MMSE Based FDD Overhead Optimization for a Multiuser Two-way System

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***Abstract*—Based on maximizing the sum of lower bounds derived for the *uplink* (UL) and the *downlink* (DL) rates with imperfect *Channel State Information* (CSI), overhead optimization i.e. finding the optimum lengths of the UL and DL pilots and feedback bits, is performed. Estimated CSI and quantized *Channel Direction Information* (CDI), which may be received erroneously, are available at the *Base Station* (BS) for computing the receive and the transmit *Wiener* filters (WF), respectively. Using large system analysis, we were able to derive analytical DL and UL *mean squared error* (MSE) expressions that give a lower bound of the DL and UL rates, respectively, needed for our optimization.**

***Index Terms*—Multiple Access Channel (MAC); Broadcast Channel (BC); Quantized Channel Direction Information; Estimated Channel State Information; limited Feedback; Wiener filter; Overhead Optimization.**

I. Introduction

Multiple antenna wireless communication systems provide considerable channel capacity assuming perfect CSI at the BS. This is a non-realistic assumption in *Frequency Division Duplex* (FDD) systems since the UL and the DL channels are subject to independent fading. Moreover, motivated by the need of simple *Mobile Stations* (MSs), transmit and receive processing are performed at the BS which thus requires the availability of transmit and receive CSI. For this, T_{UL} pilots are needed in the UL to estimate the UL channel. In the DL, each user estimates its channel with T_{DL} DL pilots, then normalizes its estimated channel and finally quantizes this normalized version with B bits, that are fed back to the BS using some resources in the UL that may be received erroneously. Thus, a quantized version of the DL channel is available at the BS.

The high complexity of the capacity-achieving non-linear transmit and receive processing schemes [1] limits the use of these methods especially with imperfect CSI cases. In this paper, we suggest that the BS performs linear receive and transmit processing since it provides a good trade-off between performance and complexity. Namely, transmit and receive Wiener (or minimum MSE) filters for the UL and DL are derived based on estimated CSI in the UL and quantized CDI in the DL, respectively. Using large system analysis, analytical UL and DL MSE expressions are derived that lead to a lower bound of the UL and DL capacities respectively since capacity with imperfect CSI is not known.

In a two-way system, besides the trade-off between training and data payload on each link, we have an additional trade-off between the UL and the DL rates due to feedback. However, only limited work considered overhead optimization (i.e. the optimum lengths of T_{UL}, T_{DL} and B) for these systems or even for one-way systems. For example, a one-way system is considered in [2] where the number of MSs is assumed to be equal the number of antennas at the BS but requires a second dedicated pilot phase in the DL. A two-way system optimization was done in [3] but for the single user case. A two-way system optimization for the multiuser case with *Zero-forcing* (ZF) linear filters was presented in [4] but has the restriction that the number of antennas at the BS is strictly greater than the number of users. In [5], the bandwidth is partitioned between the UL, DL and feedback in an optimized way for FDD single-user case.

In this paper, given a fixed packet size T, we find the optimum lengths of the UL pilots, DL pilots and feedback that maximize the sum of the lower bounds of the UL and the DL rates as a figure of merit. The lower bounds of the UL and DL rates are based on the UL and the DL WF-MSE obtained according to certain system model assumptions, i.e., estimated CSI in the UL and quantized CDI, that may be received erroneously, in the DL respectively. This MSE-based criterion is simpler than the *Signal to Interference plus Noise Ratio* (SINR) method presented in [2] and does not impose any constraint on the number of antennas at the BS and users as what was assumed in [4].

The rest of the paper is organized as follows. In Section II, we explain our UL and DL systems. Section III presents the optimization problem to be tackled. Section IV shows the results of the optimization. Finally, concluding remarks and future work are presented in Section V.

Throughout the paper, lower-case and capital bold letters are used to denote vectors and matrices, respectively. $\mathrm{E}[\cdot]$ represents the expectation of the argument. The operator $\{\cdot\}^{\mathrm{H}}$ stands for the Hermitian of a complex number. $\mathrm{tr}(\cdot)$ is the trace of a matrix. $\boldsymbol{I}_N$ is the identity matrix of size $N \times N$. The operator $\mathrm{diag}(\boldsymbol{a})$ with $\boldsymbol{a} \in \mathbb{C}^N$ results in $N \times N$ matrix with all zeros except on the diagonal, which contains the elements of $\boldsymbol{a}$.

II. System Model

We consider in a single isolated cell a BS which is equipped with M antennas and K single-antenna decentralized MSs.

A. Uplink Model

The UL, where the MSs are the transmitters and the BS is the receiver, can be modeled as a *multiuser - single input multiple output* (MU-SIMO) system since the users are decentralized. The UL channel is expressed in a matrix form as $\boldsymbol{H}_{\text{UL}} = [\boldsymbol{h}_{\text{UL},1}, \ldots, \boldsymbol{h}_{\text{UL},K}] \in \mathbb{C}^{M\times K}$, where $\boldsymbol{h}_{\text{UL},k} \in \mathbb{C}^{M}$ is the channel from each user k to the BS whose elements are i.i.d. zero-mean complex Gaussian random variables with unit variance. The BS estimates the UL channel of each user by K dedicated orthogonal UL pilots of length $T_{\text{UL}} \geq K$. The BS has thus access to the estimated channel of each user k denoted as $\hat{\boldsymbol{h}}_{\text{UL,k}} \in \mathbb{C}^{M}$. The UL channel, called also the *Multiple Access Channel* (MAC), can be thus written after being estimated as:

$$\boldsymbol{H}_{\text{UL}} = \hat{\boldsymbol{H}}_{\text{UL}} + \boldsymbol{E}_{\text{UL}},$$

where $\hat{\boldsymbol{H}}_{\text{UL}} = [\hat{\boldsymbol{h}}_{\text{UL,1}}, \ldots, \hat{\boldsymbol{h}}_{\text{UL,K}}] \in \mathbb{C}^{M\times K}$ is the estimated UL channel available at the BS and $\boldsymbol{E}_{\text{UL}} = [\boldsymbol{e}_{\text{UL},1}, \ldots, \boldsymbol{e}_{\text{UL},K}] \in \mathbb{C}^{M\times K}$ is the estimation error matrix such that $\boldsymbol{e}_{\text{UL},k}$ represents the estimation error vector of the channel for user k which has zero mean and the following variance [6]:

$$\sigma^2_{e_{\text{UL}}} = \frac{1}{1 + \frac{P_{\text{UL}}}{\sigma^2_{n,\text{UL}}} T_{\text{UL}}},$$

where P_{UL} is the available transmit power at each user and $\sigma^2_{n,\text{UL}}$ is the variance of the *Additive White Gaussian Noise* (AWGN) at each antenna at the BS in the uplink.

Having access to $\hat{\boldsymbol{H}}_{\text{UL}}$, the BS is able to perform receive processing such that the receive WF filter $\boldsymbol{G} \in \mathbb{C}^{K\times M}$ at the BS and the scalar transmit filter p at each MS are computed by minimizing the MSE between the transmitted and received symbols under a total transmit power constraint. These filters can be given as:

$$\boldsymbol{G} = \sqrt{\sigma^2_{s,\text{UL}} P_{\text{UL}}}\, \hat{\boldsymbol{H}}^{\text{H}}_{\text{UL}} \left(P_{\text{UL}} \hat{\boldsymbol{H}}_{\text{UL}} \hat{\boldsymbol{H}}^{\text{H}}_{\text{UL}} + (K P_{\text{UL}} \sigma^2_{e_{\text{UL}}} + \sigma^2_{n,\text{UL}}) \boldsymbol{I}_M \right)^{-1},$$

$$p = \sqrt{\frac{P_{\text{UL}}}{\sigma^2_{s,\text{UL}}}} \in \mathbb{R}_+,$$

where $\sigma^2_{s,\text{UL}}$ is the variance of the transmitted symbols. The MSE in the UL can be then expressed as [1]:

$$\text{MSE}_{\text{UL}} = \sigma^2_{s,\text{UL}} \text{tr}\left(\left(\boldsymbol{I}_K + P_{\text{UL}} (K P_{\text{UL}} \sigma^2_{e_{\text{UL}}} + \sigma^2_{n,\text{UL}})^{-1} \hat{\boldsymbol{H}}^{\text{H}}_{\text{UL}} \hat{\boldsymbol{H}}_{\text{UL}} \right)^{-1} \right). \tag{1}$$

For large systems (large M and K), MSE_{UL} becomes deterministic variable (independent of the special realization of $\boldsymbol{H}_{\text{UL}}$) and can be approximated as follows using large system derivation in the Appendix:

$$\text{MSE}_{\text{UL}} \approx K \sigma^2_{s,\text{UL}} \text{MSE}_{\text{LS}}, \tag{2}$$

where MSE_{LS} is analytically given in (23) such that $\text{snr} = M(1-\sigma^2_{e_{\text{UL}}}) P_{\text{UL}} (K P_{\text{UL}} \sigma^2_{e_{\text{UL}}} + \sigma^2_{n,\text{UL}})^{-1}$. Simulations show that this approximation is quite accurate even for moderate number of antennas.

[1] A detailed derivation of the MSE is not shown here due to lack of space, but it can be still derived by substituting the transmit and receive filter expressions in Eq. (22) of [7].

B. Downlink Model

In the DL, the BS communicates with K decentralized MSs. Thus, the DL can be modeled as a *multiuser - multiple input single output* (MU-MISO) system. The flat fading DL channel, which is a *Broadcast Channel*, is expressed in a matrix form as $\boldsymbol{H}_{\text{DL}} = [\boldsymbol{h}_{\text{DL},1}, \ldots, \boldsymbol{h}_{\text{DL},K}]^{\text{T}} \in \mathbb{C}^{K\times M}$, where $\boldsymbol{h}_{\text{DL},k} \in \mathbb{C}^{M}$ is the DL channel from the BS to user k whose elements are i.i.d. zero-mean complex Gaussian random variables with unit variance. In what follows, we explain how the BS gets access to the quantized version of an estimate of the CDI of each MS.

We adopt the same channel model as that in [7] i.e. the DL channel is expressed as follows:

$$\begin{aligned} \boldsymbol{H}_{\text{DL}} &\overset{(a)}{=} \hat{\boldsymbol{H}}_{\text{DL}} + \boldsymbol{E}_{\text{DL}} \\ &\overset{(b)}{=} \boldsymbol{B}\boldsymbol{H}_n + \boldsymbol{E}_{\text{DL}} \\ &\overset{(c)}{=} \boldsymbol{B}(\boldsymbol{C}\boldsymbol{H}_q + \boldsymbol{E}_q) + \boldsymbol{E}_{\text{DL}}. \end{aligned} \tag{3}$$

(a) is the estimation step such that $\hat{\boldsymbol{H}}_{\text{DL}} = [\hat{\boldsymbol{h}}_{\text{DL},1}, \ldots, \hat{\boldsymbol{h}}_{\text{DL},K}]^{\text{T}} \in \mathbb{C}^{K\times M}$ is the estimated DL channel where $\hat{\boldsymbol{h}}_{\text{DL},k}$ is the estimated channel for MS k obtained using a common pilot of length $T_{\text{DL}} \geq M$ emitted from the BS. $\boldsymbol{E}_{\text{DL}} = [\boldsymbol{e}_{\text{DL},1}, \ldots, \boldsymbol{e}_{\text{DL},K}]^{\text{T}} \in \mathbb{C}^{K\times M}$ is the DL estimation error matrix where $\boldsymbol{e}_{\text{DL},k}$ is the estimation error vector when estimating the DL channel for MS k, which is of zero mean and covariance $\text{E}\left[\boldsymbol{e}_{\text{DL},k}\boldsymbol{e}^{\text{H}}_{\text{DL},k}\right] = \sigma^2_{e_{\text{DL}}} \boldsymbol{I}_K$, where [6]

$$\sigma^2_{e_{\text{DL}}} = \frac{1}{1 + \frac{P_{\text{DL}}}{M\sigma^2_{n,\text{DL}}} T_{\text{DL}}}, \tag{4}$$

P_{DL} represents the available total transmit power at the BS and $\sigma^2_{n,\text{DL}}$ is the variance of the AWGN at each MS.

(b) is the normalization step, where $\boldsymbol{H}_n = [\boldsymbol{h}_{n,1}, \ldots, \boldsymbol{h}_{n,K}]^{\text{T}} \in \mathbb{C}^{K\times M}$ is the normalized estimated DL channel such that $\boldsymbol{h}_{n,k}$ is that for MS k. $\boldsymbol{B} = \text{diag}\big([||\hat{\boldsymbol{h}}_{\text{DL},1}||_2, \ldots, ||\hat{\boldsymbol{h}}_{\text{DL},K}||_2]\big) \in \mathbb{R}^{K\times K}$ represents the estimated *Channel Magnitude Information* (CMI) matrix that has the following statistics available at the BS [8]:

$$\text{E}\left[\boldsymbol{B}^{-2}\right] = \frac{1}{(M-1)(1-\sigma^2_{e_{\text{DL}}})} \boldsymbol{I}_K. \tag{5}$$

(c) is the quantization step where $\boldsymbol{H}_q = [\boldsymbol{h}_{q,1}, \ldots, \boldsymbol{h}_{q,K}]^{\text{T}} \in \mathbb{C}^{K\times M}$ is the quantized channel matrix made available at the BS as follows. We consider *Random Vector Quantization* scheme [9] such that each MS k has a codebook $\mathcal{C}_k$, also available at the BS and is unique for each user, consisting of 2^B unit-norm random code vectors $\boldsymbol{t}_{j,k} \in \mathcal{C}_k$ uniformly distributed on the complex unit sphere. Each user k chooses from $\mathcal{C}_k$ the vector with the minimum chordal distance to its normalized estimated channel $\boldsymbol{h}_{n,k}$ to get its quantized channel:

$$\boldsymbol{h}_{q,k} = \underset{\boldsymbol{t}_{j,k} \in \mathcal{C}_k}{\text{argmax}} \left| \boldsymbol{t}^{\text{H}}_{j,k} \boldsymbol{h}_{n,k} \right|.$$

Note that $\boldsymbol{h}_{q,k}$ represents a quantized version of the estimated CDI of MS k. With $c_k = \boldsymbol{h}^{\text{H}}_{q,k}\boldsymbol{h}_{n,k} \in \mathbb{C}$, we can express the normalized estimated channel in terms of the quantized channel as:

$$\boldsymbol{h}_{n,k} = c_k \boldsymbol{h}_{q,k} + \boldsymbol{e}_{q,k},$$

so that $\boldsymbol{h}_{q,k}$ is orthogonal to the quantization error vector $\boldsymbol{e}_{q,k}$. We collect the quantization coefficient of each user c_k in the matrix $\boldsymbol{C} = \mathrm{diag}([c_1, \ldots, c_K]) \in \mathbb{C}^{K\times K}$. We note here that $|c_k| = \cos\theta_k \leq 1$, θ_k being the angle between $\boldsymbol{h}_{q,k}$ and $\boldsymbol{h}_{n,k}$, and

$$\mathrm{E}[\cos^{-2}\theta_k] \approx \frac{1}{\mathrm{E}[\cos^2\theta_k]} \overset{[10]}{=} \frac{1}{1-2^B\mathrm{Beta}(2^B, \frac{M}{M-1})}, \quad (6)$$

where the approximation becomes tight for high number of quantization bits ($B > 6$). Moreover, $\arg(c_k) \neq 0$ which has also to be considered in the design of the receiver. $\boldsymbol{E}_q = [\boldsymbol{e}_{q,1}, \ldots, \boldsymbol{e}_{q,K}]$ represents the quantization error matrix such that $\boldsymbol{e}_{q,k}$ represents the quantization error vector of MS k. The covariance matrix of $\boldsymbol{E}_q$ as given in [7] Eq. (62) is:

$$\mathrm{E}\left[\boldsymbol{E}_q^{\mathrm{H}}\boldsymbol{E}_q|\boldsymbol{H}_q\right] = \frac{\mathrm{E}[\sin^2\theta]}{M-1}(K\boldsymbol{I}_M - \boldsymbol{H}_q^{\mathrm{H}}\boldsymbol{H}_q), \quad (7)$$

where $\mathrm{E}[\sin^2\theta] = 2^B\mathrm{Beta}\left(2^B, \frac{M}{M-1}\right)$ [10].

Having quantized its DL channel, each MS k feeds back its quantized channel $\boldsymbol{h}_{q,k}$ with $B/2$ uncoded QPSK symbols to the BS in the UL, which may be received erroneously. Thus, the BS has access to the quantized channel $\boldsymbol{H}_q$.

Motivated by the need of simple MSs, transmit processing is performed at the BS which has access to the quantized channel $\boldsymbol{H}_q$ and to the statistics of the CMI given in (5). The optimum transmit WF precoder $\boldsymbol{P} \in \mathbb{C}^{M\times K}$ and the scalar WF receive filter $g \in \mathbb{C}$ at each MS are found by minimizing the MSE between the transmitted and the received symbols under a total transmit power at the BS P_{DL}. With the assumption of correct feedback of $\boldsymbol{H}_q$, the receive and transmit filters read as [7]:

$$g = \sqrt{\frac{\sigma^2_{s,\mathrm{DL}}\mathrm{tr}\left(\left((1-\kappa)\boldsymbol{H}_q^{\mathrm{H}}\boldsymbol{H}_q + \xi\boldsymbol{I}_M\right)^{-2}\boldsymbol{H}_q^{\mathrm{H}}\boldsymbol{H}_q\right)}{P_{\mathrm{DL}}}}, \quad (8)$$

$$\boldsymbol{P} = \frac{1}{g}\left((1-\kappa)\boldsymbol{H}_q^{\mathrm{H}}\boldsymbol{H}_q + \xi\boldsymbol{I}_M\right)^{-1}\boldsymbol{H}_q^{\mathrm{H}}, \quad (9)$$

where $\sigma^2_{s,\mathrm{DL}}$ is the variance of the transmitted symbols and

$$\kappa = \frac{\mathrm{E}[\tan^2\theta]}{M-1} = \frac{\mathrm{E}[\sin^2\theta]\mathrm{E}[\cos^{-2}\theta]}{M-1}, \quad \text{and} \quad (10)$$

$$\xi = K\kappa + \frac{\mathrm{E}[\cos^{-2}\theta]}{(M-1)(1-\sigma^2_{e_{\mathrm{DL}}})}\left(\sigma^2_{e_{\mathrm{DL}}} + \frac{\sigma^2_{n,\mathrm{DL}}}{P_{\mathrm{DL}}}\right). \quad (11)$$

This leads to the following expression of the MSE in the DL:

$$\mathrm{MSE}_{\mathrm{DL}} = \sigma^2_{s,\mathrm{DL}}\Bigg[K - \frac{K}{1-\kappa} + \frac{1}{1-\kappa}\mathrm{tr}\left(\left(\frac{1-\kappa}{\xi}\boldsymbol{H}_q\boldsymbol{H}_q^{\mathrm{H}} + \boldsymbol{I}_K\right)^{-1}\right)\Bigg], \quad (12)$$

which can be approximated as follows when using the large system analysis in the Appendix:

$$\mathrm{MSE}_{\mathrm{DL}} \approx \sigma^2_{s,\mathrm{DL}}\left(K - \frac{K}{1-\kappa} + \frac{K}{1-\kappa}\mathrm{MSE}_{\mathrm{LS}}\right), \quad (13)$$

where $\mathrm{MSE}_{\mathrm{LS}}$ is given in (23) such that $\mathrm{snr} = \frac{M(1-\kappa)}{(M-1)\xi}$.

III. Overhead Optimization

We find the optimum T_{UL}, T_{DL} and B that maximize the sum of the lower bounds of the per-user DL and UL rates since true rates with imperfect CSI are unknown. We tackle thus the following optimization problem:

$$\max_{T_{\mathrm{UL}},T_{\mathrm{DL}},B}\left(\frac{T-T_{\mathrm{DL}}}{T}\right)p_c C_{\mathrm{DL,lb}} + \left(\frac{T-T_{\mathrm{UL}}-B/2}{T}\right)C_{\mathrm{UL,lb}},$$

$$\text{s.t. } 0 \leq B \leq (2T-2K),\ M \leq T_{\mathrm{DL}} \leq T,\ K \leq T_{\mathrm{UL}} \leq (T-B/2), \quad (14)$$

where $T = T_{\mathrm{DL}} + T_{\mathrm{data,DL}} = T_{\mathrm{UL}} + \frac{B}{2} + T_{\mathrm{data,UL}}$ is the duration of a packet for which we assume that the channel is constant. $T_{\mathrm{data,DL/UL}}$ is the number of symbols for the data in DL and UL, respectively. $C_{\mathrm{DL/UL,lb}}$ represents the lower bound of the DL and the UL ergodic capacities:

$$C_{\mathrm{DL/UL,lb}} = \log_2\left(\frac{\sigma^2_{s,\mathrm{DL/UL}}}{\mathrm{MSE}_{\mathrm{DL/UL}}/K}\right), \quad (15)$$

which is attained assuming Gaussian signals and worst-case (i.e. Gaussian) noise that has a variance given by $\mathrm{MSE}_{\mathrm{DL/UL}}$ in (13) and (2). The division by the factor K is done because (13) and (2) represent a total MSE. For the lower bound of the DL capacity i.e. $C_{\mathrm{DL,lb}}$, we distinguish the following two cases:

- $B \neq 0$: This is the case that we already explained in section III-B. $C_{\mathrm{DL,lb}}$ is that given in (15).
- $B = 0$: This is the case when no feedback is employed. We propose that the transmitter uses a fixed precoder which is also known at each MS. Therefore the system can be regarded as K SISO interfering channels. Each user's channel is Rayleigh faded with a power of $(1-\sigma^2_{e\mathrm{DL,NoFB}})P_{\mathrm{DL}}/K$: and it is perturbated by a total interference and noise. Thus the SINR denoted as $\gamma_{\mathrm{NoFB,DL}}$:

$$\gamma_{\mathrm{NoFB,DL}} = \frac{(1-\sigma^2_{e\mathrm{DL,NoFB}})\frac{P_{\mathrm{DL}}}{K}}{(K-1)(1-\sigma^2_{e\mathrm{DL,NoFB}})\frac{P_{\mathrm{DL}}}{K} + \sigma^2_{e\mathrm{DL,NoFB}}\frac{P_{\mathrm{DL}}}{K} + \sigma^2_{n,\mathrm{DL}}},$$

  where $\sigma^2_{e\mathrm{DL,NoFB}}$ is obtained from (4) by substituting $M = 1$. Since large system analysis can not be applied (since SISO), we use the formula known for the ergodic capacity for a Rayleigh faded SISO channel [12]:

$$C_{\mathrm{DL,lb,NoFB}} = \log_2(\mathrm{e})\mathrm{e}^{1/\gamma_{\mathrm{NoFB,DL}}}\mathrm{E}_1(1/\gamma_{\mathrm{NoFB,DL}}), \quad (16)$$

  where e is the exponential number and $\mathrm{E}_u(z)$ is the generalized exponential integral defined as $\mathrm{E}_u(z) = \int_1^\infty \frac{\mathrm{e}^{-zt}}{t^u}\mathrm{d}t$.

On the other hand, p_c in (14) is the probability that the quantized CDI is received correctly at the BS obtained as follows. At high SNR values, the expectation of the symbol error probability in a Rayleigh faded environment can be approximated as [11]:

$$\mathrm{E}\left[P_s(\chi)\right] \approx \frac{1}{(1+\frac{\gamma_o}{2})^{M-K+1}}, \quad (17)$$

where

$$\gamma_o = \frac{K\sigma^2_{s,\mathrm{UL}}}{\mathrm{MSE}_{\mathrm{UL}}} - 1,$$

which represents the SINR in the UL with WF precoding. The per-user feedback is received error-free with an upper-bound probability [13]:

$$p_{\epsilon,k} = 1 - \left(1 - \mathrm{E}\left[P_s(\chi)\right]\right)^{\frac{B}{2}}. \quad (18)$$

The feedback link is a MAC, since all users are transmitting simultaneously their feedback symbols. The feedback message is thus composed of the feedback symbols of all users. We assume that if at least one user feeds back erroneously its symbols, a total loss of the message is incurred and thus the probability that the CDI is received correctly at the BS is:

$$p_c = (1-p_\epsilon)^K, \quad (19)$$

where p_ϵ is given in (18) omitting the per user subscript because we assume that all users have the same channel statistics. p_c is an upper bound probability since (18) is an upper bound to p_ϵ. Although the BS and the users are unaware of the feedback errors, we assume that both entities know the feedback error probability, since p_ϵ remains constant over many transmission blocks [13].

IV. Simulation Results

The optimization in (14) is done in two steps: first we find the optimum T_{UL} and T_{DL} for a given B and then we perform a search over the number of feedback bits to find the optimum B. For a fixed B, $R_{\mathrm{UL,lb}}$ i.e. $\left(\frac{T-T_{\mathrm{UL}}-B/2}{T}\right) C_{\mathrm{UL,lb}}$, only depends on T_{UL}. However, we have an additional trade off in two-way systems between the UL and the DL due to feedback. The DL rate i.e. $p_c\left(\frac{T-T_{\mathrm{DL}}}{T}\right) C_{\mathrm{DL,lb}}$, not only depends on T_{DL} and B, but also on the UL channel estimation, i.e. on T_{UL}. A poor channel estimation leads to a larger p_ϵ in (18) and a smaller p_c in (19) which in turn decreases the DL rate. The DL rate is thus a function of T_{UL}. The first step in the optimization includes finding for each T_{UL}, the DL training length $T_{\mathrm{DL}}(T_{\mathrm{UL}})$, as a function of T_{UL} which maximizes the $R_{\mathrm{DL,lb}}(T_{\mathrm{UL}})$. Then, we find T_{UL} which maximizes $R_{\mathrm{DL,lb}}(T_{\mathrm{UL}}) + R_{\mathrm{UL,lb}}(T_{\mathrm{UL}})$. The second step in the optimization requires finding the optimum B, which maximizes $R_{\mathrm{DL,lb}} + R_{\mathrm{UL,lb}}$ while the optimum training lengths are those found in the first step. The optimum B is searched over $0 \leq B \leq (2T-2K)$. Note that at $B=0$, (16) is used. If the figure of merit were solely the UL rate, the optimum value would be at $B=0$. If the figure of merit were the DL rate, the optimum value of B is found while the rest of the symbols are employed for training i.e. $R_{\mathrm{UL,lb}} = 0$.

Before we proceed to present the numerical results for the optimization in (14), we note that the following modes of access are adopted in the following regions for a system with $M=5$ antennas and for the settings: $T=100$, $\sigma^2_{n,\mathrm{DL}} = \sigma^2_{n,\mathrm{UL}} = 1$, and $\sigma^2_{s,\mathrm{DL}} = \sigma^2_{s,\mathrm{UL}} = 1$ vrs. $10\log_{10}\left(\frac{P_{\mathrm{DL}}}{K\sigma^2_{n,\mathrm{DL}}}\right)$ and $P_{\mathrm{UL}} = \frac{P_{\mathrm{DL}}}{M}$ so that we have the same power per transmit antenna in the UL and in the DL.:

- K=5: SDMA in UL while in the DL since no feedback is employed (see Figure 2) i.e. there is a fixed precoder, it is treated as K SISO interference channels for $10\log_{10}\frac{P_{\mathrm{DL}}}{(K\sigma^2_{n,\mathrm{DL}})} \leq 8$ dB,
- K=2: SDMA in UL and in DL for 8 dB $< 10\log_{10}\frac{P_{\mathrm{DL}}}{(K\sigma^2_{n,\mathrm{DL}})} \leq 12$ dB,
- K=3: SDMA in UL and in DL for 12 dB $< 10\log_{10}\frac{P_{\mathrm{DL}}}{(K\sigma^2_{n,\mathrm{DL}})} < 25$ dB, and
- K=4: SDMA in UL and in DL for $10\log_{10}\frac{P_{\mathrm{DL}}}{(K\sigma^2_{n,\mathrm{DL}})} \geq 25$ dB.

This is the case since in each region the corresponding sum of the rates (i.e. Eq. (14)) achieves the highest values among the others as it is shown and plotted in Figure 1. The optimum

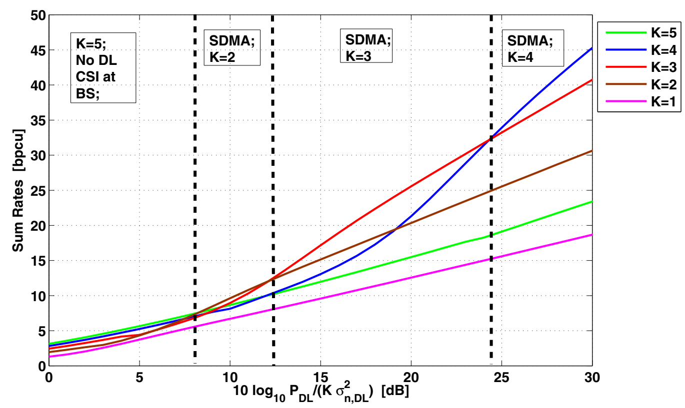

Fig. 1. Sum Rates vrs. $10\log_{10}\frac{P_{\mathrm{DL}}}{(K\sigma^2_{n,\mathrm{DL}})}$.

values for (14) are plotted in Figure 2 for the same above settings and modes of access. The optimum values associated with each region are plotted. As expected, B increases with increasing the transmit power for all regions and dominates the overhead. The abrupt change of T_{UL} when switching between the modes is due to the increase in the feedback error probability, i.e. $(1-p_c)$. Since (14) is an integer optimization and since we perform a full search to find the optimum lengths of T_{UL}, T_{DL} and B, convergence to optimum values is guaranteed.

In Figures 3 and 4, we plot the lower bounds of the DL and the UL rates of the optimization in (14) respectively. As expected, as K increases for fixed M, the UL rate increases. This is not the case for the DL rate since it is affected by p_c. This can be seen in Figure 3 for the case $M=K=5$.

V. Conclusions and Future Work

In this paper, we found the optimum lengths of T_{UL}, T_{DL} and B for the MU-SIMO system in the UL and the MU-MISO system in the DL. Estimated UL channel obtained with T_{UL}

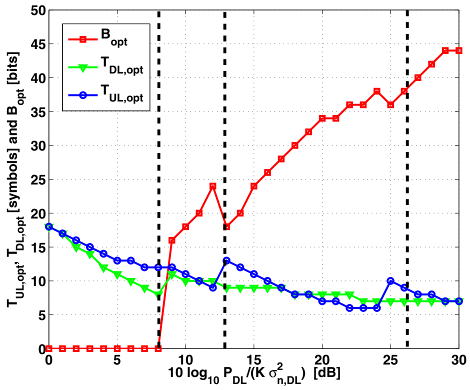

Fig. 2. Optimum Overhead Values vrs. 10 $\log_{10} P_{\mathrm{DL}}/\left(K\sigma_{n,\mathrm{DL}}^2\right)$.

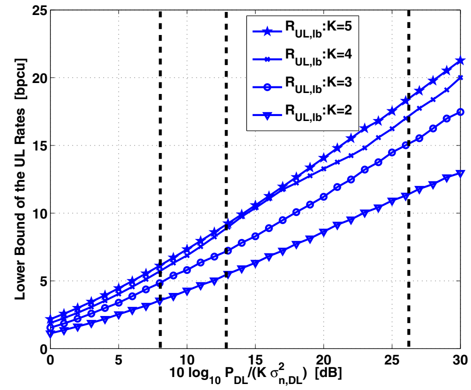

Fig. 4. $R_{\mathrm{UL,lb}}$ vrs. 10 $\log_{10} P_{\mathrm{DL}}/\left(K\sigma_{n,\mathrm{DL}}^2\right)$.

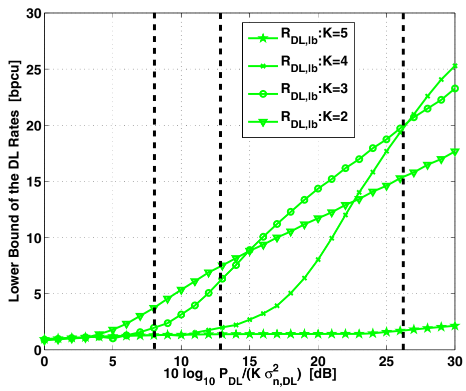

Fig. 3. $R_{\mathrm{DL,lb}}$ vrs. 10 $\log_{10} P_{\mathrm{DL}}/\left(K\sigma_{n,\mathrm{DL}}^2\right)$.

dedicated symbols and quantized DL channel obtained first by estimation with T_{DL} common symbols, then normalized and finally quantized with B bits, which are fed back with $B/2$ symbols and may be received erroneously are both made available at the BS for receive and transmit processing respectively. WF transmit and receive filters are then derived based on the available CSI in both links. Using large system analysis, we were able to derive analytical MSE expressions in the UL and DL that give lower bound of the UL and DL capacities respectively. These lower bounds are necessary for our optimization problem. As future work, we can extend our system analysis and optimization to MU-MIMO such that more than one antenna is employed at each MS. For the DL rate, coding of the feedback bits for higher reliability can be employed which is expected to lead to better DL rates.

Appendix A: Large System Analysis

The per-user MSE can be expressed as function of the MMSE multiuser efficiency η [14] as:

$$\mathrm{MSE}_k = \frac{1}{1+\mathrm{snr}_k \eta}, \tag{20}$$

such that snr_k represents the received SNR of user k which differs for the UL and the DL, but we assume that it is the same for all users i.e. $\mathrm{snr}_k = \mathrm{snr}\ \forall\, k$. According to Theorem 2.39 in [14], η is the solution to the fixed-point equation:

$$1-\eta = \beta(1-\eta_{\mathrm{snr}}(\eta)), \tag{21}$$

where $\beta = \frac{K}{M}$ and $\eta_{\mathrm{snr}}(X)$ represents the η-transform given as

$$\eta_{\mathrm{snr}}(X) \overset{[14]}{=} \mathrm{E}_{\mathrm{snr}}\left[\frac{1}{1+\mathrm{snr}X}\right] = \frac{1}{1+\mathrm{snr}X}, \tag{22}$$

since snr is the same for all users for the second equality. We can thus solve (21) for MSE_k by using (20) and (22):

$$\mathrm{MSE}_{\mathrm{LS}} = \left(\frac{\beta-1-1/\mathrm{snr}+\sqrt{(1/\mathrm{snr}+1-\beta)^2+4\beta/\mathrm{snr}}}{2\beta}\right). \tag{23}$$

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