Quantized CDI Based Tomlinson Harashima Precoding for Broadcast Channels

Israa Slim, Amine Mezghani and Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Technische Universität München, Munich, Germany
Email: {issl,amme,jono}@nws.ei.tum.de

***Abstract*—For the *Multiuser Multiple-Input Single-Output* (MU-MISO) system in a *downlink* (DL) scenario, we design a non-linear precoding scheme, namely the *spatial Tomlinson-Harashima precoder* (S-THP) based on the total *mean squared error* (MSE) criterion. The *channel state information* (CSI) available at the transmitter for precoding is the quantized *channel direction information* (CDI) relayed back in a *frequency division duplex* (FDD) system from each receiver using B feedback bits received error-free and quasi-instantaneous. The proposed MMSE-S-THP with a suboptimum precoding order clearly outperforms the linear precoding scheme given in [1] starting from a certain *Signal-to-Noise Ratio* (SNR) and a number of feedback bits in terms of *Bit Error Ratio* (BER).**

***Index Terms*—MU-MISO; Tomlinson Harashima Precoding; BC channels; Quantized and estimated CSI.**

I. Introduction

Systems with multiple transmit and receive antennas can increase the spectral efficiency and the supported data rates dramatically. In a *broadcast* (BC) scenario, i.e., the DL in a MU-MISO system, employing M antennas at the transmitter (e.g. the Base Station) and K decentralized receivers (e.g. mobile user terminals) with a single antenna each, and motivated by the need of simple receivers, transmit processing, i.e. precoding, has to be performed at the transmitter.

Linear precoding, e.g. *Zero Forcing* (ZF) and *Minimum MSE* (MMSE), has been investigated for such systems due to its simplicity [2]. Alternatively, non-linear precoding has received attention due to the increased throughput obtained as compared to the linear precoding. Tomlinson [3] Harashima [4] precoding belongs to a class of successive and non-linear precoding schemes where the already precoded symbols are fed back and the spatial interference is suppressed with a non-linear modulo operator.

As many precoding schemes, S-THP requires that perfect CSI has to be available at the transmitter so that the precoder is matched to the channel. However, perfect CSI is an unrealistic assumption in FDD systems. Due to independent channels in the DL and in the *uplink* (UL) in FDD systems, CSI is relayed to the transmitter through a dedicated limited feedback channel [5]. The K users first estimate their DL channels with a common DL pilot. Each of the K users then quantizes its CDI - for instance using *random vector quantization* (RVQ) [6] scheme - with B feedback bits which are relayed back to the transmitter. Hence, the latter has only access to a quantized version of the estimated CDI and to the statistics of the *Channel Magnitude Information* (CMI) of each user as transmit CSI [7].

In this work, we design a spatial TH-precoder based on minimizing the sum-MSE of the individual users (MMSE-S-THP) due to its mathematical tractability. Based on the available statistics of the CMI and the quantized CDI, received error-free but quasi-instantaneously, the optimum feedforward filter as well as the optimum feedback filter are computed at the transmitter. Additionally, a near optimal precoding order is found with a reduced complexity based on the derived filters. In [8] and [9], an MMSE-S-THP based on imperfect CSI where the quantization is performed using scalar quantization has been proposed. In [10], the MMSE-THP was derived based on an estimated CSI. To the best of our knowledge, an MMSE-S-THP based on quantized CDI using the RVQ scheme has not been yet investigated.

The rest of the paper is organized as follows. In Section II, we explain our system model and our channel model. In Section III, a review of the design of the MMSE linear precoder based on quantized CDI is provided. In Section IV, the design of the MMSE-S-THP based on quantized CDI is presented. Section V includes the simulation results and finally conclusions and future work are drawn in Section VI.

Throughout the paper $\mathrm{E}[\cdot]$ represents the expectation of the argument. The operators $\{\cdot\}^{\mathrm{T}}$, $\{\cdot\}^{*}$, $\{\cdot\}^{\mathrm{H}}$, $\mathrm{Re}\{\cdot\}$, $\mathrm{Im}\{\cdot\}$ stand for the transpose, conjugate, Hermitian, real part, and imaginary part of a complex number respectively. $\mathbf{1}_N$ is a column vector of identity elements of size N and $\boldsymbol{I}_N$ is the identity matrix of size $N \times N$. $\mathrm{tr}(\cdot)$ is the trace of a matrix. $\lfloor \cdot \rfloor$ denotes the floor operator which gives the largest integer smaller than or equal to the argument.

II. System and Channel Model

A. System Model

We consider, in a single isolated cell, a BS equipped with M transmit antennas and K decentralized users each having a single antenna such that $K \leq M$. The general form of this MU-MISO system with S-THP is given in Fig. 1, where $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_K]^{\mathrm{T}} \in \mathbb{C}^{K \times M}$ is the flat fading channel matrix. $\boldsymbol{h}_k \in \mathbb{C}^M$ is the channel from the BS to user k whose elements are i.i.d. zero-mean complex Gaussian random variables with variance γ_k^2.

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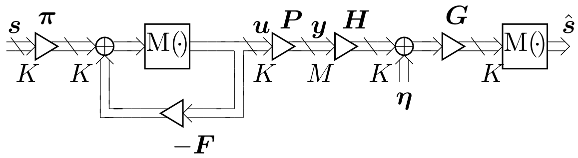

Fig. 1. Downlink System with THP

The input symbols $\boldsymbol{s} \in \mathbb{C}^K$ are drawn from a QAM modulation alphabet with zero mean and covariance matrix $\boldsymbol{R}_s = \mathrm{E}[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}] = \sigma_s^2 \boldsymbol{I}_K$ and are transformed by the precoding order $\boldsymbol{\pi}$, i.e., user π_k sees the interference caused by the data streams of users $\pi_{k+1} \ldots \pi_K$. The permutated symbols are then passed through the nonlinear feedback loop, which consists of the modulo operator $\mathrm{M}(\cdot)$ and the feedback filter $\boldsymbol{F}$, to get the symbols $\boldsymbol{u} \in \mathbb{C}^K$. The modulo operation is given in [2]

$$\mathrm{M}(x) = x - \left\lfloor \frac{\mathrm{Re}(x)}{\tau} + \frac{1}{2} \right\rfloor \tau - j \left\lfloor \frac{\mathrm{Im}(x)}{\tau} + \frac{1}{2} \right\rfloor \tau.$$

with $x \in \mathbb{C}$ and $\tau \in \mathbb{R}_+$ which is chosen depending on the modulation alphabet; for example $\tau = 2\sqrt{2}$ for QPSK symbols ($s_k \in \{\pm 1 \pm \mathrm{j}\}$) and $\tau = 8/\sqrt{10}$ for 16-QAM symbols. Since only already precoded symbols can be fed back, the structure of $\boldsymbol{F}$ has to be lower triangular with zero main diagonal:

$$\boldsymbol{F} = \begin{bmatrix} 0 & 0 & \ldots & 0 \\ f_{2,1} & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ f_{K,1} & \ldots & f_{K,K-1} & 0 \end{bmatrix} \in \mathbb{C}^{K \times K},$$

where $f_{k,j}$ is the entry at the k-th row and the j-th column ($k > j$) of $\boldsymbol{F}$. The outputs of the feedback loop $\boldsymbol{u}$ are computed successively as:

$$u_k = \mathrm{M}\Big(s_{\pi_k} - \sum_{j<k} f_{k,j} u_j \Big). \tag{1}$$

Then, we construct the transmit symbols:

$$\boldsymbol{y} = \boldsymbol{P}\boldsymbol{u} \in \mathbb{C}^M,$$

where $\boldsymbol{P} = [\boldsymbol{p}_1, \ldots, \boldsymbol{p}_K] \in \mathbb{C}^{M \times K}$ is the feedforward filter such that $\boldsymbol{p}_k$ is the precoder for the π_k-th user. The transmit symbols propagate over the flat fading channel and are perturbed by the additive white Gaussian noise (AWGN) $\boldsymbol{\eta} \in \mathbb{C}^K$ that has zero mean and covariance matrix $\boldsymbol{R}_\eta = E[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}] = \sigma_n^2 \boldsymbol{I}_K$. Since the receivers are decentralized, the receive matrix $\boldsymbol{G}$ is restricted to be diagonal with diagonal elements denoted as g_k. Each estimated symbol is thus given by:

$$\hat{s}_{\pi_k} = \mathrm{M}\left(g_k \boldsymbol{h}_{\pi_k}^{\mathrm{T}} \boldsymbol{y} + g_k \eta_{\pi_k} \right). \tag{2}$$

B. Channel Model

We adopt the same channel model as given in [1], where the entries of $\boldsymbol{h}_k$ are i.i.d complex Gaussian with variance γ_k^2. No detailed explanation is included in this section due to lack of space, and in [1] the model is explained in lengthiness. As given in (48) from [1], the DL channel of user k is:

$$\begin{aligned} \boldsymbol{h}_k &\overset{(a)}{=} \hat{\boldsymbol{h}}_k + \boldsymbol{e}_k \\ &\overset{(b)}{=} b_k \boldsymbol{h}_{n,k} + \boldsymbol{e}_k \\ &\overset{(c)}{=} b_k (c_k \boldsymbol{h}_{q,k} + \boldsymbol{e}_{q,k}) + \boldsymbol{e}_k \\ &= b_k c_k \boldsymbol{h}_{q,k} + b_k \boldsymbol{e}_{q,k} + \boldsymbol{e}_k. \end{aligned} \tag{3}$$

Briefly, (a) is the estimation step such that $\hat{\boldsymbol{h}}_k$ is the estimated channel obtained using a common pilot of length $T_{\mathrm{DL}} \geq M$ emitted from the BS. The variance of the estimation error is $\mathrm{E}\left[\boldsymbol{e}_k \boldsymbol{e}_k^{\mathrm{H}}\right] = \sigma_{e_k}^2 \boldsymbol{I}_K$, where [11]

$$\sigma_{e_k}^2 = \frac{\gamma_k^2}{1 + \gamma_k^2 \frac{P_{\mathrm{DL}}}{M \sigma_\eta^2} T_{\mathrm{DL}}},$$

P_{DL} represents the total transmit power available at the BS. (b) is the normalization step where $\boldsymbol{h}_{n,k}$ is the normalized channel and $b_k = ||\hat{\boldsymbol{h}}_k||_2$ that has the following statistics available at the BS [7]:

$$\mathrm{E}\left[b_k^{-2}\right] = \frac{\gamma_k^{-2}}{(M-1)(1-\sigma_{e_k}^2)}. \tag{4}$$

(c) is the quantization step where $\boldsymbol{h}_{q,k}$ is the quantized channel and $c_k = \boldsymbol{h}_{q,k}^{\mathrm{H}} \boldsymbol{h}_{n,k} \in \mathbb{C}$ such that $|c_k| = \cos\theta_k$, θ_k being the angle between $\boldsymbol{h}_{q,k}$ and $\boldsymbol{h}_{n,k}$. We have that:

$$\mathrm{E}[\cos^{-2}\theta_k] \approx \frac{1}{\mathrm{E}[\cos^2\theta_k]} \overset{[12]}{=} \frac{1}{1 - 2^B \mathrm{Beta}(2^B, \frac{M}{M-1})}, \tag{5}$$

where the approximation becomes tight for high number of feedback bits ($B > 6$). Moreover, $\arg(c_k) \neq 0$ [13] which has also to be considered in the design of the receiver.

For the quantization, we consider a RVQ scheme [6] where each user k has a codebook $\mathcal{C}_k$, also available at the BS, consisting of 2^B random vectors uniformly distributed on the complex unit sphere. The vector with the minimum chordal distance to $\boldsymbol{h}_{n,k}$ is chosen from $\mathcal{C}_k$ for $\boldsymbol{h}_{q,k}$. It is also worth mentioning that the quantization error $\boldsymbol{e}_{q,k}$ is orthogonal to $\boldsymbol{h}_{q,k}$, i.e., $\boldsymbol{h}_{q,k}^{\mathrm{H}} \boldsymbol{e}_{q,k} = 0$. The covariance matrix of $\boldsymbol{e}_{q,k}$ as given in [1] Eq. (62) is:

$$\mathrm{E}[\boldsymbol{e}_{q,k}^{\mathrm{H}} \boldsymbol{e}_{q,k} | \boldsymbol{h}_{q,k}] = \frac{\mathrm{E}[\sin^2\theta_k]}{M-1} (\mathbf{1}_M - \boldsymbol{h}_{q,k}^* \boldsymbol{h}_{q,k}^{\mathrm{T}}), \tag{6}$$

where

$$\mathrm{E}[\sin^2\theta_k] = 1 - \mathrm{E}[\cos^2\theta_k].$$

As already mentioned, each receiver k knows its own estimated channel $\hat{\boldsymbol{h}}_k$ while the transmitter knows all quantized channel vectors $\boldsymbol{h}_{q,k}$ which don't include information about the channel norms. In other words, the quantization coefficient c_k and the amplitude b_k are not known at the transmitter. To get rid of these unknown variables at the transmitter, it has been proposed in [1] to take the receivers g_k as follows:

$$g_k = c_k^{-1} b_k^{-1} g_{\text{THP/lin}}, \tag{7}$$

where a common scalar $g_{\text{THP/lin}}$ for all users is introduced as an additional degree of freedom for the precoder optimization.

III. Review of MMSE Filter for Quantized CDI

In this section, we review the MMSE linear precoding from [1]. Note that a MU-MISO system in Fig. 1 without the precoding order $\boldsymbol{\pi}$, the feedback loop at the transmitter, and the modulo operator $\mathrm{M}(\cdot)$ at the receiver is that for a linear precoding system. Based on the quantized CDI and the CMI statistics available at the BS, the optimum MMSE receive and precoding filters as given in (25) and (26) in [1] are respectively:

$$g_{\text{lin}} = \sqrt{\frac{\operatorname{tr}\left(\left((1-\kappa)\boldsymbol{H}_q^{\mathrm{H}}\boldsymbol{H}_q+\xi\mathbf{I}_M\right)^{-2}\boldsymbol{H}_q^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{s}}\boldsymbol{H}_q\right)}{P_{\mathrm{DL}}}} \quad (8)$$

$$\boldsymbol{P}_{\text{lin}} = \frac{1}{g_{\text{lin}}}\left((1-\kappa)\boldsymbol{H}_q^{\mathrm{H}}\boldsymbol{H}_q+\xi\mathbf{I}_M\right)^{-1}\boldsymbol{H}_q^{\mathrm{H}}, \quad (9)$$

where

$$\kappa = \frac{\mathrm{E}[\tan^2\theta]}{M-1}, \quad \text{and} \quad (10)$$

$$\xi = K\kappa + \sum_k \frac{\mathrm{E}[\cos^{-2}\theta]\gamma_k^{-2}}{(M-1)(1-\sigma_{e_k}^2)}\left(\sigma_{e_k}^2+\frac{\sigma_\eta^2}{P_{\mathrm{DL}}}\right). \quad (11)$$

$\boldsymbol{H}_q = [\boldsymbol{h}_{q,1},\boldsymbol{h}_{q,2},\ldots,\boldsymbol{h}_{q,K}]^{\mathrm{T}} \in \mathbb{C}^{K\times M}$ is the quantized channel matrix available at the transmitter.

IV. Spatial MMSE THP design for Quantized CDI

The optimum $\boldsymbol{P}_{\text{THP}}$, g_{THP}, $\boldsymbol{F}$, and $\boldsymbol{\pi}$ for the system in Fig. 1 are found by minimizing the sum of the MSEs of the individual users, ε_{π_k}, subject to a total transmit power constraint, i.e.,

$$\min_{\boldsymbol{P}_{\text{THP}},g_{\text{THP}},\boldsymbol{F}}\sum_{k=1}^{K}\varepsilon_{\pi_k} \ \text{ s.t. } \operatorname{tr}(\boldsymbol{P}_{\text{THP}}\boldsymbol{R}_u\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \le P_{\mathrm{DL}}, \quad (12)$$

with $\boldsymbol{R}_u = \mathrm{E}[\boldsymbol{u}\boldsymbol{u}^{\mathrm{H}}] = \sigma_u^2\boldsymbol{I}_K$. Because of the modulo operator at the transmitter, the statistics of s_{π_k} differ from that of u_k which has a variance $\sigma_u^2 = \frac{\tau^2}{6} \ge \sigma_s^2$ [2]. The outputs of the modulo operators are as well uncorrelated.

To solve this optimization problem, we need first to compute the individual MSE of the π_k-th user denoted by ε_{π_k} in what follows.

When substituting (3) in (2), the estimated symbol of the π_k-th user becomes:

$$\begin{aligned}\hat{s}_{\pi_k} &= \mathrm{M}\Big(g_k b_{\pi_k} c_{\pi_k}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} + g_k b_{\pi_k}\boldsymbol{e}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} \\ &\quad + g_k\boldsymbol{e}_{\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} + g_k\eta_{\pi_k}\Big). \quad (13)\end{aligned}$$

However, due to (7), (13) results in:

$$\begin{aligned}\hat{s}_{\pi_k} &= \mathrm{M}\Big(g_{\text{THP}}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} + g_{\text{THP}}c_{\pi_k}^{-1}\boldsymbol{e}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} \\ &\quad + g_{\text{THP}}c_{\pi_k}^{-1}b_{\pi_k}^{-1}\boldsymbol{e}_{\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} + g_{\text{THP}}c_{\pi_k}^{-1}b_{\pi_k}^{-1}\eta_{\pi_k}\Big),\end{aligned}$$

which when adding and subtracting the term $\sum_{j<k} f_{k,j}u_j$ becomes:

$$\begin{aligned}\hat{s}_{\pi_k} &= \mathrm{M}\Big(s_{\pi_k} - u_k - \sum_{j<k} f_{k,j}u_j \\ &\quad + g_{\text{THP}}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} + g_{\text{THP}}c_{\pi_k}^{-1}\boldsymbol{e}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} \\ &\quad + g_{\text{THP}}c_{\pi_k}^{-1}b_{\pi_k}^{-1}\boldsymbol{e}_{\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} + g_{\text{THP}}c_{\pi_k}^{-1}b_{\pi_k}^{-1}\eta_{\pi_k}\Big),\end{aligned}$$

where (1) was used. Neglecting the modulo-loss, the error $e_{\pi_k} = \hat{s}_{\pi_k} - s_{\pi_k}$ can be given as:

$$\begin{aligned}e_{\pi_k} &= -u_k - \sum_{j<k} f_{k,j}u_j + g_{\text{THP}}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} \\ &\quad + g_{\text{THP}}c_{\pi_k}^{-1}\boldsymbol{e}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} + g_{\text{THP}}c_{\pi_k}^{-1}b_{\pi_k}^{-1}\boldsymbol{e}_{\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{u} \\ &\quad + g_{\text{THP}}c_{\pi_k}^{-1}b_{\pi_k}^{-1}\eta_{\pi_k}. \quad (14)\end{aligned}$$

Based on (14), the individual MSE ε_{π_k} can be expressed as:

$$\begin{aligned}\varepsilon_{\pi_k} &= \mathrm{E}[e_{\pi_k}e_{\pi_k}^{\mathrm{H}}|\boldsymbol{H}_q] = \sigma_u^2\Big(1 + \sum_{j<k}|f_{k,j}u_j|^2 \\ &\quad + |g_{\text{THP}}|^2\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}\boldsymbol{h}_{q,\pi_k}^{*} - 2\,\mathrm{Re}(g_{\text{THP}}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{p}_k) \\ &\quad - 2\,\mathrm{Re}(g_{\text{THP}}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\sum_{j<k}\boldsymbol{p}_j f_{k,j}^{*}) + |g_{\text{THP}}|^2\kappa\operatorname{tr}(\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \\ &\quad - |g_{\text{THP}}|^2\kappa\operatorname{tr}(\boldsymbol{h}_{q,\pi_k}^{*}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \\ &\quad + |g_{\text{THP}}|^2\sigma_{e_k}^2\mathrm{E}[\cos^{-2}\theta_{\pi_k}]\mathrm{E}[b_{\pi_k}^{-2}]\operatorname{tr}(\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}})\Big) \\ &\quad + |g_{\text{THP}}|^2\mathrm{E}[\cos^{-2}\theta_{\pi_k}]\mathrm{E}[b_{\pi_k}^{-2}]\sigma_\eta^2, \quad (15)\end{aligned}$$

where the linearity property of the trace operator has been used for the computation of:

$$\begin{aligned}\mathrm{E}[\boldsymbol{e}_{\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}\boldsymbol{e}_{\pi_k}^{*}] &= \sigma_{e_k}^2\operatorname{tr}(\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \quad \text{and,} \\ \mathrm{E}[\boldsymbol{e}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}\boldsymbol{e}_{q,\pi_k}^{*}|\boldsymbol{H}_q] &= \operatorname{tr}\left(\frac{\mathrm{E}[\sin^2\theta_{\pi_k}]}{M-1}\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}\right) - \\ &\quad \operatorname{tr}\left(\frac{\mathrm{E}[\sin^2\theta_{\pi_k}]}{M-1}\boldsymbol{h}_{q,\pi_k}^{*}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}\right).\end{aligned}$$

$\mathrm{E}[b_{\pi_k}^{-2}]$, $\mathrm{E}[\cos^{-2}\theta_{\pi_k}]$, and κ are given in (4), (5) and (10) respectively.

Based on (15), we can formulate the Lagrangian function for the optimization problem (12) with the Lagrangian multiplier $\lambda \in \mathbb{R}_+$

$$\mathrm{L}(\boldsymbol{p}_k, g_{\text{THP}}, f_{k,j}) = \sum_k \varepsilon_{\pi_k} + \lambda(\sigma_u^2\operatorname{tr}(\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) - P_{\mathrm{DL}}). \quad (16)$$

When differentiating (16) with respect to $f_{k,j}$, and setting it to zero, we get the optimal feedback matrix elements

$$f_{k,j} = g_{\text{THP}}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{p}_j. \quad (17)$$

Upon substituting (17) in (15), we get the individual MSE expression:

$$\begin{aligned}\varepsilon_{\pi_k} &= \sigma_u^2\Big(1 + |g_{\text{THP}}|^2\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\sum_{j\ge k}\boldsymbol{p}_j\boldsymbol{p}_j^{\mathrm{H}}\boldsymbol{h}_{q,\pi_k}^{*} \\ &\quad - 2\,\mathrm{Re}(g_{\text{THP}}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{p}_k) + |g_{\text{THP}}|^2\kappa\operatorname{tr}(\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \\ &\quad - |g_{\text{THP}}|^2\kappa\operatorname{tr}(\boldsymbol{h}_{q,\pi_k}^{*}\boldsymbol{h}_{q,\pi_k}^{\mathrm{T}}\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \\ &\quad + |g_{\text{THP}}|^2\sigma_{e_k}^2\mathrm{E}[\cos^{-2}\theta_{\pi_k}]\mathrm{E}[b_{\pi_k}^{-2}]\operatorname{tr}(\boldsymbol{P}_{\text{THP}}\boldsymbol{P}_{\text{THP}}^{\mathrm{H}})\Big) \\ &\quad + |g_{\text{THP}}|^2\mathrm{E}[\cos^{-2}\theta_{\pi_k}]\mathrm{E}[b_{\pi_k}^{-2}]\sigma_\eta^2. \quad (18)\end{aligned}$$

We formulate the KKT equations with respect to g_{THP} and $\boldsymbol{p}_k$ as:

$$\begin{aligned}\frac{\partial \mathrm{L}(\boldsymbol{P}_{\text{THP}}, g_{\text{THP}}, \lambda)}{\partial g_{\text{THP}}} = & \sum_k \Big[\sigma_u^2 \Big(g_{\text{THP}}^* \boldsymbol{h}_{q,\pi_k}^{\mathrm{T}} \sum_{j\geq k} \boldsymbol{p}_j \boldsymbol{p}_j^{\mathrm{H}} \boldsymbol{h}_{q,\pi_k}^* \\ & - \boldsymbol{h}_{q,\pi_k}^{\mathrm{T}} \boldsymbol{p}_k + g_{\text{THP}}^* \kappa \mathrm{tr}(\boldsymbol{P}_{\text{THP}} \boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \\ & - g_{\text{THP}}^* \kappa \mathrm{tr}(\boldsymbol{h}_{q,\pi_k}^* \boldsymbol{h}_{q,\pi_k}^{\mathrm{T}} \boldsymbol{P}_{\text{THP}} \boldsymbol{P}_{\text{THP}}^{\mathrm{H}}) \\ & + g_{\text{THP}}^* \sigma_{e_k}^2 \mathrm{E}[\cos^{-2}\theta_{\pi_k}] \mathrm{E}[b_{\pi_k}^{-2}] \mathrm{tr}(\boldsymbol{P}_{\text{THP}} \boldsymbol{P}_{\text{THP}}^{\mathrm{H}})\Big) \\ & + g_{\text{THP}}^* \mathrm{E}[\cos^{-2}\theta_{\pi_k}] \mathrm{E}[b_{\pi_k}^{-2}] \sigma_\eta^2\Big] = 0, \qquad (19)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathrm{L}(\boldsymbol{P}_{\text{THP}}, g_{\text{THP}}, \lambda)}{\partial \boldsymbol{p}_k} = & \sigma_u^2 \Big(|g_{\text{THP}}|^2 \sum_{j\leq k} \boldsymbol{h}_{q,\pi_j} \boldsymbol{h}_{q,\pi_j}^{\mathrm{H}} \boldsymbol{p}_k^* - g_{\text{THP}} \boldsymbol{h}_{q,\pi_k} \\ & + |g_{\text{THP}}|^2 K \kappa \boldsymbol{p}_k^* - |g_{\text{THP}}|^2 \kappa \boldsymbol{H}_q^{\mathrm{T}} \boldsymbol{H}_q^* \boldsymbol{p}_k^* \\ & + \sum_j |g_{\text{THP}}|^2 \sigma_{e_j}^2 \mathrm{E}[\cos^{-2}\theta_{\pi_j}] \mathrm{E}[b_{\pi_j}^{-2}] \boldsymbol{p}_k^* \\ & + \lambda \boldsymbol{p}_k^*\Big) = \boldsymbol{0}_M. \qquad (20)\end{aligned}$$

Summing the complex conjugate of the derivative with respect to $\boldsymbol{p}_k$ from (20), then multiplying it with $\boldsymbol{P}_{\text{THP}}^{\mathrm{H}}$ from the right, and finally applying the trace operator, while multiplying (19) with g_{THP}, we obtain the Lagrangian multiplier:

$$\lambda = |g_{\text{THP}}|^2 \frac{K\sigma_\eta^2}{\sigma_u^2 \mathrm{tr}(\boldsymbol{P}_{\text{THP}} \boldsymbol{P}_{\text{THP}}^{\mathrm{H}})} = |g_{\text{THP}}|^2 \frac{\mathrm{tr}(\boldsymbol{R}_\eta)}{P_{\text{DL}}},$$

which when substituted in the second KKT condition leads to the following expression for $\boldsymbol{p}_k$:

$$\boldsymbol{p}_k = \frac{1}{g_{\text{THP}}} \left(\sum_{j\leq k} \boldsymbol{h}_{q,\pi_j}^* \boldsymbol{h}_{q,\pi_j}^{\mathrm{T}} - \kappa \boldsymbol{H}_q^{\mathrm{H}} \boldsymbol{H}_q + \xi \boldsymbol{I}_M\right)^{-1} \boldsymbol{h}_{q,\pi_k}^*, \qquad (21)$$

where κ and ξ are given in (10) and (11) respectively. Due to the transmit power constraint, we finally get

$$g_{\text{THP}} = \sqrt{\frac{\sigma_u^2}{P_{\text{DL}}} \sum_k \boldsymbol{h}_{q,\pi_k}^{\mathrm{T}} \left(\sum_{j\leq k} \boldsymbol{h}_{q,\pi_j}^* \boldsymbol{h}_{q,\pi_j}^{\mathrm{T}} - \kappa \boldsymbol{H}_q^{\mathrm{H}} \boldsymbol{H}_q + \xi \boldsymbol{I}_M\right)^{-2} \boldsymbol{h}_{q,\pi_k}^*}. \qquad (22)$$

With these optimum filter expressions, the resulting sum-MSE $\mathrm{MSE}_{\text{THP}}$ reads as:

$$\mathrm{MSE}_{\text{THP}} = \sigma_u^2 \left[K - \sum_k \boldsymbol{h}_{q,\pi_k}^{\mathrm{T}} \Big(\sum_{j\leq k} \boldsymbol{h}_{q,\pi_j}^* \boldsymbol{h}_{q,\pi_j}^{\mathrm{T}} - \kappa \boldsymbol{H}_q^{\mathrm{H}} \boldsymbol{H}_q + \xi \boldsymbol{I}_M\Big)^{-1} \boldsymbol{h}_{q,\pi_k}^*\right]. \qquad (23)$$

The problem of finding the optimum ordering $\boldsymbol{\pi}$ minimizing $\mathrm{MSE}_{\text{THP}}$ is NP hard, because we must check all $K!$ possible permutations. To reduce complexity, we minimize each summand successively, i.e. π_k is chosen under the assumption that $\pi_{k+1}, ..., \pi_K$ are fixed:

$$\pi_k = \underset{i \notin \pi_{k+1},...,\pi_K}{\mathrm{argmax}}\ \boldsymbol{h}_{q,i}^{\mathrm{T}} \left(\sum_{j\leq k} \boldsymbol{h}_{q,\pi_j}^* \boldsymbol{h}_{q,\pi_j}^{\mathrm{T}} - \kappa \boldsymbol{H}_q^{\mathrm{H}} \boldsymbol{H}_q + \xi \boldsymbol{I}_M\right)^{-1} \boldsymbol{h}_{q,i}^*. \qquad (24)$$

V. Simulation Results

For a MU-MISO system with 4 transmit antenna, i.e., $M = 4$ and 4 users, i.e., $K = 4$ and based on the proposed MMSE-S-THP with quantized CDI, we plot the uncoded BER vrs. different SNR values which is defined as $\frac{P_{\text{DL}}}{\mathrm{tr}(\boldsymbol{R}_\eta)}$ in dB. The results are the average of 10000 i.i.d. complex Gaussian channel realizations ($\gamma_k^2 = 1, \forall k$) where 100 QAM symbols are sent per channel. Other system parameters include the length of the DL pilots which is $T_{\text{DL}} = 10$ symbols and the assumption of white signals and white noise i.e. $\sigma_s^2 = 1$ and $\sigma_\eta^2 = 1$ respectively. For performance comparison, the BER of the systems with linear MMSE precoder based on quantized CDI (c.f. (8) and (9)), linear MMSE precoder based on perfect CDI ($B \to \infty$, $\kappa \to 0$ in (10) and $\xi \to \frac{K}{P_{\text{DL}}(M-1)}$ in (11)) and MMSE-S-THP with perfect CDI are as well plotted for QPSK and 16-QAM in Fig. 2 and Fig. 3 respectively. As expected, MMSE-S-THP approaches clearly outperform the linear ones for any CSI available above a certain SNR threshold in both figures. One can see, as well for both modulation schemes, how the performance increases with the increase of B but on the expense of increased feedback resources. The BER saturation for different values of B at high SNR is due to the quantization errors. In Fig. 2, for low SNR values, this performance loss of the MMSE-S-THP is due to both the modulo operator at the receiver (implicating the generation of new neighbors) and due to the increased power at the output of the modulo operators incurring a transmit power loss of 1.25 dB ($10\log_{10}\frac{\tau^2}{6}$) for QPSK modulation scheme. However, due to the higher number of constellation points in the case of 16-QAM modulation scheme, the modulo operator at the receiver has less effect than that for the QPSK symbols case as can be seen in Fig. 3. This reinforces the fact that the THP performance depends as well on the modulation scheme.

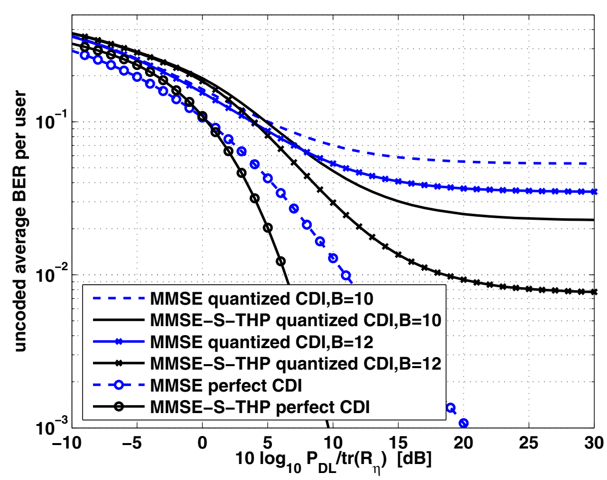

Fig. 2. QPSK: uncoded BER vrs. $10\log_{10}\frac{P_{\text{DL}}}{\mathrm{tr}(\boldsymbol{R}_\eta)}$ for different transmit precoding schemes and different transmit CSI

For the QPSK case, the SNR range where the linear MMSE precoder scheme outperforms the MMSE-S-THP precoder

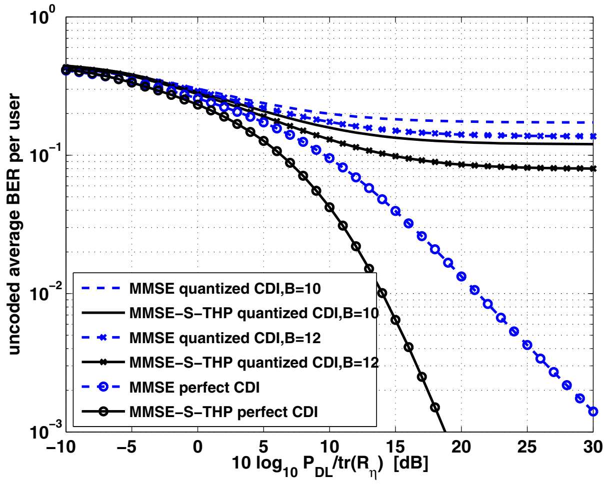

Fig. 3. 16-QAM: uncoded BER vrs. $10\log_{10}\frac{P_{\mathrm{DL}}}{\mathrm{tr}(\boldsymbol{R}_\eta)}$ for different transmit precoding schemes and different transmit CSI

scheme highly depends on B. In Fig. 4, we specify these regions where for some specific B and SNR values, linear schemes outperform the non-linear ones. For the 16-QAM case, this dependency becomes weaker since the modulo operator has less impact in that case.

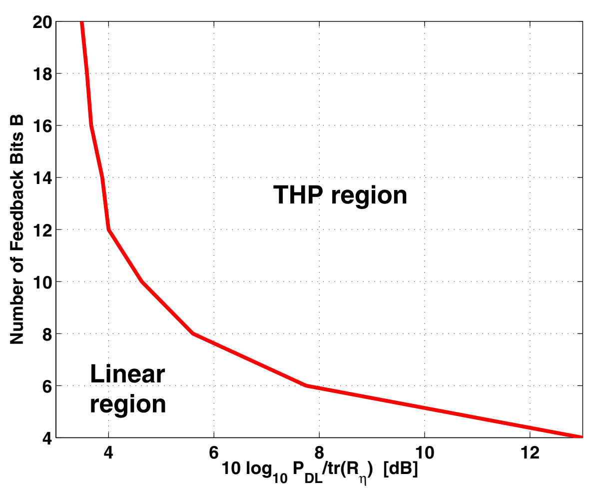

Fig. 4. Number of Feedback Bits B vrs. $10\log_{10}\frac{P_{\mathrm{DL}}}{\mathrm{tr}(\boldsymbol{R}_\eta)}$: The MMSE and the MMSE-S-THP regions

VI. Conclusions and Future Work

We proposed an MMSE-S-THP for the DL of a MU-MISO system based on quantized CDI and on the statistics of the CMI as transmit CSI using the total MSE criterion. The quantized CDI, which is chosen from a RVQ codebook, are relayed back from each user to the BS using B feedback bits on a dedicated feedback channel. The BS then calculates the feedforward filter and the feedback filter necessary for a S-THP based on the quantized CDI. We also showed how the non-linearities of the modulo operators at the transmitter and the receiver degrade the performance of the MMSE-S-THP as compared to the linear MMSE precoder at low SNR values for the case of QPSK modulation scheme. However, in moderate to high SNR regions, and depending on B, MMSE-S-THP clearly outperforms the linear MMSE.

For this system and as a future work, resource allocation, i.e., the optimum number of B, T_{DL} and T_{UL} (the length of UL pilots) can be found based on maximizing the capacity lower bound which can be easily derived based on the MSE expression. In this work, we considered just the case $K \leq M$ with single antenna at the receiver. Future work can include $K > M$ cases and the MIMO cases where a scheduler might be included at the transmitter. Both scalar (uniform and non-uniform) quantizers and the optimum vector quantizer are also a matter of investigations for MU-MIMO systems.

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