INTRODUCTORY COURSE ON SATELLITE NAVIGATION

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Abstract

Satellite navigation is widely used for personal navigation and more and more also in precise and safety-critical applications. Thus, the subject is suited for attracting the interest of young people in science and engineering. The practical applications allow catching the students' attention for the theoretical background. Educational material on the subject is sparse, especially with respect to the practical side. This paper describes a combined approach based on experiments and theory. It was tested during a two days course for college students visiting the Technische Universität München called "Girls do Research - the Autumn University at TUM". The positive feedback the supervisors received led to the conclusion that the approach is supportive for raising the students' interest in science and engineering.

Keywords: Distance measurement, Educational activities, Global Positioning System, Navigation, Satellites, Satellite navigation systems, Student experiments.

1 INTRODUCTION

In the last decade GPS received considerable attention in service, industrial and research communities. Furthermore, the GLONASS constellation was restored, and the European Galileo and Chinese BeiDou Global Navigation Satellite Systems (GNSS) are making rapid progress. The increased accuracy and availability offered by the combined use of these constellations promises breakthroughs in a number of application areas. They range from mass market with hand-held devices (see e.g. [1]), best effort services to precision approaches of aircraft (see e.g. [2]).

Although satellite navigation is broadly used, training and tutorial courses are sparse as compared to other radio technologies [3]. Furthermore, most of the published material targets at a highly educated audience already familiar with concepts such as statistical signal processing. The existing material includes software tools such as the NAVKIT [4] and online courses [5, 6]. Other publications just state results, without aiming at deeper explanations [7].

The visit of a group of young students during a two days course in the framework of "Girls do Research - the Autumn University at TUM" motivated the development of an approach that combines an introduction to the theoretical background of GNSS with hands-on experiments. An important goal was to capture the girls' interest in science and engineering. Further aims were to initiate them in aspects of scientific thinking, including important elements such as curiosity, thinking in analogies, and reaching increasing levels of abstraction.

The paper is organized as follows: Section 2 gives an overview about the two days course schedule and the pedagogical concept applied in the development of the experiments. In Section 3 the experiments are detailed. The evaluation of the course is discussed in Section 4. Finally the paper is concluded in Section 5.

2 OVERVIEW

2.1 Course Schedule

The course intends to teach school graduates or freshman students in the field of satellite navigation. The focus was on understanding distance measurements, positioning and finally differential navigation. It includes theoretical introductions and hands-on experiments. The schedule is based on an elaborate pedagogical concept (see Section 2.2).

The two days were organized as follows:

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First day:

• Introduction to the institute's interests and research topics

• Visit of a lecture and guided tour through laboratories

• Introduction to GNSS with a focus on distance measurement

• Experiment: Positioning in 1D, 2D and 3D

  • Theory behind 1D and 2D equations

  • Introduction to 3D positioning with satellites

  • STK[1] simulations about GNSS constellations

• Experiment: Distance measurement

  • Manual outdoor sound propagation time measurement

  • Digital indoor sound propagation time measurement

• Debriefing of the first day

• Homework: Handling of clock and atmospheric effects?

Second day:

• Discussion of the homework

• Introduction to differential navigation

• Experiment: Precise positioning

  • Derivation of 3D system of equations

  • Programming of an equation solver in C++

  • Introduction to the paper chase

  • Paper chase including outdoor GPS measurements

• Debriefing of the second day

• Evaluation of the course

2.2 Pedagogical Concept

During the development of the concept the focus lay on the target audience - in general a group with only basic knowledge about scientific and engineering work. In this specific case it was about a group of young college girls. The course aimed at the following educational targets:

• Increase the interest in engineering

• Give insight into scientific work and methods

• Show the scientific foundations beyond products/services

• Practice team work

As the course is not part of a mandatory lecture or other educational program finishing with an exam, the motivation for attention has to be based on interest. Therefore one of the main goals during the design of the course schedule was to look for a constant high level of enthusiasm. The means applied are a balance between theoretical and experimental periods as well as a sequence of diversified teaching methods. Among them are discussions, group work, pairwise work, ex-cathedra teaching, simulations, sketches on the black board, etc... As the course members possess not yet a deep theoretical knowledge a compromise had to be found between popular science explanations and necessary theoretical facts. A key solution used was to motivate new content by introducing it based on common experience.

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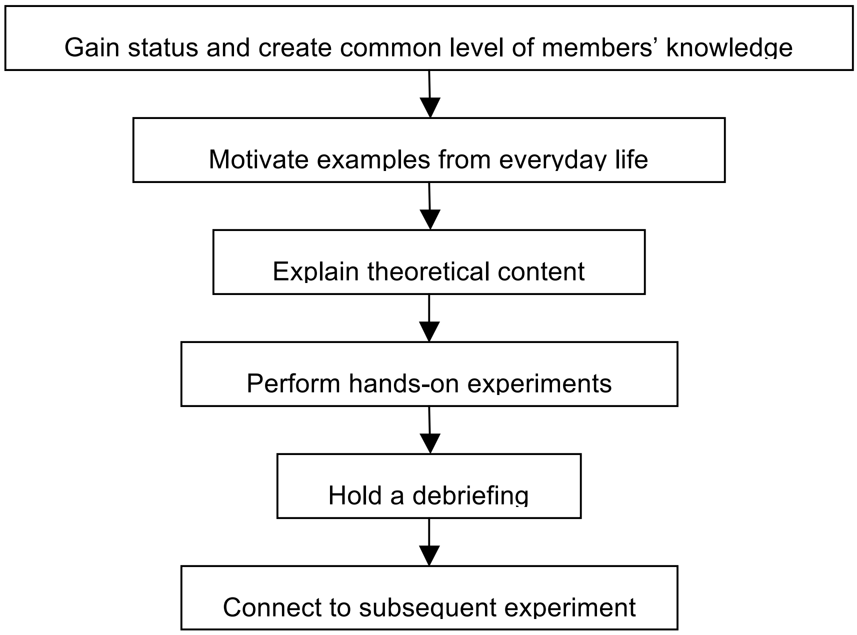
[1] The Satellite Tool Kit (STK) is a software package from Analytical Graphics, Inc. (http://www.agi.com).

The content itself was separated in single but connected topics (called experiments) which makes it easier to explain the complex matter. Periods for discussions and recapitulation lead to a deceleration and prevent overexertion which often results in discouragement. Debriefing provides feedback on the learning progress to the tutors and helps in deepening the understanding. The homework was intended to create a bridge between the two days. The overall concept was to start with basics and raise the level of knowledge step by step. Finally, the course closed with an outdoor measurement campaign in the form of a paper chase for which a general understanding about satellite navigation is required. This was seen as an additional driver for attention during the course.

Each of the experiments can be regarded as separated units which are mainly structured in the following way:



The detailed realization of this concept will be explained during the following sections on the single experiments.

3 EXPERIMENTS

3.1 Experiment 1: Positioning in 1D, 2D and 3D

a) Introduction

The intent of any positioning algorithm is to determine the coordinates of the receiver. From basic calculus it is known that to solve for the three components of a vector, one needs at least three (independent) equations. In a group-discussion with the students distance measurements can be identified as the common observations in most navigation systems. And so the goal of the first experiment is

- to understand the mathematical and geometrical principles of positioning, and
- to introduce the distance measurement equations.

b) Method

First, the positioning in one dimension is discussed: If the position of the *k*-th satellite is known (say x^k), and its distance from the receiver was measured, ρ^k, then the receiver position, x, can be found by solving the equations[2]

[2] In the satellite navigation community the following notation is widely accepted: *Upper* indices denote the satellite, as they are *up* in the sky. *Lower* indices represent the receiver, as the receivers are *down* on the earth.

$$x^k - x = \rho^k \text{, or } x - x^k = \rho^k,$$

or more compact $\rho^k = \left| x^k - x \right|$. (1)

The next step is to proceed with the two-dimensional problem, visualized in Figure 1: Compared to the one-dimensional case, Equation (1) is rewritten using the norm:

$$\rho^k = \left\| \vec{r}^k - \vec{r} \right\| = \sqrt{\left(x^k - x\right)^2 + \left(y^k - y\right)^2}, \quad (2)$$

where $\vec{r}^k = \left(x^k, y^k\right)$ and $\vec{r} = (x, y)$ are the position vectors of the *k*-th satellite and the receiver.

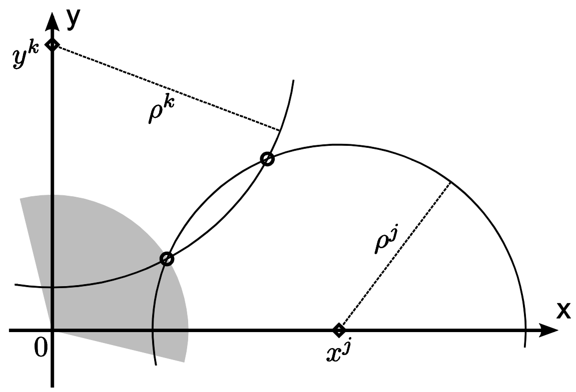


Figure 1: Positioning in two dimensions. The grey circle represents the earth the user is assumed to stand on.

c) Results

To allow the students to quickly find an integer solution, the following values have been used

$$\rho^k = 13,\ \rho^j = 17,\ \left(x^k, y^k\right) = (20{,}0),\ \left(x^j, y^j\right) = (0{,}20).$$

This resulted in the two solutions $(x, y) = (8{,}5)$ and $(x, y) = (15{,}12)$. The students also found the same solutions by setting two compasses at the locations of the satellites and intersecting the corresponding circles (like in Figure 1). They figured out two answers to the question which of the two solutions is correct: either another satellite is introduced, or the knowledge about the receiver moving on the earth is applied (indicated in the figure).

d) Discussion

It could be observed that the introductory group discussion gave the students an easy access to the topic of the experiment. It also allowed them to easily follow the subsequent theory part.

Finally the students are introduced to 3D positioning with satellites. A short overview of the functioning of satellites was followed by simulations with the STK software. The students could create their own Walker constellation [8], as used for Galileo. Navigating through space and trying different points of view, gave them a good impression of the length scales satellite navigation is faced with. The concluding group discussion on distance based positioning then served as a connection to the next experiment.

3.2 Experiment 2: Distance Measurement

a) Introduction

The objective of the second experiment is to introduce the students to the distance measurement as the basic observation in satellite navigation. The experiment should make the students aware of the difficulties directly connected to these measurements:

- the atmospheric delays,

- the measurement noise, and
- the clock difference between the transmitter and receiver.

The experiment should also raise the attention of the students for peculiarities of experimental studies in general.

During a short group discussion the students worked out that distances *d* can be measured using the propagation delay τ of any kind of signals. If their speed of propagation *v* is known, the solution reads

$$d = v \cdot \tau .$$

The radio signals used in satellite navigation are not instructive due to their high speed of propagation. Therefore acoustic signals are used instead. Of course, the speed of sound depends on the local conditions. Thus, as in the case of satellite navigation, the exact value of *the speed of propagation is unknown*.

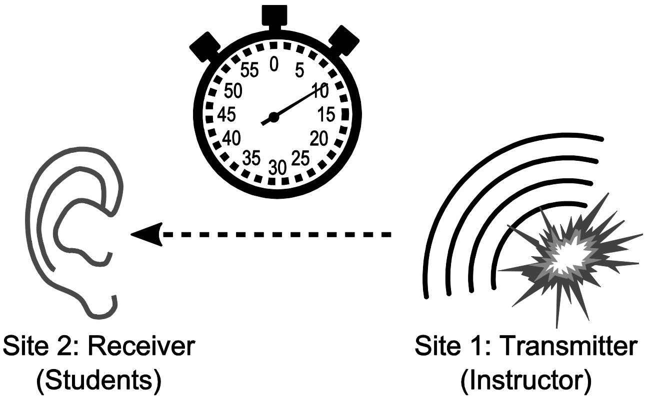


Figure 2: Experiment 1: Measuring distance using stopwatches.

b) Method

The hands-on experiment is performed outdoor and is set up at two distant places (see Figure 2). At both sites a countdown is indicating the full minutes at which the instructor emits a bang (e.g. smacking of two boards). At the receiving site every student carries a stopwatch. These timers are started manually at the full minutes and stopped as soon as the bang was recognized by the student. Since every student starts and stops its watch at slightly different time instants, they get an intuitive understanding of what we call *measurement noise*. The countdown at the transmitting site can be made to run slow, which means that the bang is initiated later than the students start their timer. And thus a measured distance results, which is too large. In this way the *problem of synchronicity* of the transmitter and receiver can directly be experienced.

c) Results

At the time of the course, the temperature of the air was around 7 °C, and so the speed of sound can be approximated as [9]

$$v_{\text{air}} = \sqrt{\frac{\gamma R T}{M}} \approx \sqrt{\frac{1.4 \cdot 8.314 \cdot (273.15 + 8)}{29 \cdot 10^{-3}}} = 335 \text{ m/s}$$

Above *R* denotes the molar gas constant, γ and *M* the adiabatic index and molar mass of air, respectively. The reaction time between the perception of the bang and the stopping of the stopwatch

has to be accounted for. It can either be determined experimentally or from literature. Due to the low sound pressure at reception, a reaction time of 280 ms was assumed [10].

During the measurements the countdown at the transmitting site was running slow by one second. The measurements faced an offset of 335 m. This led the students directly to the conclusion that the clock at the transmitting and the clock at the receiving site have been offset.

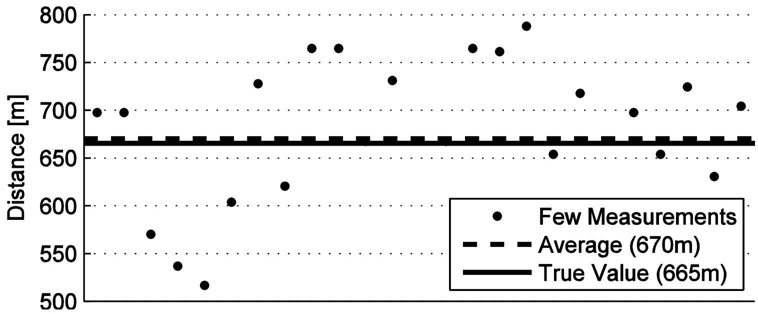


Figure 3: Distance measurements of experiment. The actual distance was 330 m and the intentional offset induced by the clock error 335 m.

Some of the measurements are shown in Figure 3. Obviously the measurement noise is very high when performing the starting and stopping of the stopwatches manually. The measurements can be enhanced by recording the bang with two (directive) microphones. The distance can then be found by comparing the times when the bang was recorded with the two microphones (see Figure 4).

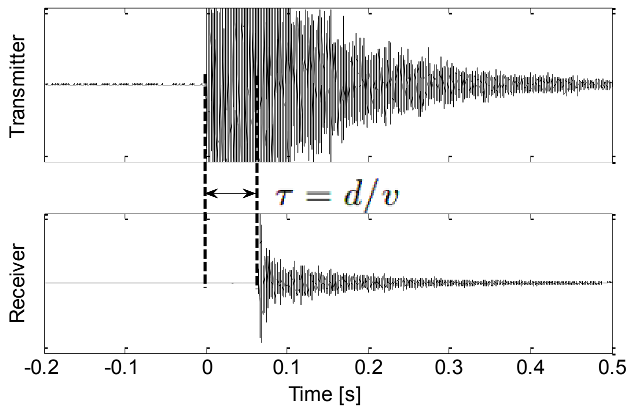


Figure 4: Distance measurements of an experiment using two microphones (the estimated distance was 22.5 m, the actual one 23 m).

d) Discussion

The students participated very actively in this experiment and its discussion. First because they enjoyed the practical work. And second because they were surprised by the outcome of the measurements: the large scatter and bias. Therefore this experiment is seen as a good educational method to learn more about the problems satellite navigation is faced with. The final digital measurements marked a close connection to the third experiment.

3.3 Experiment 2: Precise Positioning

a) Introduction

The goal of the third experiment is to find a solution to the problems observed previously. The experiment should explain how a precise positioning is possible, even if the propagation speed of the signals and the clock offset is unknown.

b) Method

The definition of the measurements were established in the last experiment:

$$\rho = d + c \cdot \left(\delta_{\mathrm{Rx}} - \delta_{\mathrm{Tx}}\right) + \Delta_{\mathrm{atm.}} + \varepsilon,$$

where the distance measurement ρ is composed of d, the geometric distance between transmitter and receiver, δRx and δTx the clock offset of the receiver and transmitter respectively, Δatm. the error induced by the uncertainty of the propagation speed through the atmosphere, and ε the measurement uncertainty, or measurement noise.

By investigating the properties of the atmospheric layers of the earth, the students will find out that only two layers affect the signal propagation in satellite navigation: the ionosphere and the troposphere (e.g. [11]). And so the atmospheric error term above is split up in an ionospheric delay term and a tropospheric delay term: $\Delta_{atm.}=I+T$. From a rough sketch of the ionospheric and tropospheric layers around the earth, it is apparent that the above introduced delays depend on the location of the satellite and the receiver. Thus corresponding indices must be appended (satellite *j*, receiver *i*):

$$\rho_i^j = d_i^j + c\left(\delta_i - \delta^j\right) + I_i^j + T_i^j + \varepsilon_i^j .$$

In the same sketch an additional satellite and receiver (close to the first one) can be introduced (Figure 5). And so the students can easily see that two closely located receivers observe the same

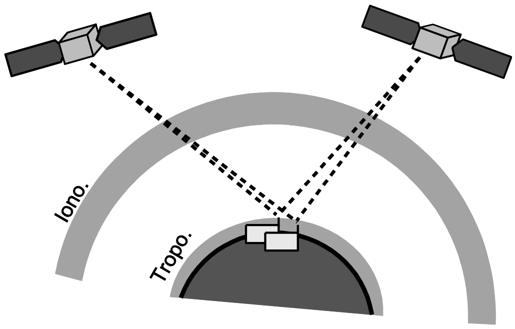


Figure 5: The earth, its ionospheric and tropospheric layer and the propagation paths of two satellites transmitting to two receivers.

atmospheric delay for one satellite. Consequently the lower index of the atmospheric delays can be removed:

$$\rho_1^j = d_1^j + c\left(\underline{\delta_1} - \underline{\delta^j}\right) + \underline{I^j} + \underline{T^j} + \varepsilon_1^j , \qquad (3)$$

$$\rho_2^j = d_2^j + c\left(\underline{\delta_2} - \underline{\delta^j}\right) + \underline{I^j} + \underline{T^j} + \varepsilon_2^j , \qquad (4)$$

In the equations above, the underlined quantities are the ones that were identified in the second experiment as nuisance parameters. By examining the two equations, the students discover that many of the parameters can be eliminated by taking the difference of Equations (3) and (4):

$$\rho_1^j - \rho_2^j = d_1^j - d_2^j + c\left(\underline{\delta_1} - \underline{\delta_2}\right) + \varepsilon_{12}^j$$

In the same way, the last remaining nuisance parameters can be eliminated by subtracting the above difference from the difference with a second satellite:

$$\left(\rho_1^j - \rho_2^j\right) - \left(\rho_1^k - \rho_2^k\right) = \left(d_1^j - d_2^j\right) - \left(d_1^k - d_2^k\right) + \varepsilon_{12}^{jk} . \qquad (5)$$

Taking a closer look at the d_i^j, it can be observed that these distances can be rewritten using vector notation $d_i^j = \left\|\vec{r}^{\,j} - \vec{r}_i\right\|$. One can avoid this nonlinearity by exploiting the fact that every GPS receiver computes the satellite locations and a rough receiver position. This information is given as the unit vector pointing from the receiver to the satellite:

$$\vec{e}_i^{\,j} = \frac{\vec{r}^{\,j} - \vec{r}_i}{\left\|\vec{r}^{\,j} - \vec{r}_i\right\|} .$$

The distances d_i^j can now be rewritten using this unit vector as[3]

$$\vec{e}_i^{\,j} \cdot \left(\vec{r}^{\,j} - \vec{r}_i\right) = \frac{\vec{r}^{\,j} - \vec{r}_i}{\left\|\vec{r}^{\,j} - \vec{r}_i\right\|} \cdot \left(\vec{r}^{\,j} - \vec{r}_i\right) = \frac{\left\|\vec{r}^{\,j} - \vec{r}_i\right\|^2}{\left\|\vec{r}^{\,j} - \vec{r}_i\right\|} = d_i^j .$$

Furthermore from Figure 5 it appears that since the satellites are very far away from the two receivers, compared to the distance between them, the unit vector is almost the same for both, i.e. $\vec{e}_1^{\,j} \approx \vec{e}_2^{\,j} = \vec{e}^{\,j}$. And so Equation (5) reduces to

$$\begin{aligned} \left(\rho_1^j - \rho_2^j\right) - \left(\rho_1^k - \rho_2^k\right) &= \left(\vec{e}^{\,j} - \vec{e}^{\,k}\right) \cdot \left(\vec{r}_2 - \vec{r}_1\right) + \varepsilon_{12}^{jk} \\ &= \left(\vec{e}_x^{\,j} - \vec{e}_x^{\,k}\right) \cdot \left(\vec{r}_{2,x} - \vec{r}_{1,y}\right) \\ &\quad + \left(\vec{e}_y^{\,j} - \vec{e}_y^{\,k}\right) \cdot \left(\vec{r}_{2,y} - \vec{r}_{1,y}\right) \\ &\quad + \left(\vec{e}_z^{\,j} - \vec{e}_z^{\,k}\right) \cdot \left(\vec{r}_{2,z} - \vec{r}_{1,z}\right) + \varepsilon_{12}^{jk} . \end{aligned}$$

Using two receivers and measurements to four satellites, a linear system of three equations and three unkowns (the three components of $\vec{r}_2 - \vec{r}_1$) arises. This linear system of equations can be solved quickly by the students using basic calculus. If the first receiver operates as a reference receiver whose location is known, the above approach can be used to precisely determine $\vec{r}_2$. E.g. in the case of landing aircraft with LAAS, the reference receiver is placed close to the runway [2].

c) Results

After solving the mentioned system of equations, the students implemented the solution in a C++ library. To test the algorithm in practice, this library was then used during a paper chase. Two teams of students got a Laptop to which two cheap GPS receivers (each about ~40$) were connected to. In contrast to the broadly known Geocaching, the goal was to find the correct direction vector pointing from one stage of the chase to the next one.

An example of the measured directional vectors is shown in Figure 7. In the first two minutes, the antennas were in line with the true direction. From 200 to 320 and 350 to 500 seconds, the antennas were offset by 45° and 90°, respectively.

Figure 6: One group of students during the paper chase.

[3] If the students are already familiar with the concept of linearization, the last few steps can be abbreviated.

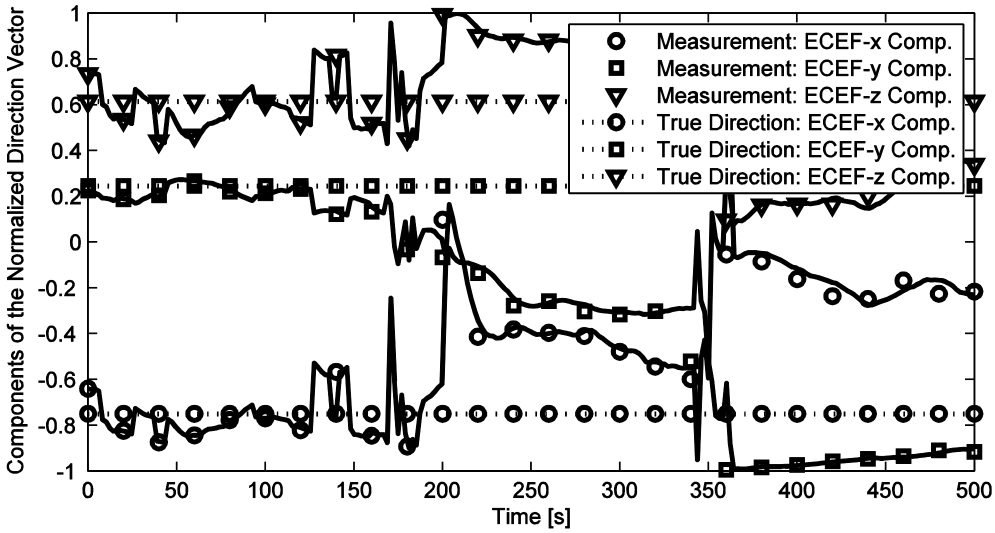


Figure 7: Component wise comparison of the obtained direction vector $\vec{r}_2 - \vec{r}_1$ (*Measurement*) and the true direction to the next stage of the chase (*True Direction*).

d) Discussion

The actual positioning solution cannot be solved by undergraduate students. Due to the nonlinearity of the measurement equations, see e.g. Equation (2), knowledge of multi-dimensional numerical solution methods together with some background in estimation theory would be needed. Even if the equations were linear, a solution for all four unknowns (position $\vec{r} = (r_x, r_y, r_z)$ and clock-error δ) would be too complicated for a quick solution by hand. Therefore the differential approach helps to render the problem analytically solvable for the students. Additionally they could figure out by themselves how the nuisance terms can be eliminated. And at last they got to know a widely used concept for precise position determination.

Regarding the questions, the students asked, it was found to be important that they performed some practical work between the theoretic derivation and the paper chase. The programming gave the students more time to realize how the algorithm works in details and to identify its inputs and outputs. Although none of the students had some prior experience of software programming, they could easily implement their found solution.

The paper chase can be seen as the highlight of the course. It has also been combined with a general quiz about satellite navigation, from which it could be observed that the students actually understood the most important aspects communicated in the course. For sure, finishing the paper chase successfully was rewarded with small presents.

In the preparation of the paper chase multipath propagation was identified to impair the evaluation of different directions. As a solution either better receivers could be used or the measurements are performed in an environment with as few buildings around as possible. The second solution was chosen for this course. And from Figure 7 it can be seen that it was not hard to distinguish between directions separated by as less as 45°.

4 EVALUATION

After the two-days course on satellite navigation, the supervisors let the seven students fill out a questionnaire. To the question *What did you particularly like about the course?* the students replied:

"interesting topic, exciting projects", "explanations", "interesting, lot of fun", "very interesting", "practical work, well implemented", "theory combined with practice", "much enjoyment".

These positive comments match with the fact that all participants answered that they would recommend this course to their friends. Also the supervisors had the feeling that all students have been motivated during the whole course. Consequently one of the pedagogical main goals had been reached. The positive feedback matches the responses to another question which shows that the students now have a higher interest in engineering and scientific work than before the course.

5 CONCLUSION

Summing up the two days, one can conclude that the course fulfills all our requirements stated in the Overview (see Section 2). The experiments are based on each other, therefore the students must have understood one experiment to be able to proceed with the next. This implicitly approved the learning progress which could also be confirmed during discussions and the debriefings.

All non-technical goals, the aim to increase interest in engineering and science among the participants and to provide a balanced, interesting course, seem also to be satisfactorily reached. The overall positive feedback from the students approved this.

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