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***Abstract*—We focus on the linear beamformer design based on *quality-of-service* (QoS) power minimization in the satellite *downlink* with perfect and statistical *channel state information* (CSI) users. Contrary to the usual rate requirements for the perfect CSI users, we consider either *ergodic* or *outage constrained* rate requirements for the statistical CSI users. Modeling the fading channels as zero-mean Gaussian vectors with rank-one covariance matrices, tractable ergodic mutual information and outage probability expressions are obtained. While the resulting outage constrained rate requirements can directly be reformulated to equivalent *signal-to-interference-and-noise-ratios* (SINRs) this is not possible for the ergodic rate constraints. However, representing the necessary useful signal power of each user as a function of the experienced interference and linearizing this interference function, the usual SINR representation is obtained. Based on this observation, a sequential approximation strategy is proposed that solves a QoS power minimization with standard SINR constraints in each iteration. In the numerical results section, the convergence properties and the achieved performance of this sequential QoS optimization are discussed.**

***Index Terms*—QoS power minimization; beamforming; statistical CSI; ergodic rates; outage constrained rate requirements; satellite communication**

I. Introduction

As mentioned in [1], the achievements in the *multi-user* and *multi-antenna* technology of terrestrial wireless systems have inspired researchers to explore the benefit of advanced physical-layer designs in *satellite communication* (SatCom). The reason is that this technology potentially offers increasing link-quality and the possibility of interference reduction just by additionally exploiting the *spatial domain* and without extra cost concerning transmit power and bandwidth. This is especially important as power is a scarce resource at the satellite. The more power is necessary to serve the users, the larger the solar pannels and batteries have to be, which leads to higher launching costs. Hence, the goal of research investigations are power efficient communications and data service coverage of future SatCom systems. In this work, we concentrate on the well known QoS power minimization in the feed forward-link, where transmission takes place through the satellite to the ground terminals. That is, given terminal-specific QoS rate requirements shall be fulfilled using minimum satellite transmit power.

A. Preliminary Work and Recent Advances

In accordance with recent publications, e.g., [2] and [1], we *give up the fixed-spotbeam technique* of state-of-the art SatCom systems in favor of modeling the forward-link as a vector *broadcast channel* (BC). Thus, all the known efficient beamforming techniques can be exploited, e.g., the linear beamforming approaches in [3] and [4]. In [2], also the linear and non-linear restrictions for per-feed transmit powers are taken into account. Note that the joint beamforming is designed at the gateway which avoids power intensive computations at the satellite.

While above approaches rely on perfect CSI for all users at the operating gateway, *robust formulations* of the considered QoS power minimization have to be considered to cope with SatCom system that serve also *statistical CSI* users, e.g., *land-mobile SatCom systems* (cf. [5]). While the channel states for static users with line of sight to the satellite are perfectly known at the gateway, only the statistical properties of the channels to mobile users, e.g., the covariance matrices, are available due to the long round trip time in *land-mobile SatCom systems* (cf. [5]). Fortunately, these covariance matrices are essentially rank one because of the large distance from the satellite to the ground and the local shadowing and scattering effects around the users in urban environments. Therefore, this work concentrates on the robust beamformer design based on *chance constrained formulations* and *ergodic requirements*.

Chance constrained programming has recently gained importance in wireless communications due to its capability of properly describing optimizations with outage rate requirements (e.g., see [6], [7], [8]). However, the tractability of these outage probabilities strongly depends on the form of the probabilistic expression. Fortunately, the considered channel statistics with zero-mean and rank-one covariance matrices leads to closed-form expressions for the outage rate probability that can directly be incorporated into the existing (perfect CSI) QoS beamformer-design strategies.

Ergodic robust formulations for the considered problem were already considered in [9], [10], [11]. The difficulty is that the ergodic rate requirements cannot be represented in terms of some SINR constraints. Thus, straightforwardly applying the usual perfect CSI optimizations is impossible. Due to this property and the fact that no convex reformulations are known for the ergodic rate requirements, a globally optimal *branch-and-bound* approach was applied in [10] that, however, has exponential complexity. The other references consider bounds on the ergodic rates which are clearly suboptimal, but lead to efficient algorithms with quadratic convergence speed. In this work, we propose a locally optimal sequential approximation strategy that has tractable complexity.

B. Contributions and Structure

We present a locally optimal sequential approximation strategy for the robust QoS power minimization in the satellite downlink with perfect CSI and statistical CSI users. To this end, we propose a robust QoS power minimization formulation with rate based chance constraints and ergodic rate requirements for the statistical CSI users in Section III. While the chance constrained QoS formulations can be equivalently reformulated into minimum SINR requirements, this is not possible for the ergodic rate constraints (see Section IV). Therefore, approximations with SINR structure are created that are based on a first order Taylor expansion w.r.t. the experienced interference of the specific user. Based on these SINR approximations for the ergodic users, we are able to approximate the original problem via a standard QoS power minimization with SINR constraints that in turn can be solved with the usual efficient methods for perfect CSI. This motivates a sequential approximation method for the initial QoS power minimization, where in each iteration a standard SINR constrained power minimization problem is solved (see Section V). The numerical simulations in Section VI indicate that this (locally optimal) sequential approximation method converges in only a few iterations when starting from an initial feasible point. Moreover, it achieves the global optimum in small satellite system setups.

II. System Model

Modeling the satellite feed forward-link as a vector BC, K users are simultaneously served by the N-antenna satellite in the considered carrier. The satellite linearly precodes the independent unit-variance data signals $s_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ with the beamforming vectors $\boldsymbol{t}_k \in \mathbb{C}^N$, $k \in \{1,\ldots,K\}$ and simultaneously transmits the superposition of the outcomes to the K receivers. User k's received signal is given by

$$y_k = \boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k + \boldsymbol{h}_k^{\mathrm{H}}\sum_{i\neq k}\boldsymbol{t}_i s_i + n_k$$

and suffers from zero-mean unit-variance additive Gaussian noise $n_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$. Here, $\boldsymbol{h}_k^{\mathrm{H}} \in \mathbb{C}^{1\times N}$ denotes the frequency flat fading channel vector corresponding to user k.

Note that the channel states $\boldsymbol{h}_k$ are only available for static users. For mobile users, the gateway is only aware of the statistics of the fading channel that are modeled as zero-mean Gaussian vectors with rank-one covariance matrices, i.e., $\boldsymbol{h}_k \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \boldsymbol{C}_{\boldsymbol{h}_k})$ with $\boldsymbol{C}_{\boldsymbol{h}_k} = \boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{H}}$. The channel signature $\boldsymbol{v}_k$ is the scaled dominant eigenvector of $\boldsymbol{C}_{\boldsymbol{h}_k}$. This model is appropriate when assuming mobile users in an urban environment and the satellite works at Ka-band (cf. [5]). That is, the spatial signatures of the channels remain essentially constant due to the large distance from the satellite to the ground and the slow movement of the mobiles. The signal of far distant large scatterers, e.g., mountains and skyscrapers, can be neglected due to the additional path loss and delays. However, the norm and phase of the channels strongly depends on the position of the mobile, the surrounding scatterers, and the heavy shadowing in urban environments.

III. Quality of Service Optimization

For perfect channel state information, the minimization of the total average transmit power under minimum *quality-of-service* (QoS) rate requirements ρ_k, $k \in \{1,\ldots,K\}$ reads as

$$\begin{aligned} &\text{minimize} \;\; P(\boldsymbol{t}_1,\ldots,\boldsymbol{t}_K) \\ &\text{subject to} \;\; r_k(\boldsymbol{t}_1,\ldots,\boldsymbol{t}_K) \geq \rho_k \quad \forall k \in \{1,\ldots,K\}, \end{aligned} \tag{$\mathcal{Q}oS$}$$

where the transmit power $P(\boldsymbol{t}_1,\ldots,\boldsymbol{t}_K) = \sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2$ and the rates $r_k(\boldsymbol{t}_1,\ldots,\boldsymbol{t}_K)$ are both functions of the precoding vectors $\boldsymbol{t}_k$, with $k \in \{1,\ldots,K\}$.

Besides the achievable data rates, common QoS metrics are the users' SINRs. Due to their bijective relationship, i.e.,

$$r_k = \log_2(1 + \mathrm{SINR}_{k,\mathrm{pCSI}}), \tag{1}$$

with the kth user's (perfect CSI) SINR given by

$$\mathrm{SINR}_{k,\mathrm{pCSI}} = \frac{|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{1 + \sum_{i\neq k}|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}, \tag{2}$$

the optimization in ($\mathcal{Q}oS$) can equivalently be written as a power minimization with the SINR constraints

$$\mathrm{SINR}_{k,\mathrm{pCSI}} \leq 2^{\rho_k} - 1. \tag{3}$$

Solutions for this equivalent formulation are given in [3], [4]. Furthermore, equivalently reformulating above SINR constraints in terms of the *minimum mean square errors* (MMSEs), i.e., $\mathrm{MMSE}_k = 1/(1+\mathrm{SINR}_k)$, feasibility of ($\mathcal{Q}oS$) can easily be verified. That is, rewriting the rate constraints as $\mathrm{MMSE}_k \leq \varepsilon_k = 2^{-\rho_k}$, $k \in \{1,\ldots,K\}$, the feasible region of the requirements-tuple $(\varepsilon_1,\ldots,\varepsilon_K)$ is a polytope with box constraints $0 \leq \varepsilon_k \leq 1 \; \forall k \in \{1,\ldots,K\}$ and the half-space constraint $\sum_{k=1}^K \varepsilon_k \geq K - N$ [12, Theorem 1].

However, this work considers that $\boldsymbol{h}_k$ is only available for perfect CSI users $k \in \mathbb{P} \subseteq \{1,\ldots,K\}$. For the statistical CSI users $k \in \mathbb{S} = \{1,\ldots,K\} \setminus \mathbb{P}$, only the spatial channel signatures $\boldsymbol{v}_k$ are known at the gateway, why we cannot rely on (1) for these users. For some of these users $k \in \mathbb{Q} \subseteq \mathbb{S}$, robust designs shall be based on the reliability that the rate targets ρ_k are achieved with some minimal probability. That is, the chance constraints $\Pr\{r_k \geq \rho_k\} \geq \tau_k$ are employed for $k \in \mathbb{Q}$, with $\tau_k \in [0,1]$ being the required minimal probability that the target is met. For the remaining users $k \in \mathbb{E} = \mathbb{S} \setminus \mathbb{Q}$, the ergodic rate constraints $R_k = \mathrm{E}_{\boldsymbol{h}_k}[r_k] \geq \rho_k$ are used (cf. [10], [11]). Given these three different rate requirements for static and mobile users, the QoS optimization reads as

$$\begin{aligned} &\text{minimize} && \sum_{i=1}^K \|\boldsymbol{t}_i\|_2^2 && \\ &\text{subject to} && r_k \geq \rho_k && k \in \mathbb{P} \\ & && \Pr\{r_k \geq \rho_k\} \geq \tau_k && k \in \mathbb{Q} \\ & && R_k \geq \rho_k && k \in \mathbb{E} \end{aligned} \tag{$\mathcal{Q}$}$$

Note that ($\mathcal{Q}oS$) is a non-convex problem in the original BC formulation and might become infeasible when $N < K$ and too large ρ_k are chosen.

The perfect CSI rates in (1), the reliability probability $\Pr\{r_k \geq \rho_k\}$, and the ergodic rates $R_k = \mathrm{E}[r_k]$ are neither convex nor concave functions of $\{\boldsymbol{t}_k\}_{k=1}^K$. To overcome this difficulty, the perfect CSI rate requirements are transformed to their SINR based counterparts as seen above. In the next section, it is shown that 'SINR' like terms are also available for the rate based chance constraints and approximations of the ergodic rate constraints in ($\mathcal{Q}$). This enables applying a *sequential approximation approach* as detailed in Section V.

IV. Constraint Set Reformulation

We show that the chance constraints in ($\mathcal{Q}$) can directly be transformed to SINR constraints and the ergodic rate constraints can be approximated with 'signal-to-interference-plus-noise-ratio' constraints.

A. Chance Constrained Rate Requirements

To reformulate the chance constraint in ($\mathcal{Q}$) as respective SINR requirement, we first rewrite the constraint as

$$\Pr\{r_k \leq \rho_k\} \leq 1 - \tau_k, \tag{4}$$

with $\Pr\{r_k \leq \rho_k\}$ being the *outage probability*—the probability that the rate requirement ρ_k is not satisfied. Since we can write $\boldsymbol{h}_k \sim \boldsymbol{v}_k w_k$ with $w_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$, above outage probability can equivalently be expressed as

$$\Pr\{r_k \leq \rho_k\} = \Pr\left\{|w_k|^2 \leq \frac{2^{\rho_k}-1}{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2 - (2^{\rho_k}-1)I_k}\right\}$$

with $I_k = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2$ denoting the average experienced interference of user k. Note that $\lambda_k = |w_k|^2$ is a χ^2-distributed random variable with two degrees of freedom and $\mathrm{E}[\lambda_k] = 1$. The *cumulative distribution function* of λ_k reads as (e.g., see [13])

$$\mathrm{F}_{\lambda_k}(\theta) = \Pr\{\lambda_k \leq \theta\} = 1 - \exp(-\theta).$$

Hence, an explicit form of (4) is given by

$$(2^{\rho_k}-1)/\big(|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2 - (2^{\rho_k}-1)I_k\big) \leq -\ln(\tau_k).$$

This inequality can in turn be written as the SINR constraint

$$\mathrm{SINR}_{k,\text{outage}} = \frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{\frac{1}{-\ln(\tau_k)} + I_k} \geq 2^{\rho_k} - 1. \tag{5}$$

Here, $\mathrm{SINR}_{k,\text{outage}}$ has obviously the same properties as $\mathrm{SINR}_{k,\text{pCSI}}$ in (2), namely, it is a fraction of the useful signal power over a positive and linearly increasing function in the interference I_k. The only difference to the perfect CSI structure is that the effective 'noise' variance $1/(-\ln(\tau_k))$ depends on the reliability probability τ_k in (5).

B. Ergodic Rate Requirements

Next, we consider the ergodic rate requirements

$$R_k = \mathrm{E}[r_k] \geq \rho_k. \tag{6}$$

Due to $\boldsymbol{C}_{\boldsymbol{h}_k} = \boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{H}}$, an explicit expression for the kth user's ergodic rate reads as

$$R_k = \frac{1}{\ln(2)}\left[h\left(|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2 + I_k\right) - h\left(I_k\right)\right]. \tag{7}$$

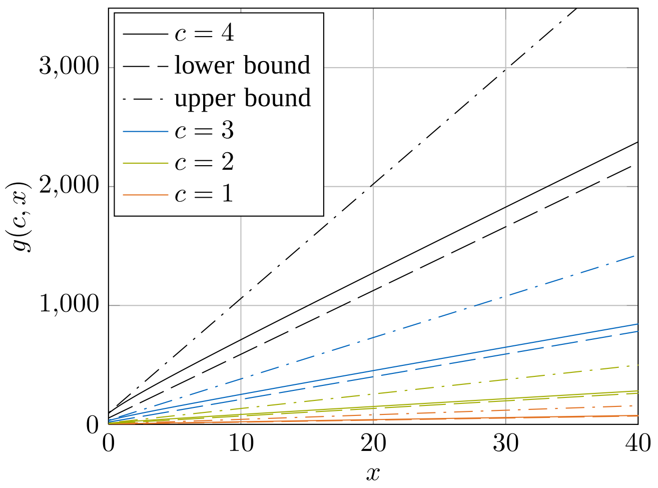

Figure 1. A plot of the function $g(c,x)$ together with the upper bound $g(c,x) \leq (\mathrm{e}^{c+\gamma}-1)(1+x)$ and the lower bound $g(c,x) \geq (\mathrm{e}^{c}-1)(1+x)$.

Here, $I_k = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2$ and the function $h : \mathbb{R}_+ \to \mathbb{R}_+$ is defined as $h(x) \triangleq \mathrm{e}^{\frac{1}{x}}\,\mathrm{E}_1\left(\frac{1}{x}\right)$, where $\mathrm{E}_1(x) = \int_x^\infty \frac{\mathrm{e}^{-t}}{t}\,\mathrm{d}t$ denotes the exponential integral function (e.g., see [14], [15]).

Unfortunately, a direct reformulation of (6) does not result in the same SINR structure as for perfect CSI in (3) and the chance constraint formulation in (5). That is, by reformulating the ergodic constraint such that the average received useful signal $|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2$ over a function of the interference I_k has to meet some minimum requirement, we obtain

$$\frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{g(\ln(2)\rho_k, I_k)} \geq 1, \tag{8}$$

where the continuous function

$$g(c,x) = h^{-1}\big(c + h(x)\big) - x$$

is non-linear and (non-convex) increasing in x (cf. Fig. 1). In fact, $g(c,x)$ appears to be concave in x, but a rigorous proof for this observation is still missing.

To obtain some SINR like version of (8), a linear approximation of $g(c,x)$ is required. For this purpose, we apply the first order Taylor expansion

$$g(c,x) \approx g(c,x') + \left.\frac{\partial g(c,x)}{\partial x}\right|_{x=x'}(x - x'),$$

with the partial derivative being

$$\frac{\partial g(c,x)}{\partial x} = \frac{\big(h^{-1}(c+h(x))\big)^2}{h^{-1}(c+h(x)) - c - h(x)}\,\frac{x-h(x)}{x^2} - 1,$$

Then, the ergodic constraint in (8) can be approximated via

$$\mathrm{SINR}_{k,\text{ergodic}} = \frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{\frac{\beta_k}{\alpha_k} + I_k} \geq \alpha_k, \tag{9}$$

with $\alpha_k = \left.\frac{\partial g(\ln(2)\rho_k, I_k)}{\partial I_k}\right|_{I_k=I_k'}$ and $\beta_k = g(\ln(2)\rho_k, I_k') - \alpha_k I_k'$. Since the function $g(c,x)$ is close to linear in x, this approximation is accurate whenever the optimal I_k is close to the linearization point I_k'. Note that the SINR target α_k in (9) depends on I_k' and the ergodic rate target ρ_k and appears to approach $\mathrm{e}^{\ln(2)\rho_k} - 1$ for $I_k' \to \infty$ (see Fig. 1).

Algorithm 1 Iterative QoS power minimization with ergodic rate requirements

Require: $\boldsymbol{h}_k\ \forall k \in \mathbb{P}$, $\boldsymbol{v}_k\ \forall k \in \mathbb{S}$, $I_k^{(0)}\ \forall k \in \mathbb{E}$, $\epsilon \leftarrow 10^{-4}$

repeat
- $n \leftarrow n+1$
- SINR approximation of the ergodic constraints (9) with $\alpha_k = \frac{\partial}{\partial x} g(\ln(2)\rho_k, x)\big|_{x=I_k^{(n-1)}}$
  $\beta_k = g\big(\ln(2)\rho_k, I_k^{(n-1)}\big) - \alpha_k I_k^{(n-1)}$
- Solving the QoS sub-problem $(\mathcal{QA})$ optimally $\big(\{\boldsymbol{t}_\kappa^{(n)}\}, P_{\text{opt}}^{(n)}\big) = \mathcal{QA}\big(\{\rho_\kappa\},\{\tau_\kappa\},\{\alpha_\kappa\},\{\beta_\kappa\}\big)$
- Update of the interference values $I_k^{(n)} = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i|^2\ \forall k \in \mathbb{E}$

until $|I_k^{(n)} - I_k^{(n-1)}| < \epsilon\ \forall k \in \mathbb{E}$

return $\boldsymbol{t}_k^{(n)}\ \forall k \in \{1,\ldots,K\}, P_{\text{opt}}$

V. Sequential Approximation Approach

Based on the SINR reformulations in (3) and (5) for the perfect CSI and the reliability rate requirements, respectively, and the SINR approximation of the ergodic constraints in (9), we can approximate $(\mathcal{Q})$ via the standard SINR formulation

$$\begin{aligned} &\text{minimize} && \sum_{i=1}^{K} \|\boldsymbol{t}_i\|_2^2 \\ &\text{subject to} && \text{SINR}_{k,\text{pCSI}} \geq 2^{\rho_k} - 1 \quad \forall k \in \mathbb{P}, \\ &&& \text{SINR}_{k,\text{outage}} \geq 2^{\rho_k} - 1 \quad \forall k \in \mathbb{Q}, \\ &&& \text{SINR}_{k,\text{ergodic}} \geq \alpha_k \quad \forall k \in \mathbb{E}. \end{aligned} \qquad (\mathcal{QA})$$

This standard power minimization with SINR constraints can in turn be solved via the usual methods and algorithms. That is, either we reformulate $(\mathcal{QA})$ into a second order cone problem and apply standard interior-point solvers as in [4], or we apply the uplink-downlink SINR duality and a fixed point algorithm in the dual uplink as proposed in [3]. Moreover, feasibility of $(\mathcal{QA})$ can be tested in the same way as for the totally perfect CSI scenario when defining the MMSE targets as $\varepsilon_k = 2^{-\rho_k}$ for $k \in \mathbb{P} \cup \mathbb{Q}$ and $\varepsilon_k = 1/(1+\alpha_k)$ for $k \in \mathbb{E}$.

We remark that the optimal solution of $(\mathcal{QA})$ is in general not a solution of the original QoS power minimization in $(\mathcal{Q})$. However, whenever the optimal solution of $(\mathcal{QA})$, denoted as $(\boldsymbol{t}_1^\star,\ldots,\boldsymbol{t}_K^\star,P_{\text{opt}}) = \mathcal{QA}\big(\{\rho_\kappa\},\{\tau_\kappa\},\{\alpha_\kappa\},\{\beta_\kappa\}\big)$, results in interference values $I_k^\star = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i^\star|^2$ that satisfy $I_k^\star = I_k'$ for all $k \in \mathbb{E}$, then $(\boldsymbol{t}_1^\star,\ldots,\boldsymbol{t}_K^\star)$ is a locally optimal solution for the initial power minimization problem $(\mathcal{Q})$.

This motivates a sequential (convex) approximation algorithm, where in the n-th iteration $(\mathcal{Q})$ is approximated with the SINR constrained power minimization in $(\mathcal{QA})$ that is based on the interference values $I_k^{(n-1)} = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i^{(n-1)}|^2$ for $k \in \mathbb{E}$, with $(\boldsymbol{t}_1^{(n-1)},\ldots,\boldsymbol{t}_K^{(n-1)})$ being the outcome of the previous iteration (see Algorithm 1). As a starting point, we use an initial feasible set $\{I_k^{(0)}\}_{k\in\mathbb{E}}$. To this end, we replace the ergodic rates in $(\mathcal{Q})$ with their upper bounds (cf. [10])

$$\overline{R}_k = \log_2\Big(1 + \tfrac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{1+\sum_{i\neq k}|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}\Big) \geq R_k,$$

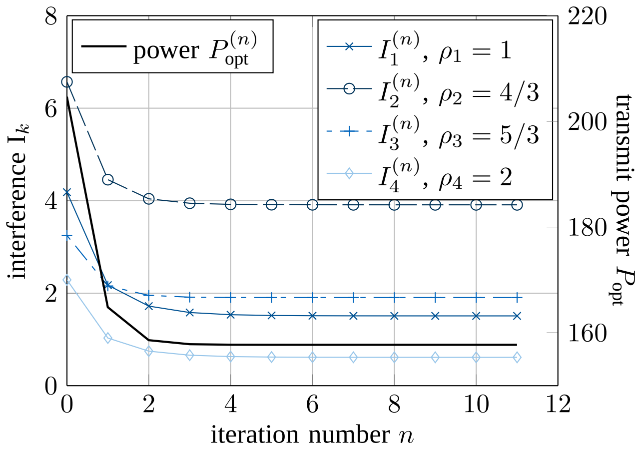

Figure 2. Minimal Power and Interference vs. number of iterations until convergence for a $K = N = 4$, $\mathbb{E} = \{1,\ldots,K\}$, and above targets.

and solve the resulting approximate optimization problem. Then, the obtained beamforming vectors are scaled with a common factor such that all rate constraints in $(\mathcal{Q})$ are satisfied. By this two step procedure we obtain an initial feasible tuple of precoders $(\boldsymbol{t}_1^{(0)},\ldots,\boldsymbol{t}_K^{(0)})$ [10, Proof of Theorem 1] and, therewith, we can calculate feasible interference values as $I_k^{(0)} = \sum_{i\neq k} |\boldsymbol{v}_k^{\mathrm{H}} \boldsymbol{t}_i^{(0)}|^2$ for $k \in \mathbb{E}$. Using this initialization, an exemplary convergence behavior for a $K = N = 4$ antenna scenario with only ergodic rate requirements, i.e., $\mathbb{E} = \{1,\ldots,4\}$, is shown in Fig. 2. We see that the transmit power as well as the individual interference values are decreasing in each iteration.

VI. Numerical Results

For numerical simulations, we consider a GEO-stationary satellite with a rectangular antenna array of N elements. The K users are randomly placed within 1° to 21°east and 40° to 56°north. Moreover, we differentiated two system setups: *fully loaded systems* with $K = N$, e.g., $K = N = 4$ and $K = N = 64$, and *overloaded systems* with $K > N$, e.g., small systems with $K = 6$ and $N = 4$ and large systems with $K = 80$ and $N = 64$. Within these geometric models, we used the free space path loss model for determining values of $\{\boldsymbol{h}_k\}_{k\in\mathbb{P}}$ and $\{\boldsymbol{v}_k\}_{k\in\mathbb{S}}$ that are normalized with a common factor.

To analyze the convergence behavior of Algorithm 1, we have performed simulations with solely statistical CSI users that have ergodic rate requirements, i.e., $\mathbb{E} = \{1,\ldots,K\}$ and $\mathbb{P} = \mathbb{Q} = \emptyset$, and 100 channel realizations. The basic targets of the users are chosen to be $\rho'_{2i-1} = 1$ and $\rho'_{2i} = 2$, $i \in \{1,\ldots,K/2\}$, which we scaled with some common factor ρ_0 such that $\rho_k = \rho_0\rho'_k$. For the histogram in Fig. 3, where the necessary iterations until convergence are depicted, we calculated the minimum transmit power for $\rho_0 \in \{0.25, 0.5, 1, 2, 4\}$ in the fully loaded system and for $\rho_0 \in \{0.25, 0.5, 1\}$ in the overloaded system. We can see that the required number of iterations slightly increases with the system dimension, but remains small (below 16) for all observed scenarios. This indicates fast convergence of the proposed algorithm.

Another aspect is the achieved performance of the proposed

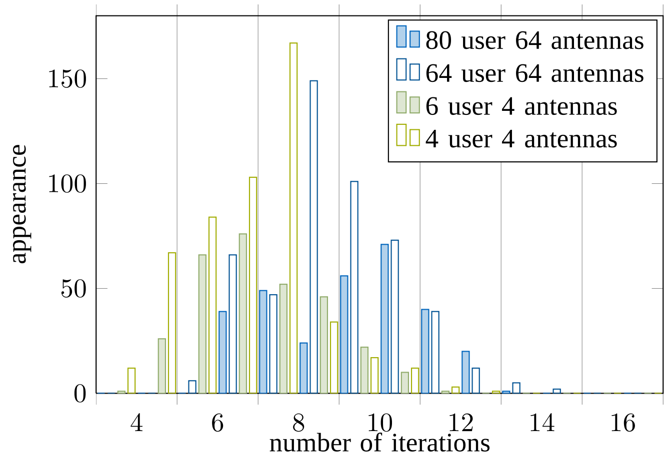

Figure 3. Histogram of the necessary number of iterations until convergence.

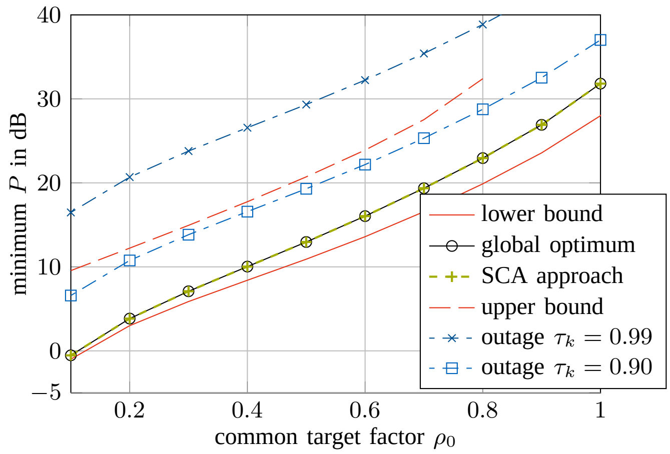

Figure 4. Minimal Power for a scenario with $K = 6$, $N = 4$, and 3 statistical CSI users with either ergodic or reliability rate requirements.

algorithm. To this end, we have performed simulations in the small systems to be able to compute the global optimal solutions, since the branch and bound method in [10] is exponential in $|\mathbb{E}|$. Here, we depict only the results for an *overloaded system* with $K = 6$, $N = 4$, and $K/2$ statistical CSI users. All these statistical CSI users had either reliability or ergodic rate requirements and above basic targets ρ'_k.

In Fig. 4, the minimal transmit power is plotted over the common target factor ρ_0. We remark that all depicted points ρ_0 were feasible for the robust power minimization ($\mathcal{Q}$). For the ergodic constraint case, we depict the *upper bound* and the *lower bound* for the minimum transmit power from [11], the globally optimal solution, and the *sequential (convex) approximation* (SCA) approach. Note that each point of the bound curves is obtained via a single SINR constrained power minimization, contrary to the SCA curve points that require a sequence of these power minimizations. However, with this increased complexity, we exactly meet the global optimum in the considered scenario. This indicates that the SCA method has the potential to achieve the global optimum with tractable complexity. Similar observations were made for the fully loaded scenario with $K = N = 4$ and $|\mathbb{E}| = 2$.

For the case with only reliability rate constraints for the $|\mathbb{S}| = 3$ statistical CSI users, we depicted curves with $\tau_k = \tau$ for all $k \in \mathbb{Q}$ and $\tau \in \{0.99, 0.90\}$ in Fig. 4. We see that the minimum achievable transmit power strongly depends on $\tau \in (0, 1)$ as expected, since the effective noise term in (5) is increasing with τ. That means, the more reliable the transmission has to be, the more transmit power is required.

VII. Conclusion and Outlook

We considered the stochastically robust beamformer design in the satellite downlink based on QoS power minimization with reliability and ergodic rate constraints. To get along with the ergodic requirements, we proposed a sequential (convex) approximation procedure that solves a standard SINR constrained power minimization in each iteration. Starting from an initial feasible point, this procedure locally optimally solves the initial problem and, thus, potentially achieves the global optimum with reasonable complexity. The performance and the fast convergence motivate a detailed analysis of the ergodic 'SINR' constraints and its approximation in future work.

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