Precise Positioning of a Geostationary Data Relay using LEO Satellites

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Abstract – Earth observation uses a large number of Low Earth Orbiting (LEO) satellites (ca. 400). The increasing resolution of their instruments has inflated the data volumes that need to be transmitted. The low altitudes limit the contact times, which is challenging both with respect to the data volumes and delay until data can be transmitted. Geostationary satellite (GEO) relaying is thus a promising alternative. In order to allow pre-compensation of Doppler Shift at the LEO side and to synchronize the transmission from the LEOs to the GEO, the positions and relative velocities of the satellites have to be known. LEOs can be positioned today with centimetre accuracy by GPS. GEOs orbit on higher altitudes than GPS satellites and therefore face problems using GPS signals. Conventional methods allow GEO positioning from ground with accuracies in the km range. In this paper, a new concept of GEO precise positioning using communication channels of LEO satellites is presented. Simulations show that Kalman filtered pseudoranges lead to positioning errors in cm range. A new method based on Newton algorithm allows to determine the Keplerian parameters and their linear drifts in order to predict the GEO position.

Keywords – Orbit determination, GEO satellite, Positioning

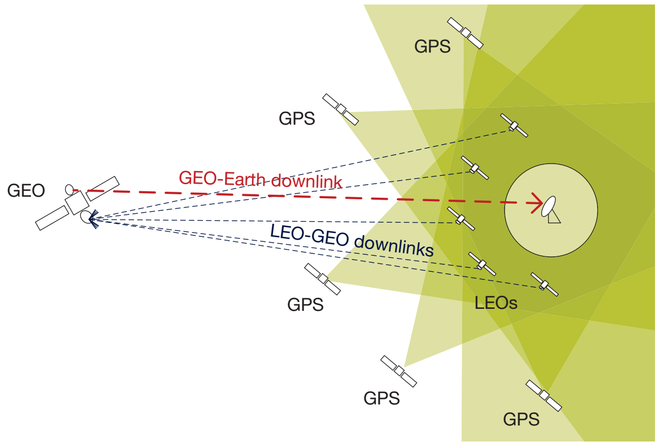

Fig. 1. LEO satellites can be positioned with GPS achieving centimetre accuracy. The transmission of signals from LEO satellites to a GEO satellite allows the precise determination of the GEO position.

I. Introduction

Conventional data download from Low Earth Orbit (LEO) satellites via a direct link can only be performed when the satellite appears in the ground station antenna's field of view. Depending on the orbit and location of the ground station, this can lead to large delays between gain of data on the LEO satellite and transmission to earth. This problem can be solved by using geostationary data relays. The American Tracking and Data Relay Satellite System (TDRSS) [2] and the recently developed European Data Relay Satellite (EDRS) [3] are examples for such approaches. An alternative concept is currently designed under the lead of the German Aerospace Centre (DLR) in the GeReLEO project. Thereby a multibeam array antenna consisting of approximately 400 single spot beams serves as receiving antenna for LEO signals (see also [4], [5]). In order to avoid interference on neighbouring spot beams, the LEO channels are separated by a Frequency Division Multiple Access (FDMA) scheme. If two or more signals from different LEO satellites are simultaneously passing the same spot beam, an additional Time Division Multiple Access (TDMA) scheme is foreseen as a baseline for alternating transmission. Due to the relative movement of GEO and LEO satellites, Doppler Shift occurs and has to be compensated for. This can be done for example directly on the LEO satellite by adjustment of the local oscillator. Therefore, the exact LEO and GEO position and velocity has to be known. The LEO position easily can be determined with the help of GPS, where an accuracy in position of a few centimetres can be reached [6]. The GEO position so far is determined via ranging and ground station tracking and only leads to accuracies in the km range [7]. The estimated worst case error after Doppler Shift compensation based on these conditions can be calculated to ± 60 Hz. This could be tremendously improved by precise positioning of the geostationary satellite. Additionally synchronization becomes easier and the timing of switching from one spot beam to the neighbouring one can be defined more accurate by knowledge about the GEO precise position. Therefore a new concept of GEO positioning is presented in this paper based on communication signals of the LEO satellites.

II. Positioning concept for a GEO data relay

The geostationary data relay developed in the GeReLEO project is meant to receive Ka band signals from up to 12 LEO satellites simultaneously and forward them to ground. Today LEO positions can be determined on board the LEO satellites in real time with accuracies of a few centimetres according to [6]. The concept is now to use the LEO satellites as "navigation satellites" for precise GEO positioning. Thereby the LEO satellites are foreseen to transmit navigation signals containing their position to the GEO satellite at certain time

slots between the communication data. On board the GEO satellite this information is used to estimate its position. This concept faces different conditions compared to positioning of receivers on ground with the help of Global Navigation Satellite Systems (GNSS) like GPS:
Some of the main sources for positioning errors are not present along the LEO-GEO links, like tropospheric effects and multipath from ground reflections. Other errors are much weaker, like the delay induced by ionosphere which thins out in density above 600 km - a typical LEO altitude. Additionally, the Ka band signals at around 27 GHz are delayed less by a factor of $\sim$ 300 than GPS signals at around 1.5 GHz due to the $1/f^2$ frequency dependency. In the GeReLEO project, the LEO satellites are assumed to transmit via a 30 cm reflector antenna with a power of 30 W and the signals are received on the LEO side with a 120 cm reflector antenna. This leads to a high antenna gain at both the transmitter and receiver side. However, the free space loss of the LEO/GEO-system is significantly stronger than for GPS based positioning on Earth due to the increased distance and higher frequency. Nevertheless, link-budget calculations show a large C/N_0 of 77 dB-Hz at 36 MHz leading to a Cramer-Rao Bound [8] of around 1 mm.

A. Estimation of Keplerian elements and drifts with Newton algorithm and coordinate transformation

The basic concept foresees to estimate the GEO position on board the GEO satellite. This position data can then be broadcasted to the LEO satellites where it is used to perform the Doppler pre-compensation by detuning their local oscillators. An update of the GEO position at high frequency, e.g. several times per second, leads to an extensive overhead in LEO telecommand link allocation. As these links typically only allow a few kbit/s methods have to be found to predict the GEO position on board the LEO satellites for a certain period of time. The conventional way of describing satellite movement is to provide the Keplerian orbit parameters (i.e. a semi-major axis, e eccentricity, i inclination, Ω RAAN, ω perigee and ν true anomaly) for the trajectory. However, a Kepler orbit always underlies certain perturbations based on solar radiation pressure, gravity anomalies due to the Moon or other planets and also changes in the Earth gravity field at inclined orbits. Good determination of all these effects leads to highly realistic orbit models like presented in [1]. Such parameter estimation however is very complex and therefore not suited to run on simple LEO on-board processors.
For short term prediction of satellite positions in the time range of maybe one hour we want to propose an alternative concept. Fig. 2 shows a functional diagram for a new algorithm to estimate the GEO position as well as the Keplerian parameters and linear drifts of the GEO orbit. In detail it works as follows:

The GEO satellite orbit is estimated based on the measured pseudoranges from K LEO satellites to the GEO satellite. These pseudoranges are modelled as

$$\left[\rho^1, \ldots, \rho^K\right]^T = \boldsymbol{z}_t = \boldsymbol{H}_t \boldsymbol{x}_t + \boldsymbol{v}_t, \tag{1}$$

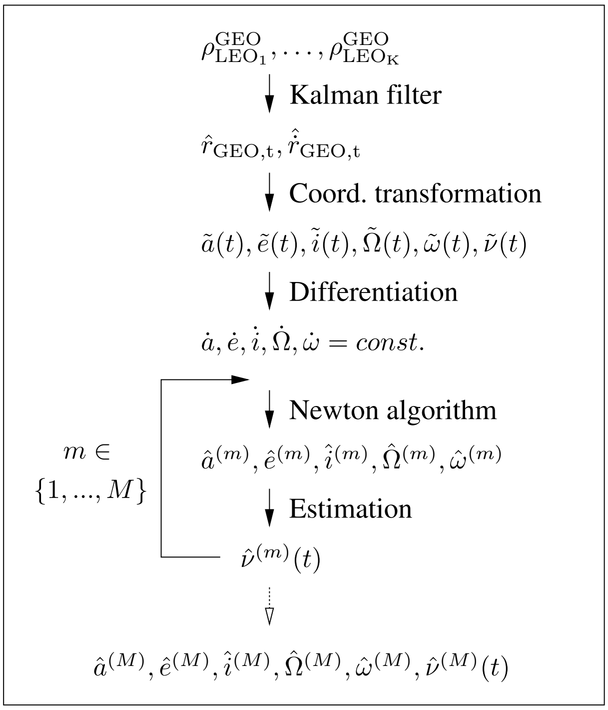

Fig. 2. Functional diagram for estimation of Keplerian parameters and their linear drifts for GEO orbits.

where $\boldsymbol{H}_t$ includes the unit vectors pointing from the LEOs to the GEO, $\boldsymbol{v}_t \sim (\boldsymbol{0}, \boldsymbol{\Sigma_v})$ is the measurement noise and $\boldsymbol{x}_t$ is the state vector that is defined as

$$\boldsymbol{x}_t = \begin{bmatrix} \boldsymbol{r}_t \\ c\delta\tau_t \\ \dot{\boldsymbol{r}}_t \\ c\delta\dot{\tau}_t \end{bmatrix}, \tag{2}$$

with the position $\boldsymbol{r}_t$ and velocity $\dot{\boldsymbol{r}}_t$ of the GEO satellite in the ECEF coordinate system. The relative movement of the GEO w.r.t. a fixed point on Earth is significantly smaller than in the case of GPS, which enables the use of a linear state space model for the satellite movement, i.e.

$$\boldsymbol{x}_{t+1} = \boldsymbol{\Phi}_t \boldsymbol{x}_t + \boldsymbol{w}_t, \tag{3}$$

with the state transition matrix $\boldsymbol{\Phi}_t$ and the white Gaussian process noise $\boldsymbol{w}_t \sim (\boldsymbol{0}, \boldsymbol{\Sigma_w})$. The GEO position and velocity are estimated with a Kalman filter, which consists of alternating state predictions and updates:

$$\begin{aligned} \hat{\boldsymbol{x}}_t^- &= \boldsymbol{\Phi}_t \hat{\boldsymbol{x}}_t^+ \\ \hat{\boldsymbol{x}}_t^+ &= \hat{\boldsymbol{x}}_t^- + \boldsymbol{K}_t \left(\boldsymbol{z}_t - \boldsymbol{H}_t \hat{\boldsymbol{x}}_t^-\right). \end{aligned} \tag{4}$$

Thereby, $\boldsymbol{K}_t = \boldsymbol{P}_{\hat{\boldsymbol{x}}_t^-} \boldsymbol{H}_t^T \left(\boldsymbol{H}_t \boldsymbol{P}_{\hat{\boldsymbol{x}}_t^-} \boldsymbol{H}_t^T + \boldsymbol{\Sigma}_v\right)^{-1}$ represents the Kalman gain. The a posteriori position and velocity estimates are then be transformed into osculating Keplerian parameters, i.e.

$$\{\boldsymbol{r}_t, \dot{\boldsymbol{r}}_t\} \Rightarrow I(t) \in \{a(t), e(t), i(t), \Omega(t), \omega(t), \nu(t)\}. \tag{5}$$

In a third step, the drifts of the Keplerian parameters are estimated as

$$\hat{\dot{I}} = \frac{1}{T}(I(t+T) - I(t)) \quad \text{with period} \quad T. \tag{6}$$

These drifts are treated as constant from now on. The time-independent Keplerian parameters are finally estimated such that the squared deviation between the a posteriori position estimates of (4) and the calculated position based on the unknown Keplerian parameters is minimized, i.e.

$$\min_{a,e,i,\Omega,\omega} \left(C = \sum_{t=1}^{T} \| \hat{\boldsymbol{r}}_t^+ - \boldsymbol{r}(a,e,i,\Omega,\omega,t) \| \right) \tag{7}$$

This minimization is performed iteratively with the Newton algorithm, where the Keplerian parameter estimates in step $n \in \{1, \ldots, N-1\}$ are given by

$$\begin{bmatrix} \hat{a} \\ \hat{e} \\ \hat{i} \\ \hat{\Omega} \\ \hat{\omega} \end{bmatrix}^{(n+1)} = \begin{bmatrix} \hat{a} \\ \hat{e} \\ \hat{i} \\ \hat{\Omega} \\ \hat{\omega} \end{bmatrix}^{(n)} - \left(\boldsymbol{H}^{(n)} \right)^{-1} \begin{bmatrix} \partial C/\partial a \\ \partial C/\partial e \\ \partial C/\partial i \\ \partial C/\partial \Omega \\ \partial C/\partial \omega \end{bmatrix}^{(n)} \tag{8}$$

with the Hessian matrix

$$\boldsymbol{H} = \begin{bmatrix} \frac{\partial^2 C}{\partial a^2} & \frac{\partial^2 C}{\partial a \partial e} & \cdots & \frac{\partial^2 C}{\partial a \partial \omega} \\ \frac{\partial^2 C}{\partial a \partial e} & \frac{\partial^2 C}{\partial e^2} & & \vdots \\ \vdots & & \ddots & \\ \frac{\partial^2 C}{\partial a \partial \omega} & & \cdots & \frac{\partial^2 C}{\partial \omega^2} \end{bmatrix}. \tag{9}$$

Note that the true anomaly is not included in the state vector due to its non-linear behaviour over time. However, it is still required to estimate the other 5 Keplerian parameters.

There are two further aspects that shall be addressed: First, the initialization of the Newton method was performed with the so-called osculating elements of (5). Secondly, the different orders of magnitude of the Keplerian parameters lead to an ill-conditioned Hessian matrix. This motivates the use of a block inversion of (9):

$$\boldsymbol{H}^{-1} = \left[\begin{array}{c|c} \cdot & \cdot \\ \hline \cdot & \left(\boldsymbol{H}_{22} - \boldsymbol{H}_{21} \boldsymbol{H}_{11}^{-1} \boldsymbol{H}_{12} \right)^{-1} \end{array} \right] \tag{10}$$

for the following partitioning:

$$\boldsymbol{H} = \left[\begin{array}{c|c} \boldsymbol{H}_{11} & \boldsymbol{H}_{12} \\ \hline \boldsymbol{H}_{21} & \boldsymbol{H}_{22} \end{array} \right] \quad \text{with} \quad \boldsymbol{H}_{11} = \frac{\partial^2 C}{\partial a^2}. \tag{11}$$

As we are still just possessing the primary osculating true anomaly we also want to improve its estimation. At first the converged Keplerian estimates of (8) are used to estimate the time dependent eccentric anomaly:

$$\hat{E}^{(N)}(t) = \operatorname{acos}\left(\frac{1}{\hat{e}^N} \cdot \left(1 - \frac{\|\boldsymbol{r}_t^+\|}{\hat{a}^N} \right) \right), \tag{12}$$

which allows us then to determine the true anomaly $\nu^N(t)$:

$$\hat{\nu}^{(N)}(t) = \operatorname{atan}\left(\frac{\sqrt{1 - (\hat{e}^{(N)})^2} \cdot \sin(\hat{E}^{(N)})}{\cos(\hat{E}^{(N)}) - \hat{e}^{(N)}} \right). \tag{13}$$

This $\hat{\nu}^{(N)}(t)$ now has an improved accuracy and, therefore, motivates a reapplication of the Newton algorithm as given by (8). As a consequence, the Newton algorithm and the estimation of the true anomaly are performed in alternating sequence until the change in all Keplerian parameters becomes negligible. In this work, the number of sequences is denoted by M as shown in Fig. 2.

B. Orbit Models for GEO and LEO satellites

In these simulations the LEO orbits have been modelled as Keplerian orbits. With a standard propagator the time dependent positions of 12 randomly chosen LEO satellites have been calculated using their Keplerian parameters derived from a database of Analytical Graphics, Inc. (see also Table I). Also the GEO orbit has been modelled as a Keplerian orbit with additional linear perturbations in inclination and RAAN. The Keplerian parameters of the geosynchronous *ARTEMIS* satellite have been used for a realistic scenario.

a [km]	$e \cdot 10^2$	i [deg]	Ω [deg]	ω [deg]	M_0 [deg]
7076.18	0.18662	98.3026	48.5885	76.8817	58.8934
7077.05	0.04436	97.9070	280.9800	287.3030	69.0252
6935.16	0.12237	35.4695	49.9273	80.0589	34.8614
6978.65	0.18093	98.1511	136.7620	335.0610	133.2550
7077.06	0.08194	97.9199	281.5070	278.7750	73.2186
7042.13	0.10100	97.9906	101.9120	76.2754	272.9050
7079.74	0.66173	72.0629	65.5132	55.0826	227.0020
7063.34	0.06704	98.0737	336.9370	82.8137	260.9120
6962.12	0.13497	2.0161	85.4328	234.5930	241.6420
6957.87	0.23378	98.0508	6.8177	126.0040	132.9780
7165.35	0.11368	98.6307	48.5850	136.3890	274.1530
7008.66	0.16296	97.9848	129.3020	150.6080	242.2400
42166.30	0.03877	8.9267	298.1700	61.0706	326.2190

TABLE I
INPUT KEPLERIAN PARAMETERS FOR 12 LEO SATELLITES AND 1 GEO SATELLITE (BOTTOM LINE) USED DURING OUR SIMULATIONS.

III. RESULTS

A communication link between the LEO and GEO satellites has been simulated for a carrier frequency of $f_0 = 26$ GHz, a BPSK modulation, a bandwidth of 36 MHz, and a carrier-to-noise power ratio of $C/N_0 = 77$ dB-Hz, which leads to a Cramer-Rao bound [8] of approximately 1 mm. The process noises of the GEO position and clock were assumed to be white Gaussian noise with standard deviations of 1 cm.

Fig. 3 shows that the proposed method allows to estimate the position of a GEO data relay with a Kalman filter with centimetre accuracy. This position error is several orders of magnitude smaller than the currently achievable accuracy [7] although the Cramer-Rao bound is significantly amplified due to the poor geometry. Moreover, the positioning error also includes process noise for perturbations of an ideal GEO orbit.

Fig. 4 shows the difference between the osculating and true Keplerian elements. The linear perturbations of the Keplerian elements (here: inclination and RAAN) transfer to harmonic perturbations in the estimated osculating elements. However, the Newton algorithm in combination with the true anomaly estimation leads to a convergent method of iterations (Fig. 5).

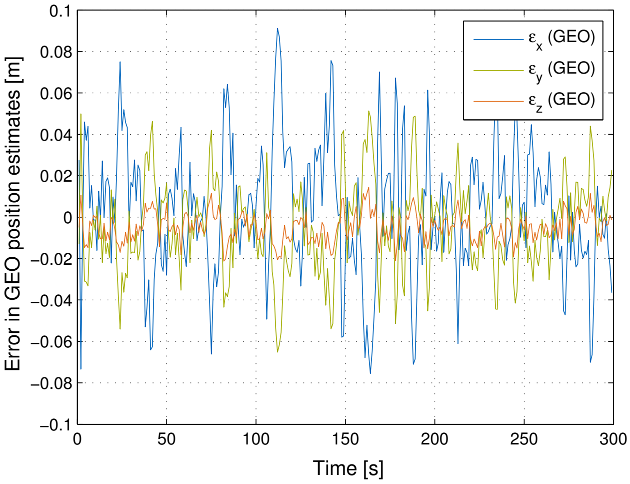

Fig. 3. Estimation of GEO position with a Kalman filter enables centimetre accuracy: The position error arises from the measurement error being heavily amplified by the poor geometry as well as from the process noise describing the perturbations of an ideal GEO orbit.

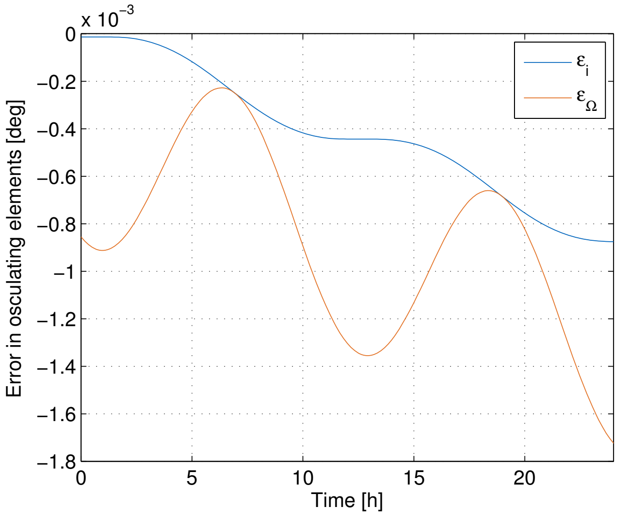

Fig. 4. Difference between osculating and true Keplerian elements: The linear perturbations of the Keplerian elements (here: inclination and RAAN) transfer to harmonic perturbations in the estimated osculating elements.

IV. Conclusion

The aim of our method presented in this paper was to give an accurate GEO position estimation and prediction possibility with the help of LEO satellites. In order to additionally allow position propagation, the Keplerian orbit parameters and their linear perturbations were determined. The new concept consists of two steps to estimate the Keplerian parameters: the first step includes a Newton algorithm to estimate all Keplerian parameters except for the true anomaly, which is determined in the second step. Both steps were applied in an alternating manner until convergence was achieved. The different orders of magnitude of the semi-major axis and the

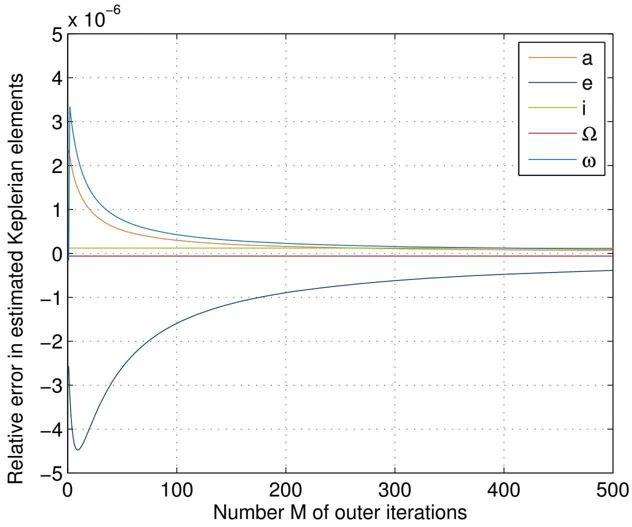

Fig. 5. Convergent behaviour of Keplerian parameter estimation: The alternating application of the Newton algorithm and true anomaly estimation leads to an absolute convergence of the final Keplerian parameter estimation.

other Keplerian elements lead to an ill-conditioned Hessian matrix. This numerical problem was efficiently solved by applying a block inversion of the Hessian matrix.

This concept should enable the positioning of a geostationary data relay with a sufficiently high accuracy in order to allow a Doppler pre-compensation on-board the LEO satellites. Future steps will consist of including further orbital perturbations and of testing the algorithm with real data.

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