Mathematical Modeling of Discrete Sports Behaviors as Continuous Topologies -
Applications in Golf and Field Hockey

Michael Stöckl
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Chapter 1

Introduction

Since people began competing in sports they coincidentally have tried to optimize their chances to win (e.g. Ioannidis et al., 2008). Thus, athletes and their coaches started to analyze the human being itself and the performance of an athlete in order to improve certain skills or abilities so that the chances to win the next competition(s) are increased. These analyses eventually were developed further and further and finally resulted in a branch of science - sport science. Sport Science is interdisciplinary and uses methods and approaches from other research disciplines, e.g. psychology, biomechanics, medicine, and performance analysis. All those research areas analyze athletes’ skills & abilities and their performances from different viewpoints and, among other goals, all the disciplines want to improve athletes’ performance.

This thesis describes the development of a new performance analysis method, the ISOPAR method, and demonstrates its use in theoretical and practical applications. The following articles represent the work involved in this thesis:


Because of space limitations an outline of the respective background of performance analysis and of the ISOPAR method’s underlying framework is not provided in these articles. This thesis is written at a German university and thus it is written from a German viewpoint on performance analysis. Whereas sports performance analysis is part of several disciplines internationally (Hughes & Bartlett, 2002), performance analysis is part of the discipline ‘Trainingswissenschaft’ in German sport science. Hence, **Chapter 2** gives an overview of ‘Trainingswissenschaft’, lays out what performance analysis is, and how it is rooted in ‘Trainingswissenschaft’.

**Chapter 3** introduces two of the main concepts of the dynamical systems theory, on which the ISOPAR method is based, and the approach of the ISOPAR method. Further details on the ISOPAR algorithm are also listed in this chapter. This is done because the introduction of the algorithm in article 4.1 is golf specific and since its publication the algorithm has been developed further. Furthermore, new performance indicators provided by the ISOPAR method are introduced including the properties of these indicators which have not been shown in any of the articles.

**Chapter 4** provides a summary of each of the four articles which are included as part of this thesis.

**Chapter 5** discusses the ISOPAR method, its approach, the performance indicators, and the application of the ISOPAR method including current research.
Chapter 2

‘Trainingswissenschaft’ and Performance Analysis

In German sport science, performance analysis is part of the research area ‘Trainingswissenschaft’. Hence, in this section performance analysis will be explained and defined in context of ‘Trainingswissenschaft’. The research area ‘Trainingswissenschaft’ will be introduced first. Second, performance analysis will be introduced and located in ‘Trainingswissenschaft’.

2.1 ‘Trainingswissenschaft’

‘Trainingswissenschaft’ is the central research area in German sport science (Schindler et al., 1970). Generally, ‘Trainingswissenschaft’ deals with the scientific foundation of training and competition with respect to sports from a holistic and applied perspective (Hohmann et al., 2010, p. 17, own translation). These authors also state that ‘Trainingswissenschaft’ is an integrative research discipline. Therefore, this discipline is hallmarked by collecting, applying, comparing them to each other, and validating known theories from several other research areas. Those research areas are biomechanics, medicine, sociology, psychology, kinesiology, and in a wider sense also pedagogy (Hohmann et al., 2010, p. 19).

choice of using skills & abilities that they clearly want to distinguish between the
skills & abilities of an athlete and the performance an athlete realized in a compe-
tition. This is necessary because, on the one hand, athletes’ performance usually
varies from competition to competition, but, on the other hand, every athlete has
his or her own skills & abilities which are characteristic of the respective athlete.

Figure 2.1: Visualization of the interaction of the three subject areas of ‘Train-
ingswissenschaft’ (adapted from Hohmann et al., 2010, p. 30, own translation)

Hohmann et al. (2010); Schnabel et al. (2011); Hottenrott & Neumann (2010)
agree that the three subject areas of ‘Trainingswissenschaft’ cannot be studied sep-
arately since they are tightly coupled (see Figure 2.1). However, ‘Training’ is the
pivotal area of these three subject areas of ‘Trainingswissenschaft’ (Hohmann et al.
2010).

Figure 2.1 illustrates the interaction of ‘Training’, ‘Competition’, and ‘Skills &
Abilities’. ‘Skills & Abilities’ are influenced and developed by the training process
and, coincidently, the current level of skills & abilities is basis for the goal of de-
velopment which should be achieved by the upcoming training phase. Furthermore, ‘Training’ interacts with ‘Competition’ since the preceding training process proves itself in the competition, because the result of the competition is determined by the performance during the competition and this performance is limited to the skills & abilities. Moreover, the outcome of a competition, successful or unsuccessful, is feedback for the preceding training phase. Finally, ‘Competition’, in particular the structure of the competition, determines generally which skills & abilities are required and, additionally, how an athlete should be able to perform to win. On the other hand, the skills & abilities of an athlete form the basis for performance and hence determine the performance in a competition to a great extent. For this reason, Hohmann et al. (2010) emphasize that the analyses of the coupling of ‘Training’, ‘Competition’, and ‘Skills & Abilities’ plays a prominent role for improving an athlete’s or team’s performance.

2.2 The special role of ‘Competition’ in ‘Trainingswissenschaft’

This thesis focuses on competition analysis, hence the special role of the subject area ‘Competition’ in ‘Trainingswissenschaft’ will be explained in this section. A (sport) competition is a comparison which is determined by the book of rules of the respective sport and by the performance of individual athletes or teams in order to determine a winner or a ranking (Lühnenschloss 1995, own translation). However, this definition is only a framework and needs to be refined for different categories of sports.

In this thesis two sports were studied, field hockey and golf which both belong to game sports according to Döbler (1984). Döbler was the first to categorize game sports and eventually tried to provide a formal definition of game sports. The cate-
Categories are (Döbler, 1984, p. 27)

a) Tor-, Mal und Korbspiele (e.g. football, basketball, field hockey, etc.)

b) Rückschlagspiele (e.g. tennis, squash, volleyball, etc.)

c) Schlagball- oder Abwurfspiele (e.g. baseball, cricket, softball, etc.)

d) Ziel- und Treibballs Spiele (e.g. golf, billiard, curling, etc.).

These categories are used internationally as well; [a] is Invasion Games, [b] is Net and Wall Games, [c] is Striking and Fielding Games, and [d] is Target Games (Read & Edwards, 1992). Döbler (1984) defined game sports as the following:

Game sports are competitions which are characterized by performance, are focusing on fun, and the process of a game is not predefined. Teams or individual athletes compete against each other at the same time, which means their actions depend on each other’s actions, and according to game specific rules (p. 29, own translation).

This definition is not very precise, but it is on the lowest possible level that covers elementary characteristics of sports of these four categories, which can be quite different like, for example, golf and field hockey. However, Döbler’s definition does not match with the structure of the game sport golf completely. In this sport the actions of the athletes do not really depend on each other. This is only true in a wider sense when, for example, the athletes of the last group play for the win of the tournament. Then maybe these two athletes react to the others’ play by playing more or less risky in order to win. However, Döbler’s definition is the best matching one for the game sport golf (for an overview see Lames, 1991). Lames (1991) refined the definition of game sports in the following way:
Game sports are sports which have an internationally used book of rules, in which individual athletes or teams interact with each other in order to achieve the goal of the game and, coincidently, to try preventing the opponent from achieving its goal of the game. The goal of the game is a symbolic action and is determined by game specific rules (p. 33, own translation).

Lames’ definition is more precise than the one of Döbler (1984). However, it only fits well to invasion games, like field hockey which was studied in this work, and net and wall games and in a further sense to striking and fielding games as well. It is not suitable for sports of the category ‘Ziel- und Treibballspiele’ such as golf because these sports lack direct interaction between the participants.

In context of ‘Trainingswissenschaft’, ‘Competition’ can be separated into three subject areas: Modeling the competition, controlling the competition, and analyzing the competition (Hohmann et al., 2010). The following paragraphs introduce the subject areas of ‘modeling the competition’ and ‘controlling the competition’. The subject area ‘analyzing the competition’ will not be dwelled on here because it is part of performance analysis which will be introduced more detailed in the next section.

**Modeling the competition** As mentioned in the previous section, the tight coupling between ‘Competition’ and ‘Training’ plays an important role in ‘Trainingswissenschaft’, especially with respect to elite sports, on which this thesis focuses. The competition gives feedback for the training and the aspects of performance which explicitly were trained are realized on some level in the competition. Hohmann et al. (2010) separate the feedback process into three steps. First, the performance in a competition needs to be modeled with respect to a specific purpose, for example resulting in a performance indicator. Second, the developed model is used to analyze and interpret the performance of a competition in order to identify strengths and
weaknesses or whether planned tactics or behaviors were put into practice. In this step, it is important to consider the constraints which influenced a measured performance, especially when abnormal values of performance indicators are interpreted. Finally, the outcomes of the previous step reveal possible aims for the next training phase, depending on whether those aims are achievable and depending on whether they are worthwhile. Eventually, the new aims are integrated into the upcoming training process.

Controlling the competition Generally, competition controlling consists of three phases: the preparation for the competition, the coaching during the competition, and the analysis of the competition (Hohmann, 1997, as cited in Hohmann et al. (2010)). Schnabel et al. (2011) report the same phases of competition controlling, with the exception that they emphasize the consequences the competition analyses have on the following training phase explicitly. Hohmann et al. (2010) include this process in the step ‘analysis of the competition’ implicitly.

The preparation for the competition includes two steps; first, analyzing the current skills & abilities and, based on that, specifying a strategy for the upcoming competition, and, second, implementing this idea in the training. Coaching during a competition means that the coach influences or supports the decision making of an athlete during a competition based on live-analyses of the competition or the game strategy. During analysis of the competition the performance of an athlete shown in a competition is analyzed with respect to the result and the aimed performance. Noteworthy, in all three phases different kinds of measurements of performance play an important role.
2.3 Performance analysis

In German sport science performance analysis is part of the discipline ‘Trainingswissenschaft’, albeit different techniques from other sport science disciplines are used to analyze performance. Internationally, performance analysis in sport is a collective term and evolved from two different kinds of analyses, biomechanical analysis and notational analysis (Hughes & Bartlett, 2002). Generally, performance analysis covers assessing performance of a whole competition, assessing performance of parts of a competition, and assessing skills & abilities using different techniques and with respect to different purposes (Hohmann et al., 2010). Thus, performance analysis takes place aiming at the subject areas ‘Skills & Abilities’ and ‘Competition’ as they were introduced in Section 2.1. In this thesis, the focus is put on performance analysis in competitions, although performance analysis covers more than that (Hohmann et al., 2010; Hughes & Bartlett, 2002; Schnabel et al., 2011; Hottenrott & Neumann, 2010). Hence, the following description of performance analysis is tied to this viewpoint.

To be able to analyze performance, norms are necessary to which measured performance can be compared. According to Hohmann et al. (2010) and Schnabel et al. (2011) there are three different kinds of norms:

- **Ideal norms** are based on top ranked athletes or models.

- **Statistical norms** are means and variances of samples and are valid for a certain peer group of athletes/teams.

- **Functional norms** are requirements which need to be fulfilled at least in order to realize a certain level of performance.

To analyze performance it needs to be measured somehow first. Different techniques to ‘measure’ performance are assessments by either sport specific experts,
video analyzes, or observations of competitions using standardized observation systems. Mostly from the latter, indicators are dedicated which allow quantifying a measurement of performance or a measurement of part of a performance. Hughes & Bartlett (2002, p. 739) provide the following definition of performance indicator: “A performance indicator is a selection, or combination, of action variables that aims to define some or all aspects of a performance.” It is important for the use and, especially, an objective interpretation of performance indicators that they are based on statistical norms because success of a performance in a competition always depends on the opponent and is independent of previous performances (Hughes & Bartlett, 2002).

Finally, performance analysis with respect to competitions can be distinguished into two subject areas - theoretical performance analysis and practical performance analysis (Lames & McGarry, 2007). These two subject areas are distinct from each other with respect to the respective aims but they are not independent from each other. The two subject areas will be discussed further in the next two subsections.

2.3.1 Theoretical performance analysis

The general aim of theoretical performance analysis is to explain and understand sports behavior of a sport in order to put this knowledge into practice (Lames & McGarry, 2007). In order to understand sports behavior it is important to study the structure of the sport. Investigating the structure of a sport involves

- identifying performance variables or patterns of performance which are common and important for a sport,

- analyzing and quantifying the relation between different performance variables, and
• identifying key performance variables; such performance variables which are important for and, hence, are highly correlated with being successful and for winning competitions in this sport, respectively.

Theoretical performance analyses mostly use general models (e.g. dynamical systems theory, see Section 3.1) to analyze empirical data (Lames & McGarry, 2007). The findings from these theoretical analyses are very important for sports practice as well as provide a database for individual (practical) performance analyses. The findings of theoretical analyses are important for sports practice because they reveal which performance variables are important for being successful, the aim of every athlete/team in every sport, and consequently provide useful information about on what athletes should focus in their (long-term) training process. The above mentioned empirical databases, which were created during theoretical performance studies, can be used for developing statistical norms for assessing measured performance or for identifying the level of skills & abilities which is necessary for a certain sport (Lames & McGarry, 2007). This links theoretical performance analysis and practical performance analysis.

2.3.2 Practical performance analysis

Generally, practical performance analysis aims at analyzing individual athletes or single teams. Thus, the tight coupling between performance in sport competitions and the underlying training process (see Section 2.2) is the main goal of practical performance analyses (Lames & McGarry, 2007). Performance of athletes or teams in competitions is analyzed in order to gain information on which the upcoming training process should focus. Therefore, performance in a competition is analyzed from two viewpoints: Analyzing the athletes’ or teams’ performance and analyzing
the opponents’ performance.

The goal of analyzing the athletes’ performance is to identify their weaknesses and strengths (Hohmann et al., 2010; Lames & McGarry, 2007; Schnabel et al., 2011). The results of these analyses indicate the most crucial aims for the upcoming training phase. Furthermore, this kind of performance analysis also allows one to monitor whether a training phase leads to a planned target or not (Hohmann et al., 2010; Schnabel et al., 2011).

Analyzing the performance of opponents is only useful for practical performance analysis of game sports which include interaction of the participants. From this viewpoint of practical performance analysis, the goal is to detect strengths and weaknesses as well, but with the aim of being able to create an optimized strategy for the upcoming competition, a game against this opponent. However, in games like golf it does not matter how the opponents perform since these games lack interaction of the participating athletes and the athletes only have to perform as well as possible based on their respective skills & abilities.

Practical performance analysis is connected to theoretical performance analysis as well. Identifying abnormalities when performance of individuals or single teams is analyzed, practical performance analysis creates hypotheses about (important) performance variables in a sport. Eventually, studying these hypotheses is the task of theoretical performance analysis.
Chapter 3

Modeling

In this thesis, the modeling is based on a systems approach which is based on concepts of dynamical systems theory. The systems approach which serves as framework for the developed model will be introduced first. Eventually, the underlying concepts of the new model will be outlined. Finally, the new model will be introduced, the resulting algorithm explained in detail, and the respective applications to golf and field hockey described.

3.1 The systems approach

In this thesis, performance in sports competitions is analyzed using a dynamical systems approach. A dynamical complex system can be described as a system consisting of many subsystems which are connected to each other and helps understanding spatial-temporal patterns. The interaction of the different subsystems with each other determines the collective behavior of the system. The changing collective behavior over time is characterized by a sort of communication between the subsystems, the so called self-organization which is one of the main concepts of dynamical systems theory. Another very important concept of dynamical systems theory is the constraints approach which describes different kinds of influences on the evolving collective behavior of a system. These two concepts will be explained in detail in the next two subsections.

The systems approach has already been used to describe and analyze performance in game sports and, therefore, prompts the non-linearity of performance in these
sports because of the interaction of many elements of the respective systems. The approach was applied to analyze intra-individual behavior, thus the performance of athletes in individual sports like golf (e.g. Lames 1992). Moreover, the approach was also used to analyze inter-individual behavior of interacting athletes, on the one hand, in sports in which single athletes are interacting with each other like in tennis or squash (e.g. McGarry et al. 1999, Palut & Zanone 2005) and, on the other hand, athletes who are interacting in teams which are interacting with each other like football (e.g. Gréhaigne et al. 1997, Hughes et al. 1998, McGarry et al. 2002, Frencken et al. 2011) or basketball (e.g. Bourbousson et al. 2010a,b).

Whereas in all these applications dynamical systems theory was used in order to study the behavior of the complex systems directly by analyzing order parameters, the application in this thesis is slightly different.

As Glazier (2010) suggests, in this thesis dynamical systems theory is used as a framework to explain sports behavior or performance rather than analyzing the complex systems behavior directly using tools like the relative phase. Thus, in this thesis a mathematical model is provided to analyze athletes’ performance in individual sports (golf) as well as team sports (field hockey) and the analyzed performances are explained using dynamical systems theory as a framework in an intra-individual application (golf) and in an inter-individual application (field hockey).

3.1.1 The concept of self-organization

Self-organization is a concept of the dynamical systems theory and was introduced among others by Haken (1977) in context of his work about synergetics. There are several definitions of self-organization from viewpoints of different research areas. In this thesis two well-known and quite similar definitions will be provided, Haken’s
definition which is from a physicist’s viewpoint and Camazine’s definition which is from a biologist’s viewpoint. Haken (1988) defined self-organization in context of his work about synergetics as the following:

a system is self-organizing if it acquires a spatial, temporal or functional structure without specific interference from the outside. By ‘specific’ we mean that the structure or functioning is not impressed on the system, but that the system is acted upon from the outside in a nonspecific fashion (p. 11).

Camazine et al. (2001) defined self-organization as

a process in which pattern at the global level of a system emerges solely from numerous interactions among the lower-level components of the system. Moreover, the rules specifying interactions among the system’s components are executed using only local information, without reference to the global pattern (p. 8).

Both definitions state that self-organization within a complex system is the process by which patterns emerge without any external influence on the complex system, for example by a coach in sports.

It is important to mention the coupling between the concept of self-organization and the concept of constraints which will be introduced in the next section. Stable states of a complex system usually are perturbed by internal (individual) constraints as well as external constraints and eventually force a system into a self-organizing process in order to shape an emergent stable pattern. Furthermore, the self-organizing process itself is accompanied by another set of internal and external constraints which guide the system into a new stable state (Newell 1986).

The concept of self-organization was brought to human movement science first by Kugler et al. (1980, 1982). From this branch of science, self-organization was
adopted and applied to analyze performance of game sports which can be modeled
as dynamical complex systems (e.g. Lames, 1992; Gréghaine et al., 1997; Hughes et
al., 1998; McGarry et al., 2002).

3.1.2 The concept of constraints

The theory of constraints is tightly coupled with Gibson’s theory of affordances
(Gibson, 1966, 1977, 1986). Affordances are opportunities for actions which are
provided to an individual by the environment in which this individual is planning
and performing an action. Furthermore, affordances provided by an environment are
relative to individuals and differ from person to person. An individual’s action is
supported as well as constrained by the constitution of the environment.

This leads to the idea of the constraints approach, which is widely used in differ-
ent branches of science. The difference between affordances and constraints is that
the former offer opportunities for actions, whereas the latter render actions impos-
sible. Hence, constraints describe boundaries for actions which are possible in an
environment for an individual. Kugler et al. (1980, p. 9) emphasized “it is not that
actions are caused by constraints; it is, rather, that some actions are excluded by
them”. Hence, constraints are factors which guide a dynamical complex system.

The constraints approach which is used in this thesis originally was outlined by
Newell (1986) and refined by Newell & Jordan (2007). There are three different
types of constraints: organismic constraints, environmental constraints, and task
constraints. Since humans are studied in this thesis, the organismic constraints will
be called individual constraints. The three types of constraints cover the following:

• **Individual constraints** are constraints which are relative to the individual
  and describe the boundaries of an individual. Individual constraints can be
separated into two groups: Structural constraints and functional constraints. Structural constraints are usually physical constraints which change very little over time. For example, they include height, body mass, genetic structure, and so on. Functional constraints are less static over time than structural constraints. Typical functional constraints are physiological, e.g. heart rate or lactate concentration, and psychological, e.g. anxiety, motivation, or the intention of an individual.

- **Environmental constraints** are constraints which are external of individuals and which belong to the respective surrounding environment in which individuals perform. Examples are temperature, ambient light, or, with respect to the sports which are studied in this thesis, friction of the ground on which a field hockey ball rolls or the direction of wind which influences the flight of a golf ball. Sociocultural constraints like the acoustic atmosphere of the spectators, peer pressure, or family expectations are environmental constraints as well (Clark [1995]).

- **Task constraints** are constraints which are specific to the task which is performed and are related to the goal of the task. In game sports task constraints are the game specific rules which usually determine the goal of the game as well as how a game sport is played. In field hockey, for example, task constraints are to score goals which means that the field hockey ball needs to be moved into the goal by moving it with a field hockey specific stick. Moreover, the ball can only be touched with the flattened side of the stick. In golf task constraints include moving the ball from the tee into a hole with as few shots as possible. The ball movement needs to be performed by hitting the ball with a club. Coaching also can be treated as a type of task constraint (Davids et
These three types of constraints do not describe the constraints in detail which influence an individual’s performance by, for example, listing all the different variables of the respective constraint type. They only describe the source of the different constraints which influence an individual’s performance (Newell et al., 1989). As well as it is for affordances, constraints are relative to individuals (Newell, 1989). Newell (1986) also highlights that not only single types of constraints influence an individual’s performance, it is the combination and confluence of constraints of the three sources which influences an individual’s performance. He also points out that small changes in any constraint can have a large impact on performance, as well as, that a large change in any constraint can have nearly no effect on performance.

### 3.2 The ISOPAR method

#### 3.2.1 The ISOPAR approach

In analyses of sports performance, an important task is to identify and measure key performance variables which statistically are highly correlated with successful outcomes of the respective sport. The problem of the conventional, statistical performance indicators is that they mostly neglect the underlying process of a sport, a factor which influences the resulting sports performance outcomes to a large extent (Glazier, 2010; McGarry, 2009).

This thesis focuses on sports whose underlying process can be described using dynamical systems theory. In particular, it focuses on the application of the concept of constraints and the concept of self-organization and their connection. As already explained, in this thesis the variables which describe a dynamical complex system
are not analyzed directly, dynamical systems theory is only used to explain sports performance and resulting outcomes, respectively.

When athletes perform an action at a location of a field of play they face similar conditions. The conditions are determined by the environmental constraints, the task constraints, which both usually are very similar for the same location, and the individual constraints, the type of constraints which vary most between athletes. It is also important how athletes perceive constraints and how they use this information for their upcoming performance, for example, which tactics they choose based on the given information (Fajen et al., 2009). Since performance of elite athletes is analyzed in this thesis, it can be argued that each athlete’s ability to perceive constraints is similar. However, the subsequent respective decision making is relative to the athlete and also the respective individual constraints. Eventually, the performance itself is guided by the confluence of the environmental, individual, and task constraints together with self-organization processes within the individual movement system.

In sum, sports performance should be analyzed with respect to the location where the action took place because only at this location are the conditions comparable. Thus, the model developed in this thesis focuses on relating sports performance to the location where it is performed.

The idea of the mathematical model in this thesis is to map sports performance of a competition on a map of the field of play of the respective sport as a continuous topology of representative performance. The goal of relating sports performance to the location where it takes place is not completely new (McGarry, 2009). In the past this approach was realized by artificially creating discrete zones to describe performance (e.g. Alcock, 2010; Sunderland et al., 2006; Yiannakos & Armatas, 2006) or by reducing the description of the location of performances to distance to
the target based categories (e.g. James 2007, 2009). Eventually, performance was assessed to statistical norms considering the respective zones or categories. However, this approach forces the comparison of performances which were performed under different conditions.

The model in this thesis allows one to compare the performance at a certain location on the field of play to a calculated ‘average’ performance at exactly the same location. The implementation of the model is based on measurements of performance, \( z_p \), and the respective location where it took place, \((x_p, y_p)\). Based on these measurements of performance an ‘average’ performance across the entire field of play is calculated. The calculation of this ‘average’ performance is based on all the measurements of a dataset, but those measurements which are closer to the location for which the ‘average’ performance is calculated should have greater influence on the resulting ‘average’ performance than those which are further away. The reason for this constraint is that the closer measurements are, the more similar the represented collection of constraints are. The resulting algorithm \( f \) needs to map the set of discrete measurements \( M \) as a continuous topology of ‘average’ performance \( AP \) for the complete field of play

\[
f : M \rightarrow AP
\]

where \( M, AP \in \mathbb{R}^3 \) and \( AP \) is limited to the range of the field of play.

### 3.2.2 The ISOPAR algorithm

Originally, the method described above was developed with respect to golf. The general idea was to calculate two dimensional maps of a golf hole on which the performance of the field is visualized in form of the lines of equal performance. In golf, the lines represent equal number of remaining shots until the ball was holed. The
idea for these maps is analogous to the concept of *isobar* (*iso* - meaning equal and *bar* - meaning pressure) maps in meteorology on which isobar lines represent equal barometric pressure. Small diameter, closed lines represent minima and maxima by which areas of low-pressure and high-pressure can be identified. Densely packed isobar lines indicate a steep gradient of air pressure. Because of this analogy and because of the application to golf the method has been named the ISOPAR method and was developed for calculating a gradient of difficulty, represented by the number of remaining shots, for a golf hole. Thus, the resulting two-dimensional maps with the *iso*-lines of golf performance superimposed are called ISOPAR maps.

Overall, the ISOPAR algorithm consists of four steps which will be described in detail in the following. Depending on the sports to which the method is applied additional constraints have to be considered and will be described in the following subsections about the application to golf and field hockey. As mentioned above, the ISOPAR method is based on triplets \((x_p, y_p, z_p)\), \(p = 1, \ldots, q\), where \((x_p, y_p)\) represents the location of a performance and \(z_p\) represents a quantitative or qualitative assessment of this performance. Generally, the ISOPAR algorithm consists of the following four steps to calculate a three-dimensional performance topology and eventually two-dimensional ISOPAR maps:

1. An area needs to be determined for which ISOPAR values are calculated, usually the field of play. A grid with a specified mesh size is assigned to this area. The ISOPAR values are computed at the grid nodes represented by the set \((x_{ij}, y_{ij})\), \(i = 1, \ldots, m, j = 1, \ldots, n\). For positions which lie between grid nodes the ISOPAR values must be estimated. Therefore, a grid with an extremely small mesh size represents the data very well, while a very large mesh size does not. However, there is a trade-off between representational power and
computational intensity.

2. ISOPAR values $z_{ij}$ are calculated at the grid nodes $(x_{ij}, y_{ij})$ based on the measurements of performance $(x_p, y_p, z_p)$. Since the measurements which are closer to a grid node for which an ISOPAR value is calculated should have greater impact on this ISOPAR value, the measurements of performance are sorted in ascending order (the nearest point first) with respect to their Euclidean distance

$$d_{ijp} = \sqrt{(x_{ij} - x_p)^2 + (y_{ij} - y_p)^2}$$

(3.1)

to the respective grid node. This allows the triplets from above to be written as pairs $(d_{ijp}, z_p)$. Based on the sorted pairings, $(d_{ijr}, z_r)$ represents the measurement with the shortest distance to the respective grid node and $(d_{ij1}, z_1)$ represents the measurement with the largest distance to the grid node, the ISOPAR value is calculated using an exponential smoothing (Hamilton, 1994)

$$z_{ij} = \alpha \sum_{k=0}^{r-2} (1 - \alpha)^k z_{r-k} + (1 - \alpha)^{r-1} z_1,$$

(3.2)

where $0 \leq \alpha \leq 1$ is the smoothing parameter.

3. For the estimation of ISOPAR values between the grid nodes and in order to achieve a satisfactorily smooth surface a slightly smoothing spline interpolation is used. The cubic smoothing spline interpolation (Fahrmeir et al., 2009) which was used is

$$\min_f \beta \sum_{i=1}^{n} \sum_{j=1}^{m} (z_{ij} - f(v_{ij}))^2 + (1 - \beta) \lambda \iint (D^2 f(x, y))^2 dxdy$$

(3.3)

where

$$D^2 = \frac{\partial^2}{\partial^2 x} + 2 \frac{\partial^2}{\partial x \partial y} + \frac{\partial^2}{\partial^2 y},$$
\( \mathbf{v}_{ij} \) denotes the vector with entries \((x_{ij}, y_{ij})\), \( \lambda = 1 \) in our case and \( \beta \) is the smoothing parameter.

4. The ISOPAR map is created by calculating ISOPAR lines and superimposing them on a map of the area determined in step 1. The ISOPAR lines are lines of intersection between the smoothed surface and planes which are parallel to the \( x, y \)-plane in certain intervals. For example, the line for level \( l_t \) is calculated as a set of pairs \((x_s, y_s)\) which fulfill

\[
\begin{pmatrix}
  x_s \\
  y_s \\
  f(v)
\end{pmatrix}
= \begin{pmatrix}
  0 \\
  0 \\
  l_t
\end{pmatrix}
+ \mu \begin{pmatrix}
  1 \\
  0 \\
  0
\end{pmatrix}
+ \theta \begin{pmatrix}
  0 \\
  1 \\
  0
\end{pmatrix}
\]

(3.4)

where \( \mathbf{v} \) denotes the vector with entries \((x_s, y_s)\) and \( \mu, \theta \in \mathbb{R} \). The value for the interval depends on the objectives of and resources available to the user.

**Application to golf**

As mentioned above, the ISOPAR method was originally developed for performance analysis in golf. In this subsection the refinements of the algorithm for golf are shown as well as a new opportunity is introduced for post-hoc assessment of performance of individual shots considering the underlying process of the game sport golf.

**The ISOPAR algorithm in golf**  At the beginning, the method was constrained to calculations of ISOPAR values and ISOPAR maps of greens (see paper [4.1]). Eventually, the method was enhanced so that ISOPAR values and ISOPAR maps can be calculated for entire holes (see paper [4.3]). Generally, the calculation of ISOPAR values and maps in golf is based on triplets which represent the location of a golf ball and the respective number of remaining shots until the ball was holed. For golf
the first two steps of the ISOPAR algorithm are refined slightly. The refinements are:

in 1: The area for which ISOPAR values are calculated is defined as a non-convex hull of all measured ball locations because it does not make sense to extrapolate the empirical information of the measurements to an area where there was no ball location on that particular hole. An exception is the ISOPAR value at the tee box because in this thesis the used data is from the ShotLink™ database of the PGA Tour and in this database all tee shots recorded on the same hole (and the same round) are assigned the same $x,y$ coordinates – a single point. For this reason, the average score ($S_{\text{ave}}$) for the hole is used as the ISOPAR value ($IPV$) at the tee

$$IPV_{\text{Tee}} = S_{\text{ave}} = \frac{1}{p} \sum_{j=1}^{p} S_j,$$

(3.5)

where $S_j$ are the hole scores for all $p$ different players on the hole.

in 2: From the practical viewpoint of golf it does not make sense to include ball locations which are on the opposite side of the hole for an ISOPAR value calculation at a grid node. To explain, the assumption for the sorting of the ball locations was that similar ball locations provide similar information about constraints. Therefore, the further a ball location is from a grid node, the less influence it has on the calculation of the ISOPAR value at the grid node. In golf there is an exception, however, ball locations which are on the other side of the hole often represent opposing constraints (opposite break and slope). For this reason they are not included in the calculation of the ISOPAR value at the grid node. Empirically the decision was made that ball locations which are
considered for computing an ISOPAR value need to be in an area of 60 degrees left and 60 degrees right from the straight line between the pin location and the respective grid node (the red data points in Figure 3.1).

Figure 3.1: The mesh grid shown on the green. Green line represents the edge of the green. \((x_{ij}, y_{ij})\) represents coordinates for a grid point, blue dots represent ball positions, and red dots represent ball positions which are used for calculating the ISOPAR value at \((x_{ij}, y_{ij})\). The black, solid lines form a 60° angle which marks the boundary within which ball locations are used in the calculation.

Performance Analysis in golf using ISOPAR maps and ISOPAR values

The ISOPAR maps can be used to visually investigate the performance of the field (see paper 4.3). Moreover, the ISOPAR maps can also be used to reveal the influence of constraints on golfers’ performance on the green (see paper 4.2).

The ISOPAR method also allows the definition of a performance indicator which
considers the underlying process of golf. In golf every golfers’ final outcome is the hole score. The hole score is made up of a chain of events, the shot sequence starting with the tee shot and ending with the shot holing the ball. Thus, the finishing position of a golfer’s shot determines the starting conditions of the following shot.

The new performance indicator is called Shot Quality ($SQ$) and is a post-hoc assessment of a shot taken. Similar to the shot value concept of [Broadie 2008, 2012], Shot Quality is the difference in ISOPAR value at the starting position ($IPV_{before}$) and the ISOPAR value at the finishing position ($IPV_{after}$) of the shot

$$SQ = IPV_{before} - IPV_{after}. \quad (3.6)$$

Shot Quality, as its name implies, represents the quality of a shot played and has some important properties:

- Like the additivity property of the model of [Broadie 2012], a unique property of Shot Quality allows consecutive shots, which are performed in sequence $(1, \ldots, n_p)$ ending with the ball being holed, by a given player $p$ to be weighted so that the sum of their Shot Quality scores ($SQ_j$) equals the ISOPAR value of the beginning position ($IPV_{Tee}$) of the sequence:

$$\sum_{j=1}^{n_p} SQ_j = \sum_{j=1}^{n_p-1} (IPV_j - IPV_{j+1}) + IPV_{n_p} - 0$$

$$= IPV_{Tee} - IPV_2 + IPV_3 - IPV_4 + \ldots$$

$$+ IPV_{n_p-1} - IPV_{n_p} + IPV_{n_p} - 0$$

$$= IPV_{Tee}. \quad (3.7)$$

0 is included in the the final term $IPV_{n_p} - 0$ to make clear that it represents the Shot Quality of the final shot played on the hole (zero shots are required once the ball is holed).
The average Shot Quality of all shots played on a hole \((SQ_{ave})\) must be 1 and can now be shown by

\[
SQ_{aveTotal} = \frac{1}{p} \cdot \sum_{j=1}^{p} \sum_{i=1}^{n_j} SQ_i
\]

\[(3.7)\]

\[
= \frac{1}{p} \cdot \sum_{j=1}^{p} IPV_{Tee}
\]

\[(3.8)\]

where \(p\) is the number of different players on the hole and \(n_j\) is the number of shots played on the hole by each player.

Additionally, a new concept can be derived from Shot Quality. Similar to strokes gained \([\text{Broadie} 2012, \text{Fearing et al.} 2011]\), already in use by the PGA TOUR, the advantage gained relative to the average by a well played shot (or vice versa) can be assessed. Terminologically, the term \textit{Shots Saved} is preferred instead of \textit{strokes gained} because a long putt made saves instead of gains the player shots. Similar to the strokes gained concept \([\text{Broadie} 2012]\), \textit{Shots Saved} is defined as

\[
\text{Shots Saved} = SQ - SQ_{ave},
\]

\[(3.9)\]

where \(SQ_{ave}\) denotes the average Shot Quality of certain shot types \((SQ_{aveType})\), e.g. drives, or the average Shot Quality of all shots \((SQ_{aveTotal})\). The latter equals
the strokes gained definition of Broadie (2012).

Performance analysis using these performance indicators is demonstrated in the papers in Section 4.1 and in Section 4.3.

**Application to field hockey**

The application of the ISOPAR method to field hockey is the first application of this new approach to an invasion game. In field hockey, the goal was to use the resulting ISOPAR maps to study the distribution of plays considering the respective outcomes (for details see paper 4.4). The input triplets for the ISOPAR algorithm consisted of the location of a ball movement and a quantification of the ball movement related to all other ball movements and with respect to the outcome of the play. The quantification of the plays was achieved using a data mining association rule technique which originally was developed for market basket analyses (Agrawal et al., 1993). The area for which ISOPAR values are calculated is the field hockey pitch.

The ISOPAR algorithm itself was applied as it is described in Section 3.2.2. No further refinements were necessary with respect to field hockey and the purpose of this study.
Chapter 4

Articles

In this chapter, a summary of the four articles is provided which are included in this thesis. All the articles introduce or apply the ISOPAR method and are connected to the provided theoretical background and framework as the following:

- Paper 4.1: Introduction of the ISOPAR method
- Paper 4.2: Theoretical performance analysis with focus on the the influence of constraints
- Paper 4.3: Theoretical and practical performance analysis
- Paper 4.4: Theoretical and practical performance analysis

4.1 The ISOPAR Method: A New Approach to Performance Analysis in Golf

Conventional performance analysis in golf is focused on performance indicators, most of which are categories based on distance; either the distance of a shot, the distance to the hole before a shot, or the left over distance to the hole after a shot. These performance indicators usually are shot type specific, but often describe the performance of more than only one shot. Furthermore, they neglect the context of a golfer playing a hole represented by the shot sequence starting with the tee shot and ending with the shot that holes the ball. For this reason the ISOPAR method is developed to model the performance of the participating golfers continuously and to account for
difficulty on greens of golf holes at first glance. Additionally, the ISOPAR method allows defining a new performance indicator that accounts for each unique shot - Shot Quality, which is the difference between the ISOPAR value before a shot and the ISOPAR value after a shot. In this study data from the 2009 Bavarian Junior Championship in Burgwalden, Germany, are analyzed. Resulting ISOPAR maps show the difficulty on greens represented by an ‘average’ remaining number of shots until the ball is holed. The ISOPAR maps can be used to characterize greens in terms of difficulty as well as a tool for making tactical decisions. The Shot Quality should be used by coaches or teachers for assessing shot outcomes.

My contribution to this paper was the literature review, developing the ISOPAR method, analyzing the data, writing of the Section Introduction, the Section Methods, and the Section Results. The Section discussion was written together with my co-authors Peter Lamb and Martin Lames.

4.2 Modeling Constraints in Putting: The ISOPAR Method

The goal of this study was to investigate the potential of the ISOPAR method to visualize the influence of environmental constraints on the performance of the field. Every environment provides individuals opportunities for actions, the so-called affordances, but also provides constraints which render actions impossible. Both, affordances and constraints are relative to the individual. The environment also provides affordances and constraints in context of sports, in this study applied to golfers. Golfers face similar environmental constraints such as the distance to the hole, the undulation of the green, slopes and breaks, the length of the grass, and so on. In this study, data of the 2009 Bavarian Junior Championship in Burgwalden, Germany, at the ninth green, measured by professional surveyors, and data of the 2010 Augsburg Classic
(EPD) at the ninth green were analyzed. Using the ISOPAR method iso-lines were calculated and superimposed on a contour map of the ninth green. The results show that the pattern of the iso-lines reveals the influence of constraints on the field and, additionally, allows a quantification of this influence. The density and the respective shape of the iso-lines show how the field was constrained more or less by, among others, identifiable environmental constraints - slope, break, and distance.

My contribution to this paper was the literature review, analyzing the data, and writing the whole paper. My co-author Martin Lames supported me with minor changes in the Section Introduction and the Section Results & Discussion.

4.3 A Model for Visualizing Difficulty in Golf and Subsequent Performance Rankings on the PGA Tour

The study’s goal was to analyze the data from 2011 ShotLink™ database, which contains data of PGA Tour tournaments, using the ISOPAR method. For this purpose the ISOPAR method from the article in section 4.1 was enhanced so that it can be used to calculate ISOPAR values and ISOPAR maps for entire holes. It was shown that the performance indicator Shots Saved, which is provided by the ISOPAR method, allows analyses of individual shots of different types of shots which, however, is not possible using the performance indicators used by the PGA Tour. The conventional PGA Tour statistics either artificially classify shots based on distance or they describe the performance of a sequence of shots. Therefore, they lack information concerning the difficulty of a shot or they do not extract information for one single shot of the chain of shots. The five analyzed shot types were drives, long approach shots, short approach shots, around the green shots, and putts. Furthermore, ISOPAR maps for entire holes allow the visualization and analysis of the
performance of the field. ISOPAR maps can identify the impact of hazards (bunkers, trees, rough, etc.) on the field’s play as well as the advantage or disadvantage of a shot, for example, a tee shot that provides a good or a bad angle for the approach shot.

My contribution to this paper was the literature review, further developing the ISOPAR method, analyzing the data, writing of the Section Introduction, the Section Methods, and the main part of the Section Results. The rest of the Section Results and the Section Discussion was written together with my co-authors Peter Lamb and Martin Lames.

4.4 Visualization and Analysis of Spatial Characteristics of Attacks in Field Hockey

Goals of this study were to investigate if the spatial distribution of attacks in field hockey are influenced by specific field hockey rules and which consequences this distribution has on the outcome, successful or unsuccessful, on the respective plays. The crucial rule in field hockey is that players are only allowed to move the ball with the flattened side of a right-handed stick. Thus, players should have advantage moving the ball or retaining ball possession on the right side of a player’s body since the ball is easier protect. In this study, the characteristics of ball possession, passing, and attacking behavior of world-class teams were analyzed using association rule mining techniques to generate the different input datasets for the ISOPAR method. The ISOPAR method was used to create maps to visualize the respective characteristics. The dataset for this study consisted of the women’s preliminary round matches of Pool A of the 2008 Olympic Games. The results show:
• Teams tend to carry the ball on the right side of the pitch.

• Ball possessions resulting in goal shots were more likely to be neutral or left sided, although left sided plays were less frequent in total.

My contributions to this paper were the literature review, writing of the Section Introduction, the subsections about the ISOPAR method and the data analysis in the Section Methods, and the Section Results. The Section Discussion was written in cooperation with my co-author Stuart Morgan.
Chapter 5

Discussion

Results from applications of the ISOPAR method and the ISOPAR method itself will be discussed in this chapter. First, the ISOPAR method will be discussed from a technical viewpoint and from a viewpoint considering the underlying systems approach. Afterwards, ISOPAR maps and the performance indicators resulting from the ISOPAR method will be reviewed with respect to current studies.

5.1 Discussion of the ISOPAR method

The ISOPAR method provides a new opportunity for performance analysis by analyzing performance with respect to the location where it took place. For this purpose the ISOPAR method creates a continuous representative performance based on discrete measurements of performance of the participating athletes/teams and, hence, provides a topology of performance subject to a predefined area, mostly the field of play of the respective sport.

5.1.1 Using the systems approach as theoretical framework

The approach of explaining sports performance using theoretical concepts of dynamical systems theory was suggested by Glazier (2010). In this thesis this approach was applied to golf and field hockey. The following paragraphs outline the respective applications.

Golf is an individual sport and, hence, to explain the golfers’ performance an intra-individual application of dynamical systems theory is applied. The performance
of each shot of a golfer is guided by a set of constraints and during the performance his or her behavior can be modeled as a self-organizing dynamical system. The paper in Section 4.2 demonstrates that in golf the ISOPAR values can even be used to visualize the influence of the different types of interacting constraints on putting performance as the pattern of the iso-lines. The environmental constraints at a location on a hole are quite stable during a round in golf. The individual constraints differ from golfer to golfer, also on an elite level, and probably make up most of the variability in performance at a location. However, the mentioned paper reveals that the pattern and density of iso-lines allow one to identify certain constraints (slope, break, distance) as factors which influence golfers’ ‘average’ performance and to quantify that influence.

Field hockey is a team sport and an invasion game. The application of dynamical systems theory is, on the one hand, inter-individual when explaining a team’s performance, but also intra-individual when explaining a team member’s performance. A team’s performance is always determined by its members’ performance and is result of self-organization of the team members under a set of constraints which guide the team’s performance. From this viewpoint, performances at a location usually are performed by different team members each facing at least slightly different sets of constraints which vary over time depending on the own team’s pattern as well as the opponent’s pattern. Hence, such performances can be analyzed and explained similar to golf from an intra-individual viewpoint as well. As mentioned, the constraints at a location are similar at least, but also differ over time more than in golf. The individual constraints vary as much as in golf. As in golf the environmental constraints are quite similar at every location. However, the task constraints vary more over time than in golf. Task constraints such as the sidelines of the field hockey pitch or in
general that there are teammates and opponent players stay the same. However, the patterns of the own and the opponent team vary over time because of self-organizing processes within and between the teams. Thus, although important information is still missing by only relating a sports performance to the location where it occurs for invasion games, the ISOPAR approach improves on conventional, statistical approaches by interpreting sports performance in the context of its location (McGarry, 2009).

The application of the systems approach to explain performance in invasion games is supported by the study of Vilar et al. (2012). They also used the systems approach suggested by Glazier (2010) and enhanced it by combining those theories with ideas from ecological psychology in order to explain performance in invasion games. Whereas the ISOPAR method analyzes athletes’ performance relative to its location, Vilar et al. (2012) rather analyzed and successfully showed “how performance of individuals emerged from self-organized processes under the constraining influence of the locations of their opponents, the ball and the goal” (p. 8), independent of the exact location of the respective individual’s performance.

5.1.2 Validity of ISOPAR values

In some situations the continuous topology of performance provided by the ISOPAR method can cause degradation of the validity of the ISOPAR values. The area for which ISOPAR values are calculated is the field of play, or in the application to golf, it is a non-convex hull of the measured ball locations. In such an area there can be subareas where there are limited measured performances or even single outliers. In those areas the calculation of ISOPAR values is over influenced by measurements of performance from other locations. These measurements do not contribute any
information or at least very little to performance in those areas. Thus, there may be areas on the topology of performance which suffer from lack of information about performance and where the validity of the ISOPAR values is questionable.

However, there are usually dense ‘clouds’ of measurements of performance in the area for which ISOPAR values are calculated. For these areas the ISOPAR values are valid with respect to the participating athletes. Those clouds of measurements of performance are results of environmental, task, and individual constraints. For example, in golf such a cloud of measurements is the landing area for drives at par 4 and par 5 holes. The paper 4.3 reveals how the location and shape of the landing area is influenced by the interaction of environmental (hole design, trees, coast line, etc.), task (hitting the ball with the driver), and individual (skills and ability of driving) constraints.

5.1.3 Similar models used in golf

Besides the ISOPAR method there are two additional models in golf which provide an ‘average’ performance. These two models are based on statistical approaches; one was developed by Broadie (2008, 2012) and the other model was developed by Fearing et al. (2011) building upon Broadie’s approach. Both models are based on the left over distance to the hole and provide statistical benchmarks describing an average number of remaining shots until the ball is expected to be holed. The model of Broadie (2012) additionally takes into account information about the ball lie (rough, fairway, green, etc.) and provides benchmarks for all distances to the hole with respect to the ball lie at the starting location. Moreover, the model of Broadie (2012) is not hole or course specific. Besides the distance to the hole, the model of Fearing et al. (2011) additionally considers the golfers’ skills and the difficulty of the
course in order to calculate their benchmarks. Furthermore, their model is limited to greens and the benchmarks are adjusted using a variable describing the difficulty of each green. Overall, the benchmarks of both models describe expectations for average hole scores, are still distance based, and are independent of the hole in case of Broadie’s model. The ISOPAR method also establishes ‘benchmarks’, the ISOPAR values, however, they are independent of the left over distance to the hole, only depend on the performance of the participating golfers, and are calculated for each hole in each round separately. Thus, the pattern of the ISOPAR values is even expected to vary slightly if the same players played the same hole with the same pin position again, because the ISOPAR method only describes an ‘average’ performance of players post-hoc.

The strength of the statistical models is that these models do not have the validity problem which the ISOPAR method has when there are only few or no ball locations in an area. The weakness of the statistical models compared to the ISOPAR method is that they cannot account for certain environmental constraints which obviously influence golfers’ performance. For example, the ISOPAR method accounts for the tree in the landing area of the drives at the 18th hole in Pebble Beach (see paper 4.3). In contrast, the statistical benchmarks only account for the left over distance to the hole independent whether the ball lies behind this tree.

5.2 Discussion of analyzing performance using the ISOPAR method

Based on the systems approach the ISOPAR method specifically provides new opportunities to analyze performance post-hoc. First, there are ISOPAR maps which are visualizations of a representative performance of the participating athletes or teams.
Second, there are new, however, in this thesis golf-specific performance indicators Shot Quality and Shots Saved which allow analyses of individual shots. As shown in the papers, both techniques allow theoretical and practical performance analysis.

5.2.1 ISOPAR maps

ISOPAR maps are two-dimensional visualizations of ‘average’ performance. These maps show certain levels of performance, the iso-lines, superimposed on a map of the area for which ISOPAR values were calculated. The pattern of the iso-lines and the density reveal areas of minimum and areas of maximum performance. Furthermore, the density of these lines indicate the amount of change of the ‘average’ performance and the change in performance can be quantified by the difference of ISOPAR values. Hence, the ISOPAR maps allow a novel visual analysis of performance considering the underlying process of the respective sport, and is a new approach in golf. The following paragraphs reveal how ISOPAR maps enhance visual performance analysis compared to conventional performance analyses which are limited to discrete zones.

The paper about visualization of spatial aspects of attacks in field hockey (see Section 4.4) is a good example of how ISOPAR maps can be used to analyze a sport from the viewpoint of theoretical performance analysis. Based on the location of ball movements and information of the respective outcomes, ISOPAR maps are appropriate to show field hockey specific behavior. On the one hand, the maps reveal that in field hockey, generally, because of the right handed stick rule plays are performed more often on the right side of the field of play. On the other hand, there are not more plays resulting in a goal shot when they are performed on the right side. With respect to the number of plays the chance that a play results in a goal shot is even greater when an attack is played on the left side of the field of
play. Former notational analyses showed similar findings, a right-sided field play, but goal scorings were more frequently observed on the left side (e.g. Hughes & Billingham, 1986; Wilson, 1987a,b; Sunderland et al., 2006). However, those studies were limited to attacks resulting in goals. Compared to the ISOPAR maps, which show continuous pattern of performance, those performance analyses were limited to discrete zones. The discrete zones are an artifact of the data collection in notational analyses and, hence, performance can only be analyzed with respect those discrete zones. Overall, the ISOPAR maps allow a more detailed analysis of attacks in field hockey than the conventional notational analyses.

A further example for a visual analysis of sports performance using ISOPAR maps is a study about free kicks and the respective outcomes in women’s soccer (Stöckl & Lames, Accepted) which were analyzed by Alcock (2010) as well. The ISOPAR map in this study reveals the pattern of performance of direct free kicks with respect to the outcome of the 2007 World Cup. The shape of the iso-lines indicates that the most dangerous areas are slightly skewed to the left side, and outside, but close to the penalty box (see Figure 5.1). However, the topology of ‘average’ performance does not show circular pattern as Alcock (2010) artificially defined the zones of different levels of performance (the levels were determined by the grey circles in Figure 5.1). The threat of scoring from the area of best performance drops off in all directions, in some directions faster, in some directions more slowly. Compared to the notational analyses about field hockey mentioned above, Alcock (2010) collected the data using a coordinate system rather than allocating it to discrete zones. Alcock (2010) cut down the positional information of performance post-hoc and defined discrete zones of different levels of performance based on the measurements. In sum, the ISOPAR maps allow a more detailed analysis of performance than the approach of artificially
creating discrete zones which describe different levels of performance.

Figure 5.1: Two-dimensional illustration of the topology of ‘average’ performance of direct free kicks at the women’s FIFA World Cup 2007 with discrete categories of performance (within the 7 m line, between the 7 m and 12 m line, and outside the 12 m line) superimposed as suggested by Alcock (2010)

5.2.2 The performance indicators Shot Quality and Shots Saved

The new performance indicators Shot Quality and Shots Saved, a concept which is derived from Shot Quality, are golf-specific and also allow theoretical performance analysis and practical performance analysis (see papers in Sections 4.1, 4.3 and also Stöckl et al., Accepted). The following shows the development of performance indicators in golf in general and the advantage of the performance indicators provided by the ISOPAR method.
Conventional performance indicators

The conventional analysis of performance is usually reduced to the distance of a shot, left over distance to the pin, starting distance to the pin, or a combination of the latter two (James, 2007, 2009). Thus, most of the constraints which make each shot unique and also make up the difficulty of a shot are ignored. In particular, the difficulty of a shot is determined by environmental constraints to a great extent (e.g. the ball lie (fairway, rough, bunker), whether the direct line to the pin blocked by an obstacle, weather conditions). Also artificially creating categories of shots within which shots are compared to each other neglects important constraints under which a shot was performed. For example, approach shots from 50-125 yards is such a category whose interval is selected arbitrarily. It includes comparing shots within this interval. Hence, a shot from 50 yards can be compared to a shot from 125 yards, but not to a shot from 49 yards. Other categories like putts per Greens in Regulation (GIR) measure the performance of a combination of shots rather than the performance of a particular shot. Overall, conventional performance indicators usually do not account for individual shots. There are performance indicators focusing on a particular shot, like Driving Distance or Driving Accuracy. However, they usually only take into account one aspect of the outcome of the shot, but ignore the constraints under which the shot was performed.

Performance indicator strokes gained

The performance indicator strokes gained is a shot value concept and was originally developed by Broadie (2008, 2012) and further developed for putting by Fearing et al. (2011). Strokes gained is defined as \( \text{Shot Value}_{\text{before}} - \text{Shot Value}_{\text{after}} - 1 \). The respective ‘Shot Value’ depends on the benchmarks of the model of Broadie (2012) and of Fearing et al. (2011), respectively. Strokes gained was the first performance
indicator used by the PGA Tour which allowed one to analyze individual shots within the context of the sequence of all shots at one hole. However, strokes gained is generally still based on information about the left over distance to the hole, but includes more detail about other constraints, as mentioned above. For example, strokes gained cannot be used to study golfers’ performances of uphill putts because the statistical models do not account for such extra difficulty, which is added to the difficulty caused by the distance to the hole.

**Performance indicators Shot Quality and Shots Saved**

Shot Quality and Shots Saved are performance indicators which break up the distance based categorization of shots which is used for the conventional performance indicators. They account for each unique shot and provide an opportunity to analyze performance of individual shots considering the context of the performance, the sequence of all shots at one hole. As demonstrated in the papers in Sections 4.1, 4.2, and 1.3 the ISOPAR method provides a continuous representative performance of participating players from which inferences about the difficulty can be drawn. Hence, Shot Quality, as the name implies, reveals the quality of a shot with respect to the difficulty of the starting location and the difficulty of the finishing position. Shots Saved shows how a golfer gains or loses advantage on the rest of the field with each shot.

The idea of Shots Saved is similar to the shot value approach defined by Broadie (2008, 2012). On the one hand, Shots Saved is defined differently from strokes gained as Shot Quality – $SQ_{Ave}$, where $SQ_{Ave}$ denotes the average Shot Quality of a certain type of shot (drive, approach shot, putt, etc.) or denotes the average of all shots, in which case Shots Saved and strokes gained are equivalent. As outlined above the underlying models of the respective performance indicators represent another and more
important difference between these performance indicators. The advantage of Shot Quality and Shots Saved is that they can be used to analyze specific performances of golfers. For example, the above mentioned performances of uphill putts can be studied using these performance indicators, because the ISOPAR method accounts for such constraints implicitly. A disadvantage of these performance indicators is that they cannot appropriately assess performance of shots which are single outliers or are located in areas where there are only few measurements, since the validity of the ISOPAR values in such areas are questionable as mentioned above. Therefore, the performance indicators based on the ISOPAR method, Shot Quality and Shots Saved, should be used for different purposes than the performance indicator strokes gained which is based on statistical benchmarks.

When comparing performances from different tournaments to each other using Shot Quality or Shots Saved, the ‘average’ performance can lead to misinterpretation. If in tournament A better golfers participate than in tournament B, a good golfer gets greater Shot Quality values and Shots Saved values for his or her performance in tournament B than he or she gets for the same performance in tournament A. The performance indicators Shot Quality and Shots Saved only can assess performance with respect to the participating golfers. However, on an elite level like the PGA Tour one can argue that the strength of the field is relatively constant across all tournaments.
Chapter 6

Conclusion

This thesis provides a new method to model and analyze sports performance. The approach of the ISOPAR method is based on a systems approach and relates performance to the location where it took place. The systems approach which includes the concepts of constraints and self-organization, which are two main concepts of dynamical systems theory, provides an opportunity to analyze and to explain performance in context of a sport’s underlying process. The articles which are included in this thesis revealed that the ISOPAR method, more precisely the resulting ISOPAR maps and the performance indicators Shot Quality and Shots Saved, are appropriate for performance analysis considering the limitation of the ISOPAR method. Results in the articles included in this thesis show the potential of using this method for theoretical and practical performance analysis in golf and in field hockey. In particular, the application of the performance indicators Shot Quality and Shots Saved allow new insight into golf performance because they account for each unique shot.

The application of the ISOPAR method to field hockey revealed that the method is appropriate for performance analysis of invasion games as well, although the explanation of the results is slightly different from an individual sport like golf with respect to the concepts of dynamical systems theory. However, the ISOPAR method can provide new insight in performance of invasion games and should be applied to other invasion games for which positional data for aspects of performance exist. This allows to identify lawful structures of performance of the respective game or to analyze teams’ performances.
References


