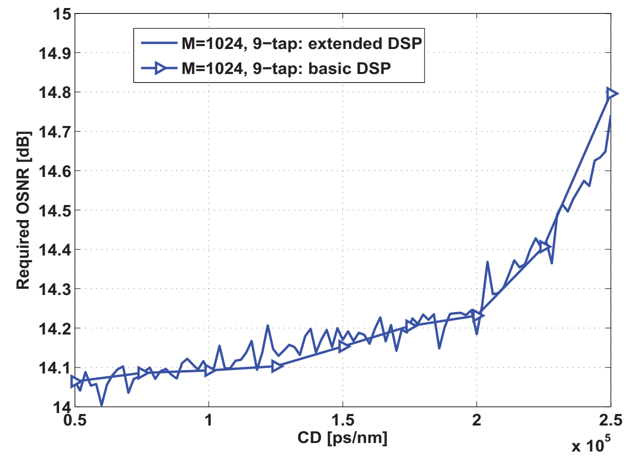

Fig. 9. Comparison of basic CD model with extended channel model for widely considered FFT size of M =1024 and N_t =9.

The simulations in Fig. 8 show the required OSNR for the same N_t but at different FFT-size M. As a benchmark, we choose the FFT size $M = 1024$ and $N_t = 1$. Significant smaller FFT sizes as compared to the benchmark for $N_t = 5$ is needed to tolerate the same CD value. For example, to tolerate a CD value of 32,000 ps/nm, it is sufficient to have an FFT size of 256 with $N_t = 5$ taps with less than 1-dB OSNR penalty. With $M = 1024$ and $N_t = 5$, a negligible OSNR penalty is observed even outperforming the benchmark.

Extending the signal processing by an 11-tap ($T/2$ tap spacing) 2×2 multi-input multi-output (MIMO) time-domain equalizer with blind convergence and acquisition by constant modulus algorithm (CMA), Viterbi & Viterbi 4-th power carrier phase estimation and carrier recovery (no differential decoding) does not largely alter the results of Figs. 7 and 8. No laser phase noise and no carrier frequency offset were assumed in the simulation. In Fig. 9, the required OSNR is plotted for the extended model including CD and PMD equalization and phase recovery with $M = 1024$ and $N_t = 9$, while still assuming ideal synchronization. It is clearly seen that the results are essentially the same. The linear increase of filtering penalty up to 200 000 ps/nm results from the decomposition of the signal into an increasing number of sub-bands. It results from slight phase imperfections between adjacent sub-bands after signal decomposition. Above 200 000 ps/nm the channel memory starts to exceed the filter memory with the typical rapid penalty accumulation. For an OSNR filtering penalty of 0.5 dB more than 240 000 ps/nm can be equalized which refers to a trans-Pacific distance around 15 000 km with standard single-mode fiber.

VI. Conclusion

In this letter, FD CD compensation is proposed based on a non-maximally decimated DFT FB with trivial prototype filters and a delayed single-tap equalizer per sub-band. The design of the delayed single-tap equalizer takes into account the group delay due to the nature of the inverse of CD channel in each sub-band. The well known overlap and discard method with 50% overlap applied so far for CD compensation can be as well interpreted as a non-maximally decimated FB with trivial prototype filters with a single tap equalizer per sub-band. With our design larger CD values can be compensated with a smaller FFT size by increasing the number of taps of the equalizer in each sub-band as long as maximum number of delay taps remains negligible as compared to the FFT size. With our technique uncompensated trans-Pacific transmission becomes feasible with digital CD compensation i.e. more than 240 000 ps/nm CD tolerance with only 0.5 dB OSNR filtering penalty can be achieved by use of 1024 FFT.

References

[1] G. Gavioli, *et al.*, “NRZ-PM-QPSK 16×100 Gb/s transmission over installed fiber with different dispersion maps,” *IEEE Photon. Technol. Lett.*, vol. 22, no. 6, pp. 371–373, Mar. 15, 2010.

[2] B. Spinnler, “Equalizer design and complexity for digital coherent receivers,” *IEEE J. Sel. Topics Quantum Electron.*, vol. 16, no. 5, pp. 1180–1192, Sep./Oct. 2010.

[3] K. Ishihara, *et al.*, “Frequency-domain equalization for optical transmission systems,” *Electron. Lett.*, vol. 44, no. 14, pp. 870–871, Jul. 2008.

[4] I. Slim, *et al.*, “CD equalization with non-maximally decimated DFT filter bank,” in *Proc. IPC 2012*, Burlingame, CA, Sep. 2012, p. 173.

[5] T. Karp, *et al.*, “Modified DFT filter banks with perfect reconstruction,” *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.*, vol. 46, no. 11, pp. 1404–1414, Nov. 1999.