Upper Bound to Interference Coordination with Channel State Information Outdating

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Abstract—We consider the network sum rate maximization in the downlink of a cellular system based on the spatial channel model of the 3GPP MIMO urban macro cell with multiple antenna base stations and single antenna mobile devices. We formulate an upper bound to the kind of cooperation, where each mobile device is served by only one base station, but base stations try to control the interference they produce. The benefits of higher possible data rates through interference mitigation are traded-off against a minimum overhead needed for realizing the required cooperation. The outdating of the channel state information is used to calculate an upper bound to the efficiency of the system. We show, that an increase in cooperation above a boundary does not lead to an increase in possible data rates.

I. INTRODUCTION

Inter-cell interference (ICI) is the dominant effect limiting the performance of today’s cellular networks. It is assumed, that this limitation can be overcome by letting base stations (BS) cooperate. Whereas a strong research focus is set on how cooperation should be organized, it is not proved so far that cooperation will be beneficial, if all implementation issues are taken into account. It is discussed intensely, whether cooperation should be included in future standards. The benefits of cooperation sound very promising, but very little is known about the drawbacks of cooperation. In this contribution, we will give a loose upper bound to cooperation by including the costs of channel state information (CSI) measurements. We restrict cooperation to synchronized scheduling, a predefined mobile device (MD) selection, and interference coordination, where each MD is only served by one BS.

Algorithms for cellular systems need to be designed for the specific model. Algorithms, which work very well for independent and identically-distributed channels, can completely fail for channels, which are strongly influenced by path-loss. The used system and channel model in this paper, are described in Section II.

In Section III, we describe the CSI measurements for information and interference channels and how we can assure interference awareness. To account for the costs of cooperation, we take the limitation of the block length by outdating into account, which is discussed in Section IV. The upper bound to interference coordination is derived in Section V and, in Section VI, we demonstrate that there is a limit to how much cooperation is beneficial.

II. SYSTEM MODEL

A cellular system with 19 sites, which employ three faced sectorization, is considered. Therefore, the system consists of 57 BSs. Each BS serves the MDs of the hexagonal shaped cell it covers. An MD in the set $K$ of all MDs is specified by the tuple $(b, k) \in K$, where $b \in B$ identifies the BS in the set $B$ of all BSs and $k \in K_b$ the MD in the set $K_b$ of all MDs in the cell of BS $b$.

In order to treat all cells equally, the wrap-around method is used. The 57 BSs are copied, including their beamforming, and placed six times around the central cluster. Each MD only sees the 57 BSs, which are closest by Euclidean distance. The cellular layout can be seen in Figure 1. The central cluster is inked slightly darker than the wrap-around clusters. The placement and orientation of the BSs is indicated by small arrows.

![Cellular Cluster with Wrap-Around](image)

The spatial channel model of the 3GPP MIMO urban macro cell with a distance of $500\,\text{m}$ between the two closest sites and a center frequency of $f_c = 2\,\text{GHz}$ is utilized [1].
In this paper, cooperation between the BSs is limited to interference coordination. Each MD is only served by one BS. But, BSs do not only try to adapt their beamforming to serve their own MDs with the maximum possible rate as in [2], they also try to avoid or limit the interference caused at other MDs. With this kind of cooperation the interference limitation of the system could be overcome and higher data rates might be possible.

Cooperation with a BS far away by distance will surely not improve the performance, as the produced interference rarely harms the rates. Therefore, we include a background interference in our system, which originates from the BSs further away than the 57 closest BSs. If we do not use the wrap-around method and extend the system to an unlimited number of BSs, this background interference will be the mean sum interference at a MD at the central site, from BS 58 to infinity. This background interference variance converges due to the pathloss and a fixed transmit power. With this background interference, the system is again subject to an interference limitation. The system is assumed to be operated in the interference limited region, where an increase in transmit power would not lead to an increase in the sum rate.

We assume that each BS has $N$ antennas and serves $K = |\mathcal{K}_b|$ single antenna MDs, respectively. The vectors $h_{b,b,k} \in \mathbb{C}^N$ contain the channel coefficients between the antennas of BS $b$ and MD $(b,k)$. With $(\cdot)^T$ and $(\cdot)^H$ we denote the transposition and complex conjugate transposition, respectively. The achievable, normalized rates of MD $(b,k)$ can be expressed as

$$r_{b,k} = \log_2 \left( 1 + \frac{|h_{b,b,k}^H p_{b,k}|^2}{\sigma_{b,k}^2 + |h_{b,b,k}^H p_{b,k}|^2 + \theta_{b,k}^2} \right), \quad (1)$$

$$\theta_{b,k}^2 = \sum_{b \in \mathcal{D}_b} |h_{b,b,k}^H q_{b,k}|^2 + \theta_{bg}^2, \quad (2)$$

where $p_{b,k} \in \mathbb{C}^N$ is the beamforming vector for MD $(b,k)$ and $Q_b = \sum_{b \in \mathcal{D}_b} p_{b,k} p_{b,k}^H \in \mathbb{C}^{N \times N}$ is the sum transmit covariance matrix of BS $b$. $\sigma_{b,k}^2$ is the variance of the noise, $|\sum_{c < k} h_{b,b,c}^H p_{b,k}|^2$ is the variance of the intracell interference if dirty paper coding is applied, and $\theta_{bg}^2$ is the variance of the received intercell interference. $\theta_{bg}^2$ denotes the background interference for a given signal variance per transmit antenna. Furthermore, a transmit power constraint $\text{tr}(Q_b) \leq P$ is imposed for every BS.

### III. CSI AND INTERFERENCE AWARENESS

To perform adaptive signal processing, instantaneous knowledge of CSI is required. For a non-cooperative sum rate maximization in each cell, knowledge of the channels from each BS to its own MDs is sufficient, respectively. If the BSs want to cooperate, additional information about interference channels has to be known.

The measurement of CSI differs in *time division duplex* (TDD) and *frequency division duplex* (FDD) systems. In both systems it is divided into two steps. In the first step, the channel vectors and in the second step the intercell interference plus noise (IIPN) are measured. After the first step, the BSs calculate their beamforming according to the measured channels. A measured or assumed IIPN variance at this stage can only indicate possible rates for the MDs. The moment the beamforming is applied, the IIPN and assumed rates are outdated. This effect distorts the DPC optimization and only with reliable IIPN and rates efficient coding can be applied.

In the second step, all BSs send orthogonal pilots with the calculated beamforming vectors. The number of pilot symbols $T_{2\text{nd}}$ can be considerably lower compared to the first phase, because the MDs have to estimate only a positive, real scalar instead of a complex vector. Now, the MDs can measure the actual IIPN variance. This value is fed back to the associated BS. Also, the associated feedback resources $T_{\text{sim fb}}$ for the second step are less than for the first step. After this, the BSs do not alter their beamforming anymore. But, with the 2nd pilot they can serve the MDs with ICI-aware rates.

#### A. TDD

In TDD systems, the reciprocity of the propagation channels is exploited. In the first step, the channels are measured in the uplink and the gained information is then utilized in the downlink. The MDs are split into equally sized subsets $\mathcal{K}_c$. The MDs within a subset use pilot sequences, which are orthogonal to each other. But, these pilot sequences are reused in all other subsets. Therefore, the pilot length is at least as large as the number of users in a subset $T_{\text{UL pilots}} \geq K + L = |\mathcal{K}_c|$, $\forall c$. Each BS can measure the channels to its own $K$ MDs and $L$ interference channels, additionally.

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<thead>
<tr>
<th>UL pilots</th>
<th>SNR fb</th>
<th>data</th>
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<tr>
<td>DL</td>
<td>2nd pilot</td>
<td>T_{2\text{nd}}</td>
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*Figure 2. TDD Signaling*

In Figure 2 the signaling scheme of TDD with the first and second step can be seen. $T_{\text{TDD data}}$ corresponds to the resources available for transmitting data, while $T_{\text{block}}$ are the resources within on block. In each block the channels are measured and the beamforming is adapted. Therefore, the block length is defined by the recurrence of piloting. The efficiency of this scheme is $\frac{T_{\text{TDD data}}}{T_{\text{block}}}$.

#### B. FDD

The signaling scheme for FDD can be seen in Figure 3. In the first step, the BSs are split into subclusters $B_c$. The BSs within a subcluster use pilots, which are orthogonal to each other, but these pilots are reused in all other subclusters. This is organized in such a way, that each MD can measure $C = |B_c|$, $\forall c$, channel vectors. The pilot length has to be at least as large as the number of transmit antennas in a
cluster $T_{DL,\text{pilots}} \geq NC$. Now, each MD could return all these channels, but to keep TDD and FDD comparable, we assume that only $K + L$ channels are fed back to every BS, respectively. Each MD feeds back the channel, over which it will be served, to its serving BS. Additionally, each BS receives the CSI of $L$ interference channels. The CSI is distributed either directly over the air to the interfering BS or via the serving BS and a backhaul network. This limitation to $L$ interference channels reduces the required feedback symbols $T_{\text{chan fb}}$, which are usually the dominant part of this signaling scheme.

**C. Efficiency Upper Bound**

In [3] it was shown, that there will be always less training symbols in a TDD system than in a FDD system, if further implementation issues are neglected. Therefore, the efficiency of the TDD system is higher and we will concentrate in the rest of the paper on TDD with $T_{\text{data}} = T_{\text{TDD data}}$.

Unlike [4], [5], we neglect interference, noise, quantization, and feedback errors during the pilot phase and assume all channel and IIPN measurements to be perfect. Therefore, the minimum pilot length can be used. For the rest of the paper, we will drop the pilot costs for the 2nd pilot and the IIPN feedback and set

$$T_{\text{data}} = T_{\text{block}} - (K + L).$$

(3)

The selection of the subsets $K_c$ in TDD or $B_c$ in FDD, the design of the pilot sequences, and the selection of the $L$ interference channels, which should be known to each BS, respectively, are formidable tasks. For the upper bound, we assume that all these problems are solved optimally by omisciently choosing the $L$ interference channels for each BS, respectively, which have the largest impact on the network sum rate.

![Figure 3. FDD Signaling](image)

**IV. CSI OUTDATING**

On the one hand, an increase in $L$ allows more complex cooperation and, therefore, allows higher rates during the data phase. On the other hand, this reduces the efficiency of the system, as $T_{\text{pilots}}$ has to increase. We include this effect by connecting the block length with the error, which is induced by the outdated of the CSI.

If we model the time and frequency variation as a first-order Markov process, the normalized error variance can be found as

$$\sigma_e^2(t, f) = \frac{E\left[\|\hat{h}_{b,b,k} - h_{b,b,k}(t, f)\|^2\right]}{E\left[\|\hat{h}_{b,b,k}\|^2\right]} = 1 - \rho^2(t, f).$$

(4)

$\hat{h}_{b,b,k}$ is the measured channel at a reference time $t_0$ and frequency $f_0$, $h_{b,b,k}(t, f)$ is the outdated channel at a slightly different time $t_0 + t$ and frequency $f_0 + f$, and $\rho(t, f)$ is the correlation coefficient between the two channels. This assumes that the precoding is adapted continuously to the outdated of the channel. The influence of this beamforming adaption will be ignored for the interference.

According to Jakes [6], the correlation can be approximated as

$$\rho(t, f) = \frac{J_0(2\pi f_D t)}{1 + (2\pi \sigma_{DS} f)^2},$$

(5)

where $J_0(\bullet)$ is the Bessel function of the first kind and zero-th order, $f_D$ is the maximum Doppler frequency, and $\sigma_{DS}$ is the root mean square delay spread.

For the upper bound, we calculate the minimum mean error variance

$$\sigma_e^2 = \frac{E}{T_{\text{block}}},$$

(6)

which is the minimum sum error $E$ during a block divided by the block length. The block length, the total amount of transmit symbols, is a virtual area in the time-frequency domain and the sum error is the integral over the error in this area. The beamforming is based on only one channel estimation per block. To minimize the sum error, the reference must lie in the center of the block length area. For a given block length this error minimizing area is enclosed by the curve $\hat{\rho}(T_{\text{block}}) = \rho(t, \hat{f})$, where $\hat{\rho}(T_{\text{block}})$ is the smallest, occurring correlation for the given block length. Respectively, the block length and minimum sum error can be calculated as

$$T_{\text{block}} = 4 \int_{0}^{\hat{f}_0} \int_{0}^{t_0} 1 \, df \, dt,$$

(7)

$$E = 4 \int_{0}^{\hat{f}_0} \int_{0}^{f_0} \sigma_e^2(t, f) \, df \, dt.$$  

(8)

In Figure 4 the block length area is shown, where $\hat{t}_0$ and $\hat{f}_0$ denote the maximum time and frequency distance to the reference, respectively.

**V. MAXIMUM RATE UNDER GIVEN CSI**

The cooperative network sum rate maximization can be formulated with (1) as

$$R_{\text{coop}} = \max_{(P_{b,k}, Q_{b,k}) \in \mathcal{K}} \sum_{(b,k) \in \mathcal{K}} r_{b,k},$$

(9)

s.t. $\text{tr}(Q_{b}) \leq P \forall b$.

Here, the beamforming vectors of all BSs in the central cluster are optimized jointly.

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<thead>
<tr>
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![Figure 3. FDD Signaling](image)
As described in Section III, only the $L$ interference channels per BS with the strongest influence on the network sum rate are known. Therefore, we can rewrite the intercell interference from (2) as

$$\theta_{b,k}^2 = \sum_{b \in C_{bh,k} \setminus b} h_{b,b,k}^H Q_b h_{b,b,k} + \sum_{b \in C_{bh,k}} h_{b,b,k}^H Q_b h_{b,b,k} + \theta_{bg}^2 = \sum_{b \in C_{bh,k} \setminus b} h_{b,b,k}^H Q_b h_{b,b,k} + \theta_{bg}^2$$

(10)

The first term in (11) is the sum of the interference over all measured and, therefore, known interference channels. The set $C_{bh,k}$ contains all BSs, which know the channel to the MD $(b, k)$. The second term in (11) is the sum interference over the unknown interference channels of the central cluster and $\theta_{bg}^2$ is the background interference. The channels in the last two terms are not known. Assuming a central node optimizing (9), we can say that $\theta_{mo,b,k}^2 = \sum_{b \in \mathcal{W} \setminus C_{bh,k}} h_{b,b,k}^H Q_b h_{b,b,k} + \theta_{bg}^2$ is known. It can be calculated by subtracting only known terms from $\theta_{b,k}^2$, which is known from the measurements in the second pilot phase. As we only know $\theta_{mo,b,k}^2$, we cannot optimize over the precoding vectors transmitting over the unknown interference channels.

Based on (9) and (12), we can formulate the upper bound as

$$\rho_{coop} \leq \rho_{upper} = \max_{\{p_{b,k} \forall (b,k) \in K\}} \sum_{(b,k) \in K} \hat{r}_{b,k},$$

(13)

with

$$\hat{r}_{b,k} = \log_2 \left( 1 + \frac{|h_{b,b,k}^T p_{b,k}|^2}{\sigma_{b,k}^2 + \sum_{k < b} |h_{b,b,k}^T p_{b,k}|^2 + \theta_{mo,b,k}} \right),$$

(14)

$$\text{s.t. } \text{tr}(Q_b) \leq P \forall b,$$

where we use the rates from (1), but set all measured interference channels to zero. This is clearly a non achievable upper bound as attenuating interference does normally not come for free. In contrast to interference coordination techniques like [7], [8], here, after nulling the interference channels each BS has still all degrees of freedom left for serving its MDs with $N$ antennas and DPC, respectively.

As the transmit vectors in $\theta_{mo,b,k}^2$ cannot be optimized without further channel measurements, problem (13) is convex and can be solved distributed at all BSs independently. There exist precoding vectors, which would result in a higher sum rate. But, these vectors cannot be found without further measurements. With the given CSI, $R_{upper}$ is a loose upper bound, which is always maximized with the maximum transmit power.

To include the cost of the interference channel measurements, we extend (13) with the efficiency

$$R_{sum} = \frac{T_{data}}{T_{block}} \max_{\{p_{b,k} \forall (b,k) \in K\}} \sum_{(b,k) \in K} \hat{r}_{b,k},$$

s.t. $\text{tr}(Q_b) \leq P \forall b$.

The outdating is included in the noise term

$$\sigma_{b,k}^2 = \sigma_e^2 + \sigma_{bg}^2 h_{b,b,k}^H Q_b h_{b,b,k} + \sigma_e^2 \sum_{b \in C_{bh,k} \setminus b} h_{b,b,k}^H Q_b h_{b,b,k},$$

(17)

where $\sigma_e^2$ is the thermal noise. The second term is the error due to the outdating of the serving channel and the third term is the error due to the outdating of the nulled interference channels.

Still, each BS uses all degrees of freedom to serve its MDs with DPC. But, the outdating of the serving channel is now included. The measured leakage channels are not set to zero, they are scaled down with the mean outdating error variance. Problem (16) is convex and can be solved, because the transmit vectors in $\theta_{mo,b,k}^2$ and $\sigma_{b,k}^2$ cannot be optimized with the given CSI. The maximum transmit power is always maximizing $R_{sum}$.

VI. SIMULATIONS

The following results are obtained with Monte Carlo simulations. The normalized average cell sum rate is plotted over the block length for different MD speeds. Every BS has $N = 4$ transmit antennas and transmits with $P = 83$ W. In every cell, $K = 10$ MDs are placed uniformly distributed and suffer from a thermal noise variance of $\sigma_e^2 = 8.3 \cdot 10^{-14}$ W at their receive antenna, respectively.

By not using the wrap-around method and extending the BS number to 500, we calculate a background interference of $\theta_{bg}^2 = 1.98 \cdot 10^{-11}$ W. At this point, the sum interference from BS 58 to 500 is converged. For this, all BSs use a scaled identity matrix as transmit covariance and we average over the MDs at the central site. For the simulations, we select the $L$ interference channels per BS sub-optimally, but still upper bounding because of the omnisciently given selection. The initial supported rates of each MD are computed with maximum ratio transmission and scaled identity matrices as transmit covariances for the interference producing BSs. In parallel, each BS selects the $L$ MDs, which will have the largest gain in rate, if the produced interference is nulled. The
optimization in (16) is solved with the worst case Gaussian noise assumption for $\theta_{n,h,k}$ and $\sigma^2_{n,k}$.

We fix the root mean square delay spread to $\sigma_{DS} = 0.5 \mu$s. The maximum Doppler frequency $f_D = f_C V_c / c$, with the center frequency $f_C$ and the speed of light $c$, directly depends on the MD speed $v$. In Figure 5 we show a low mobility scenario with a common MD device speed of $v = 3 \text{km/h}$. The different curves display the upper bound $R_{sum}$ per cell over the block length for different choices of $L$. $L = 0$ stands for no cooperation, an increase in $L$ represents an increase in cooperation. All curves ascend in the beginning for longer block lengths, because the efficiency of the system improves. At some point, each curve starts to descend, because the outdating of the channel degrades the possible rates. For each channel model, we can find a $L$ and a block length which maximizes the sum network throughput. In the low mobility scenario, the optimum lies in the range of $25 \leq L \leq 30$ and $250 \leq T_{block} \leq 350$. The upper bound with cooperation lies with almost 17 bits per channel usage (bpcu) much higher than the almost 11 bpcu without cooperation. We only take the costs for measuring the channels into account, but not the costs for mitigating the interference over these interference channels. Therefore, this upper bound cannot be reached, especially for a large $L$. But, already $L = 5$ shows significant improvements compared to no cooperation with 14 bpcu. In average, 4 MDs are served in every cell simultaneously. $L = 24$ could correspond to limiting the interference to the MDs of the surrounding 6 cells. With the optimal block length, additional cooperation to this leads to worse rates.

A high mobility scenario with a common MD device speed of $v = 30 \text{km/h}$ is shown in Figure 6. Here, the optimum lies around $L = 15$ and $T_{block} = 150$. The possible improvements through cooperation are much smaller than in the low mobility scenario. The upper bound with $L = 0$ is above 9 bpcu and with the best selection of $L$ around 11.5 bpcu. If cooperation can enhance the rate achieved in such a scenario, should be investigated in-depth.

VII. CONCLUSION

A loose, but new upper bound to the possible network sum rate in a cellular network could be formulated with perfect, but to a user subset restricted CSI. By nulling or scaling the known interference and treating the unknown interference as noise, the network sum rate maximization could be transformed to a convex problem, which can be solved. Based on this upper bound, on minimum overhead, on minimum outdating error, and on optimum CSI measurement and selection, we analyzed a limit to interference coordination. With this upper bound, cooperation can only be increased by additional CSI measurements. The necessary increase in the overhead and the associated lower efficiency of the system absorbs the gain of cooperation. Depending on the channel characteristics and a fixed block length, we showed that there is a limit for cooperation to be beneficial. An increase beyond this limit results in lower data rates due to the large overhead.

REFERENCES