

TUM

INSTITUT FÜR INFORMATIK

A Denotational Model for Mobile Point-to-Point Dataflow Networks

Radu Grosu, Ketil Stølen



TUM-I9527
Oktober 1995

TECHNISCHE UNIVERSITÄT MÜNCHEN

TUM-INFO-10-1995-I9527-350/1.-FI
Alle Rechte vorbehalten
Nachdruck auch auszugsweise verboten

©1995 MATHEMATISCHES INSTITUT UND
INSTITUT FÜR INFORMATIK
TECHNISCHE UNIVERSITÄT MÜNCHEN

Typescript: ---

Druck: Mathematisches Institut und
 Institut für Informatik der
 Technischen Universität München



TECHNISCHE
UNIVERSITÄT
MÜNCHEN

INSTITUT FÜR INFORMATIK

**Sonderforschungsbereich 342:
Methoden und Werkzeuge für die Nutzung
paralleler Rechnerarchitekturen**

A Denotational Model for Mobile Point-to-Point Dataflow Networks

Radu Grosu, Ketil Stølen

**TUM-19527
SFB-Bericht Nr.342/14/95 A
Oktober 1995**

TUM-INFO-10-95-127-0/1.-FI

Alle Rechte vorbehalten

Nachdruck auch auszugsweise verboten

©1995 SFB 342 Methoden und Werkzeuge für
die Nutzung paralleler Architekturen

Anforderungen an: Prof. Dr. A. Bode
Sprecher SFB 342
Institut für Informatik
Technische Universität München
Arcisstr. 21 / Postfach 20 24 20
D-80290 München, Germany

Druck: Fakultät für Informatik der
Technischen Universität München

A Denotational Model for Mobile Point-to-Point Dataflow Networks*

Radu Grosu, Ketil Stølen

Institut für Informatik, TU München, D-80290 München
email:grosu,stoelen@informatik.tu-muenchen.de

October 10, 1995

*The first author has been financially supported by the DFG project SPECTRUM. The second author has been financially supported by the Sonderforschungsbereich 342 “Werkzeuge und Methoden für die Nutzung paralleler Rechnerarchitekturen”.

Abstract

We present a denotational model for mobile, timed, unbounded nondeterministic dataflow networks whose components communicate in a point-to-point fashion. We first introduce a model for static, point-to-point dataflow networks. In this model components and networks of components are represented by sets of strongly pulse-driven stream processing functions. A stream processing function is strongly pulse-driven if its output until time $j + 1$ is completely determined by its input until time j . This model is then extended to support mobility by allowing the components to communicate ports. In the mobile case, the functions are not only required to be strongly pulse-driven, but also to be generic in the sense that they do not read or write on channels whose ports they have not received or generated themselves. The model is shown to be fully abstract. We demonstrate the power and applicability of our model by specifying a mobile communication central.

Contents

1	Introduction	1
2	Static Dataflow Networks	3
2.1	Communication Histories	3
2.2	Operations on Named Communication Histories	4
2.3	Static Components	4
2.4	Point-to-Point Composition	6
3	Mobile Dataflow Networks	8
3.1	Communication Histories	8
3.2	Operations on Named Communication Histories	10
3.3	Mobile Components	11
3.4	Point-to-Point Composition	14
4	Communication Central	19
5	Full Abstractness	21
6	Discussion	23
7	Acknowledgments	25
A	Metric Space Basics	28
B	Streams and Named Stream Tuples	31

Chapter 1

Introduction

Mobile systems are systems in which every component may change its communication partners on the basis of computation and interaction. In the last few years the formal modeling of mobile systems has received considerable interest. Examples of models expressing mobility are the Actor Model [HBS73, AMST92], the π -Calculus [EN86, MPW92a, MPW92b], the Chemical Abstract Machine [BB90], the Rewriting Logic [Mes91], the Higher Order CCS [Tho89] and the Concurrent Constraint Programming Languages [SRP91].

Most approaches published so far are operational. For example, with the exception of [SRP91], all the approaches mentioned above are of a very operational nature. In this paper we give a denotational model for timed, unbounded nondeterministic, mobile dataflow networks whose components communicate in a point-to-point fashion. Our model can be seen as an extension of the model for static deterministic dataflow networks proposed by Kahn [Kah74], and the models for static nondeterministic dataflow networks proposed by Park [Par83], Kok [Kok87] and Broy [Bro87].

In our model, a complete communication history for a channel is represented by an infinite stream of finite streams of messages. A complete communication history for a set of channels is represented by a named communication history. In the style of [BD92], a named communication history is a function mapping channel names to complete communication histories. We represent components by sets of stream processing functions mapping named communication histories for the input channels to named communication histories for the output channels.

These functions are required to satisfy two properties. First of all, they have to be strongly pulse-driven in the sense that the input until time j completely determines the output until time $j + 1$. Due to this constraint, any network of deterministic components has a unique fixed point with respect to a complete communication history for its external input channels. Secondly, since mobility is achieved by allowing the functions to communicate ports, we have to make sure that the functions do not write or read on channels whose ports they do not know or have not created themselves. More explicitly, the functions are only allowed to read and write on channels dynamically generated on the basis of their initial wiring. A function which is well-behaved in this sense is said to be generic.

Our model is compositional with respect to the operator for network composition. We also prove that our model is fully abstract with respect to a notion of observation.

As we see it, the contribution of this paper is two-fold. Firstly, we provide a denotational understanding of mobility. This in contrast to the very operational nature of most other models. Secondly, point-to-point communication is imposed semantically. Thus, contrary to for example [SRP91], we do not need additional syntactic constraints to avoid inconsistencies.

Although we could have formulated our semantics in a cpo context, we decided to base it on the topological tradition of metric spaces [Niv82, dBZ82, AdBKR89]. Firstly, we wanted to understand the exact relationship between our approach and those based on metric spaces. Secondly, the use of metric spaces seems more natural since our approach is based on infinite streams, and since our pulse-drivenness constraints — strong and weak pulse-drivenness — correspond straightforwardly to contractiveness and non-expansiveness.

The rest of the paper is split into six chapters. Chapter 2 introduces our basic formalism and briefly shows how it can be used to model static point-to-point dataflow networks. Then, in Chapter 3 we extend our static model to deal with mobility. An example is given in Chapter 4, and in Chapter 5 we define a notion of observation and prove full abstractness with respect to this notion. Chapter 6 relates our approach to other approaches known from the literature. Finally, there is an appendix reviewing some basic stuff on metric spaces and streams.

Chapter 2

Static Dataflow Networks

The objective of this chapter is twofold. First of all, we want to introduce our underlying formalism. Secondly, to strengthen the reader's intuition, we want to use this formalism to model static point-to-point dataflow networks. For the basic definitions of metric spaces, streams and named stream tuples we refer to the appendix.

2.1 Communication Histories

A dataflow network is a network of components which exchange messages asynchronously via directed channels. We model the communication histories of directed channels by infinite streams of finite streams of messages. Each finite stream represents the communication history within a least unit of time. The first finite stream contains the messages received within the first time unit, the second the messages received within the second time unit, and so on (see Figure 2.1). Since time never halts, any such stream of finite streams representing a complete communication history must contain infinitely many finite sequences.

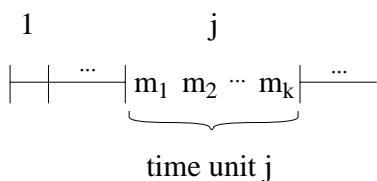


Figure 2.1: Communication History

Let D be the set of all messages. This means that $[D^*]$ is the set of all complete communication histories, and $(D^*)^*$ is the set of all partial communication histories. In this paper all channels are untyped, i.e. they can be used to communicate any action in D . It is of course straightforward to generalize our model to deal with typed channels.

A named communication history $\theta \in N \rightarrow [D^*]$ is a named stream tuple mapping channel names to complete communication histories. Intuitively, θ assigns a complete communication history to each channel named by the names in N . Thus if N names the set of input

channels of some component, then θ gives a complete communication history for its input channels.

2.2 Operations on Named Communication Histories

In this section we define two non-expansive operators on named communication histories, namely *summation* and *projection*. The summation operator constructs the union of two named communication histories with disjoint domains. The projection operator projects a named communication history on a given set of channel names.

Definition 1 (Summation) For any I, J such that $I \cap J = \emptyset$, we define

$$+ \in (I \rightarrow [D^*]) \times (J \rightarrow [D^*]) \rightarrow ((I \cup J) \rightarrow [D^*])$$

as follows:

$$(\theta + \varphi)(i) = \begin{cases} \theta(i) & \text{if } i \in I, \\ \varphi(i) & \text{if } i \in J. \end{cases}$$

□

Theorem 1 The summation operator is non-expansive.

Proof: Given $\theta_1, \theta_2 \in I \rightarrow [D^*]$ and $\varphi_1, \varphi_2 \in J \rightarrow [D^*]$ with $I \cap J = \emptyset$. Then:

$$d(\theta_1 + \varphi_1, \theta_2 + \varphi_2) = \inf \{2^{-j} \mid \theta_1 \downarrow_j = \theta_2 \downarrow_j \wedge \varphi_1 \downarrow_j = \varphi_2 \downarrow_j\} \leq \max\{d(\theta_1, \theta_2), d(\varphi_1, \varphi_2)\}.$$

□

Definition 2 (Projection) We define

$$| \in (I \rightarrow [D^*]) \times O \rightarrow ((I \cap O) \rightarrow [D^*])$$

as follows:

$$(\theta|_O)(i) = \theta(i) \quad \text{if } i \in (I \cap O).$$

□

Theorem 2 The projection operator is non-expansive in its first argument.

Proof: Given $\theta, \varphi \in I \rightarrow [D^*]$ and some set of channel names J . Then:

$$d(\theta|_J, \varphi|_J) = \inf \{2^{-j} \mid (\theta|_J) \downarrow_j = (\varphi|_J) \downarrow_j\} \leq \inf \{2^{-j} \mid \theta \downarrow_j = \varphi \downarrow_j\}.$$

□

2.3 Static Components

We model components by sets of *stream processing functions*. A deterministic component, whose input and output channels are named by respectively I and O , is modeled by a stream processing function

$$f \in (I \rightarrow [D^*]) \rightarrow (O \rightarrow [D^*])$$

mapping named communication histories for its input channels to named communication histories for its output channels. Note that if no message is communicated along an input

channel within a time unit then the empty stream occurs in the communication history for that channel informing the function that time has progressed. The lack of this information causes the famous fair merge anomaly [Kel78].

A stream processing function is said to be *weakly pulse-driven* if its input until time j completely determines its output until time j . It is said to be *strongly pulse-driven* if its input until time j completely determines its output until time $j + 1$. Formally:

Definition 3 (Pulse-driven functions) *A function $f \in (I \rightarrow [D^*]) \rightarrow (O \rightarrow [D^*])$ is weakly pulse-driven if*

$$\forall \theta, \varphi, j : \theta \downarrow_j = \varphi \downarrow_j \Rightarrow f(\theta) \downarrow_j = f(\varphi) \downarrow_j,$$

and strongly pulse-driven if

$$\forall \theta, \varphi, j : \theta \downarrow_j = \varphi \downarrow_j \Rightarrow f(\theta) \downarrow_{j+1} = f(\varphi) \downarrow_{j+1}.$$

□

We use the arrow \rightarrow to distinguish strongly pulse-driven functions from functions that are not strongly pulse-driven.

A weakly pulse-driven function is non-expansive and a strongly pulse-driven function is contractive with respect to the metric on streams. As a consequence, strong pulse-drivenness not only replaces the usual monotonicity and continuity constraints of domain theory but also guarantees unique fixed points of feedback loops.

Theorem 3 *A stream processing function is strongly pulse-driven iff it is contractive with respect to the metric of streams. A stream processing function is weakly pulse-driven iff it is non-expansive with respect to the metric of streams.*

Proof:

(\Rightarrow) : Suppose that $d(x, y) = 2^{-k}$ and that f is strongly pulse-driven. Then by definition:

$$d(f(x), f(y)) = \inf \{ 2^{-m} \mid f(x) \downarrow_m = f(y) \downarrow_m \} \leq 2^{-(k+1)} = (1/2) \cdot d(x, y).$$

(\Leftarrow) : Suppose that $d(f(x), f(y)) = 2^{-l}$ and that f is contractive i.e.

$$\exists c < 1 : \forall x, y : d(f(x), f(y)) \leq c \cdot d(x, y).$$

Then $2^{k-l} \leq c < 1 = 2^0$. This implies that $k-l \leq -1$ i.e. that $l \geq k+1$. As a consequence

$$x \downarrow_k = y \downarrow_k \Rightarrow f(x) \downarrow_{k+1} = f(y) \downarrow_{k+1}.$$

In other words, f is strongly pulse driven. The second equivalence is proven accordingly. □

Definition 4 (Static components) *A static component whose input and output channels are named by I and O , respectively, is modeled by a nonempty set of stream processing functions*

$$F \subseteq (I \rightarrow [D^*]) \rightarrow (O \rightarrow [D^*]),$$

that is closed in the sense that for any strongly pulse-driven function f of the same sig-

nature

$$(\forall \theta \in (I \rightarrow [D^*]) : \exists f' \in F : f(\theta) = f'(\theta)) \Rightarrow f \in F.$$

□

The above definition is very powerful. It not only makes our model *fully abstract* (see Chapter 5), but it also allows us to handle *unbounded nondeterminism*.

2.4 Point-to-Point Composition

We now introduce an operator \otimes which allows components to be composed into networks of components — networks, which when observed from the outside, themselves behave as components.

Definition 5 (Point-to-point composition) *Given two static components:*

$$F_1 \subseteq (I_1 \rightarrow [D^*]) \rightarrow (O_1 \rightarrow [D^*]), \quad F_2 \subseteq (I_2 \rightarrow [D^*]) \rightarrow (O_2 \rightarrow [D^*]),$$

where $I_1 \cap O_1 = I_2 \cap O_2 = I_1 \cap I_2 = O_1 \cap O_2 = \emptyset$. Let

$$I = (I_1 \setminus O_2) \cup (I_2 \setminus O_1), \quad O = (O_1 \setminus I_2) \cup (O_2 \setminus I_1),$$

we define

$$F_1 \otimes F_2 \subseteq (I \rightarrow [D^*]) \rightarrow (O \rightarrow [D^*]),$$

$$F_1 \otimes F_2 = \{f \in (I \rightarrow [D^*]) \rightarrow (O \rightarrow [D^*]) \mid \forall \theta : \exists f_1 \in F_1, f_2 \in F_2 :$$

$$f(\theta) = (\varphi + \psi)|_O \text{ where } \varphi = f_1((\theta + \psi)|_{I_1}), \psi = f_2((\theta + \varphi)|_{I_2})\} \quad \square$$

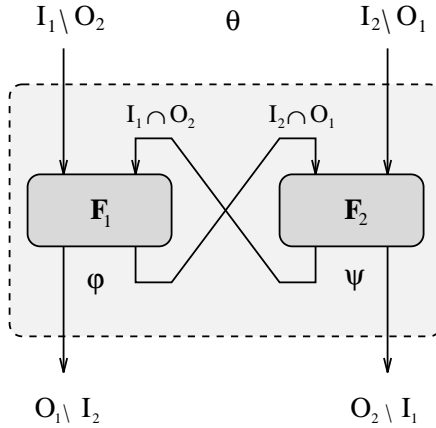


Figure 2.2: Point-to-Point communication

Note the close correlation between this definition and Figure 2.2. Any input channel of F_1 which is also an output channel of F_2 , and any input channel of F_2 which is also an input

channel of F_1 are connected and hidden. Any channel that cannot be connected in this way remains external. We refer to the resulting network as the *point-to-point* composition of F_1 and F_2 .

Theorem 4 $F_1 \otimes F_2 \neq \emptyset$.

Proof: Since F_1 and F_2 are static components we may find functions f_1, f_2 such that $f_1 \in F_1$ and $f_2 \in F_2$. The function:

$$g((\varphi, \psi), \theta) = (f_1((\theta + \psi)|_{I_1}), f_2((\theta + \varphi)|_{I_2}))$$

where $\varphi \in (O_1 \rightarrow [D^*])$ and $\psi \in (O_2 \rightarrow [D^*])$ is strongly pulse-driven as the composition of strongly pulse-driven and weakly pulse-driven functions¹. By Theorem 18 (in the appendix) so is μg . Moreover, by non-expansiveness of Cartesian projection, sum and stream tuple projection, f defined by:

$$f(\theta) = (\varphi + \psi)|_O \quad \text{where} \quad (\varphi, \psi) = (\mu g)(\theta)$$

is also strongly pulse-driven. Since for any predicate P :

$$\exists f_1 \in F_1, f_2 \in F_2 : \forall \theta : P \Rightarrow \forall \theta : \exists f_1 \in F_1, f_2 \in F_2 : P,$$

we obtain $F_1 \otimes F_2 \neq \emptyset$. □

Theorem 5 $F_1 \otimes F_2$ is a static component.

Proof: By Theorem 4 it is enough to prove that $F_1 \otimes F_2$ is closed. Suppose $f \in (I \rightarrow [D^*]) \rightarrow (O \rightarrow [D^*])$ and

$$\forall \theta : \exists f' \in F_1 \otimes F_2 : f(\theta) = f'(\theta).$$

Then for a given θ there is an $f' \in F_1 \otimes F_2$ an $f_1 \in F_1$ and an $f_2 \in F_2$ such that:

$$f'(\theta) = (\varphi + \psi)|_O \quad \text{where} \quad \varphi = f_1((\theta + \psi)|_{I_1}), \quad \psi = f_2((\theta + \varphi)|_{I_2}).$$

Hence, for every θ there are functions $f_1 \in F_1$ and $f_2 \in F_2$ such that:

$$f(\theta) = (\varphi + \psi)|_O \quad \text{where} \quad \varphi = f_1((\theta + \psi)|_{I_1}), \quad \psi = f_2((\theta + \varphi)|_{I_2}).$$

In other words $f \in F_1 \otimes F_2$. □

¹Tupling and projection are non-expansive in the finitary product metric space. See for example [Eng77].

Chapter 3

Mobile Dataflow Networks

The denotational model presented so far is a model for *static* dataflow networks in the tradition of [Par83], [Kok87], [Bro87]. The main drawback of this model is that it does not allow the components to change their communication partners on the basis of computation and interaction. In this chapter we extend our model to deal with mobility.

3.1 Communication Histories

As before, we use named communication histories to represent the communication histories of sets of channels. However, these named communication histories are extended in two ways to support mobility.

Firstly, the set of messages is extended with *ports*. A port is a *channel name* together with an *access right*, which is either a read right (represented by $?$) or a write right (represented by $!$). Hence, if M is a set of channel names, then $?M = \{?m \mid m \in M\}$ is the corresponding set of read ports, and $!M = \{!m \mid m \in M\}$ is the corresponding set of write ports. When no ambiguity occurs, we use $[M^*]$ as a short-hand for $[(D \cup ?M \cup !M)^*]$. This is justified by the convention that D is fixed.

Secondly, to allow a stream processing function to access a channel, whose port it has received or sent, we extend the domain of each named communication history to N , the *set of all channel names*. Although this means that any named communication history has the set of all channel names as its domain, stream processing functions are nevertheless only allowed to access the ports they have received or created themselves. As explained in Chapter 3.3, this is ensured by requiring stream processing functions to be generic.

In the presence of ports, the behavior of a stream processing function, with input history θ and output history φ , can be described, with respect to Figure 3.1, as follows.

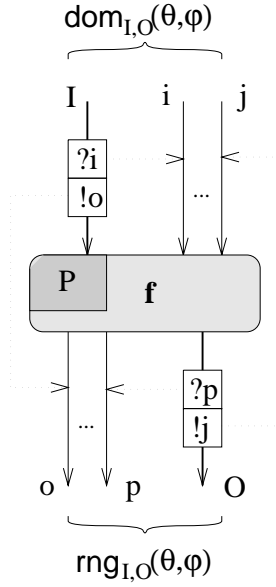


Figure 3.1: Mobile Stream Processing Function

Initially, each mobile function reads from a designated set of input channels I and writes on a designated set of output channels O . These two sets name the *static* channels or the initial wiring. To make sure that the channels *created* by the different components in a network are given different names, each mobile function is assigned a set of *private names* P . Obviously, this set should be disjoint from the static interface. Thus we require that $(I \cup O) \cap P = \emptyset$.

At any point of time n the sets of accessible input and output channels are characterized by respectively $\text{dom}_{I,O}(\theta, \varphi)(n)$ and $\text{rng}_{I,O}(\theta, \varphi)(n)$. These sets are formally defined in Chapter 3.3.

During the computation, the sizes of these sets gradually grow. For example, if the function receives a read port $?i$ then it may read from the channel i , and if it receives a write port $!o$ then it may write on the channel o . Similarly, whenever the function sends an output port $!j$, whose channel $j \in P$ it has created itself, it may later on read what is sent along j , or whenever it sends an input port $?p$, whose channel $p \in P$ it has created itself, it may itself send messages along p which eventually are read by the component which receives the input port.

Based on the discussion above, it should be clear that in the case of mobility, we need named communication histories of the following signature:

$$\theta \in N \rightarrow [M^*],$$

where $M \subseteq N$.

3.2 Operations on Named Communication Histories

The new definition of named communication histories requires that the summation and projection operators are redefined. In the static case, the sum $\varphi + \psi$ was defined for φ and ψ with disjoint domains. In the mobile case we need a weaker condition, because all named communication histories have the same domain N . Let $\text{act}(\varphi) = \{i \mid \varphi(i) \neq \epsilon^\infty\}$ be the set of active channels in φ , i.e. the set of channel names whose corresponding communication histories contain at least one message. The sum $\varphi + \psi$ is then defined if $\text{act}(\varphi)$ is disjoint from $\text{act}(\psi)$.

Definition 6 (Partial Sum) *Given two named communication histories $\varphi \in (N \rightarrow [M^*])$ and $\psi \in (N \rightarrow [P^*])$ such that $\text{act}(\varphi) \cap \text{act}(\psi) = \emptyset$. We define their partial sum $\varphi + \psi$ to denote the element of $N \rightarrow [(M \cup P)^*]$ such that for all $i \in N$:*

$$(\varphi + \psi)(i) = \begin{cases} \varphi(i) & \text{if } i \in \text{act}(\varphi), \\ \psi(i) & \text{if } i \notin \text{act}(\varphi). \end{cases} \quad \square$$

Although analogous to the summation operator defined in Section 2.2, partial summation has no syntactic conditions assuring its well-definedness. We therefore define a total version $\varphi \leftrightarrow \psi$. This simplifies the use of the Banach's fixed point theorem. Totalisation is achieved by defining $(\varphi \leftrightarrow \psi)(i)$ to consist of only ϵ 's from the first moment n in which both $\varphi \downarrow_n$ and $\psi \downarrow_n$ are active on i , i.e. different from ϵ^n .

Definition 7 (Total sum) *Given two named communication histories $\varphi \in (N \rightarrow [M^*])$ and $\psi \in (N \rightarrow [P^*])$. We define their total sum $\varphi \leftrightarrow \psi$ to denote the element of $N \rightarrow [(M \cup P)^*]$ such that for all $i \in N$:*

$$(\varphi \leftrightarrow \psi)(i)(n) = \begin{cases} \psi(i)(n) & \text{if } \varphi(i) \downarrow_n = \epsilon^n, \\ \varphi(i)(n) & \text{if } \varphi(i) \downarrow_n \neq \epsilon^n \wedge \psi(i) \downarrow_n = \epsilon^n, \\ \epsilon & \text{if } \varphi(i) \downarrow_n \neq \epsilon^n \wedge \psi(i) \downarrow_n \neq \epsilon^n. \end{cases} \quad \square$$

Note that $\varphi \leftrightarrow \psi$ has a hiding effect if $\text{act}(\varphi) \cap \text{act}(\psi) \neq \emptyset$, and that $\varphi \leftrightarrow \psi$ is equal to $\varphi + \psi$, otherwise.

Theorem 6 *The total summation operator is weakly pulse-driven.*

Proof: Suppose that $\varphi_1, \varphi_2 \in N \rightarrow [M^*]$ and that $\psi_1, \psi_2 \in N \rightarrow [P^*]$ such that $\max\{d(\varphi_1, \varphi_2), d(\psi_1, \psi_2)\} = 2^{-n}$, i.e. such that $\varphi_1 \downarrow_n = \varphi_2 \downarrow_n$ and $\psi_1 \downarrow_n = \psi_2 \downarrow_n$. Since the definition of $(\varphi + \psi) \downarrow_n$ depends only on $\varphi \downarrow_n$ and $\psi \downarrow_n$, it follows that $d(\varphi_1 + \psi_1, \varphi_2 + \psi_2) \leq 2^{-n}$. \square

In the static case, the sets of input and output channels of a component were constant. This was also reflected by the projection operator, which did a constant projection over time. In the mobile case, however, the sets of known input and output channels $\text{dom}_{I,O}(\theta, \varphi)(n)$ and $\text{rng}_{I,O}(\theta, \varphi)(n)$, respectively, may differ with n . Since the unknown channels should be hidden, we employ a projection operator which for each n selects only the known channels. This is easily achieved by replacing the second argument by a stream of sets of channel identifiers. The static case is recovered by taking a constant set of channels for each n .

Definition 8 (Projection) For any named communication history $\theta \in (N \rightarrow [M^*])$, its projection $\theta|_O$ on $O \in [\mathcal{P}(N)]$ denotes the element of $N \rightarrow [M^*]$ such that for all i, k :

$$\theta|_O(i)(k) = \begin{cases} \theta(i)(k) & \text{if } i \in O(k), \\ \epsilon & \text{otherwise.} \end{cases}$$

□

Theorem 7 The projection operator is weakly pulse-driven.

Proof: Suppose that $\theta_1, \theta_2 \in (N \rightarrow [M^*])$, that $O_1, O_2 \in [\mathcal{P}(N)]$ and that $\max\{d(\theta_1, \theta_2), d(O_1, O_2)\} = 2^{-n}$, i.e. that $\theta_1 \downarrow_n = \theta_2 \downarrow_n$ and $O_1 \downarrow_n = O_2 \downarrow_n$. Since $(\theta|_O) \downarrow_n$ depends only on $\theta \downarrow_n$ and $O \downarrow_n$ it follows that $d(\theta_1|_{O_1}, \theta_2|_{O_2}) \leq 2^{-n}$. □

3.3 Mobile Components

Also in the case of mobile dataflow networks we model components by sets of stream processing functions. However, before we can give a formal characterization of these sets, we have to be more explicit with respect to what sort of networks we actually want to model.

Firstly, as in the static case, we want to model point-to-point communication. As a consequence, any network of components maintains the following invariant: each channel can be accessed by at most two components — the reader and the writer.

Secondly, the components can create channels and forward their ports. Each component has a set of private channel names P . Any channel created by a component is assigned a new name p from this set P . Any other component can only gain access to p by receiving either $?p$ or $!p$ along one of its input channels.

Thirdly, to ensure that the same channel is not accessed by more than two components, a port is sent by its creator along *only one channel*. Otherwise, it may happen that the same port is received and used by two different components. To simplify things, we assume the same port is never sent more than *once*. In addition, the creator is not allowed to send both ports of a channel. Thus for instance if it sends the write port of a channel then it is not allowed to send the read port for the same channel.

In a network where all components have disjoint sets of private names, and where all components behave in accordance with the communication constraints imposed above, we may restrict ourselves to named communication histories in which the same port occurs not more than once and where two different ports are assigned different channel names. We say that a named communication history is *port-unique* if it has these two properties. Moreover, we use the arrow \xrightarrow{u} to distinguish named communication histories that are port-unique from other named communication histories.

Port uniqueness is preserved by projection, and also by summation as long as the named communication histories have disjoint sets of ports.

Theorem 8 If $\theta \in N \xrightarrow{u} [P^*]$ we also have that $\theta|_O \in N \xrightarrow{u} [P^*]$. Moreover, if $\varphi \in N \xrightarrow{u} [P_1^*]$, $\psi \in N \xrightarrow{u} [P_2^*]$ and $P_1 \cap P_2 = \emptyset$, we also have that $\varphi \uplus \psi \in N \xrightarrow{u} [(P_1 \cup P_2)^*]$.

Proof: Trivial. □

Note that a received port can be forwarded without violating privacy as long as it is not read or written by the component forwarding it. Nevertheless, to keep it simple, we do not allow received ports to be forwarded. This constraint does not reduce the number of networks that can be expressed, because whenever the forwarding component receives a port $?i$, it can always forward a copy $?p$ by generating a new channel p and then transmitting anything received on i along p . We also restrict components from communicating the ports used for their initial wiring.

It follows from these constraints that any port sent by a component has to belong to a channel created by the component.

For simplicity, we split the set of names N into two disjoint sets – a set of static channel names S , which can only be used for the initial wiring, and a set of dynamic channel names A , which can only be assigned channels created during the computation.

Based on the discussion above, when modeling a component with private names P , we only have to consider stream processing functions of the following signature:

$$(N \xrightarrow{u} [\overline{P}^*]) \rightarrow (N \xrightarrow{u} [P^*]),$$

where $P \subseteq A$ and $\overline{P} = A \setminus P$.

So far we have fixed the syntactic signature of our stream processing functions. The next step is to characterize their semantic properties. As in the static case, any stream processing function is required to be strongly pulse-driven. However, contrary to earlier, it is also required to have a second property — as already mentioned, it is required to be *generic* in the sense that it only accesses a channel via a port it has already received or created itself.

Before giving a formal definition of what it means for a stream processing function to be generic, we have to define the two sets $\text{dom}_{I,O}(\theta, \varphi)(n)$ and $\text{rng}_{I,O}(\theta, \varphi)(n)$, mentioned earlier, characterizing respectively the sets of accessible input and output channels at time n with respect to the input/output histories θ/φ and the initial wiring I/O . In the definition below the operator \in is overloaded to test for containment in a list.

Definition 9 (Domain and range) *Given I, O, θ and φ , we define:*

$$\begin{aligned} D_1 &= I, \\ R_1 &= O, \end{aligned}$$

$$\begin{aligned} D_{n+1} &= D_n \cup \bigcup_{i \in D_n} \{p \mid ?p \in \theta(i)(n)\} \cup \bigcup_{i \in R_n} \{p \mid !p \in \varphi(i)(n)\}, \\ R_{n+1} &= R_n \cup \bigcup_{i \in D_n} \{p \mid !p \in \theta(i)(n)\} \cup \bigcup_{i \in R_n} \{p \mid ?p \in \varphi(i)(n)\}. \end{aligned}$$

The definitions of $\text{dom}_{I,O}(\theta, \varphi)$ and $\text{rng}_{I,O}(\theta, \varphi)$ follow immediately:

$$\text{dom}_{I,O}(\theta, \varphi)(n) = D_n, \quad \text{rng}_{I,O}(\theta, \varphi)(n) = R_n. \quad \square$$

Theorem 9 *The functions $\text{dom}_{I,O}$ and $\text{rng}_{I,O}$ are strongly pulse-driven.*

Proof: Suppose that $\theta_1, \theta_2 \in N \rightarrow [M^*]$ and that $\varphi_1, \varphi_2 \in N \rightarrow [Q^*]$ such that $\max\{d(\theta_1, \theta_2), d(\varphi_1, \varphi_2)\} = 2^{-n}$, i.e. that $\theta_1 \downarrow_n = \theta_2 \downarrow_n$ and $\varphi_1 \downarrow_n = \varphi_2 \downarrow_n$. Since the definitions of $\text{dom}_{I,O}(\theta, \varphi) \downarrow_{n+1}$ and $\text{rng}_{I,O}(\theta, \varphi) \downarrow_{n+1}$ depend only on $\varphi \downarrow_n$ and $\psi \downarrow_n$, it follows that

$d(\text{dom}_{I,O}(\theta_1, \varphi_1), \text{dom}_{I,O}(\theta_2, \varphi_2)) \leq 2^{-(n+1)}$ and $d(\text{rng}_{I,O}(\theta_1, \varphi_1), \text{rng}_{I,O}(\theta_2, \varphi_2)) \leq 2^{-(n+1)}$. \square

Theorem 10 *The functions $\text{dom}_{I,O}$ and $\text{rng}_{I,O}$ have the following properties:*

$$\begin{aligned} \text{dom}_{I,O}(\theta, \varphi) &= \text{dom}_{I,O}(\theta|_{\text{dom}_{I,O}(\theta, \varphi)}, \varphi) = \text{dom}_{I,O}(\theta, \varphi|_{\text{rng}_{I,O}(\theta, \varphi)}), \\ \text{rng}_{I,O}(\theta, \varphi) &= \text{rng}_{I,O}(\theta|_{\text{dom}_{I,O}(\theta, \varphi)}, \varphi) = \text{rng}_{I,O}(\theta, \varphi|_{\text{rng}_{I,O}(\theta, \varphi)}). \end{aligned}$$

Proof: The proof is based on the inductive definition of $\text{dom}_{I,O}$ and $\text{rng}_{I,O}$.

Induction hypothesis:

$$\begin{aligned} \text{dom}_{I,O}(\theta, \varphi)(n) &= \text{dom}_{I,O}(\theta|_{\text{dom}_{I,O}(\theta, \varphi)}, \varphi)(n) = \text{dom}_{I,O}(\theta, \varphi|_{\text{rng}_{I,O}(\theta, \varphi)})(n), \\ \text{rng}_{I,O}(\theta, \varphi)(n) &= \text{rng}_{I,O}(\theta|_{\text{dom}_{I,O}(\theta, \varphi)}, \varphi)(n) = \text{rng}_{I,O}(\theta, \varphi|_{\text{rng}_{I,O}(\theta, \varphi)})(n). \end{aligned}$$

To simplify the notation we write:

$$\begin{aligned} D_n &= \text{dom}_{I,O}(\theta, \varphi)(n), & R_n &= \text{rng}_{I,O}(\theta, \varphi)(n), \\ D'_n &= \text{dom}_{I,O}(\theta|_{\text{dom}_{I,O}(\theta, \varphi)}, \varphi)(n), & R'_n &= \text{rng}_{I,O}(\theta|_{\text{dom}_{I,O}(\theta, \varphi)}, \varphi)(n), \\ D''_n &= \text{dom}_{I,O}(\theta, \varphi|_{\text{rng}_{I,O}(\theta, \varphi)})(n), & R''_n &= \text{rng}_{I,O}(\theta, \varphi|_{\text{rng}_{I,O}(\theta, \varphi)})(n). \end{aligned}$$

Base case: $D_1 = D'_1 = D''_1 = I$ and $R_1 = R'_1 = R''_1 = O$.

Induction Step: By induction hypothesis $D_n = D'_n = D''_n$ and $R_n = R'_n = R''_n$. By definition of $\text{dom}_{I,O}$ and $\text{rng}_{I,O}$:

$$\begin{aligned} D_{n+1} &= D_n \cup \bigcup_{i \in D_n} \{p \mid ?p \in \theta(i)(n)\} && \cup \bigcup_{i \in R_n} \{p \mid !p \in \varphi(i)(n)\}, \\ D'_{n+1} &= D'_n \cup \bigcup_{i \in D'_n} \{p \mid ?p \in \theta|_{\text{dom}_{I,O}(\theta, \varphi)}(i)(n)\} && \cup \bigcup_{i \in R'_n} \{p \mid !p \in \varphi(i)(n)\}, \\ D''_{n+1} &= D''_n \cup \bigcup_{i \in R''_n} \{p \mid !p \in \varphi|_{\text{rng}_{I,O}(\theta, \varphi)}(i)(n)\} && \cup \bigcup_{i \in D''_n} \{p \mid ?p \in \theta(i)(n)\}, \\ R_{n+1} &= R_n \cup \bigcup_{i \in D_n} \{p \mid !p \in \theta(i)(n)\} && \cup \bigcup_{i \in R_n} \{p \mid ?p \in \varphi(i)(n)\}, \\ R'_{n+1} &= R'_n \cup \bigcup_{i \in D'_n} \{p \mid !p \in \theta|_{\text{dom}_{I,O}(\theta, \varphi)}(i)(n)\} && \cup \bigcup_{i \in R'_n} \{p \mid ?p \in \varphi(i)(n)\}, \\ R''_{n+1} &= R''_n \cup \bigcup_{i \in R''_n} \{p \mid ?p \in \varphi|_{\text{rng}_{I,O}(\theta, \varphi)}(i)(n)\} && \cup \bigcup_{i \in D''_n} \{p \mid !p \in \theta(i)(n)\}. \end{aligned}$$

By definition of projection:

$$\begin{aligned} \theta|_{\text{dom}_{I,O}(\theta, \varphi)}(i)(n) &= \theta(i)(n) && \text{if } i \in \text{dom}_{I,O}(\theta, \varphi)(n) = D_n = D'_n = D''_n, \\ \varphi|_{\text{rng}_{I,O}(\theta, \varphi)}(i)(n) &= \varphi(i)(n) && \text{if } i \in \text{rng}_{I,O}(\theta, \varphi)(n) = R_n = R'_n = R''_n. \end{aligned}$$

So $D_{n+1} = D'_{n+1} = D''_{n+1}$ and $R_{n+1} = R'_{n+1} = R''_{n+1}$. \square

Now we are ready to formally state what it means for stream processing functions to be generic.

Definition 10 (Generic functions) *A function $f \in (N \xrightarrow{u} [\overline{P}^*]) \rightarrow (N \xrightarrow{u} [P^*])$ is said to be generic with respect to the initial wiring (I, O) iff:*

$$\forall \theta : f(\theta) = f(\theta|_{\text{dom}_{I,O}(\theta, f(\theta))}) = f(\theta)|_{\text{rng}_{I,O}(\theta, f(\theta))}. \quad \square$$

We use the arrow $\xrightarrow{I,O}$ to distinguish stream processing functions that are strongly pulse-driven and in addition generic with respect to the initial wiring (I, O) from other functions.

Definition 11 (Mobile components) *A mobile component, whose initial wiring is characterized by (I, O) and private names are characterized by P , is modeled by a nonempty*

set of stream processing functions

$$F \subseteq (N \rightarrow [\overline{P}^*]) \xrightarrow{I, O} (N \rightarrow [P^*])$$

that is closed in the sense that for any strongly pulse-driven function f of the same signature

$$(\forall \theta \in (N \rightarrow [\overline{P}^*]) : \exists f' \in F : f(\theta) = f'(\theta)) \Rightarrow f \in F.$$

□

3.4 Point-to-Point Composition

We now redefine the composition operator \otimes for mobile components.

Definition 12 (Point-to-point composition) *Given two mobile components:*

$$F_1 \subseteq (N \xrightarrow{u} [\overline{P}_1^*]) \xrightarrow{I_1, O_1} (N \xrightarrow{u} [P_1^*]), \quad F_2 \subseteq (N \xrightarrow{u} [\overline{P}_2^*]) \xrightarrow{I_2, O_2} (N \xrightarrow{u} [P_2^*]).$$

where $I_1 \cap O_1 = I_2 \cap O_2 = I_1 \cap I_2 = O_1 \cap O_2 = P_1 \cap P_2 = \emptyset$. Let

$$I = (I_1 \setminus O_2) \cup (I_2 \setminus O_1), \quad O = (O_1 \setminus I_2) \cup (O_2 \setminus I_1), \quad P = P_1 \cup P_2,$$

we define

$$F_1 \otimes F_2 \subseteq (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I, O} (N \xrightarrow{u} [P^*])$$

$$F_1 \otimes F_2 = \{f \in (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I, O} (N \xrightarrow{u} [P^*]) \mid \forall \theta : \exists f_1 \in F_1, f_2 \in F_2 :$$

$$f(\theta) = (\varphi \leftrightarrow \psi) \Big|_{\text{rng}_{I, O}(\theta, \varphi \leftrightarrow \psi)} \quad \text{where} \\ \varphi = f_1(\delta \leftrightarrow \psi), \quad \psi = f_2(\delta \leftrightarrow \varphi), \quad \delta = \theta \Big|_{\text{dom}_{I, O}(\theta, \varphi \leftrightarrow \psi)}\}$$

□

Theorem 11 $F_1 \otimes F_2 \neq \emptyset$.

Proof: Since F_1 and F_2 are mobile components we may find functions f_1, f_2 such that $f_1 \in F_1$ and $f_2 \in F_2$. Based on these functions we construct a function f which is strongly pulse-driven, generic and satisfies the recursive definition above.

Let

$$g \in ((N \xrightarrow{u} [P_1^*]) \times (N \xrightarrow{u} [P_2^*])) \times (N \xrightarrow{u} [\overline{P}^*]) \rightarrow (N \xrightarrow{u} [P_1^*]) \times (N \xrightarrow{u} [P_2^*])$$

be defined as follows:

$$g((\varphi, \psi), \theta) = (f_1(\delta \leftrightarrow \psi), f_2(\delta \leftrightarrow \varphi)) \quad \text{where} \quad \delta = \theta \Big|_{\text{dom}_{I, O}(\theta, \varphi \leftrightarrow \psi)}.$$

Lemma 1 g is well defined.

Proof: Since $\theta \in (N \xrightarrow{u} [\overline{P}^*])$ it follows by Theorem 8 that $\delta \in (N \xrightarrow{u} [\overline{P}^*])$. Since $\psi \in (N \xrightarrow{u} [P_2^*])$ and $P_2 \cap \overline{P} = \emptyset$, and since $\varphi \in (N \xrightarrow{u} [P_1^*])$ and $P_1 \cap \overline{P} = \emptyset$, it follows by Theorem 8 that $\delta \leftrightarrow \psi \in (N \xrightarrow{u} [(P_2 \cup \overline{P})^*])$ and that $\delta \leftrightarrow \varphi \in (N \xrightarrow{u} [(P_1 \cup \overline{P})^*])$. Since $P_2 \cup \overline{P} \subseteq \overline{P}_1$ and $P_1 \cup \overline{P} \subseteq \overline{P}_2$, we have that

$$\delta \leftrightarrow \psi \in (N \xrightarrow{u} [\overline{P}_1^*]), \quad \delta \leftrightarrow \varphi \in (N \xrightarrow{u} [\overline{P}_2^*]).$$

As a consequence $f_1(\delta \leftrightarrow \psi) \in (N \xrightarrow{u} [P_1^*])$ and $f_2(\delta \leftrightarrow \varphi) \in (N \xrightarrow{u} [P_2^*])$. □

Theorem 16 (in the appendix) and the way g is defined in terms of strongly and weakly pulse-driven functions imply that g is strongly pulse-driven. Thus μg is well-defined, in which case Theorem 18 (in the appendix) implies that μg is strongly pulse-driven. Let

$$f \in (N \xrightarrow{u} [\overline{P^*}]) \rightarrow (N \xrightarrow{u} [P^*])$$

be defined as follows:

$$f(\theta) = (\varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi)} \quad \text{where} \quad (\varphi, \psi) = (\mu g)(\theta).$$

Lemma 2 f is well defined.

Proof: It is enough to show that $\varphi \leftrightarrow \psi \in N \xrightarrow{u} [P^*]$. This follows straightforwardly by the signature of g and Theorem 8 since $P_1 \cap P_2 = \emptyset$. \square

Theorem 16 and the way f is defined in terms of strongly and weakly pulse-driven functions imply that f is strongly pulse-driven.

That f is generic is a consequence of the next two lemmas.

Lemma 3 $f(\theta) = f(\theta |_{\text{dom}_{I,O}(\theta, f(\theta))})$.

Proof: First, note that $f(\theta) = f(\theta |_{\text{dom}_{I,O}(\theta, \varphi \leftrightarrow \psi)})$ because

$$\begin{aligned} \text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi) &= \text{rng}_{I,O}(\theta |_{\text{dom}_{I,O}(\theta, \varphi \leftrightarrow \psi)}, \varphi \leftrightarrow \psi) && \{\text{by Theorem 10}\} \\ \delta = \theta |_{\text{dom}_{I,O}(\theta, \varphi \leftrightarrow \psi)} &= \theta |_{\text{dom}_{I,O}(\theta, \varphi \leftrightarrow \psi) |_{\text{dom}_{I,O}(\theta, \varphi \leftrightarrow \psi)}} && \{\text{by idempotence of projection.}\} \end{aligned}$$

Now

$$\begin{aligned} f(\theta |_{\text{dom}_{I,O}(\theta, f(\theta))}) &= \\ f(\theta |_{\text{dom}_{I,O}(\theta, (\varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi)}})) &= \{\text{by definition of } f\} \\ f(\theta |_{\text{dom}_{I,O}(\theta, \varphi \leftrightarrow \psi)}) &= \{\text{by Theorem 10}\} \\ f(\theta) &= \{\text{by above remark}\} \end{aligned}$$

\square

Lemma 4 $f(\theta) = f(\theta) |_{\text{rng}_{I,O}(\theta, f(\theta))}$.

Proof:

$$\begin{aligned} f(\theta) |_{\text{rng}_{I,O}(\theta, f(\theta))} &= \\ (\varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, (\varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi)}})} &= \{\text{by definition of } f\} \\ (\varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi)}} &= \{\text{by Theorem 10}\} \\ (\varphi \leftrightarrow \psi) |_{\text{rng}_{I,O}(\theta, \varphi \leftrightarrow \psi)} &= \{\text{by idempotence of projection}\} \\ f(\theta) &= \{\text{by definition of } f\} \end{aligned}$$

\square

Finally, since $\exists f_1, f_2 : \forall \theta : P$ implies $\forall \theta : \exists f_1, f_2 : P$ it follows that $f \in F_1 \otimes F_2$. \square

Theorem 12 $F_1 \otimes F_2$ is a mobile component.

Proof: That $F_1 \otimes F_2 \neq \emptyset$ follows from Theorem 11. To see that $F_1 \otimes F_2$ is closed, let $f \in (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I, O} (N \rightarrow [P^*])$, and assume that

$$\forall \theta : \exists f' \in F_1 \otimes F_2 : f(\theta) = f'(\theta).$$

The definition of \otimes implies that for any θ there are $f_1 \in F_1, f_2 \in F_2$ such that:

$$\begin{aligned} f(\theta) &= (\varphi \leftrightarrow \psi) \big|_{\text{rng}_{I, O}(\theta, \varphi \leftrightarrow \psi)} \quad \text{where} \\ \varphi &= f_1(\delta \leftrightarrow \psi), \quad \psi = f_2(\delta \leftrightarrow \varphi), \quad \delta = \theta \big|_{\text{dom}_{I, O}(\theta, \varphi \leftrightarrow \psi)} \end{aligned}$$

By the definition of \otimes , it follows that $f \in F_1 \otimes F_2$. □

The definition of \otimes depends on the total operator for summation. This operator is a bit strange since it results in hiding if its arguments are active on the same channels. Our composition operator \otimes , on the other hand, should only hide the feedback channels. This means that all messages sent or received by the components along the external channels should be visible also after the composition. As a consequence, in the definition of \otimes it should be possible to replace the total operator for summation by the partial one.

Theorem 13 *The operator \leftrightarrow can be replaced by $+$ in the definition of \otimes .*

Proof: Given that

$$\varphi = f_1(\delta \leftrightarrow \psi), \quad \psi = f_2(\delta \leftrightarrow \varphi), \quad \delta = \theta \big|_{\text{dom}_{I, O}(\theta, \varphi \leftrightarrow \psi)}.$$

It is enough to show that

$$\text{act}(\varphi) \cap \text{act}(\psi) = \text{act}(\varphi) \cap \text{act}(\delta) = \text{act}(\psi) \cap \text{act}(\delta) = \emptyset.$$

Since the genericity of f_1 and f_2 imply

$$\varphi = \varphi \big|_{\text{rng}_{I_1, O_1}(\delta \leftrightarrow \psi, \varphi)}, \quad \psi = \psi \big|_{\text{rng}_{I_2, O_2}(\delta \leftrightarrow \varphi, \psi)},$$

this is equivalent to showing that for all $n \in \mathbb{N}$, the sets

$$\text{rng}_{I_1, O_1}(\delta \leftrightarrow \psi, \varphi)(n), \quad \text{rng}_{I_2, O_2}(\delta \leftrightarrow \varphi, \psi)(n), \quad \text{dom}_{I, O}(\theta, \varphi \leftrightarrow \psi)(n)$$

are mutually disjoint. The proof is by induction on n . Let

$$\begin{aligned} D_1^n &= \text{dom}_{I_1, O_1}(\delta \leftrightarrow \psi, \varphi)(n), & R_1^n &= \text{rng}_{I_1, O_1}(\delta \leftrightarrow \psi, \varphi)(n), \\ D_2^n &= \text{dom}_{I_2, O_2}(\delta \leftrightarrow \varphi, \psi)(n), & R_2^n &= \text{rng}_{I_2, O_2}(\delta \leftrightarrow \varphi, \psi)(n), \\ D^n &= \text{dom}_{I, O}(\theta, \varphi \leftrightarrow \psi)(n), & R^n &= \text{rng}_{I, O}(\theta, \varphi \leftrightarrow \psi)(n). \end{aligned}$$

Induction hypotheses:

$$\begin{aligned} R_1^n \cap R_2^n &= \emptyset, & D_1^n \cap D_2^n &= \emptyset, \\ R_1^n \cap D^n &= \emptyset, & D_1^n \cap R^n &= \emptyset, \\ R_2^n \cap D^n &= \emptyset, & D_2^n \cap R^n &= \emptyset. \end{aligned}$$

Base case: by definition of dom and rng

$$\begin{aligned} D_1^1 &= I_1, & D_2^1 &= I_2, & D^1 &= I, \\ R_1^1 &= O_1, & R_2^1 &= O_2, & R^1 &= O, \quad \text{so} \\ R_1^1 \cap R_2^1 &= R_1^1 \cap D^1 = R_2^1 \cap D^1 = \emptyset, \\ D_1^1 \cap D_2^1 &= D_1^1 \cap R^1 = D_2^1 \cap R^1 = \emptyset. \end{aligned}$$

Induction step: by definition of dom and rng

$$\begin{aligned}
R_1^{n+1} &= R_1^n \cup \bigcup_{i \in D_1^n} \{p \mid p \in (\delta \leftrightarrow \psi)(i)(n)\} \cup \bigcup_{i \in R_1^n} \{p \mid ?p \in \varphi(i)(n)\} \\
D_1^{n+1} &= D_1^n \cup \bigcup_{i \in D_1^n} \{p \mid ?p \in (\delta \leftrightarrow \psi)(i)(n)\} \cup \bigcup_{i \in R_1^n} \{p \mid !p \in \varphi(i)(n)\} \\
R_2^{n+1} &= R_2^n \cup \bigcup_{i \in D_2^n} \{p \mid p \in (\delta \leftrightarrow \varphi)(i)(n)\} \cup \bigcup_{i \in R_2^n} \{p \mid ?p \in \psi(i)(n)\} \\
D_2^{n+1} &= D_2^n \cup \bigcup_{i \in D_2^n} \{p \mid ?p \in (\delta \leftrightarrow \varphi)(i)(n)\} \cup \bigcup_{i \in R_2^n} \{p \mid !p \in \psi(i)(n)\} \\
R^{n+1} &= R^n \cup \bigcup_{i \in D^n} \{p \mid !p \in \theta(i)(n)\} \cup \bigcup_{i \in R^n} \{p \mid ?p \in (\varphi \leftrightarrow \psi)(i)(n)\} \\
D^{n+1} &= D^n \cup \bigcup_{i \in D^n} \{p \mid ?p \in \theta(i)(n)\} \cup \bigcup_{i \in R^n} \{p \mid !p \in (\varphi \leftrightarrow \psi)(i)(n)\}.
\end{aligned}$$

We show that $R_1^{n+1} \cap R_2^{n+1} = \emptyset$. The justification of a proof step is given in curly brackets: $pU(\varphi, \psi)$ means “by port uniqueness” of φ and ψ , $dJ(\varphi, \psi, \delta)$ means “by mutually disjointness” of φ, ψ and δ , and iH means “by induction hypothesis”. If two cases are symmetric only one is presented.

$$\begin{aligned}
R_1^n \cap R_2^n &= \emptyset && \{iH\} \\
R_1^n \cap \bigcup_{i \in D_2^n} \{p \mid !p \in (\delta \leftrightarrow \varphi)(i)(n)\} &= \emptyset && \{pU(\varphi, \delta), dJ(P_1, P_2, \overline{P})\} \\
R_1^n \cap \bigcup_{i \in R_2^n} \{p \mid ?p \in \psi(i)(n)\} &= \emptyset && \{pU(\psi), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid !p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in D_2^n} \{p \mid !p \in (\delta \leftrightarrow \varphi)(i)(n)\} &= \emptyset && \{dJ(P_1, P_2, \overline{P}), pU(\delta), iH\} \\
\bigcup_{i \in D_1^n} \{p \mid !p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in R_2^n} \{p \mid ?p \in \psi(i)(n)\} &= \emptyset && \{pU(\psi), dJ(P_2, \overline{P})\} \\
\bigcup_{i \in R_1^n} \{p \mid ?p \in (\varphi)(i)(n)\} \cap \bigcup_{i \in R_2^n} \{p \mid ?p \in \psi(i)(n)\} &= \emptyset && \{dJ(P_1, P_2)\}
\end{aligned}$$

We show that $R_1^{n+1} \cap D^{n+1} = \emptyset$. The proof that $R_2^{n+1} \cap D^{n+1} = \emptyset$ is symmetric and therefore left out.

$$\begin{aligned}
R_1^n \cap D^n &= \emptyset && \{iH\} \\
R_1^n \cap \bigcup_{i \in D^n} \{p \mid ?p \in \theta(i)(n)\} &= \emptyset && \{pU(\theta), dJ(P_1, P_2, \overline{P})\} \\
R_1^n \cap \bigcup_{i \in R^n} \{p \mid !p \in (\varphi \leftrightarrow \psi)(i)(n)\} &= \emptyset && \{pU(\varphi, \psi), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid !p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap D^n &= \emptyset && \{pU(\psi, \theta), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid !p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in D^n} \{p \mid ?p \in \theta(i)(n)\} &= \emptyset && \{pU(\theta), dJ(P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid !p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in R^n} \{p \mid !p \in (\varphi \leftrightarrow \psi)(i)(n)\} &= \emptyset && \{dJ(P_1, P_2, \overline{P}), pU(\psi), iH\} \\
\bigcup_{i \in R_1^n} \{p \mid ?p \in (\varphi)(i)(n)\} \cap D^n &= \emptyset && \{pU(\varphi), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in R_1^n} \{p \mid ?p \in (\varphi)(i)(n)\} \cap \bigcup_{i \in D^n} \{p \mid ?p \in \theta(i)(n)\} &= \emptyset && \{dJ(P_1, \overline{P})\} \\
\bigcup_{i \in R_1^n} \{p \mid ?p \in (\varphi)(i)(n)\} \cap \bigcup_{i \in R^n} \{p \mid !p \in (\varphi \leftrightarrow \psi)(i)(n)\} &= \emptyset && \{pU(\varphi), dJ(P_1, P_2)\}
\end{aligned}$$

We show that $D_1^{n+1} \cap D_2^{n+1} = \emptyset$. If two cases are symmetric only one is presented.

$$\begin{aligned}
D_1^n \cap D_2^n &= \emptyset && \{iH\} \\
D_1^n \cap \bigcup_{i \in D_2^n} \{p \mid ?p \in (\delta \leftrightarrow \varphi)(i)(n)\} &= \emptyset && \{pU(\varphi, \delta), dJ(P_1, P_2, \overline{P})\} \\
D_1^n \cap \bigcup_{i \in R_2^n} \{p \mid !p \in \psi(i)(n)\} &= \emptyset && \{pU(\psi), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid ?p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in D_2^n} \{p \mid ?p \in (\delta \leftrightarrow \varphi)(i)(n)\} &= \emptyset && \{dJ(P_1, P_2, \overline{P}), pU(\delta), iH\} \\
\bigcup_{i \in D_1^n} \{p \mid ?p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in R_2^n} \{p \mid !p \in \psi(i)(n)\} &= \emptyset && \{pU(\psi), dJ(P_2, \overline{P})\} \\
\bigcup_{i \in R_1^n} \{p \mid !p \in (\varphi)(i)(n)\} \cap \bigcup_{i \in R_2^n} \{p \mid !p \in \psi(i)(n)\} &= \emptyset && \{dJ(P_1, P_2)\}
\end{aligned}$$

We show that $D_1^{n+1} \cap R^{n+1} = \emptyset$. The proof that $D_2^{n+1} \cap R^{n+1} = \emptyset$ is symmetric and therefore left out.

$$\begin{aligned}
D_1^n \cap R^n &= \emptyset && \{\mathbf{iH}\} \\
D_1^n \cap \bigcup_{i \in D^n} \{p \mid p \in \theta(i)(n)\} &= \emptyset && \{pU(\theta), dJ(P_1, P_2, \overline{P})\} \\
D_1^n \cap \bigcup_{i \in R^n} \{p \mid p \in (\varphi \leftrightarrow \psi)(i)(n)\} &= \emptyset && \{pU(\varphi, \psi), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap R^n &= \emptyset && \{pU(\psi, \theta), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in D^n} \{p \mid p \in \theta(i)(n)\} &= \emptyset && \{pU(\theta), dJ(P_2, \overline{P})\} \\
\bigcup_{i \in D_1^n} \{p \mid p \in (\delta \leftrightarrow \psi)(i)(n)\} \cap \bigcup_{i \in R^n} \{p \mid p \in (\varphi \leftrightarrow \psi)(i)(n)\} &= \emptyset && \{dJ(P_1, P_2, \overline{P}), pU(\psi), \mathbf{iH}\} \\
\bigcup_{i \in R_1^n} \{p \mid p \in (\varphi)(i)(n)\} \cap R^n &= \emptyset && \{pU(\varphi), dJ(P_1, P_2, \overline{P})\} \\
\bigcup_{i \in R_1^n} \{p \mid p \in (\varphi)(i)(n)\} \cap \bigcup_{i \in D^n} \{p \mid p \in \theta(i)(n)\} &= \emptyset && \{dJ(P_1, \overline{P})\} \\
\bigcup_{i \in R_1^n} \{p \mid p \in (\varphi)(i)(n)\} \cap \bigcup_{i \in R^n} \{p \mid p \in (\varphi \leftrightarrow \psi)(i)(n)\} &= \emptyset && \{pU(\varphi), dJ(P_1, P_2)\}
\end{aligned}$$

□

Chapter 4

Communication Central

As an example we specify a communication central (see Figure 4.1). Its task is to build up connections between station_1 and station_2 . The initial “wires” are a_1 and a_2 . station_1 can send ports to be connected (both read and write) along a_1 , and station_2 can send ports to be connected (both read and write) along a_2 .

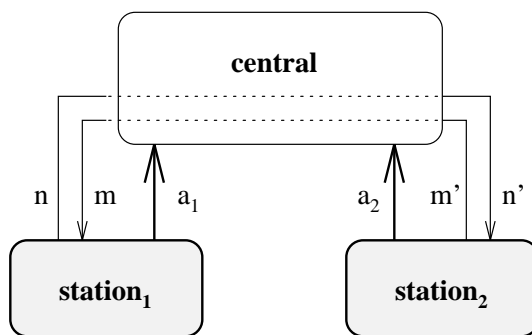


Figure 4.1: Communication central

Let $?n$ be the j 'th read port sent along a_1 by station_1 . The central is allowed to read from n as soon as this port is received. Moreover, let $!n'$ be the j 'th write port sent along a_2 by station_2 . The central is allowed to write on n' as soon as this port is received. The central “connects” these two channels by forwarding each message it receives on the channel n along the channel n' . Symmetrically, if $?m'$ is the k 'th read port sent along a_2 by station_2 and $!m$ is the k 'th write port sent along a_1 by station_1 , the messages received on m' are forwarded along m .

In order to specify this component, let us introduce three basic operators. The first one is the *filter* operator: for any set of messages M and stream of messages s , $M \odot s$ denotes the stream we get by removing any message in s not contained in M . The second one is the *length* operator: for any stream s , $\#s$ yields its length. This means that $\#s = \infty$ if s is infinite. Finally, we need a *time abstraction operator*: for any named communication history β , $\bar{\beta}$ denotes the result of removing all time information in β . For any i , this is achieved by concatenating all the finite streams in $\beta(i)$ into one stream. Thus each

communication history consisting of infinitely many finite streams of messages is replaced by the result of concatenating its finite streams into one stream of messages. As a result the timing information is abstracted away.

$$\text{Central} \in ((N \xrightarrow{u} [\overline{P}]) \xrightarrow{\{a_1, a_2\}, \emptyset} (N \xrightarrow{u} [P])) \rightarrow \text{Bool}$$

$$\text{Central}(f) \stackrel{\text{def}}{=} \forall \alpha : f(\alpha) = \beta \quad \text{where}$$

$$\begin{aligned} \forall (n, n') \in \text{acon} : \overline{\beta}(n') &= \overline{\alpha}(n) && \text{-- the forwarding mechanism} \\ \text{acon} &= \text{con}(a_1, a_2) \cup \text{con}(a_2, a_1) && \text{-- the set of all connections} \\ \text{con}(a, b) &= \{(n, n') \mid && \text{-- the set of connections from "a to b"} \\ &\exists k \in [1.. \min\{\#\text{rr}(a), \#\text{wr}(b)\}] : \\ &\text{rr}(a)(k) = ?n \wedge \text{wr}(b)(k) = !n'\} \\ \text{rr}(a) &= ?N \odot \overline{\alpha}(a) && \text{-- the set of read ports from a} \\ \text{wr}(b) &= !N \odot \alpha(b) && \text{-- the set of write ports from b} \end{aligned}$$

Note that this specification does not say anything about the timing of the output. It may be argued that the same behavior could have been obtained in a static network, where an infinite number of channels connect the **stations** with the **central**. However, in that case both the **central** and the **stations** would be allowed to observe anything that is sent along the channels. This should be contrasted with our model, where the components are allowed to observe only the channels they have received or created themselves. In our opinion, it is exactly this privacy that, not only captures the essence of mobility, but also simplifies the conceptual reasoning about mobile reconfiguration.

Chapter 5

Full Abstractness

Syntactically, a dataflow network is either a basic component or the point-to-point composition of two networks. The context free syntax is given by the following productions:

$$net ::= x \mid net \otimes net$$

Network terms defined in this way are also called *rough terms*, because they do not contain any information about their syntactic interface i.e. information about their input and output channels. By adding interface (or type) information we define the *context sensitive syntax* for networks as a typing system. As axioms we have typing declarations of the form:

$$x :: (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I,O} (N \xrightarrow{u} [P^*]).$$

We have only one typing rule:

$$\frac{\begin{array}{l} x_1 :: (N \xrightarrow{u} [\overline{P}_1^*]) \xrightarrow{I_1,O_1} (N \xrightarrow{u} [P_1^*]) \\ x_2 :: (N \xrightarrow{u} [\overline{P}_2^*]) \xrightarrow{I_2,O_2} (N \xrightarrow{u} [P_2^*]) \end{array}}{x_1 \otimes x_2 :: (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I,O} (N \xrightarrow{u} [P^*])} \{ I_1 \cap I_2 = O_1 \cap O_2 = P_1 \cap P_2 = \emptyset$$

where $I = (I_1 \setminus O_2) \cup (I_2 \setminus O_1)$, $O = (O_1 \setminus I_2) \cup (O_2 \setminus I_1)$ and $P = P_1 \cup P_2$. It states that we can compose two networks if their input and output channels are disjoint. The composition operator hides the channels connecting the output of one network with the input of the other one.

A context $C(\cdot) :: (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I,O} (N \xrightarrow{u} [P^*])$ is a network with exactly one hole of type $(N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I,O} (N \xrightarrow{u} [P^*])$. A network of this type is allowed to be substituted for the hole.

The denotational semantics of a network expression is defined as follows

$$\begin{aligned} \mathcal{D}[n] &\subseteq (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I,O} (N \xrightarrow{u} [P^*]) \quad \text{if } n :: (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I,O} (N \xrightarrow{u} [P^*]) \\ \mathcal{D}[n \otimes m] &= \mathcal{D}[n] \otimes \mathcal{D}[m] \end{aligned}$$

We define the observation semantics of a network as the input output behavior of its semantic denotation as follows:

$$\mathcal{O}[n] = \{(x, y) \mid \exists f \in \mathcal{D}[n] : y = f(x)\}$$

Now, we define full abstraction as in [Jon89].

Definition 13 Full abstraction *The model \mathcal{D} is said to be fully abstract with respect to \mathcal{O} if for all networks $n, m :: (N \xrightarrow{u} [\overline{P}^*]) \xrightarrow{I, \mathcal{O}} (N \xrightarrow{u} [P^*])$*

- (1) $\mathcal{D}[n] = \mathcal{D}[m] \Rightarrow \mathcal{O}[n] = \mathcal{O}[m] \quad \{\mathcal{D} \text{ is more distinguishing}\}$
- (2) $\forall C(\cdot) : \mathcal{D}[n] = \mathcal{D}[m] \Rightarrow \mathcal{D}[C(n)] = \mathcal{D}[C(m)] \quad \{\mathcal{D} \text{ is compositional}\}$
- (3) $\mathcal{D}[n] \neq \mathcal{D}[m] \Rightarrow \exists C(\cdot) : \mathcal{O}[C(n)] \neq \mathcal{O}[C(m)] \quad \square$

Theorem 14 *Our denotational semantics \mathcal{D} is fully abstract with respect to \mathcal{O} .*

Proof: Property (1) follows from the definition of \mathcal{O} . Property (2) follows straightforwardly from the definition of \mathcal{D} . To prove (3) suppose that $\mathcal{D}[n] \neq \mathcal{D}[m]$. This means, there is an f which is contained in the first set but not in the second. Since $\mathcal{D}[m]$ is closed there is a θ such that $\forall g \in \mathcal{D}[m] : f(\theta) \neq g(\theta)$. Otherwise, closeness would require that f is also in $\mathcal{D}[m]$. As a consequence $(\theta, f(\theta)) \notin \mathcal{O}[m]$. Hence $\mathcal{O}[n] \neq \mathcal{O}[m] \quad \square$

Note that the full abstractness property is lost if the components are not required to be closed.

Chapter 6

Discussion

We have had several sources of inspiration. First of all, the semantic model for static networks is inspired by [Par83], [Kok87], [Bro87]. Park models components by sets of functions in the same way as we do. However, he models time with time ticks \surd and his functions are defined also for finite streams. Moreover, infinite streams are not required to have infinitely many ticks. Kok models nondeterministic components by functions mapping communication histories to sets of communication histories. We use instead a closed set of strongly pulse-driven functions. This allows us to model unbounded nondeterminism without having to introduce a more complex metric. [Bro87] employs sets of functions as we do, but these functions work on untimed finite and infinite streams. This makes the model more abstract but at the same time more complex with respect to its theoretical basis. The formulation of pulse-drivenness has been taken from [Bro95a], and the use of named communication histories is based on [BD92]. The use of closed sets of functions to model dataflow components is not new — see for example [Rus90].

Our ideas on mobility have also had several sources. [Bro95b] and [Gro94] give an equational characterization of dynamic reconfiguration. [Gro94] also presents a semantic model for mobile, deterministic networks. However, that model is higher-order and mobility is achieved by communicating channels and functions instead of ports.

The idea of communicating names (ports) was inspired by [MPW92a, MPW92b]. The action structures [Mil92] are also related to our model. The idea of associating an access right with each channel in order to control interference was taken from [SKL90]. Our semantic conditions which assure point-to-point communication were also inspired by the syntactic conditions given in that paper. The semantic model of [SRP91] has also many similarities to our model. However, the intentions are complementary. [SRP91] aims at generality. We, on the other hand, have developed a particular model based on traditional stream processing functions. In [SRP91] inconsistencies can only be avoided by syntactic constraints. In our model, inconsistencies can easily be avoided at the semantic level without additional syntactic constraints.

There are many directions for future research. We believe that this work represents a first step towards a compositional development method for mobile systems in the tradition of Focus [BDD⁺93]. We also intend to extend our model to handle many-to-many composition, recursive (infinite) networks and parameterized components. Since these issues are

well understood in the case of the static networks, we do not expect these extensions to be very difficult.

Chapter 7

Acknowledgments

We thank Manfred Broy for stimulating discussions and valuable feedback. Thanks go also to Thomas Streicher who pointed out the close connection between pulse-driven and contractive functions. Christian Prehofer read an early version of this paper and made helpful comments. The first author has been financially supported by the DFG project SPECTRUM. The second author has been financially supported by the Sonderforschungsbereich 342 “Werkzeuge und Methoden für die Nutzung paralleler Rechnerarchitekturen”.

Bibliography

- [AdBKR89] P. America, J. de Bakker, J. N. Kok, and J. Rutten. Denotational semantics of a parallel object-oriented language. *Information and Computation*, 83:152–205, 1989.
- [AMST92] G. Agha, I. A. Mason, S. F. Smith, and C. L. Talcott. Towards a theory of actor computation. In *Proc. CONCUR'92, Lecture Notes in Computer Science 630*, pages 565–579, 1992.
- [BB90] G. Berry and G. Boudol. The chemical abstract machine. In *Proc. POPL'90*, pages 81–94, 1990.
- [BD92] M. Broy and C. Dendorfer. Modelling operating system structures by timed stream processing functions. *Journal of Functional Programming*, 2:1–21, 1992.
- [BDD⁺93] M. Broy, F. Dederichs, C. Dendorfer, M. Fuchs, T. F. Gritzner, and R. Weber. The design of distributed systems — an introduction to Focus (revised version). Technical Report SFB 342/2/92 A, Technische Universität München, 1993.
- [Bro87] M. Broy. Semantics of finite and infinite networks of concurrent communicating agents. *Distributed Computing*, 2:13–31, 1987.
- [Bro95a] M. Broy. Advanced component interface specification. In *Proc. TPPP'94, Lecture Notes in Computer Science 907*, pages 369–392, 1995.
- [Bro95b] M. Broy. Equations for describing dynamic nets of communicating systems. In *Proc. 5th COMPASS Workshop, Lecture Notes in Computer Science 906*, pages 170–187, 1995.
- [dBZ82] J. W. de Bakker and J. I. Zucker. Denotational semantics of concurrency. In *Proc. 14 ACM Symposium on Theory of Computing*, pages 153–158, 1982.
- [EN86] U. Engberg and M Nielsen. A calculus of communicating systems with label-passing. Technical Report DAIMI PB-208, University of Aarhus, 1986.
- [Eng77] R. Engelking. *General Topology*. PWN – Polish Scientific Publishers, 1977.
- [Gro94] R. Grosu. *A Formal Foundation for Concurrent Object Oriented Programming*. PhD thesis, Technische Universität München, 1994. Also available as report TUM-I9444, Technische Universität München.
- [HBS73] C. Hewitt, P. Bishop, and R. Steiger. A universal modular actor formalism for artificial intelligence. In *Proc. IJCAI'73*, pages 235–245, 1973.
- [Jon89] B. Jonsson. A fully abstract trace model for dataflow networks. In *Proc. 16th Annual ACM Symposium on Principles of Programming Languages*, pages 155–165, 1989.
- [Kah74] G. Kahn. The semantics of a simple language for parallel programming. In *Proc. Information Processing 74*, pages 471–475. North-Holland, 1974.

- [Kel78] R. M. Keller. Denotational models for parallel programs with indeterminate operators. In *Proc. Formal Description of Programming Concepts*, pages 337–366. North-Holland, 1978.
- [Kok87] J. N. Kok. A fully abstract semantics for data flow nets. In *Proc. PARLE’87, Lecture Notes in Computer Science 259*, pages 351–368, 1987.
- [Mes91] J. Meseguer. Conditional rewriting logic as a unified model of concurrency. Technical Report SRI-CSL-91-05, SRI, 1991.
- [Mil92] R. Milner. Action structures. Technical Report ECS-LFCS-92-249, University of Edinburgh, 1992.
- [MPS86] D. MacQueen, G. Plotkin, and R. Sethi. An Ideal Model for Recursive Polymorphic Types. *Information and Control*, 71:95–130, 1986.
- [MPW92a] R. Milner, J. Parrow, and D. Walker. A calculus of mobile processes, part I. *Information and Computation*, 100:1–40, 1992.
- [MPW92b] R. Milner, J. Parrow, and D. Walker. A calculus of mobile processes, part II. *Information and Computation*, 100:41–77, 1992.
- [Niv82] M Nivat. Behaviours of processes and synchronized systems of processes. In *Proc. Theoretical Foundations of Programming Methodology*, pages 473–551, 1982.
- [Par83] D. Park. The “fairness” problem and nondeterministic computing networks. In *Proc. 4th Foundations of Computer Science, Mathematical Centre Tracts 159*, pages 133–161. Mathematisch Centrum Amsterdam, 1983.
- [Rus90] J. R. Russell. On oraclizable networks and Kahn’s principle. In *Proc. POPL’90*, pages 320–328, 1990.
- [SKL90] V. A. Saraswat, K. Kahn, and J. Levy. Janus: A step towards distributed constraint programming. In *Proc. North American Conference on Logic Programming*, 1990.
- [SRP91] V. A. Saraswat, M. Rinard, and P Panangaden. Semantic foundations of concurrent constraint programming. In *Proc. POPL’91*, pages 333–352, 1991.
- [Sut75] W.A. Sutherland. *Introduction to metric and topological spaces*. Claredon Press - Oxford, 1975.
- [Tho89] B. Thomsen. A calculus of higher order communicating systems. In *Proc. POPL’89*, 1989.

Appendix A

Metric Space Basics

The fundamental concept in metric spaces is the concept of distance.

Definition 14 (Metric Space) A metric space is a pair (D, d) consisting of a nonempty set D and a mapping $d \in D \times D \rightarrow \mathbb{R}$, called a metric or distance, which has the following properties:

- (1) $\forall x, y \in D : d(x, y) = 0 \Leftrightarrow x = y$
- (2) $\forall x, y \in D : d(x, y) = d(y, x)$
- (3) $\forall x, y, z \in D : d(x, y) \leq d(x, z) + d(z, y)$

□

A very simple example of a metric is the discrete metric.

Definition 15 (The discrete metric) The discrete metric (D, d) over a set D is defined as follows:

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$

□

Measuring the distance between the elements of a sequence $(x_i)_{i \in \mathbb{N}}$ in D we obtain the familiar definitions for convergence and limits.

Definition 16 (Convergence and limits) Let (D, d) be a metric space and let $(x_i)_{i \in \mathbb{N}}$ be a sequence in D .

- (1) We say that $(x_i)_{i \in \mathbb{N}}$ is a Cauchy sequence whenever we have:

$$\forall \epsilon > 0 : \exists N \in \mathbb{N} : \forall n, m > N : d(x_n, x_m) < \epsilon.$$

- (2) We say that $(x_i)_{i \in \mathbb{N}}$ converges to $x \in D$ denoted by $x = \lim_{n \rightarrow \infty} x_i$ and call x the limit of $(x_i)_{i \in \mathbb{N}}$ whenever we have:

$$\forall \epsilon > 0 : \exists N \in \mathbb{N} : \forall n > N : d(x_n, x) < \epsilon.$$

- (3) The metric space (D, d) is called complete whenever each Cauchy sequence converges to an element of D .

□

Theorem 15 *The discrete metric is complete.*

Proof: Each Cauchy sequence is constant from a given N . □

A very important class of functions over metric spaces is the class of *Lipschitz functions*.

Definition 17 (Lipschitz functions) *Let (D_1, d_1) and (D_2, d_2) be metric spaces and let $f \in D_1 \rightarrow D_2$ be a function. We call f Lipschitz function with constant c if there is a constant $c \geq 0$ such that the following condition is satisfied:*

$$d(f(x), f(y)) \leq c \cdot d(x, y)$$

For a function f with arity n the above condition generalizes to:

$$d(f(x_1, \dots, x_n), f(y_1, \dots, y_n)) \leq c \cdot \max\{d(x_i, y_i) \mid i \in [1..n]\}$$

If $c = 1$ we call f non-expansive. If $c < 1$ we call f contractive. □

Theorem 16 *The composition of two Lipschitz functions $f \in D_1 \rightarrow D_2$ and $g \in D_2 \rightarrow D_3$ is a Lipschitz function with constant $c_1 \cdot c_2$.*

Proof: $d(g(f(x_1)), g(f(x_2))) \leq c_2 \cdot d(f(x_1), f(x_2)) \leq c_2 \cdot c_1 \cdot d(x_1, x_2)$ □

Lemma 1 *The composition of a contractive and a non-expansive function is contractive. The composition of two non-expansive functions is non-expansive. Identity is non-expansive.* □

The main tool for handling recursion in metric spaces is the Banach's fixed point theorem. It guarantees the existence of a unique fixed point for every contractive function.

Theorem 17 (Banach's fixed point theorem) *Let (D, d) be a complete metric space and $f \in D \rightarrow D$ a contractive function. Then there exists an $x \in D$, such that the following holds:*

- (1) $x = f(x)$ *(x is a fixed point of f)*
- (2) $\forall y \in D : y = f(y) \Rightarrow y = x$ *(x is unique)*
- (3) $\forall z \in D : x = \lim_{n \rightarrow \infty} f^n(z)$ *where*
 - $f^0(z) = z$
 - $f^{n+1}(z) = f(f^n(z))$

Proof: See [Eng77] or [Sut75]. □

Usually we want to use a parameterized version of this theorem.

Definition 18 (Parameterized fixed point) *Let $f \in D \times D_1 \times \dots \times D_n \rightarrow D$ be a function of non-empty complete metric spaces that is contractive in its first argument. We define the parameterized fixed point function μf as follows:*

$$\begin{aligned} (\mu f) &\in D_1 \times \dots \times D_n \rightarrow D \\ (\mu f)(y_1, \dots, y_n) &= x \end{aligned}$$

where x is the unique element of D such that $x = f(x, y_1, \dots, y_n)$ as guaranteed by Banach's fixed point theorem. □

Theorem 18 *If f is contractive (non-expansive) so is μf .*

Proof: See for example [MPS86] pages 114–115. □

Appendix B

Streams and Named Stream Tuples

A stream is a finite or infinite sequence of elements. For any set of elements E , we use E^* to denote the set of all finite streams over E , and $[E]$ to denote the set of all infinite streams over E . For any infinite stream s , we use $s\downarrow_j$ to denote the prefix of s containing exactly j elements. We use ϵ to denote the empty stream.

We define the metric of streams generically with respect to an arbitrary discrete metric (E, ρ) .

Definition 19 (The metric of streams) *The metric of streams $([E], d)$ over a discrete metric (E, ρ) is defined as follows:*

$$[E] = \prod_{i \in \mathbb{N}} E$$
$$d(s, t) = \inf\{2^{-j} \mid s\downarrow_j = t\downarrow_j\} \quad \square$$

This metric is also known as the Baire metric [Eng77].

Theorem 19 *The metric space of streams $([E], d)$ is complete.*

Proof: See for example [Eng77]. □

A *named stream tuple* is a mapping $\theta \in (I \rightarrow [E])$ from a set of names to infinite streams. \downarrow is overloaded to named stream tuples in a point-wise style, i.e. $\theta\downarrow_j$ denotes the result of applying \downarrow_j to each component of θ .

Definition 20 (The metric of named stream tuples) *The metric of named stream tuples $(I \rightarrow [E], d)$ with names in I and elements in (E, ρ) is defined as follows:*

$$I \rightarrow [E] \text{ is the set of functions from the countable set } I \text{ to the metric } [E],$$
$$d(s, t) = \inf\{2^{-j} \mid s\downarrow_j = t\downarrow_j\} \quad \square$$

Theorem 20 *The metric space of named stream tuples $(I \rightarrow [E], d)$ is complete.*

Proof: This metric is equivalent to the Cartesian product metric $\prod_{i \in I} [E]$ which is complete because $[E]$ is [Eng77]. □

SFB 342: Methoden und Werkzeuge für die Nutzung paralleler
Rechnerarchitekturen

bisher erschienen :

Reihe A

- 342/1/90 A Robert Gold, Walter Vogler: Quality Criteria for Partial Order Semantics of Place/Transition-Nets, Januar 1990
- 342/2/90 A Reinhard Fößmeier: Die Rolle der Lastverteilung bei der numerischen Parallelprogrammierung, Februar 1990
- 342/3/90 A Klaus-Jörn Lange, Peter Rossmanith: Two Results on Unambiguous Circuits, Februar 1990
- 342/4/90 A Michael Griebel: Zur Lösung von Finite-Differenzen- und Finite-Element-Gleichungen mittels der Hierarchischen Transformations-Mehrgitter-Methode
- 342/5/90 A Reinhold Letz, Johann Schumann, Stephan Bayerl, Wolfgang Bibel: SETHEO: A High-Performance Theorem Prover
- 342/6/90 A Johann Schumann, Reinhold Letz: PARTHEO: A High Performance Parallel Theorem Prover
- 342/7/90 A Johann Schumann, Norbert Trapp, Martin van der Koelen: SETHEO/PARTHEO Users Manual
- 342/8/90 A Christian Suttner, Wolfgang Ertel: Using Connectionist Networks for Guiding the Search of a Theorem Prover
- 342/9/90 A Hans-Jörg Beier, Thomas Bemmerl, Arndt Bode, Hubert Ertl, Olav Hansen, Josef Haunerding, Paul Hofstetter, Jaroslav Kremenek, Robert Lindhof, Thomas Ludwig, Peter Luksch, Thomas Treml: TOP-SYS, Tools for Parallel Systems (Artikelsammlung)
- 342/10/90 A Walter Vogler: Bisimulation and Action Refinement
- 342/11/90 A Jörg Desel, Javier Esparza: Reachability in Reversible Free-Choice Systems
- 342/12/90 A Rob van Glabbeek, Ursula Goltz: Equivalences and Refinement
- 342/13/90 A Rob van Glabbeek: The Linear Time - Branching Time Spectrum
- 342/14/90 A Johannes Bauer, Thomas Bemmerl, Thomas Treml: Leistungsanalyse von verteilten Beobachtungs- und Bewertungswerkzeugen
- 342/15/90 A Peter Rossmanith: The Owner Concept for PRAMs
- 342/16/90 A G. Böckle, S. Trosch: A Simulator for VLIW-Architectures
- 342/17/90 A P. Slavkovsky, U. Rüde: Schnellere Berechnung klassischer Matrix-Multiplikationen
- 342/18/90 A Christoph Zenger: SPARSE GRIDS

Reihe A

- 342/19/90 A Michael Griebel, Michael Schneider, Christoph Zenger: A combination technique for the solution of sparse grid problems
- 342/20/90 A Michael Griebel: A Parallelizable and Vectorizable Multi- Level-Algorithm on Sparse Grids
- 342/21/90 A V. Diekert, E. Ochmanski, K. Reinhardt: On confluent semi-commutations-decidability and complexity results
- 342/22/90 A Manfred Broy, Claus Dendorfer: Functional Modelling of Operating System Structures by Timed Higher Order Stream Processing Functions
- 342/23/90 A Rob van Glabbeek, Ursula Goltz: A Deadlock-sensitive Congruence for Action Refinement
- 342/24/90 A Manfred Broy: On the Design and Verification of a Simple Distributed Spanning Tree Algorithm
- 342/25/90 A Thomas Bemmerl, Arndt Bode, Peter Braun, Olav Hansen, Peter Luksch, Roland Wismüller: TOPSYS - Tools for Parallel Systems (User's Overview and User's Manuals)
- 342/26/90 A Thomas Bemmerl, Arndt Bode, Thomas Ludwig, Stefan Tritscher: MMK - Multiprocessor Multitasking Kernel (User's Guide and User's Reference Manual)
- 342/27/90 A Wolfgang Ertel: Random Competition: A Simple, but Efficient Method for Parallelizing Inference Systems
- 342/28/90 A Rob van Glabbeek, Frits Vaandrager: Modular Specification of Process Algebras
- 342/29/90 A Rob van Glabbeek, Peter Weijland: Branching Time and Abstraction in Bisimulation Semantics
- 342/30/90 A Michael Griebel: Parallel Multigrid Methods on Sparse Grids
- 342/31/90 A Rolf Niedermeier, Peter Rossmanith: Unambiguous Simulations of Auxiliary Pushdown Automata and Circuits
- 342/32/90 A Inga Niepel, Peter Rossmanith: Uniform Circuits and Exclusive Read PRAMs
- 342/33/90 A Dr. Hermann Hellwagner: A Survey of Virtually Shared Memory Schemes
- 342/1/91 A Walter Vogler: Is Partial Order Semantics Necessary for Action Refinement?
- 342/2/91 A Manfred Broy, Frank Dederichs, Claus Dendorfer, Rainer Weber: Characterizing the Behaviour of Reactive Systems by Trace Sets
- 342/3/91 A Ulrich Furbach, Christian Suttner, Bertram Fronhöfer: Massively Parallel Inference Systems
- 342/4/91 A Rudolf Bayer: Non-deterministic Computing, Transactions and Recursive Atomicity
- 342/5/91 A Robert Gold: Dataflow semantics for Petri nets
- 342/6/91 A A. Heise; C. Dimitrovici: Transformation und Komposition von P/T-Netzen unter Erhaltung wesentlicher Eigenschaften

Reihe A

- 342/7/91 A Walter Vogler: Asynchronous Communication of Petri Nets and the Refinement of Transitions
- 342/8/91 A Walter Vogler: Generalized OM-Bisimulation
- 342/9/91 A Christoph Zenger, Klaus Hallatschek: Fouriertransformation auf dünnen Gittern mit hierarchischen Basen
- 342/10/91 A Erwin Loibl, Hans Obermaier, Markus Pawlowski: Towards Parallelism in a Relational Database System
- 342/11/91 A Michael Werner: Implementierung von Algorithmen zur Kompaktifizierung von Programmen für VLIW-Architekturen
- 342/12/91 A Reiner Müller: Implementierung von Algorithmen zur Optimierung von Schleifen mit Hilfe von Software-Pipelining Techniken
- 342/13/91 A Sally Baker, Hans-Jörg Beier, Thomas Bemmerl, Arndt Bode, Hubert Ertl, Udo Graf, Olav Hansen, Josef Haunerding, Paul Hofstetter, Rainer Knödlseher, Jaroslav Kremenek, Siegfried Langenbuch, Robert Lindhof, Thomas Ludwig, Peter Luksch, Roy Milner, Bernhard Ries, Thomas Treml: TOPSYS - Tools for Parallel Systems (Artikelsammlung); 2., erweiterte Auflage
- 342/14/91 A Michael Griebel: The combination technique for the sparse grid solution of PDE's on multiprocessor machines
- 342/15/91 A Thomas F. Gritzner, Manfred Broy: A Link Between Process Algebras and Abstract Relation Algebras?
- 342/16/91 A Thomas Bemmerl, Arndt Bode, Peter Braun, Olav Hansen, Thomas Treml, Roland Wismüller: The Design and Implementation of TOPSYS
- 342/17/91 A Ulrich Furbach: Answers for disjunctive logic programs
- 342/18/91 A Ulrich Furbach: Splitting as a source of parallelism in disjunctive logic programs
- 342/19/91 A Gerhard W. Zumbusch: Adaptive parallele Multilevel-Methoden zur Lösung elliptischer Randwertprobleme
- 342/20/91 A M. Jobmann, J. Schumann: Modelling and Performance Analysis of a Parallel Theorem Prover
- 342/21/91 A Hans-Joachim Bungartz: An Adaptive Poisson Solver Using Hierarchical Bases and Sparse Grids
- 342/22/91 A Wolfgang Ertel, Theodor Gemenis, Johann M. Ph. Schumann, Christian B. Suttner, Rainer Weber, Zongyan Qiu: Formalisms and Languages for Specifying Parallel Inference Systems
- 342/23/91 A Astrid Kiehn: Local and Global Causes
- 342/24/91 A Johann M.Ph. Schumann: Parallelization of Inference Systems by using an Abstract Machine
- 342/25/91 A Eike Jessen: Speedup Analysis by Hierarchical Load Decomposition
- 342/26/91 A Thomas F. Gritzner: A Simple Toy Example of a Distributed System: On the Design of a Connecting Switch
- 342/27/91 A Thomas Schnekenburger, Andreas Weininger, Michael Friedrich: Introduction to the Parallel and Distributed Programming Language ParMod-C

Reihe A

- 342/28/91 A Claus Dendorfer: Funktionale Modellierung eines Postsystems
- 342/29/91 A Michael Griebel: Multilevel algorithms considered as iterative methods on indefinite systems
- 342/30/91 A W. Reisig: Parallel Composition of Liveness
- 342/31/91 A Thomas Bemmerl, Christian Kasperbauer, Martin Mairandres, Bernhard Ries: Programming Tools for Distributed Multiprocessor Computing Environments
- 342/32/91 A Frank Leßke: On constructive specifications of abstract data types using temporal logic
- 342/1/92 A L. Kanal, C.B. Suttner (Editors): Informal Proceedings of the Workshop on Parallel Processing for AI
- 342/2/92 A Manfred Broy, Frank Dederichs, Claus Dendorfer, Max Fuchs, Thomas F. Gritzner, Rainer Weber: The Design of Distributed Systems - An Introduction to FOCUS
- 342/2-2/92 A Manfred Broy, Frank Dederichs, Claus Dendorfer, Max Fuchs, Thomas F. Gritzner, Rainer Weber: The Design of Distributed Systems - An Introduction to FOCUS - Revised Version (erschienen im Januar 1993)
- 342/3/92 A Manfred Broy, Frank Dederichs, Claus Dendorfer, Max Fuchs, Thomas F. Gritzner, Rainer Weber: Summary of Case Studies in FOCUS - a Design Method for Distributed Systems
- 342/4/92 A Claus Dendorfer, Rainer Weber: Development and Implementation of a Communication Protocol - An Exercise in FOCUS
- 342/5/92 A Michael Friedrich: Sprachmittel und Werkzeuge zur Unterstützung paralleler und verteilter Programmierung
- 342/6/92 A Thomas F. Gritzner: The Action Graph Model as a Link between Abstract Relation Algebras and Process-Algebraic Specifications
- 342/7/92 A Sergei Gorlatch: Parallel Program Development for a Recursive Numerical Algorithm: a Case Study
- 342/8/92 A Henning Spruth, Georg Sigl, Frank Johannes: Parallel Algorithms for Slicing Based Final Placement
- 342/9/92 A Herbert Bauer, Christian Sporrer, Thomas Krodel: On Distributed Logic Simulation Using Time Warp
- 342/10/92 A H. Bungartz, M. Griebel, U. Rüde: Extrapolation, Combination and Sparse Grid Techniques for Elliptic Boundary Value Problems
- 342/11/92 A M. Griebel, W. Huber, U. Rüde, T. Störckuhl: The Combination Technique for Parallel Sparse-Grid-Preconditioning and -Solution of PDEs on Multiprocessor Machines and Workstation Networks
- 342/12/92 A Rolf Niedermeier, Peter Rossmanith: Optimal Parallel Algorithms for Computing Recursively Defined Functions
- 342/13/92 A Rainer Weber: Eine Methodik für die formale Anforderungsspezifikation verteilter Systeme
- 342/14/92 A Michael Griebel: Grid- and point-oriented multilevel algorithms

Reihe A

- 342/15/92 A M. Griebel, C. Zenger, S. Zimmer: Improved multilevel algorithms for full and sparse grid problems
- 342/16/92 A J. Desel, D. Gomm, E. Kindler, B. Paech, R. Walter: Bausteine eines kompositionalen Beweiskalküls für netzmodellerte Systeme
- 342/17/92 A Frank Dederichs: Transformation verteilter Systeme: Von applikativen zu prozeduralen Darstellungen
- 342/18/92 A Andreas Listl, Markus Pawlowski: Parallel Cache Management of a RDBMS
- 342/19/92 A Erwin Loibl, Markus Pawlowski, Christian Roth: PART: A Parallel Relational Toolbox as Basis for the Optimization and Interpretation of Parallel Queries
- 342/20/92 A Jörg Desel, Wolfgang Reisig: The Synthesis Problem of Petri Nets
- 342/21/92 A Robert Balder, Christoph Zenger: The d-dimensional Helmholtz equation on sparse Grids
- 342/22/92 A Ilko Michler: Neuronale Netzwerk-Paradigmen zum Erlernen von Heuristiken
- 342/23/92 A Wolfgang Reisig: Elements of a Temporal Logic. Coping with Concurrency
- 342/24/92 A T. Störtkuhl, Chr. Zenger, S. Zimmer: An asymptotic solution for the singularity at the angular point of the lid driven cavity
- 342/25/92 A Ekkart Kindler: Invariants, Compositionality and Substitution
- 342/26/92 A Thomas Bonk, Ulrich Rüde: Performance Analysis and Optimization of Numerically Intensive Programs
- 342/1/93 A M. Griebel, V. Thurner: The Efficient Solution of Fluid Dynamics Problems by the Combination Technique
- 342/2/93 A Ketil Stølen, Frank Dederichs, Rainer Weber: Assumption / Commitment Rules for Networks of Asynchronously Communicating Agents
- 342/3/93 A Thomas Schnekenburger: A Definition of Efficiency of Parallel Programs in Multi-Tasking Environments
- 342/4/93 A Hans-Joachim Bungartz, Michael Griebel, Dierk Röschke, Christoph Zenger: A Proof of Convergence for the Combination Technique for the Laplace Equation Using Tools of Symbolic Computation
- 342/5/93 A Manfred Kunde, Rolf Niedermeier, Peter Rossmanith: Faster Sorting and Routing on Grids with Diagonals
- 342/6/93 A Michael Griebel, Peter Oswald: Remarks on the Abstract Theory of Additive and Multiplicative Schwarz Algorithms
- 342/7/93 A Christian Sporrer, Herbert Bauer: Corolla Partitioning for Distributed Logic Simulation of VLSI Circuits
- 342/8/93 A Herbert Bauer, Christian Sporrer: Reducing Rollback Overhead in Time-Warp Based Distributed Simulation with Optimized Incremental State Saving
- 342/9/93 A Peter Slavkovsky: The Visibility Problem for Single-Valued Surface ($z = f(x,y)$): The Analysis and the Parallelization of Algorithms

Reihe A

- 342/10/93 A Ulrich Rde: Multilevel, Extrapolation, and Sparse Grid Methods
- 342/11/93 A Hans Regler, Ulrich Rde: Layout Optimization with Algebraic Multigrid Methods
- 342/12/93 A Dieter Barnard, Angelika Mader: Model Checking for the Modal Mu-Calculus using Gau Elimination
- 342/13/93 A Christoph Pflaum, Ulrich Rde: Gau' Adaptive Relaxation for the Multilevel Solution of Partial Differential Equations on Sparse Grids
- 342/14/93 A Christoph Pflaum: Convergence of the Combination Technique for the Finite Element Solution of Poisson's Equation
- 342/15/93 A Michael Luby, Wolfgang Ertel: Optimal Parallelization of Las Vegas Algorithms
- 342/16/93 A Hans-Joachim Bungartz, Michael Griebel, Dierk Rschke, Christoph Zenger: Pointwise Convergence of the Combination Technique for Laplace's Equation
- 342/17/93 A Georg Stellner, Matthias Schumann, Stefan Lamberts, Thomas Ludwig, Arndt Bode, Martin Kiehl und Rainer Mehlhorn: Developing Multicomputer Applications on Networks of Workstations Using NXLib
- 342/18/93 A Max Fuchs, Ketil Stlen: Development of a Distributed Min/Max Component
- 342/19/93 A Johann K. Obermaier: Recovery and Transaction Management in Write-optimized Database Systems
- 342/20/93 A Sergej Gorlatch: Deriving Efficient Parallel Programs by Systemating Coarsing Specification Parallelism
- 342/01/94 A Reiner Httl, Michael Schneider: Parallel Adaptive Numerical Simulation
- 342/02/94 A Henning Spruth, Frank Johannes: Parallel Routing of VLSI Circuits Based on Net Independency
- 342/03/94 A Henning Spruth, Frank Johannes, Kurt Antreich: PHRoute: A Parallel Hierarchical Sea-of-Gates Router
- 342/04/94 A Martin Kiehl, Rainer Mehlhorn, Matthias Schumann: Parallel Multiple Shooting for Optimal Control Problems Under NX/2
- 342/05/94 A Christian Suttner, Christoph Goller, Peter Krauss, Klaus-Jrn Lange, Ludwig Thomas, Thomas Schnekenburger: Heuristic Optimization of Parallel Computations
- 342/06/94 A Andreas Listl: Using Subpages for Cache Coherency Control in Parallel Database Systems
- 342/07/94 A Manfred Broy, Ketil Stlen: Specification and Refinement of Finite Dataflow Networks - a Relational Approach
- 342/08/94 A Katharina Spies: Funktionale Spezifikation eines Kommunikationsprotokolls
- 342/09/94 A Peter A. Krauss: Applying a New Search Space Partitioning Method to Parallel Test Generation for Sequential Circuits

Reihe A

- 342/10/94 A Manfred Broy: A Functional Rephrasing of the Assumption/Commitment Specification Style
- 342/11/94 A Eckhardt Holz, Ketil Stølen: An Attempt to Embed a Restricted Version of SDL as a Target Language in Focus
- 342/12/94 A Christoph Pflaum: A Multi-Level-Algorithm for the Finite-Element-Solution of General Second Order Elliptic Differential Equations on Adaptive Sparse Grids
- 342/13/94 A Manfred Broy, Max Fuchs, Thomas F. Gritzner, Bernhard Schätz, Katharina Spies, Ketil Stølen: Summary of Case Studies in FOCUS - a Design Method for Distributed Systems
- 342/14/94 A Maximilian Fuchs: Technologieabhängigkeit von Spezifikationen digitaler Hardware
- 342/15/94 A M. Griebel, P. Oswald: Tensor Product Type Subspace Splittings And Multilevel Iterative Methods For Anisotropic Problems
- 342/16/94 A Gheorghe Ștefănescu: Algebra of Flownomials
- 342/17/94 A Ketil Stølen: A Refinement Relation Supporting the Transition from Unbounded to Bounded Communication Buffers
- 342/18/94 A Michael Griebel, Tilman Neuhoeffer: A Domain-Oriented Multilevel Algorithm-Implementation and Parallelization
- 342/19/94 A Michael Griebel, Walter Huber: Turbulence Simulation on Sparse Grids Using the Combination Method
- 342/20/94 A Johann Schumann: Using the Theorem Prover SETHEO for verifying the development of a Communication Protocol in FOCUS - A Case Study -
- 342/01/95 A Hans-Joachim Bungartz: Higher Order Finite Elements on Sparse Grids
- 342/02/95 A Tao Zhang, Seonglim Kang, Lester R. Lipsky: The Performance of Parallel Computers: Order Statistics and Amdahl's Law
- 342/03/95 A Lester R. Lipsky, Appie van de Liefvoort: Transformation of the Kronecker Product of Identical Servers to a Reduced Product Space
- 342/04/95 A Pierre Fiorini, Lester R. Lipsky, Wen-Jung Hsin, Appie van de Liefvoort: Auto-Correlation of Lag-k For Customers Departing From Semi-Markov Processes
- 342/05/95 A Sascha Hilgenfeldt, Robert Balder, Christoph Zenger: Sparse Grids: Applications to Multi-dimensional Schrödinger Problems
- 342/06/95 A Maximilian Fuchs: Formal Design of a Model-N Counter
- 342/07/95 A Hans-Joachim Bungartz, Stefan Schulte: Coupled Problems in Microsystem Technology
- 342/08/95 A Alexander Pfaffinger: Parallel Communication on Workstation Networks with Complex Topologies
- 342/09/95 A Ketil Stølen: Assumption/Commitment Rules for Data-flow Networks - with an Emphasis on Completeness
- 342/10/95 A Ketil Stølen, Max Fuchs: A Formal Method for Hardware/Software Co-Design

Reihe A

- 342/11/95 A Thomas Schnekenburger: The ALDY Load Distribution System
- 342/12/95 A Javier Esparza, Stefan Römer, Walter Vogler: An Improvement of McMillan's Unfolding Algorithm
- 342/13/95 A Stephan Melzer, Javier Esparza: Checking System Properties via Integer Programming
- 342/14/95 A Radu Grosu, Ketil Stølen: A Denotational Model for Mobile Point-to-Point Dataflow Networks

SFB 342 : Methoden und Werkzeuge für die Nutzung paralleler
Rechnerarchitekturen

Reihe B

- 342/1/90 B Wolfgang Reisig: Petri Nets and Algebraic Specifications
- 342/2/90 B Jörg Desel: On Abstraction of Nets
- 342/3/90 B Jörg Desel: Reduction and Design of Well-behaved Free-choice Systems
- 342/4/90 B Franz Abstreiter, Michael Friedrich, Hans-Jürgen Plewan: Das
Werkzeug runtime zur Beobachtung verteilter und paralleler Programme
- 342/1/91 B Barbara Paechl: Concurrency as a Modality
- 342/2/91 B Birgit Kandler, Markus Pawlowski: SAM: Eine Sortier- Toolbox -
Anwenderbeschreibung
- 342/3/91 B Erwin Loibl, Hans Obermaier, Markus Pawlowski: 2. Workshop über
Parallelisierung von Datenbanksystemen
- 342/4/91 B Werner Pohlmann: A Limitation of Distributed Simulation Methods
- 342/5/91 B Dominik Gomm, Ekkart Kindler: A Weakly Coherent Virtually Shared
Memory Scheme: Formal Specification and Analysis
- 342/6/91 B Dominik Gomm, Ekkart Kindler: Causality Based Specification and
Correctness Proof of a Virtually Shared Memory Scheme
- 342/7/91 B W. Reisig: Concurrent Temporal Logic
- 342/1/92 B Malte Grosse, Christian B. Suttner: A Parallel Algorithm for Set-of-
Support
Christian B. Suttner: Parallel Computation of Multiple Sets-of-Support
- 342/2/92 B Arndt Bode, Hartmut Wedekind: Parallelrechner: Theorie, Hardware,
Software, Anwendungen
- 342/1/93 B Max Fuchs: Funktionale Spezifikation einer Geschwindigkeitsregelung
- 342/2/93 B Ekkart Kindler: Sicherheits- und Lebendigkeitseigenschaften: Ein Lit-
eraturüberblick
- 342/1/94 B Andreas Listl; Thomas Schnekenburger; Michael Friedrich: Zum En-
twurf eines Prototypen für MIDAS