

# Modified DFT Filter Bank with One-tap per Subchannel Equalizer for Frequency Domain Chromatic Dispersion Compensation

Israa Slim<sup>1</sup>, Leonardo G. Baltar<sup>1</sup>, Amine Mezghani<sup>1</sup>, Fabian N. Hauske<sup>2</sup> and Josef A. Nossek<sup>1</sup>

<sup>1</sup> Lehrstuhl für Netzwerktheorie und Signalverarbeitung, Technische Universität München, Munich, Germany

<sup>2</sup> Huawei Technologies Duesseldorf GmbH, European Research Center, Munich, Germany

## Abstract

We perform chromatic dispersion (CD) equalization in the frequency domain based on Modified Discrete Fourier Transform (MDFT) Filter Banks (FB) with a non trivial prototype filter. The equalization is done per subchannel with a single tap equalizer placed between the analysis and the synthesis filter banks. Our main contribution is to design and insert the equalizer such that the output is alias free. We compare the MDFT FB with per subchannel single tap alias free equalization with the so called overlap and discard frequency domain equalization, which is nothing else than a MDFT FB with a trivial prototype filter. Larger CD values are tolerated with our approach at the cost of slightly increased complexity.

## 1 Introduction

When carriers began to migrate from 2.5 Gb/s to 10 Gb/s transmission rates, the performance of fiber optic links in long haul, metro and enterprise networks became limited by dispersion causing a short optical pulse to broaden as it travels along the fiber leading to intersymbol interference (ISI). This is largely because group velocity dispersion (GVD) or CD in optical fibers increases with the square of the data rate and becomes a serious impairment at 10 Gb/s data rates and above. Thus, dispersion compensation is a feature required in optical fiber communication system.

Dispersion compensation can be done either optically or electronically [1],[2]. Electronic dispersion compensation (EDC) [3] techniques are performed for CD compensation since they avoid the use of expensive and bulky optical components. Recently, a number of efforts have come out to bring digital signal processing into optical communication links specially for longer reach applications. Additionally, in optical coherent transmission systems, frequency domain equalizers (FDEs) based on fast Fourier transform (FFT) have become the most appealing scheme for CD compensation due to the low computational complexity for large dispersion and the wide applicability for different fiber distances [4],[5]. FBs [6] are digital signal processing systems that find applications in various fields in wireless communications. An important class of FBs is the discrete Fourier transform (DFT) FBs, which can be efficiently implemented based on the use of polyphase filters, FFT and inverse FFT.

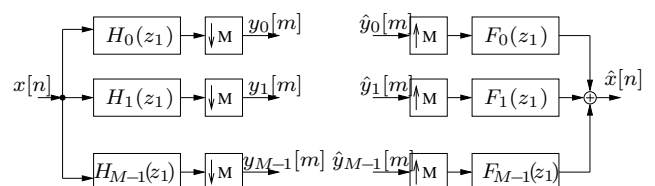
Our main interest is in the application of FBs in optical fiber communications. The efficient structure of the MDFT FBs with non trivial prototype filters is applied for FDE CD compensation. This structure has the advantage that

the signal at the output of a back-to-back connection of analysis and synthesis FB is alias free. Upon inserting a single tap CD equalizer into this structure the output is not necessarily still alias free. However, we were able to find a special arrangement that preserves an alias free output. A FB based FDE with trivial prototype filters commonly called 'overlap-and-discard' implementation of linear convolution for CD compensation serves as a benchmark. The performance of both FB based (i.e. with trivial and non-trivial prototype filters) equalization techniques are discussed from the point of their ability to compensate for different CD values.

This paper is organized as follows. In Section 2, the FB structure of interest is briefly introduced. Our approach for CD equalization is given in Section 3 along with the benchmark. Finally the results are shown in Section 4.

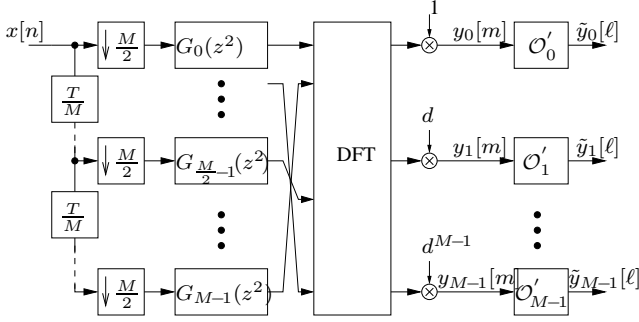
## 2 Modified Filter Bank Structure: A Review

A general overview of a maximally decimated FB is shown in **Figure 1**, where the number of subchannels  $M$  is equal to the downsampling rate. We have used the definitions  $z_1 = e^{sT/M}$  and  $z = e^{sT}$ , where  $s = \sigma + j\omega$  is the complex frequency variable.

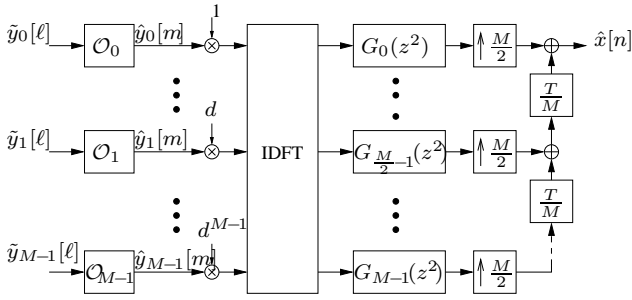


**Figure 1:** Basic complex modulated filter bank structure

Both AFB and SFB can be implemented efficiently by applying some important identities for multirate processing and by performing the complex modulation by means of IDFT and DFT [7] of size  $M$ . **Figure 2** depicts the efficient realization of the AFB and in **Figure 3** that of the SFB is shown.



**Figure 2:** AFB of an MDFT FB: Efficient Structure



**Figure 3:** SFB of an MDFT FB: Efficient Structure

Afterwards, let us introduce the transfer functions  $G_m(z)$ ,  $m = 0, \dots, M-1$ , as the type-1 polyphase components of length  $K$  of the prototype filter [6], which are derived from the relation:

$$H_0(z_1) = \sum_{m=0}^{M-1} z_1^{-m} G_m(z_1^M), \quad (1)$$

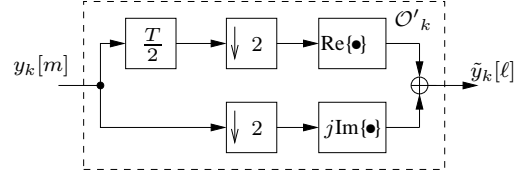
and can be calculated in the time domain as:

$$g_m[n] = h_0[nM + m], \quad m = 0, \dots, M-1. \quad (2)$$

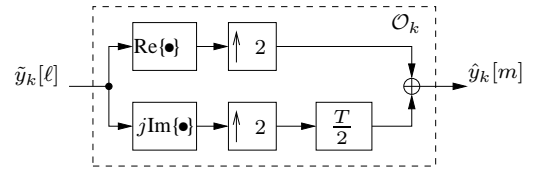
The polyphase representation is a neat tool which enables the rearrangements of the computations of the filtering operation, so as to minimize the computational load per unit time. The length of the prototype filter relates to  $K$  as  $N = KM$ .

Although DFT polyphase FB provides high computational efficiency, it suffers from the fact that it is not able to cancel alias components caused by subsampling the subband signals. This disadvantage can be overcome by introducing destaggering  $\mathcal{O}'_k$  and staggering  $\mathcal{O}_k$  operations to the

AFB and the SFB, respectively. The blocks  $\mathcal{O}_k$  perform a  $\frac{T}{2}$  staggering of the real and imaginary parts of the low rate signals  $\hat{y}_k[l]$ , while the blocks  $\mathcal{O}'_k$  perform a destaggering to generate the signals  $\tilde{y}_k[l]$ . **Figures 4** and **5** depict the internal structure of  $\mathcal{O}'_k$  and  $\mathcal{O}_k$  for  $k$  even, respectively. In the cases of  $k$  odd, the roles of the real and imaginary parts are interchanged. These blocks are necessary to guarantee the MDFT FB to be alias free.



**Figure 4:** Destaggering Operation for even  $k$



**Figure 5:** Staggering Operation for even  $k$

### 3 Filter Bank Based CD Compensation

It was shown that the use of frequency domain equalization (FDE) techniques can compensate the effects of chromatic dispersion. FDE is very attractive because it has much lower calculation complexity than time-domain equalization (TDE) when the equalizer has many taps [8],[9]. FDE has been proposed for wireless channels [10],[11] and adopted in 3rd generation long-term evolution (3G-LTE) systems [12]. Single carrier (SC) FDE improves the transmission quality with reduced calculation complexity owing that to its block wise operation by using fast Fourier transform (FFT).

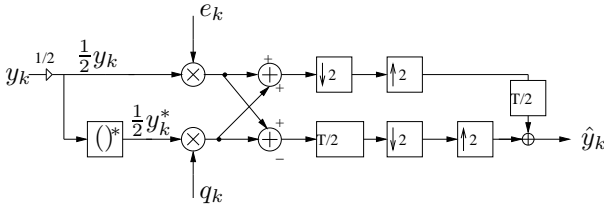
So far, the overlap-and-discard method, which is one of the SC FDE configurations for CD compensation, has been applied. As a benchmark, we realize this method as an DFT FB with trivial prototype filter. Our approach is to perform CD compensation in the frequency domain based on MDFT FBs i.e. a structure with polyphase network and FFT blocks. For this FB structure, a single tap per sub-channel equalizer is inserted in such a way that the MDFT FB with equalization keeps the alias free property.

### 3.1 Filter Bank with Non-trivial Prototype Filters Based CD Compensation

This is our approach for CD equalization where the cores for equalization are AFB, SFB, staggering/destaggering operations and an equalizer. Without intermediate processing (i.e. without an equalizer) between the AFB and the SFB, this structure of the MDFT FB is inherently alias free. In case an equalizer is introduced, it could be that the structure is no longer alias free. However, we suggest the arrangement of a per subchannel single tap equalizer shown in **Figure 6** which still leads to an overall alias free structure for the MDFT FB.

In order to get an alias free output, the equalizers  $e_k$  and  $q_k$  are designed as such:

- $e_k$  for each subband is a one-tap equalizer taken as the inverse of the channel.
- $q_k = 1$ .



**Figure 6:** Special Arrangement of a Single tap Per sub-channel Equalizer

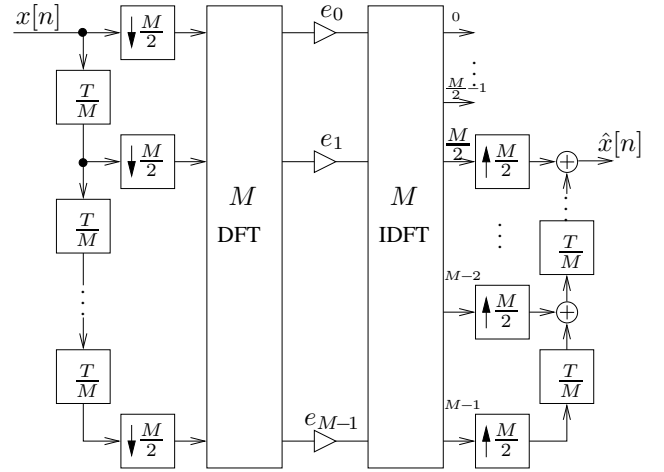
### 3.2 Filter Bank with Trivial Prototype Filters Based CD Compensation

Frequency domain CD equalization can also be done by using an overlap-and-save (OLS) FFT method also known as overlap-and-discard (OLD) FFT method [13],[5]. Overlap techniques perform linear convolution in the frequency domain, where the input signal is divided into overlapping blocks. The block length  $N_{block}$  is equal to the FFT size  $M$ . As a benchmark, the value of the overlap is taken to be half the FFT size i.e. a 50% overlap factor.

In OLD with 50% overlap, half of the output is discarded while a whole input block of length  $M$  is processed. This restricts the analysis prototype filter and the synthesis prototype filters, respectively, to be:

- $h[n]$  is a rectangular filter of length  $M$ .
- $f[n]$  is a rectangular filter of length  $M/2$ .

**Figure 7** shows OLD FFT method implemented as an DFT FB structure which is a non-maximally decimated DFT FB.



**Figure 7:** DFT Filter Banks with Trivial Prototype Filter CD compensation

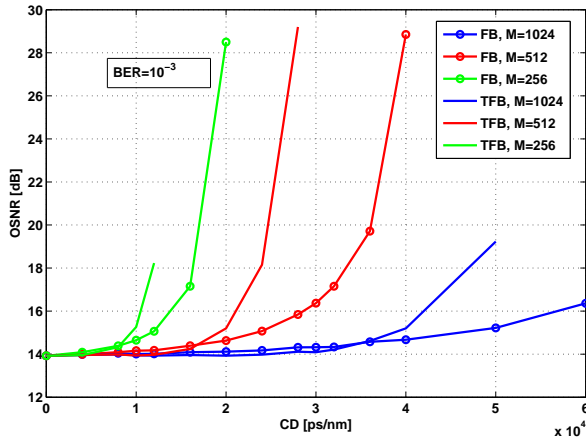
Since all  $M$  degrees of freedom are used for the design of the equalizer  $\underline{e} = [e_0, e_1, \dots, e_{M-1}]$ , it is no longer strictly the overlap-and-discard method to implement linear convolution with the aid of FFT and IFFT, but it is a FB based CD equalization with a trivial prototype filter.

## 4 Results

For a 112-Gbit/s NRZ-PDM-QPSK coherent optical transmission system, our method and the benchmark for CD compensation have been compared by evaluating their applicability for different FFT sizes and for compensating different CD values. For performance analysis, the required optical signal to noise ratio (OSNR) to tolerate different CD values (accordingly different fiber lengths) at a bit error ratio (BER) of  $10^{-3}$  is chosen to be the figure of merit. In the simulations, we employ the following for prototype filter, DFT and IDFT operations:

- The prototype filter is a real coefficient linear phase FIR lowpass filter.
- The DFT and IDFT operations are efficiently implemented with FFT and IFFT operations since we consider that the size of the DFT and IDFT  $M$  (which is also the number of subchannels) is taken as power of two.

The prototype filter is the ELT filter and  $K = 2$  has been chosen for the length of the polyphase components of the prototype filters.



**Figure 8:** Required OSNR for different CD values: MDFT FB with trivial and non-trivial prototype filters Based CD Compensation

The simulation results show that for the same OSNR, higher CD values are compensated for the same FFT size with our method. Another result revealed by the simulations is that for the same CD, less OSNR value is needed with our FB based FDE with non-trivial prototype filters. These results are shown in **Figure 8**.

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