STANDING WAVES AND THE INFLUENCE OF SPEED LIMITS

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Keywords: Traffic control automation, decentralized systems, control of complex systems, stabilization of non-linear systems, transportation systems.

Abstract

Traffic dependent speed limits have improved the security of highway traffic considerably. However, on highway sections controlled by variable speed limits large oscillations in speed can still be observed. On the basis of traffic data and a macroscopic traffic model, the influence of decentrally controlled speed limits on the states of highway traffic is analyzed. It is shown that local control of speed limits - which is typically applied - can lead to standing waves in speed. The anticipative control law proposed is able to prevent standing waves and can even damp out stop-and-go waves, such that a homogeneous traffic flow is gained.

1 Introduction

Highway traffic control by variable speed limits has reduced the number of accidents by a large amount - reports talk of about 35% for the highway A9 in the North of Munich [1]. Also the number of traffic jams and - due to a more homogeneous traffic - air pollution are reduced. This seems to indicate that there is no room for improvement. And indeed, even a perfect control system cannot prevent jams completely: once the traffic flow exceeds the capacity of a highway, traffic jams will occur.

However, even below this maximum flow congested traffic occurs due to the metastability of highway traffic (for an exact definition of stability and metastability see [3]). The border between stability and metastability is set by the flow out of a traffic jam. Below this characteristic flow, which is about 25%-30% below the maximum flow, any perturbation will disappear [3]. Above this characteristic flow, small perturbations are still damped out, but large perturbations will grow. At maximum flow even the smallest perturbations lead to congestion. It is precisely within this regime of metastability where control systems may improve the performance of highway traffic. We argue that two aspects are important for a successful control: A control law which is designed to reduce the amplitude of solitary waves and anticipation.

The paper is organized as follows: In section 2 highway data are analyzed with respect to the influence of locally controlled speed limits on the traffic state. In section 3 it is shown that the measured traffic states, in particular standing waves, can be reproduced by a locally controlled traffic model. In section 4 an anticipative control law is proposed which is able to damp out even standing waves. In section 5 concluding remarks are given.

2 Analysis of highway data

Traditionally, free and congested flow are distinguished within traffic theory. In free flow the vehicle-density along a road is small and interactions between vehicle-driver-elements rarely occur. Congested flow is characterized by strong interactions between the vehicle-driver-elements and nonlinear dynamic phenomena like jams and stop-and-go waves [3]. In jams the flow breaks down almost completely, the density increases and the speed approaches zero. In contrast, while stop-and-go waves pass a measurement site, the flow, the density, and the speed vary considerably [3]. Another type of congested traffic is given by standing waves. Here, the flow along a highway section is (almost) constant, but still the speed and the density changes considerably. Although a constant flow might be desirable for flow optimization, changes in the speed and in the density - corresponding to acceleration and deceleration actions - are disadvantageous with respect to the drivers' safety.

An analysis of traffic data from the German highway A9 from Nürnberg to Munich on November 27th 1995 reveals that a standing wave can be maintained for more than one hour. However, for the highway considered here, a standing wave exists, although speed limits (v_{limit}) are switched based on a control law[1]. The possible control actions are given by

$$v_{limit} \in \{ \oslash, \textcircled{120}, \textcircled{100}, \textcircled{80}, \textcircled{60}, \textcircled{jam} \} , \quad (1)$$

where $\oslash$ means that there is no speed limit at all. The schematic configuration of the observed highway section is given

1. Local measurements are used to determine the speed limitation in the corresponding highway segment [9].

From: ECC 2001, APCA

in fig. 1. The distance between two gantries (Δx_g) is given by almost constant 2km.

A closer look on the traffic data is given in figs. 2 and 3, thereby revealing some of the states of highway traffic. Free flow is characterized by an almost constant velocity (here about 110 km/h) and densities below some critical threshold (here about 12 cars/km). In the fundamental diagram in fig. 2, where the density is plotted against the flow (aggregated over 1 minute), free flow is represented by the straight line through the origin with the slope 110 km/h. Besides this line one observes a second cluster on a line corresponding to a velocity of about 80 km/h, a third cluster with velocities of about 20 km/h, and a scattered cloud of data at higher densities, representing congested traffic. Fully established jams would be represented by data with zero flow and high density. These data are lacking because the induction loops cannot detect zero velocities. Therefore, data at high densities and low flow, i.e. low velocities are not very reliable. The downstream front of a jam, where vehicles start accelerating again, usually moves with a velocity of about 15 km/h in upstream direction. This corresponds to data on a straight line with negative slope in the fundamental diagram. In fig. 2 this dashed line roughly corresponds to the lower border of the cloud at higher densities. Fig. 3 shows that this data cloud is due to stop-and-go waves, i.e. large oscillations in velocity and density. A comparison of the two time series in fig. 3 shows that these stop-and-go waves move upstream.

In addition, one observes a period from about 9:30 to 10:30, where the velocity at each of the two measurement sites is almost constant. This corresponds to the downstream front of the congested region, where the vehicles start accelerating again. Here this front doesn't move, i.e. a standing wave appears and one observes locally almost the same state for about one hour. In fig. 2 this is indicated by squares for the measurements taken at km 519.33 and by circles for the measurements taken at km 518.66. While the flow remains almost constant around 1000 cars/h, the difference in the speed - from about 20km/h at km 518.66 to 80km/h at km 519.33 - is significant. During the acceleration phase at km 518.66 and between 10:20 and 10:30, when the speed increases from about 20km/h to 80km/h, the flow is still almost constant, i.e. the standing wave is conserved (in fig. 2 this is indicated by triangles, the time evolution is from right to left). From 10:30 on there exists free flow, again.

Fig. (3) shows the speed measurements and the corresponding speed limits. Despite the control law, large speed oscillations are maintained for several hours, i.e. stop-and-go waves and a standing wave exist. During this time, the control actions take the values (60) and (jam)[2]. Finally, the standing wave changes to

2. Speed limits smaller than (60) should be avoided. These limits might lead to repeated switchings between different limits. Instead, when the sign (jam) is displayed, the positive effect of speed limits, i.e. the creation of a homogeneous flow is extended to higher traffic densities [7, 8].

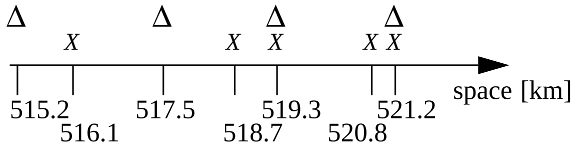

Figure 1. Schematic configuration of the chosen section of the German highway A9. The direction of flow is from left to right. Measurement sites are given by X, gantries displaying speed limits by Δ.

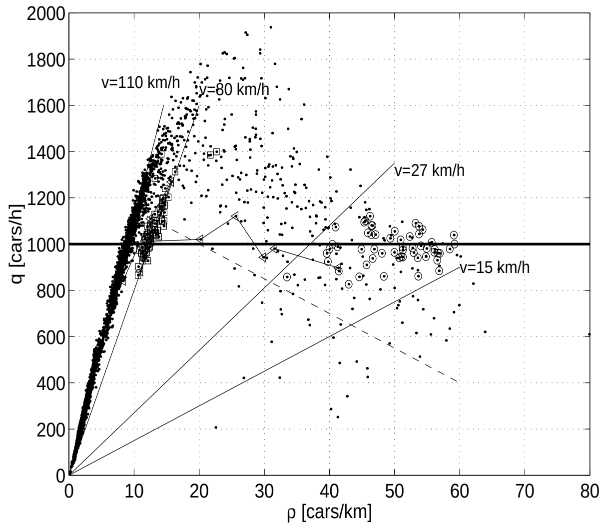

Figure 2. Measurements in the flux-density-plane for November 25th 1995 (dots). Between 9:25 and 10:30 a standing wave (squares at km 519.33, circles at km 518.66) appears, where $q \approx 1000 cars/h$ (thick horizontal line). The diagonal lines correspond to the indicated, constant speed, respectively. The dashed line corresponds to the downstream front of a traffic jam.

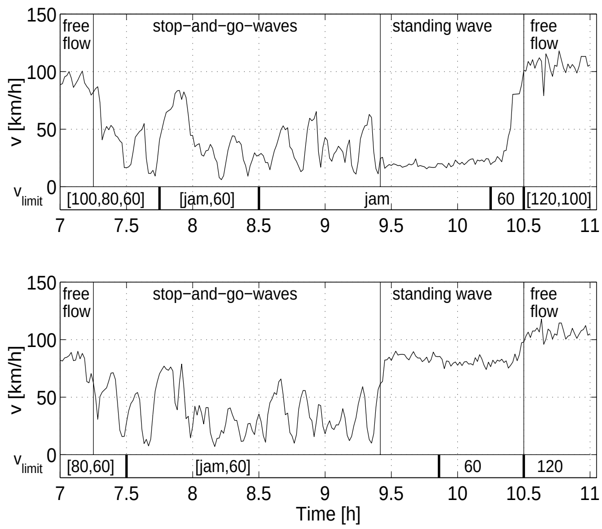

Figure 3. Speed at km 518.66 (top) and at km 519.33 (bottom) on November 25th 1995. The traffic phase and the speed limits at km 517.53 (top) and at km 519.35 (bottom) are indicated. A standing wave exists from 9:25 to 10:30.

free flow. However, it can be assumed that this step depends merely on the traffic flow, i.e. at the end of the morning rush hours at 10:30 the flow into the considered highway section decreases and free flow is observed.

3 Model based analysis

In the following, the analysis of the influence of speed limits with respect to traffic states will be continued on the basis of a macroscopic spatio-temporal continuum model [3]. The model has been chosen since it shows the characteristic properties of traffic flow, i.e. free flow and congested flow [3]. When the model is initialized by a medium density being superimposed by a small disturbance a stop-and-go-wave develops [3]. The model consists of the static, empirically determined relation $V(\rho, u)$ between the mean speed v, the density ρ, and the normalized control action u [2, 7]. Roughly speaking, u represents the scaling influence of speed limits on the $V(\rho, u)$-relationship. For a range of u see [2, 7]. In general, the mean speed v and the density ρ vary with respect to space x and time t. This is accounted for by a continuity equation [3]

$$\frac{\partial}{\partial t}\rho + \frac{\partial q}{\partial x} = \frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}[\rho v] = 0, \tag{2}$$

which holds for a circular road with periodic boundary conditions and without on- and off-ramps, and an equation for the acceleration of a volume element moving with the traffic flow [3]

$$\frac{dv}{dt} = \frac{\partial}{\partial t}v + v\frac{\partial v}{\partial x} = \frac{1}{\tau}[V(\rho, u) - v] - \frac{c_0^2}{\rho}\frac{\partial \rho}{\partial x} + \frac{\eta_0}{\rho}\frac{\partial^2 v}{\partial x^2}. \tag{3}$$

The equation of continuity describes the (local) conservation of cars. The equation of acceleration describes the dynamics out of the equilibrium given by the $V(\rho, u)$-relationship. The flow q is defined by $q = \rho v$. The parameters τ, c_0 and η_0 are required to adapt the model to measurements [3].

According to the work in [6, 7], the traffic flow defined by eqs. (2) and (3) can be stabilized by a nonlinear continuous control law such that a homogeneous flow is formed, regardless of the initial conditions. In essence, the control law opposes the process of the formation of stop-and-go waves by a two-point control law, i.e. for low densities the mean speed is reduced, whereas for high densities the mean speed increases due to the control law. An analysis of traffic data from the German highway A9 from Nürnberg to Munich for the time-span between November 1995 to January 1996 reveals that mandatory speed limits can be applied such that the nonlinear, continuous control law is approximated (see fig. 4), where $\Delta v = V(\rho, u) - V(\rho, 0)$ [7]. This leads to the discrete control law in eq. (4) and given by [7].

In general, on a highway only space-discrete measurements are available. Applying the density measured in segment i (see fig. 5) to the threshold-operation defined by eq. (4), a local (discrete) control law is obtained.

$$v_{limit} = \begin{cases} 120\frac{km}{h} & \text{if} \quad \rho \le 14\frac{cars}{km} \\ 100\frac{km}{h} & \text{if} \quad 14\frac{cars}{km} < \rho \le 17.5\frac{cars}{km} \\ 80\frac{km}{h} & \text{if} \quad 17.5\frac{cars}{km} < \rho \le 23\frac{cars}{km} \\ 60\frac{km}{h} & \text{if} \quad \rho > 23\frac{cars}{km} \end{cases} \tag{4}$$

After the traffic model has been initialized by stop-and-go waves and when the local control law is applied, a standing wave with about 1000 cars/h - as in the traffic data - is generated (cf. figs. 2 and 6). While there are stop-and-go waves in the uncontrolled model, the local control law causes a standing wave - due to the large distance of about $2km$ between gantries (cf. figs. 1 and 7). Acceptable results, i.e. small differences in the speed and in the density would only be possible, if the distance between succeeding gantries is chosen smaller than about 100 meters (see fig. 7). However, besides being cost intensive, such a high density of gantries would distract drivers' attention.

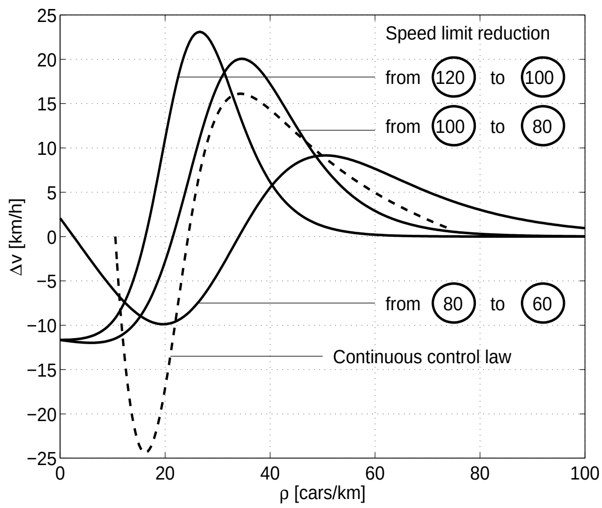

Figure 4. Control actions by the continuous control law and when the speed limit is reduced.

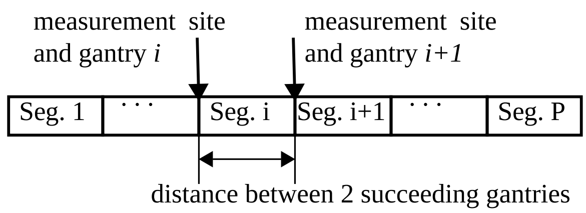

Figure 5. Dividing a highway-section into equidistant segments.

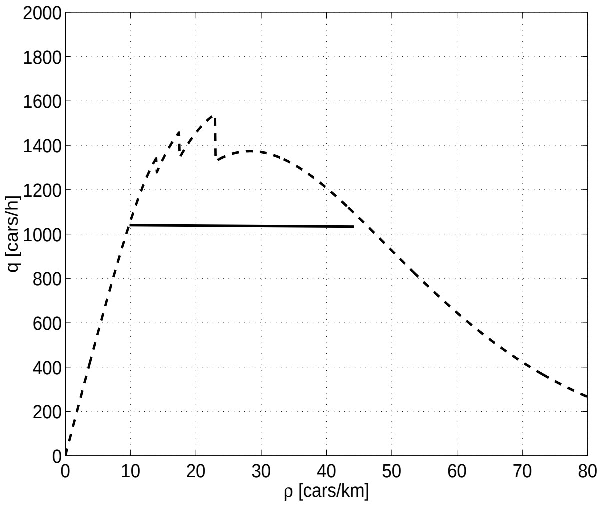

Figure 6. A standing wave (solid line) and the controlled $V(\rho, u)$ -relationship (dashed line).

4 Anticipation

Instead, a different approach is followed to overcome the problems of local control laws: making use of the fact that the direction of flow and the direction of a stop-and-go-wave are of opposite sign, an anticipative control law can be designed. Therefore a mean density $\bar{\rho}$ for segment i is determined and used within eq. (4).

$$\bar{\rho} = (\rho_i + \rho_{i+1})/2 \quad (5)$$

Hereby, the stabilizing effect of drivers observing more than one vehicle ahead is imitated [5]. In addition, losses due to the discretization of the originally space-continuous control law (see [6]) are partially compensated. Besides, the weighting of the two densities in eq. (5) could be set up more general by the use of an parameter α leading to $\bar{\rho} = \alpha\rho_i + (1-\alpha)\rho_{i+1}$. In this context, we have tested the anticipative control law only for $\alpha = 1/2$, yet, leading to successful results, as is shown, below.

On the one hand, the cost of the infrastructure for the anticipative control law are slightly bigger than the costs for the currently often applied local control law: the measurement data of the next measurement site ($i+1$) have to be made available for the current measurement site (i), which is connected with the current gantry for the display of speed limitations in segment i (see fig. 5). On the other hand, a distance of about 750 meters between succeeding gantries, i.e. about the width of a typical stop-and-go-wave, is sufficient to reach an almost homogeneous traffic flow, also with respect to speed and traffic density (see fig. 7). The anticipatively controlled flow is either homogeneous or almost homogeneous, as long as $\Delta x_g < 750\text{m}$ holds (fig. 7).

In detail, fig. 7 shows the performance of the controlled traffic flow in terms of the remaining disturbances in density, speed and flow as a function of the distance between succeeding gantries. When a local control law is applied (see upper part of fig. 7), an acceptable control performance in the density and in the speed is achieved only for very small distances between succeeding gantries in the range of less than 100 meter. However, besides being cost intensive, such a close distribution of gantries would distract drivers' attention. The control performance of the anticipative control law is shown in the lower part of fig. 7. For distances between succeeding gantries smaller than about 750 meters, the control performance of the anticipative control law is clearly superior to the control performance of the local control law. Disturbances in the density, the speed and the flow disappear completely as a result of the anticipative control actions producing appropriate speed limitations. For distances between succeeding gantries larger than about 750 meters both control laws lead to equal results with respect to the asymptotic disturbances in the density and in the speed. For the anticipative control law asymptotic perturbations in the flow stay for distances between succeeding gantries larger than about 750 meters. For these distances the local control law leads to vanishing perturbations in the flow, thereby producing the standing waves described before.

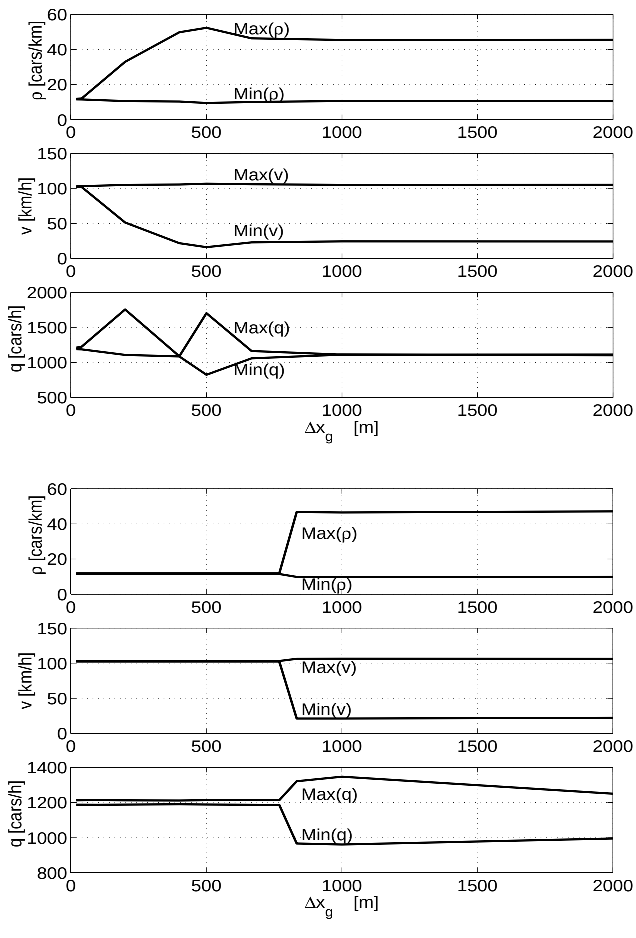

Figure 7. Asymptotic disturbances for the locally (top) and anticipatively (bottom) controlled traffic flow, after initialization by stop-and-go waves.

For a practical implementation the main conclusion from fig. 7 is the demand to apply anticipative control laws to speed limitations on highways. Besides, the distances between succeeding gantries should be regularly spaced and smaller than the critical width of stop-and-go-waves. The critical width of a stop-and-go-wave is given by the smallest width of the stop-phase of a stop-and-go-wave, which asymptotically persists on a highway. In the simulations performed, this width is about 750 meters, which is consistent with traffic data (e.g. see [4]).

The advantage of the anticipative control law is created by the gain in the reaction time of drivers and by the coupling of two measurement sites leading to a continuous flow of information. When there are perturbations on the highway section considered, the speed limits displayed based on the anticipative control law are changed earlier or at least at the same time compared to displays based on a local control law. Consequently, the drivers are informed earlier about perturbations existing in their direction of flow. The time gained is bounded by

$$T_g \le \Delta x_g / (v - v_S) , \qquad (6)$$

where v_S is the speed of the disturbance relative to the highway. It is important to note that the time gain is independent of any modelling assumptions and typically is in the range of ten seconds up to a minute (see table 1). As a consequence, several ten drivers are warned due to the anticipative control law.

Other simulations, where additional measurement sites between two gantries have been applied, show that the anticipative control law is advantageous in a second respect: the speed limits displayed at succeeding gantries are coupled by a common measurement site such that the flow of information is smooth and continuous.

speed v	**maximum time gain** $T_{g,\,max} = \Delta x_g/(v - v_S)$	**additionally warned cars on 2 lanes (3 lanes)**
105 km/h	22.5 sec.	18.8 (28.1)
80 km/h	28.4 sec.	23.7 (35.5)
60 km/h	36.0 sec.	30.0 (45.0)

Table 1. Time gain for $\Delta x_g = 750m$, $v_S = -15km/h$ and $q = 1500cars/h$.

5 Concluding Remarks

The investigations performed have shown that while a local control law is operating stop-and-go waves can be formed and they might turn into standing waves. In contrast, an anticipative control law prevents the formation of stop-and-go waves and flattens already existing stop-and-go waves such that a homogeneous traffic flow with respect to both flow as well as speed is reached. Clearly, the anticipative control law flattens the fluctuations with respect to speed and density in standing waves, too.

In essence, due to the time gain and the continuous flow of information a stabilizing effect is achieved, similar to the one due to drivers observing more than one car ahead (see [5]). Hence, when the anticipative control law is operating, perturbations are damped and a homogeneous traffic flow is gained.

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