Modelling the Value and Measuring the Risk of Private Equity

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Abstract

Private equity firms are blamed for quickly extracting all of a target company's cash, and sometimes for going even further by requesting a target company to incur additional debts – in order to be able to pay investors an additional dividend – and thus driving it into bankruptcy. Consequently, there is always a trade-off between the benefit of high extra dividends and the associated risk, due to higher debt obligations, which may cause bankruptcy. In this paper, we apply real-options theory and capital-budgeting techniques to the problem of assessing a private investor's risk. We propose a new continuous time DCF model, which incorporates the four fundamental value drivers, among others a high debt to equity ratio, and typical characteristics for private investments like a high probability to default. We also introduce different risk measures by proposing a new measurement model that is based on the cash flow process, the associated multiple process as well as on the implied IRR of the transaction. Finally, we give some details of how such a model is implemented to provide investors as well as debt lenders with a decision support regarding different investments or investments strategies.

Keywords: Private Equity, Risk Return, Business Valuation, Leveraged Finance, Risk Assessment, Credit Risk, DCF Default Model

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1 Introduction

Two worlds, which could not be more different, collided in the Swabian small town Metzingen - the workforce of the fashion label Hugo Boss and the private equity investor Permira. In May 2008 the new owner Permira announced a dividend, including a debt-financed extra dividend, of 500 million (mn) Euro in the annual general meeting, decreasing the equity stake from 50% to around 25%. Most of the successful board of directors had already left the company at that time. This kind of innovative financial engineering has recently hit the headlines. Another showcase for the stereotype of private equity investors is the history of the car wash company IMO, in which the private equity investor Carlyle invested. After two years with bad weather - car wash sales are strongly correlated to sunny days in the summer season and cold/icy days in the winter season - the optimistic sales targets for car washes could not be fulfilled. This combined with the enormous debt obligations resulted in a failure of the narrowly calculated investment plan. Senior debt lenders are now on taking over the company, due to breach of covenants. This paper contributes to the development of a risk assessment procedure for private equity investments. Currently, private equity investors simply measure an investments' risk by introducing best and worst case scenarios in the investment plan. It has become evident in the current crises that the sole consideration of rates of return without proper risk assessment is only doing well as long as no unpredicted event occurs that strikes the worst case scenario. Various publications initiated due to the introduction of the Basel II Accord provide a toolset to assess a bank's product range. However, they are not taking into account that banks provide a large portion of equity to private funds or even run their own private equity divisions that do not only provide debt to other financial sponsors, but do also purchase their own equity positions.

This paper presents a method that allows for the assessment of complex investment strategies in terms of risk and return. Cashing out the investment allows private equity companies to achieve returns for their investors in the timeliest manner. Thus upcoming risks from restructuring programmes and environmental changes can be mitigated by early payments to investors. As financial sponsors are usually judged by the internal rate of return of their investments, they prefer risk-free early cash flows as opposed to an uncertain value enhancement, whose net present value is low due to a high discount factor. Thinking beyond a capital pre-drawing the remaining equity stake becomes usually more risky, hence the overall risk effect is ambiguous.

In order to provide a decision support for different innovative financial engineering strategies we will analyse the investment strategy in terms of discounted cash flows. Especially changes in the capital structure, which affect the risk of investments, are used as levers to map a certain investment strategy to our stochastic model. Within this flexible stochastic model, on the basis of available historic data samples and expectations raised in the leveraged buyout model Monte-Carlo techniques are used to provide us with an understanding of the resulting density of the net present value or internal rate of return. A comparison of different investment strategies within a risk-return profile allows us to support both financial sponsors and debt lenders in their selection process of exclusive investments as well as in their selection process of investment strategies.

Our paper is organised as follows. In Section 2 we present the model introducing different cash flow processes in continuous time, the current value of a company is modeled involving a random multiple, which evolves also dynamically in time. Leverage sizes and default possibilities are considered as well. The section ends with the determination of the net present value of the equity exit value as well as the associated distribution of the Internal Rate of Return (IRR), which is of particular importance to the investor. Section 3 is devoted to the risk assessment of the investment focussing on the most important downside risk measures. In Section 4 we present the standard Euler discretisation scheme and list all input variables preparing thus the ground for our case study in Section 5. For various input scenarios we calculate the corresponding risk return rate. Finally, we summarize some conclusions in Section 6.

2 The Continuous-Time Model

Private equity investors value possible investments with the same techniques, which are used for portfolio decisions of liquid financial assets. Generally, cash flows (CFs) over time are typically composed of the initial equity investment (IV) at time 0, the values drawn from the Free Cash Flows to Equity (FCFE) during $(0, T)$, and the price of equity, which is realised at maturity T - the Exit Value. We will measure the performance of a private equity firm by the cash equity basis of the investment and the implied IRR.

We derive the price of equity from the Enterprise Value (EV) at time T, which is identifed by a comparable transactions analysis, which is a specific market approach: The exit value, based on a multiple (e.g. 10 times of earnings before interest, taxes, depreciation and amortisation) [1], is a simplified pricing procedure and accounts for the market value in comparison to a peer group[2].

We define an investment's peer group universe by its publicly listed competitors. A consensus of broker forecasts on peer group financial data helps us to incorporate industry specific long term growth rates in a robust manner, since on a long term perspective it is not possible for a company to outgrow its market.

The following sections in this chapter will outline each quantity of the model in detail.

2.1 The Cash Flow Process

We distinguish between two different parts of CFs. On the one hand we have operating CFs E_t like regular dividends. On the other hand, we may also have non-operating CFs, derived by extra dividends or cash injections, denoted by J_t. We model both parts independently as follows[3].

[1]SCHWARTZ/ MOON (2000) Rational Pricing of Internet Companies, p. 62.

[2]MEYER (2006) Stochastische Unternehmensbewertung. Der Wertbeitrag von Realoptionen, p. 63.

[3]We take a two-dimensional standard Brownian motion $(\mathcal{W}_t)_{t\geq 0} = (W_t, \widetilde{W}_t)_{t\geq 0}$ on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$. We assume that $\mathcal{F}_t = \sigma(m_s, E_s : 0 \leq s \leq t)$, $0 \leq t \leq T$, is the natural filtration generated by $(\mathcal{W}_t)_{t\geq 0}$, which satisfies the usual conditions. We will call $\mathcal{F}_t$-measurable quantities path dependent. As we deal only with problems on the compact interval $[0, T]$, we will also assume the usual integrability conditions.

Operating Cash Flows

We model the company's operating CFs as the solution of the stochastic differential equation

$$dE_t = E_t(\lambda_t dt + \sigma_t^E dW_t), E_0 > 0,$$

where the drift is denoted by λ_t [4] and the diffusion coefficient by σ_t^E. The chosen geometric model accounts for level-adjusted drift and volatility increments. Obviously, $E_0 > 0$ results in $E_t > 0$ almost surely. This means that we restrict our operating CF model to positive CFs [5].

Non-operating Cash Flows

Possible unexpected events are, for instance, debt-financed extra dividends (recaps), asset-sales, covenant breaches, technology purchases, unexpected equipment replacements, legal claims or cash injections. The occurrence times and their values are uncertain for all aforementioned events.

We model the event times and values separately. Usually, we will not have any preliminary knowledge concerning times and sizes. Then, as usual in such situtations, we model the inter event times by i.i.d. (independent, identically distributed) random variables G_i, which we assume to be exponentially distributed with intensity $g > 0$. The values at the event times are assumed to be independent of the inter event times and they can be positive or negative. We model the sign as being random denoted by i.i.d $\delta_i \in \{-1, 1\}$ with $\mathbb{P}(\delta_i = 1) := p \in [0, 1]$. Finally, we model the distribution of an absolute event size S_i by a lognormal distribution with parameters μ^J and $(\sigma^J)^2$ and we assume that they are independent and independent of the sign.

All these independence assumptions result then in a compound Poisson process

$$J_t := \sum_{i=1}^{\infty} \delta_i S_i 1_{\{\sum_{j=1}^{i} G_j \leq t\}}.$$

The extraordinary CFs can also be modeled by various extensions of the above compound Poisson process. Take, for instance, a sum of different compound Poisson processes, to model e.g. by J_t^1 possible recaps with intensity $g^1 > 0$, whereas J_t^2 captures possible cash injections occuring with intensity $g^2 > 0$. Obviously, the lognormal distribution can be replaced by any other distribution. Furthermore, the event times can be modeled, for instance, by a non-homogeneous Poisson process or by any other point process. All this has to be decided based on the statistical information available. It may also be advisable to introduce further conditions, for instance, recaps can only be realised if operating CFs exceed a certain level, or cash injections are only necessary, if operating CFs are below a certain level. These considerations can easily be taken into account.

[4] With reference to later sections, one may extend the idea of a time dependent drift λ_t by $\lambda_t(Debt)$. Higher debt obligations are followed by higher interest payments. Hence, not anticipated interest payments decrease the drift of future CFs. Not anticipated interests arise from the stochastic character of debt, given by $\widetilde{D}_t = \eta \int_0^t E_s \sigma_s^E dW_s + J_t$. One may take $\lambda_t = \lambda_t(Debt) = \lambda_t \left(1 - \frac{\widetilde{D}_t r_f}{E_0 \exp(\int_0^t \lambda_s ds)}\right)$ for all $t \in [0, T]$.

[5] By assuming $E_0 > 0$, we exclude the modelling of distressed investments.

Compounded Cash Flows

With respect to our CF analysis, we also incorporate in our model the possibility to repay debts at a certain percentage $\eta \in [0, 1]$ of the operating CFs. Then the compounded CF process $FCFE_t$ available for investors, including extraordinary events is given by

$$FCFE_t := (1 - \eta)E_t + J_t.$$

2.2 The Multiple Process

We shall define the multiple-process m in reference to a CF process. As the multiple is based on doubtful expectations and also incorporates market overstatements to industry growth rates, we will use a stochastic process to describe dynamically the uncertainty that underlies a value enhancement by multiple expansion or by increasing CFs at an assumed constant multiple. We use a square-root diffusion process to ensure that almost all sample paths m_t of the multiple process are positive [6], and we use a time dependent mean $\overline{m}_t$ for a more realistic modelling:

$$dm_t = \kappa(\overline{m}_t - m_t)dt + \sigma^m \sqrt{m_t} d\widetilde{W}_t, \quad m_0 \in \mathbb{R}_+.$$

The parameter κ indicates the mean-reversion rate and σ^m is the diffusion parameter of the multiple process.

2.3 Enterprise Value

We will work with an Enterprise Value/Free Cash Flow to the Firm (EV/FCFF)-multiple, as this is the most accurate multiple measure of the current value of a company [7] yielding

$$EV_t = FCFF_t \times m_t.$$

Free Cash Flow to the Firm (FCFF) can be calculated by the relation

$$\begin{aligned} FCFF_t &= FCFE_t - \Delta Debt_t + r_f Debt_t \\ &= FCFE_t + \eta E_t + dJ_t + r_f Debt_t, \end{aligned}$$

where r_f denotes the risk free rate of return [8]. Note that FCFFs are unlevered or debt-free. This is obvious, because FCFFs do not include interest and so they are independent of debt and capital structure. Therefore, the EV will also be independent of the underlying capital structure, and our modelling of the EV is consistent with the capital structure irrelevance principle of MODIGLIANI/ MILLER [9].

[6] A proof of the positivity can be found in MAGHSOODI (1996) Solution of the extended CIR term structure and bond option valuation, p. 92.

[7] JACOBS (2002) Great companies, bad stocks, p. 1.

[8] For simplicity reason we take r_f constant over time. One may also work with $r_{f_t} = r_f(l_t)$ since a higher level of leverage evolves higher interest premiums, as the company is more likely to fail to pay higher debt burdens.

[9] Cf. MODIGLIANI/ MILLER (1958) The Cost of Capital, Corporation Finance and the Theory of Investment, pp. 433-443.

Since for fixed deterministic $t > 0$ it holds that $dJ_t = 0$ [10] we may write

$$EV_t = (E_t + r_f Debt_t) \times m_t$$

with

$$\mathbb{E}(Em)_t = E_0 m_0 + \int_0^t E_s m_s \lambda_s ds + \int_0^t \kappa E_s(\overline{m}_s - m_s)ds + \int_0^t E_s \sigma^m \sigma_s^E \sqrt{m_s} ds.$$

We can see that the EV depends on the initial value Em at time 0, but also on the expected growth in CFs $Em\lambda$, which incorporates the value drivers top line growth and operational efficiency. It also depends on the multiple gap $\kappa E(\overline{m} - m)$ which can be interpreted as multiple expansion, as the exit value is pushed towards the projected multiple $\overline{m}$. Besides the leverage effect these are the fundamental value drivers for private equity firms to achieve high contributions to their investors [11]. The last term can be seen as a risk premium; a higher risk in the industry σ^m or a riskier investment σ^E is precipitating in an add-on to the EV at time t, which is what we indeed observe in equity markets. Investors in riskier assets demand higher returns as a risk compensation.

Recalling that ηE_t of CFs are at any time $t \in (0, T)$ employed to repay debt, and from the assumption that J_t is debt-financed [12] we conclude that

$$Debt_t = \max\left(Debt_0 - \eta \int_0^t E_s ds + J_t, 0\right).$$

2.4 Leverage and Default

The leverage l_t measures the current ratio of debt to total capital by

$$l_t := \min\left(\frac{Debt_t}{EV_t}, 1\right) = \min\left(\frac{\max\left(Debt_0 - \eta \int_0^t E_s ds + J_t, 0\right)}{(E_t + r_f Debt_t) m_t}, 1\right).$$

We define the default time as the time, when the EV strikes the book value of a companies' debt[13]. We work under the simplified assumption that our model assumes no recovery or cash injection at default, since cash injections are already incorporated with J_t.

$$\tau := \inf\{0 \le t \le T : l_t = 1\},$$

where we define as usual $\inf \emptyset := \infty$. If default happens before maturity T, i.e. if $\tau \le T$, then the company enters into insolvency proceedings and then we have $E_t = 0$ for all $t \in [\tau, T]$, since the company devolves to the creditors at default time τ. This also implies that the equity exit value is worthless, carried out by [14]

$$(1 - l_{t\wedge\tau}) EV_t = 0 \quad \text{for all } \tau \le t \le T.$$

[10] Note that for the stochastic jump times $T_i = \sum_{j=1}^{i} G_j$ it holds that $dJ_{T_i} \neq 0$; here we are, however, only interessted in fixed and deterministic times t.

[11] Cf. also BCG (2008) The Advantage of Persistence, pp. 12-14.

[12] Note that debt obligations can either be increased by e.g. recaps and or be decreased by e.g. cash injections.

[13] Cf. MERTON (1974) On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, p. 44.

[14] It holds that $l_{t\wedge\tau} = 1$ for all $\tau \le t \le T$.

2.5 Discount Rate

Discounting FCFE at the cost of equity will yield the value of equity in a business [15]. Working on the premises of the CAPM [16] we recall the security market line

$$r_t := r_f + (r_m - r_f)\beta_t^{lev} = r_f + (r_m - r_f)\beta^{unlev}\left(\frac{1}{1-l_t}\right),$$

with β denoting the equity beta factor, a measure for the systematic risk of a company's returns, r_m a market rate of return, and r_f the risk free rate of return. Here β^{lev} denotes the levered equity beta of the company measured by its equity return in dependence on an underlying index

$$\beta^{lev} = \frac{\sigma_e \mathrm{Corr}_{e,I}}{\sigma_I},$$

where σ_I denotes the market/ industry volatility and σ_e denotes the average equity return volatility of a publicly listed peer group universe (cf. the Introduction for details). $\mathrm{Corr}_{e,I}$ measures the correlation of the peer group universe to the market. Since we want to analyse a private company, we suggest to employ the average unlevered peer group beta in order to derive the appropriate, debt free, systematic risk. The unlevered beta factor is derived by decomposing the levered beta in an unlevered component and a leverage component [17]

$$\beta_t^{lev} = \beta^{unlev}\left(\frac{1}{1-l_t}\right).$$

2.6 Cash Flow Risk

We will motivate the equity CF risk in terms of the diffusion coefficient σ_t^E also via the definition of the equity beta in the CAPM. Stock market returns are driven by distributable FCFE, as they are the assessment base for possible payouts. Hence, in a long run perspective, stock market volatility is determined by recurring/ operating FCFE volatility [18]. Fragmenting the appropriate CF risk in a systematic component $\frac{\beta^{unlev}\sigma_I}{\mathrm{Corr}_{e,I}}$, a leverage component $\left(\frac{1}{1-l_t}\right)$, and by introducing an unsystematic component a, that accounts for market inefficencies we arrive at::

$$\sigma_t^E = \frac{\beta^{unlev}\sigma_I}{\mathrm{Corr}_{e,I}}\left(\frac{1}{1-l_t}\right)a.$$

In order to understand the motivation for the foregoing formula, consider an increase in debt at a constant EV. An increase in debt will affect CF risk in two ways. Firstly, the equity share of total capital is reduced, thus changes of a company's revenue hit a lower equity basis, and thus result in a higher volatility of CFs. Secondly, future liabilities soar as interest payments are increasing, hence the probability of not being able to repay liabilities augments, and thus the risk to default increases.

[15] See DAMODARAN (2001) Investment Valuation, Chapter 15, p. 2.

[16] Refer to SHARPE (1964) Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, pp. 425-442.

[17] For simplicity we assume a tax-free world, expressed by $\tau_C = 0$. It can, however, be incorporated easily by setting $\beta_t^{lev} = \beta^{unlev}\left(\frac{1-\tau_C l_t}{1-l_t}\right)$.

[18] Note that short term variations are caused by unexpected events, hence these are already taken into account by the modelling of CF jumps.

2.7 Valuation

The net present value of FCFE is simply derived by discounting CFs contributable to investors by the risk-adjusted rate r_t, hence the cash flow value (CFV) of the investment is modeled at time 0 by

$$CFV := \int_0^{T\wedge\tau} e^{-r_s s} FCFE_s ds,$$

putting on record that CFs are only accumulated as long as the investment is not bankrupt. The equity stake is measured by the EV minus the book value of liabilities [19]. As explained in the introduction we assume that investments are only realised at maturity T. Hence, we are not interested in discounting changes in the exit value at every time $t \leq T$. For the appropriate discount rate we employ a geometric approach [20]. The net present value of the equity exit value is denoted as terminal value (TV) and given by:

$$TV := (1 - l_{T\wedge\tau}) e^{-\int_0^T r_s ds} EV_T.$$

Hence, the net cash equity basis C, the value of the investment, is identified by introducing the initial equity investment value (IV), which constitutes the transaction price net of debt:

$$C := -IV + CFV + TV.$$

2.8 Internal Rate of Return

The Internal Rate of Return (IRR) is the yield on the invested capital, meaning that the net present value (NPV) of the investment's income stream equals zero. Recall the random variable $C = C((r_t)_{t\geq 0}) = C((E_t, m_t, J_t, Debt_t, l_t, r_t)_{t\geq 0})$ for every realisation $(E_t, m_t, J_t, Debt_t, l_t)_{t\geq 0}$, which comprises all CFs to equity. We define the IRR as the constant interest rate $r = r_t$ for all $t \in [0, T]$ such that r is the root of $C = C(r)$, i.e. the solution to

$$C(r)=0.$$

As $E_0 > 0$ holds a.s., we know that there are positive CFs, $E_t > 0$ a.s., hence investors should only be interested in investments for which at least $C(r = 0) > -IV$ holds. Further, as in general IV is positive, it holds that $\lim_{r\to\infty} C(r) < 0$. Hence, we can guarantee that such a root r, i.e. IRR exists a.s..

We denote the associated distribution function of IRR by F and note that $F(r)$ gives the probability of getting an IRR of less than or equal to r [21]. Investors can now analyse the

[19] As the company is not bankrupt, market value of debt is equal to book value of debt.

[20] One may argue, that discounting with the risk adjusted rate r_0 or the risk adjusted rate r_T is more suitable, as the investment is highly illiquid until maturity. But the reason that supports a geometric mean calculation is the fact that capital structure may change dramatically along the investment horizon. The associated risk changes are adequately taken into account by employing the average (geometric mean) risk adjusted rate as discount rate.

[21] I.e. when it comes to Monte Carlo simulation, we can identify, if a certain path implied an IRR of r by discounting CFs with r. If $C(r) < 0$ then the associated path implied an IRR of less than r.

distribution of the IRR. The distribution F expands the idea of point predictions (best/worst-case scenarios) to a probability distribution including a proper risk assessment of cash flow risk, industry (multiple) risk as well as of extra-ordinary events: cash injections, recaps, default, etc..

3 Risk Measures

As the success of private equity firms is measured by the announced IRR that is required to attract limited partners [22], the investors should and would be interested in the risk of falling below a certain IRR level. Thus in accordance with FISHBURN [23] risk is associated with returns falling below some specified target level, a hurdle h:

$$Risk(IRR) = \int_{-\infty}^{h} \varphi(h-r)dF(r),$$

where F is the distribution function of IRR and $\varphi(\cdot)$ is a nonnegative and non-decreasing function. For instance, for a risk parameter $\gamma > 0$, we can choose $\varphi(\cdot)$ so that

$$Risk(IRR) = \int_{-\infty}^{h} (h-r)^{\gamma} dF(r).$$

FISHBURN has shown congruence between this model and the expected utility model with utility function

$$U(r) = \begin{cases} r & r > h, \\ r - k(h-r)^{\gamma} & r \leq h, \end{cases}$$

and where k is a positive constant. The decision maker, here a general partner, may display various degrees of risk aversion or preference for outcomes below h, depending on the value of γ, but he/she remains risk neutral for outcomes above h [24]. After surveying a number of empirical studies of utility functions, FISHBURN concludes "that most individuals in the investment context do indeed exhibit a target return - which can be above, at, or below the point of no gain and no loss - at which there is a pronounced change in the shape of their utility functions, and that the given utility function can provide a reasonably good fit to most of these curves in the below-target region".

Since investors are in general interested in a successful track record to attract further limited partners, another adequate risk measure for private equity investors is the probability to default.

Under the requirements for the Capital Adequacy Assessment Process of Basel II, one can also assign Value at Risk or other risk measures derived from the density of the net cash equity basis C to measure required capital reserves. As C is a random variable with distribution function G, say, the probability distribution of the net present value of future CFs, in order to assess

[22] Cf. BERG (2005) What is strategy for buyout associations, p. 42.

[23] FISHBURN (1977) Mean Risk Analysis with Risk Associated Below-Target Returns, pp. 116-120.

[24] HOLTHAUSEN (1981): A Risk-Return Model with Risk and Return Measured as Deviations from a Target Return, pp. 182-185.

capital reserves one may define cash flows at risk (CFaR) [25], the maximum loss of CFs not exceeded with a given probability $\beta \in (0,1)$, i.e.

$$CFaR_\beta := \mathbb{E}C - G^{\leftarrow}(1-\beta),$$

where $G^{\leftarrow}(\beta) = \inf\{x \in \mathbb{R} : \mathbb{P}(C > x) \geq \beta\}$ is the generalized inverse or $(1-\beta)$-quantile of G.

4 Discrete Version of the Model

The model developed in the previous sections is path dependent: The EV as well as Debt at any time, which determine when bankruptcy is triggered, depend on the whole history of past CFs and multiples. Similarly, stochastic adjustments from recaps or cash injections are also path dependent. These path dependencies can easily be taken into account by using Monte Carlo simulation to calculate risk and return of a private equity investment. Hence, for the implementation of the simulation, we apply a classical EULER scheme for the CF process and the multiple process with time-dependent drift and diffusion [26]: for given $E_0 > 0$ and $m_0 > 0$, we take

$$\begin{aligned} E_{t+\Delta t} &\approx E_t + E_t\lambda_t\Delta t + E_t\sigma_t^E\Delta W_t, \\ m_{t+\Delta t} &\approx m_t + \kappa(\overline{m}_t - m_t)\Delta t + \sigma^m\sqrt{m_t}\Delta\widetilde{W}_t. \end{aligned}$$

For $n \in \mathbb{N}$ the grid $0 = t_0 < t_1 < \cdots < t_n = T$ is defined at a constant step size $t_{i+1} - t_i = \Delta t$, and $\Delta W_t, \Delta\widetilde{W}_t \sim N(0, \sqrt{\Delta t})$ and all these increments are independent [27].

In order to assign extra-ordinary CFs J_t to the grid, we recall some properties of a Poisson process. Firstly, the number of jumps $N(T)$ on the deterministic interval $[0,T]$ is distributed with $N(T) \sim Poi(gT)$. Secondly, given $N(T)$ we know that the order statistic $G_{(1)}, ..., G_{(N(T))}$ is uniformly distributed on the interval $[0,T]$ [28]. Hence, we assign the $N(T)$ events uniformly to the grid. Thirdly, we want to point out that this property is especially appropriate to model events, which are possible with a certain frequency, but for which we cannot say more about realisation dates. For instance, an investor knows that recaps are from time to time possible, but in advance he/she has no idea when such a recap fits in the investment plan. Hence, the best what we can do is to assign the recaps completely random, i.e. uniformly, over the investment period. On the basis of better statistical information one can lift this property; an example can be found in the case study.

Taking the realisations $(E_t, m_t, J_t)_{t\geq 0}$ we can, finallly, evaluate leverage l_t and risk adjusted rate of return r_t on the grid.

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[25] Cf. EMMER/ KLÜPPELBERG/ KORN (2000) Optimal portfolios with bounded downside risk, pp. 4-10.

[26] Cf. GLASSERMANN (2004) Monte Carlo Methods in Financial Engineering, p. 81 and pp. 340-34.

[27] A company's earnings growth and share price appreciation show only little correlation, hence we assume the investor's ordinary CF process and the associated multiple process are independent.

[28] Cf. EMBRECHTS/KLÜPPELBERG/MIKOSCH (1997) Modelling extremal events for insurance and finance, p. 186.

	Notation	t=-2	t=-1	t=0	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8
Net Sales		100	110	124	150	160	170	180	190	200	210	220
COGS		(35)	(39)	(43)	(53)	(56)	(60)	(63)	(67)	(70)	(74)	(77)
% sales		*35%*	*35%*	*35%*	*35%*	*35%*	*35%*	*35%*	*35%*	*35%*	*35%*	*35%*
SG&A		(14)	(14)	(15)	(17)	(16)	(15)	(14)	(13)	(14)	(15)	(15)
% sales		*14%*	*13%*	*12%*	*11%*	*10%*	*9%*	*8%*	*7%*	*7%*	*7%*	*7%*
EBITDA		51	57	66	81	88	95	103	110	116	122	128
operating FCF	E_t	18	23	30	37	44	51	58	60	66	73	80
non-operating FCF	J_t^1	(5)	3	6	-	-	2	-	-	-	-	-
Option 1 Recap	J_t^2	-	-	-	150	-	-	-	-	-	-	-
Option 2 Recap	J_t^3	-	-	-	-	-	-	-	150	-	-	-
Asset sales	J_t^4	-	-	-	-	30	-	-	-	-	-	-
Cash injections	J_t^5	-	-	-	20	-	-	-	-	-	-	-
Legal claims	J_t^6	-	-	-	-	-	-	10	-	-	-	-
Multiple	m_t	6,0	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9	7,0
Debt	$Debt_t$	300	280	256	369	295	246	198	288	221	148	68
Equity		100	100	100	120	120	120	120	120	120	120	120

Figure 1: Schematic leverage buyout (LBO) model.

Our model requires 22 parameters for its implementation. Some of the parameters are easily observable from quarterly data, others require thorough knowledge of the specific situation. Table 4.1 describes the parameters of the model and gives some suggestions about how to find appropriate realisations:

5 Case Study

This case study is based on a real world investment. Within this case study we want to analyse various investment strategies and test whether our model matches typical observations made in private equity context. Further, it is also the goal to illustrate the methodology for valuing private equity investments in terms of risk and return by applying it to one small cap investment, which is hold through three stages: investing, restructuring and exit.

5.1 Methodology

We will base our simulation upon forecasts and appraisements captured by investors in a leverage buyout (LBO) model, cf. figure 1.

The LBO model contains for all points on the grid $0 = t_0 < \cdots < t_n = T$ the expressed attitude of expectations of the investor, which CF he/she believes to realise. We distinguish the beliefs in an ordinary component denoted by $E_i := E_{t_i}$ for $i = 1, \ldots, n$, with analogous notation for all other dynamic quantities, and an extra-ordinary component derived from J_t at time points t_i with size S_i and appropriate sign.

In order to simulate the process $(E_t, m_t)_{t=t_0,\ldots,t_n}$ on the grid we derive the initial values of $\sigma_0^E, l_0, E_0, m_0$ as well as the constants $\sigma^m, \kappa, \sigma_I, Corr_{e,I}, \beta^{unlev}$ by the suggested procedures, stated in table 4.1. For a realistic modelling, we rely on the growth rates of E_i in the LBO

Parameter	Notation	Proposed Estimation Procedure
Maturity	T	Observable from LBO model
Initial ordinary CF	E_0	Observable from current CF statement
Initial multiple	m_0	Estimated from the stock data of a publicly listed peer group
Initial debt	D_0	Observable from current balance sheet
Leverage at time 0	l_0	Calculated by Debt and Equity at time 0
Exit multiple at time t	$\overline{m_t}$	Investor's future projections on peer group data
Standard deviation of multiple	σ^m	Estimated from data samples of peer group universe
Mean reversion rate of multiple	κ	Based on assumptions about the half life of process m_t, i.e. the half life of a deviation m_t from the long term mean $\overline{m_t}$
Intensity of jump	g	Investor's future projections
Associated jump height	μ^J	Investor's future projections
Standard deviation of jump height	σ^J	Investor's future projections
Probability of a positive jump	p	Investor's future projections
Degree of debt repayment	η	Investor's strategy
Growth rate of FCFE at time t	λ_t	From current CF statement and investor's future projections at time t, cf. case study
Risk free rate of return	r_f	Government bond of same maturity
Market rate of return	r_m	Estimated by applying CAPM to the publicly listed peer group
Standard deviation of market	σ_I	Inferred from volatility of assigned market index
Correlation	$Corr_{e,I}$	Inferred from stock prices of peer group against the market index
Unlevered beta factor	β^{unlev}	Estimated by applying CAPM to the publicly listed peer group
Hurdle rate of return	h	Investor's committed IRR to its fund
Investor's risk appetite	γ	Investor's self-assessment
Unsystematic risk	a	Based on assumptions about the inefficency of the company in comparison to market portfolio in the CAPM

Table 4.1: The input variables for the case study.

model, and identify the drift λ_i on the grid by linear interpolation

$$\hat{\lambda}_i = \frac{E_{i+1} - E_i}{E_i}.$$

We can now derive the first event (E_{t_1}, m_{t_1}) by using the EULER scheme provided in section 4, and can thereof update the variables l_1, σ_1^E. Repeating this procedure yields one path of $(E_t, m_t)_{t=t_0,...,t_n}$.

Further, similarly to ordinary cash flows, we assign the investors expectations to the multiple $\overline{m}_0$ and $\overline{m}_T$ to the grid by linear interpolation

$$\overline{m}_i := \overline{m}_0 + \frac{i}{n}(\overline{m}_T - \overline{m}_0).$$

Deterministic extra-ordinary CFs are simply added in any period to FCFE_i. An extra-ordinary random event S_i is simulated by taking a lognormal distribution with parameters (μ^J, σ^J) such that

$$\mathbb{E}(S_i) = e^{\mu^J + (\sigma^J)^2/2} \quad \text{and } \text{Var}(S_i) = e^{2\mu^J + (\sigma^J)^2}\left(e^{(\sigma^J)^2} - 1\right).$$

Being especially interested in analysing the cash-out behaviour of equity investors, we want to compare the effects of an early recap to the effects of a later one. We do this by taking one single jump $N(T) = 1$. For a Poisson process, given $N(T) = 1$, regardless of the intensity, this single jump occurs uniformly in $[0, T]$, which is against our intuition of the investment situation. Consequently, we invoke a different simulation scheme. Let us assume that the investor has the choice between a recap $S \sim LN(\mu^J, (\sigma^J)^2)$ that takes place in mean at $t = t_1$ and the same recap in mean at $t = t_2$. In order to analyse, which recap the investor should prefer, we do the following: We set the single inter-jump time $G_1 \sim \exp(g)$ such that $g = 1/t_1$ and $g = 1/t_2$, respectively. Therewith, we are consistent with the expectations of the investor in both cases by taking

$$J_t = S1_{\{G_1 \leq t\}},$$

as $\mathbb{E}(G_1) = t_1$ and t_2, respectively.

Taking a path $(E_t, m_t, J_t)_{t=t_0,...,t_n}$ we can calculate for all $t = t_0, ...t_n$ the events $Debt_t, EV_t$ and determine whether bankruptcy was triggered. Further, using this path, we can calculate the values CFV and TV, as the risk adjusted rate of return r_t can simply be evaluated on the grid.

We are now ready to start a Monte Carlo simulation.

5.2 Simulation Results

Before working through the selected case study we point out that some dimensions of our study, e.g. the revenue model's characteristic, had to be handled confidentially. For the accessed investment data a non-disclosure agreement was signed, so we needed to anonymize confidential information and, thus, we can only present selected or adjusted data[29]. On the other hand, since

[29] In particular, we can not show explicit calculations, currency, or fundamental data of the investment.

our study is based on real data, it has a solid real life foundation, which also reflects in the output variables, which can be interpreted like real life data. We put on record that for the present investment the investors expectations show strong growth rates of CFs (the simulation realisation will be $\lambda_t \geq 70\%$), thus we should only see small default rates. Hence, we will be more interested in analysing if the investment plan fits the bill, in terms of announced IRR to investors.

Instead of analysing a pool of possible investments, we focus our work on analysing a range of possible investment strategies for the selected small size company. Within this work we will deal with three main strategic configurations in order to show that our model is appropriate for small cap investments: The initial leverage level l_0, the equity contribution $1 - \eta$, and the possibility for the investor to recapitalise. We will determine the optimal configuration of the financing strategy in three steps [30]:

- Firstly, we determine the appropriate leverage level to match the demanded IRR of the investor
- Secondly, given the selected optimal leverage level, we identify the optimal level of equity contribution.
- And, thirdly, we will examine, whether the chosen strategy can be improved by a debt-financed recap.

Each configuration is analysed within a sample of 10.000 paths, generated by a Monte Carlo simulation using the same starting data (initial cash flow statement and balance sheet).

Further, we assume that we are dealing with an investor with $\gamma = \infty$, i.e. whose risk aversion is infinite for outcomes of IRRs below 30% ($h = 30\%$) and who is risk neutral for outcomes above 30%. Hence, we can simply measure the risk by the probability of IRR to fall below the targeted hurlde rate of 30% (%-IRR <30%). Further, we will also show default rates as a second risk measure (%-default). Returns are measured in terms of mean value of the cash equity basis C, as well as in terms of demanded IRR. The different values for IRR estimate the expected IRR for different conditions. $\overline{IRR}_1$ shows the average of all positive IRRs, given by

$$\overline{IRR}_1 = \frac{1}{\#\left\{IRR_i \mid IRR_i > 0\right\}} \sum_{i=1}^{\#\{IRR_i | IRR_i > 0\}} IRR_i \mathbf{1}_{\{IRR_i > 0\}},$$

wereas $\overline{IRR}_2$ includes also all neagative values and no defaults and thus estimates the expected IRR under the condition of a succeeding investment, in mathematical terms i.e. $\mathbb{E}(IRR|\tau > T)$. $\overline{IRR}_3$ also includes defaults, i.e. $IRR = -1$. Furthermore, we calculate an adjusted version of $\overline{C}$ which estimates the unconditional expected value of the cash equity basis C. This is given by

$$\overline{C}_{\text{adjusted}} = \left(1 - \frac{\text{default rate}}{\text{number of simulations}}\right) \overline{C \mid \tau > T}.$$

[30]Note that this approach examines each strategic event on its own, without taking interdependencies into account.

Table 5.2 shows the simulated risk-return profile in dependency on employed leverage [31]. Drawing a leverage level of 35% on the investment, we arrive at a targeted IRR of about 49%. On average, negative IRRs or defaults are not very significant. A de-leveraging of the transaction

Leverage	95.0%	85.0%	75.0%	65.0%	55.0%	45%	35%
$\overline{IRR}_1$	205.07%	113.02%	82.80%	68.60%	61.38%	56.87%	49.91%
$\overline{IRR}_2$	135.72%	84.79%	71.00%	62.21%	58.01%	54.89%	48.67%
$\overline{IRR}_3$	90.06%	59.60%	58.65%	56.57%	56.48%	54.51%	48.62%
$\overline{C \mid \tau > T}$ in mn	412.78	205.58	79.46	50.27	49.02	44.68	43.78
%-default	29.89%	24.58%	12.00%	5.50%	1.52%	0.38%	0.05%
%-IRR < 30%	71.40%	51.51%	31.64%	22.63%	15.97%	13.20%	15.90%
$\overline{C}_{\text{adjusted}}$	289.40	155.05	69.93	47.50	48.99	48.83	43.76

Table 5.2: Risks and returns in dependency on employed leverage, in terms of mean values for the internal rate of return, mean value of the cash equity basis C, the probability to default,the probability of IRR to fall below the targeted hurdle rate of 30% and an adjusted version for the mean value of the cast equity basis.

leads to a higher equity investment and translates into an even lower expected return from a successful investment. We establish that the investor, for this particular investment, should prefer a leverage level of 45%, as it dominates [32] the strategy in terms of risk and return. Taking an optimal configuration of 45% leverage, Table 5.3 shows the outcomes in terms of different equity contributions.

$1-\eta$	100.0%	90.0%	80.0%	70.0%	60.0%	50.0%	40.0%	30.0%	20.0%	10.0%	0.0%
$\overline{IRR}_1$	56.87%	55.84%	55.01%	54.11%	53.45%	52.4%	51.61%	50.95%	49.8%	49.13%	48.31%
$\overline{IRR}_2$	54.89%	53.57%	52.95%	51.89%	51.00%	49.9%	48.92%	48.17%	46.7%	46.22%	45.36%
$\overline{IRR}_3$	54.51%	53.02%	52.59%	51.46%	50.61%	49.5%	48.44%	47.68%	46.0%	45.67%	44.75%
$\overline{C \mid \tau > T}$ in mn	49.02	47.61	46.80	45.62	44.32	43.8	42.58	42.02	40.2	39.72	38.82
%-default	0.38%	0.54%	0.36%	0.43%	0.38%	0.4%	0.39%	0.44%	0.4%	0.48%	0.34%
%-IRR < 30%	13.20%	14.60%	14.72%	15.20%	16.00%	17.4%	18.01%	19.35%	20.4%	21.64%	21.82%
$\overline{C}_{\text{adjusted}}$	48.83	47.36	46.63	45.42	44.16	43.6	42.42	41.84	40.1	39.53	38.69

Table 5.3: Risks and returns in dependency on employed equity contribution, in terms of mean values for the internal rate of return, mean value of the cash equity basis C, the probability to default,the probability of IRR to fall below the targeted hurdle rate of 30% and an adjusted version for the mean value of the cast equity basis.

We see that a different equity contribution neither significantly changes the implicit return nor the associated risk. Nevertheless, the strategy with a 100% distribution to equity $(1-\eta = 1)$ dominates all other strategies in terms of risk and return. Hence, instead of repaying debt to decrease risks investors should maximise their dividends. Finally, the full dividend strategy suggests that the configuration of $1-\eta = 1$ and $l_0 = 0.55$ could be improved by an extra dividend in terms of a debt financed recap.

[31] Leverage indicates initial leverage measured by debt at time 0 divided by total capital at time 0; Table 5.2 shows results for a constant $\eta = 1$.

[32] We say a strategy i dominates a strategy j in terms of risk and return if $Risk_i < Risk_j$ or $Risk_i = Risk_j$ and $Return_i \geq Return_j$.

Table 5.4 shows the results for different recap scenarios [33]. The numbers support a maximum extra dividend (recap) of $5.0mn$ in $t = 1$ as it dominates all the scenarios, as well as the strategy without a recap. As one can see from Table 5.4 a recap decreases the probability to fail to reach the announced IRR-target.

Value in mn	in t	0.0	1.0	2.0	3.0	4.0	5.0
$\overline{IRR}$	1	56.87%	56.59%	57.52%	59.26%	61.01%	62.87 %
	2		55.93%	56.13%	57.44%	58.20%	59.41%
$\overline{IRR}_2$	1	54.89%	54.91%	56.05%	57.92%	60.00%	62.17%
	2		54.01%	54.50%	56.16%	56.96%	58.51%
$\overline{IRR}_3$	1	54.51%	54.62%	55.65%	57.60%	59.70%	61.90%
	2		53.70%	54.07%	55.85%	56.58%	58.18%
$\overline{C \mid \tau > T}$ in mn	1	49.02	48.02	47.90	49.08	50.41	52.04
	2		48.07	47.88	49.54	50.28	52.21
%-default	1	0.38%	0.26%	0.28%	0.28%	0.31%	0.39%
	2		0.31%	0.34%	0.39%	0.45%	0.49%
%-IRR $<$ 30%	1	13.20%	13.19%	11.87%	10.26%	8.53%	6.88%
	2		14.35%	13.81%	11.47%	11.12%	9.68%
$\overline{C}_{\text{adjusted}}$	1	48.83	47.90	47.77	48.94	50.26	51.84
	2		47.92	47.71	49.34	50.06	51.96

Table 5.4: Risk and returns for various recap scenarios, in terms of demanded IRR, mean value of the cash equity basis C, the probability to default and the probability of IRR to fall below the targeted hurdle rate of 30%.

An investor, taking the expectations from $t = 0$, valuing a strategy according to risk and return, in terms of IRR and the probability to fall below the exogenously given target level of 30%, should cash out this investment. With reference to MODIGLIANI/ MILLER, debt-financed extra-dividends should not increase the EV [34]. As investors value their investments on equity basis and also face the risk to default, the overall effect on the investor's dividend policy is ambiguous. Coming back to our example in the introduction, our model supports, respectively to the assumed constellation, the decisions of investors to cash out investments.

Figure 2 and 3 show histograms and kernel density estimates for simulated IRRs in different scenarios. The graphics are based on all simulated values for the IRR, including negative values and defaults. The plots are restricted to a range between -100% indicating defaults and 200%. In particular, we see a significant effect for different leverage values in the histograms. The large peak at -100% corresponds to the number of defaults, which in the case of a high leverage level is very high as well. The high leverage level also leeds to a much brider distribution of the

[33] We restrict the recap configuration to a maximum of 5mn which is due to creditor issues arising from recaps exceeding 5mn. We analyse a 100% debt-financed recap at a growth stage, $t = 1$, and at a mature stage, $t = 2$. Further, we restrict our case study to the first recap, in order to study the effects of a single action. Hence $\forall t \in [0, T]$ J_t becomes $J_t = S_1 1_{\{G_1 \leq t\}}$ with $g \in \left\{\frac{1}{t_1}, \frac{1}{t_2}\right\}$ and the lognormal S_1 such that $\mathbb{E}(S_1) = 1, ..., 5$ millions with $Var(S_1) = (10\%\mathbb{E}(S_1))^2$.

[34] MILLER et al. (1961) Dividend Policy, Growth, and the Valuation of Shares, p. 412.

simulated IRRs, where really high rates can be reached.

All other plots look more or less stable in the context of default numbers. This is due to the fact that all other simulations are done with the optimal leverage configuration of 45%. This stabilizes the number of defaults to a relatively low level and the histograms look similar.

6 Conclusion

With this work we developed a new stochastic model of CFs for private equity investors based on an ordinary discounted cash flow (DCF) approach. Thereby we have been able to analyse risks of a small cap investment in terms of different investment strategies. Our model compromises the four fundamental value drivers identified by BCG: the active value drivers top line growth and operational efficiency are accounted by the CF drift. The passive value driver multiple expansion is integrated by assuming a stochastic process to the multiple evolution, which accounts for market expectations. The leverage effect is mapped by the underlying capital structure, affecting CF risk, the risk adjusted discount rate, and risk premiums that are captured in the expected exit value. Finally, empirical characteristics like high default rates counterbalanced by superior upsides [35], could be matched.

In closing this paper, we also want at least mention some shortcomings of our model. We worked on the premises that multiple and CF processes are independent. An extended version of the model provided here could be set up without this assumption. Further, multivariate modelling for several business units or a fund of investments was not provided; this is one topic for further research in this field [36]. Also, a flexible investment horizon similar to the exercise time of American options, adapting to the dynamics of the CFs and multiple, should be investigated in the future. Nevertheless, it was the goal of this work to develop an easy-to-use tool to measure the risk of private equity investments. The paper has shown a first approach to solve such a problem by taking up several ideas and assumptions from real-options theory and capital-budgeting techniques, but also incorporating valuable suggestions made by practitioners.

We have seen in the case study that it can be optimal for investors to reduce CFs at risk by cashing out the investment via recaps. Thus at least for the studied investment, our model supports on the one hand the image of 'locusts', as discussed in the media. But on the other hand it has been shown that investment risk can be reduced by cashing out the investment. Hence, we raise the question whether the analysed risks are also crucial for social welfare or just crucial for the private equity investor to fulfil a target return level.

[35] Cf. COCHRANE, J.H. (2001) The risk and return of venture capital investments, NBER Working Paper Series No. 8066, p. 38, Tab. 1.

[36] Cf. BÖCKER (2008) Modelling and Measuring Business Risk, p.4.

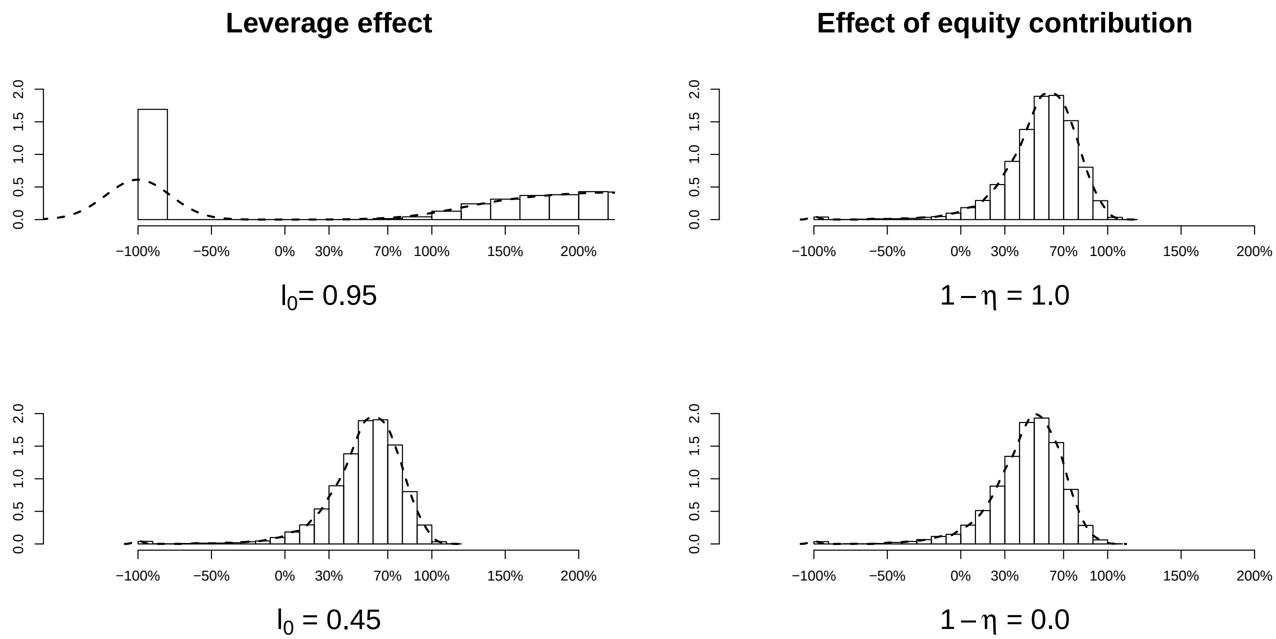

Figure 2: Histogram and kernel density plots for simulated IRRs including defaults and negative values comparing leverage and equity contribution effects.

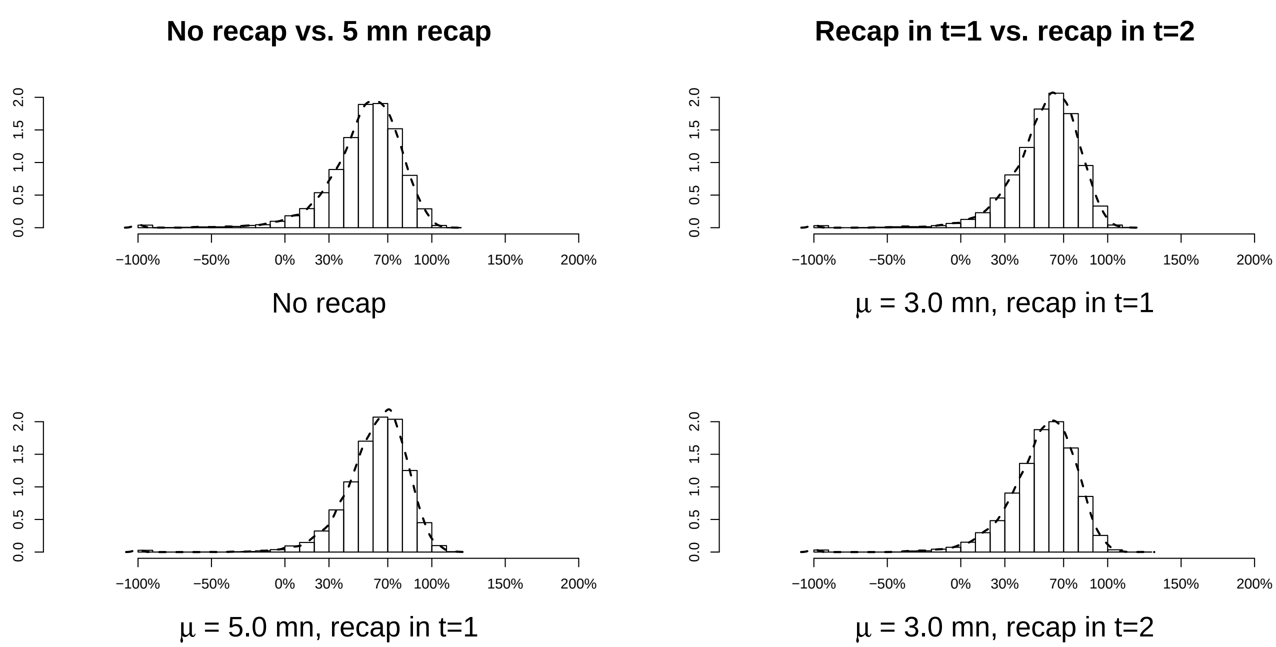

Figure 3: Histogram and kernel density plots for simulated IRRs including defaults and negative values for different recap scenarios. Recall that the recap S is lognormally distributed with $(\mu^J, (\sigma^J)^2)$.

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