Lehrstuhl für Regelungstechnik
Technical Reports on Automatic Control
Vol. TRAC-4, August 2009
Available online at: www.rt.mw.tum.de
TECHNISCHE UNIVERSITÄT MÜNCHEN

# Passivity Preserving Order Reduction of Linear Port-Hamiltonian Systems by Moment Matching

Boris Lohmann*, Thomas Wolf, Rudy Eid and Paul Kotyczka
Lehrstuhl für Regelungstechnik, Technische Universität München
Boltzmannstr. 15, D-85748 Garching, Germany

**Abstract**

In this paper, a new structure-preserving method for the reduction of linear port-Hamiltonian systems with dissipation using Krylov subspaces is presented. It is shown how to choose the projection matrices in order to guarantee the moment matching property and to obtain a passive and thus stable reduced order model in port-Hamiltonian form. The method is suitable for the reduction of large-scale systems as it employs the well-known Arnoldi algorithm and matrix-vector multiplications to compute the reduced-order model.

*Keywords:* Order Reduction; port-Hamiltonian systems; Structure Preserving; Large-Scale Systems; Moment matching; Krylov Subspace.

## 1 Introduction

Port-based modeling of physical systems leads to particularly structured models that can be represented as *port-Hamiltonian systems*. This special structure, which inherits passivity and therefore stability as key properties, is widely used to represent lumped-parameter as well as spatially discretized distributed-parameter systems. Given a complex real-world problem, its accurate modeling leads to large-scale systems having a high number of differential equations. To investigate the dynamics of these high-dimensional models, or to design a controller, it is aimed at replacing the original model with a low dimensional approximation. For this purpose, well-established model reduction methods like *Balancing and Truncation* [1] and *Krylov subspace methods* [2, 3] can be used.

However, these methods do not generally preserve the structure of the original system, which means that properties like stability and passivity, or special matrix structures can be lost during or after the reduction step. Since the original system is in port-Hamiltonian form, it is desired to obtain a reduced system that retains this structure, and therefore preserves stability and passivity. For instance, in [4], a truncated balanced realization technique, which is structure preserving for a special class of port-Hamiltonian systems, is presented.

In this paper, a structure-preserving Krylov-based order reduction method for linear port-Hamiltonian systems is introduced. By applying suitable state transformations to the original port-Hamiltonian structure and employing the classical Krylov subspace method, a reduced-order

*Corresponding author: Tel: +49 89 289 15610, Fax: +49 89 25915653, email: Lohmann@tum.de

model also in port-Hamiltonian form is obtained. Furthermore, integrating the state transformation into the computation of the projection matrices results in a computationally efficient and numerically stable algorithm for the reduction of large-scale systems.

In the following section, the definition of port-Hamiltonian systems is presented. In section 3, a short introduction into the theory of model reduction employing moment matching is given. Based on an appropriate state transformation, the problem of structure preserving model reduction is solved in section 4. The paper is concluded by a summary of the results and an outlook.

## 2 Port-Hamiltonian Systems

Consider the time invariant port-Hamiltonian system of the form

$$\begin{cases} \dot{\boldsymbol{x}} = (\boldsymbol{J} - \boldsymbol{R})\,\nabla H(\boldsymbol{x}) + \boldsymbol{G}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{G}^T \nabla H(\boldsymbol{x}), \end{cases} \tag{1}$$

where $H(\boldsymbol{x}) \geq 0$ is the total stored energy called Hamiltonian, $\boldsymbol{J} \in \mathbb{R}^{n\times n}$ the skew-symmetric interconnection matrix and $\boldsymbol{R} \in \mathbb{R}^{n\times n}$ the symmetric positive semi-definite dissipation or damping matrix. $\boldsymbol{J}$ represents the exchange of energy between the storage elements (states $\boldsymbol{x} \in \mathbb{R}^n$) of the system while dissipation is characterized by $\boldsymbol{R}$. The input matrix $\boldsymbol{G} \in \mathbb{R}^{n\times m}$ and the gradient $\nabla H(\boldsymbol{x})$ of the energy function define the collocated output $\boldsymbol{y} \in \mathbb{R}^m$, which together with the input $\boldsymbol{u} \in \mathbb{R}^m$ constitutes the power port $(\boldsymbol{u}, \boldsymbol{y})$ of the system. From the energy balance

$$\dot{H}(\boldsymbol{x}) \leq \boldsymbol{y}^T \boldsymbol{u}$$

the passivity of the port-Hamiltonian system is deduced. If $H(\boldsymbol{x})$ is positive definite, Lyapunov stability of the unforced system follows directly, however if $H(\boldsymbol{x})$ is only semi-definite, zero-state detectability is necessary in addition. Asymptotic stability of the port-Hamiltonian system can be checked by the invariance principle of Krassowskij-LaSalle. A more detailed overview on port-Hamiltonian systems can be found in [6].

In the case of linear port-Hamiltonian systems, the matrices $\boldsymbol{J}$, $\boldsymbol{R}$ and $\boldsymbol{G}$ are constant matrices, independent of the state vector $\boldsymbol{x}$. Additionally, the Hamiltonian is in fact a quadratic energy function of the form

$$H(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T \boldsymbol{Q}\boldsymbol{x}, \tag{2}$$

with $\boldsymbol{Q} \in \mathbb{R}^{n\times n}$ symmetric and positive definite. The gradient of the Hamiltonian in this case is

$$\nabla H(\boldsymbol{x}) = \boldsymbol{Q}\boldsymbol{x}, \tag{3}$$

resulting in the following state space representation (SISO case):

$$\begin{cases} \dot{\boldsymbol{x}} = (\boldsymbol{J} - \boldsymbol{R})\,\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{g}u \\ y = \boldsymbol{g}^T \boldsymbol{Q}\boldsymbol{x}, \end{cases} \tag{4}$$

with the constant vector $\boldsymbol{g} \in \mathbb{R}^m$.

Note that, in case the symmetry and definiteness properties of the matrices $\boldsymbol{J}$, $\boldsymbol{R}$ and $\boldsymbol{Q}$ are preserved while reducing the model order, the port-Hamiltonian form and therefore passivity and stability will automatically be preserved.

## 3 Order Reduction by Moment Matching

In this section, a common Krylov-based model order reduction scheme is presented very briefly.

**Definition 1.** Given a matrix $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ and a vector $\boldsymbol{v} \in \mathbb{R}^{n}$, the $q$th Krylov subspace $\mathcal{K}_q(\boldsymbol{M}, \boldsymbol{v})$ is spanned by a sequence of $q$ column vectors called basic vectors as follows [2]:

$$\mathcal{K}_q(\boldsymbol{M}, \boldsymbol{v}) = \text{span}\left\{\boldsymbol{v}, \boldsymbol{M}\boldsymbol{v}, \ldots, \boldsymbol{M}^{q-1}\boldsymbol{v}\right\}. \tag{5}$$

The system (4) can be treated as a linear time invariant state space model of the form

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u \\ y = \boldsymbol{c}^T\boldsymbol{x}, \end{cases} \tag{6}$$

having the following transfer function:

$$H(s) = \boldsymbol{c}^T(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}. \tag{7}$$

The aim of moment matching is to find a reduced order model of order $q \ll n$, whose moments (which are defined as the negative coefficients of the Taylor series expansion of the transfer function) around a certain point $s_0$ match some of those of the original one. One way to calculate this reduced order model is by applying a projection $\boldsymbol{x} = \boldsymbol{V}\boldsymbol{x}_r$ to the original model as follows:

$$\begin{cases} \boldsymbol{V}^T\boldsymbol{V}\dot{\boldsymbol{x}}_r = \boldsymbol{V}^T\boldsymbol{A}\boldsymbol{V}\boldsymbol{x}_r + \boldsymbol{V}^T\boldsymbol{b}u \\ y = \boldsymbol{c}^T\boldsymbol{V}\boldsymbol{x}_r, \end{cases} \tag{8}$$

with $\boldsymbol{V} \in \mathbb{R}^{n\times q}$. The projection matrix $\boldsymbol{V}$ is chosen to form a basis either of the input or of the output Krylov subspaces $\mathcal{K}_q\left(\boldsymbol{A}^{-1}, \boldsymbol{A}^{-1}\boldsymbol{b}\right)$ or $\mathcal{K}_q\left(\boldsymbol{A}^{-T}, \boldsymbol{A}^{-T}\boldsymbol{c}^T\right)$ respectively, leading to matching the first $q$ moments around $s_0 = 0$. Matching the moments around zero preserves the low-frequency behavior of the original system, whereas, in order to compute a reduced model that approximates the middle or high frequency behavior, matching a certain number of moments around $s_0 \neq 0$ is to be preferred. This is achieved by choosing $\boldsymbol{V}$ as basis of the shifted input or output Krylov subspace $\mathcal{K}_q\left((\boldsymbol{A} - s_0\boldsymbol{I})^{-1}, (\boldsymbol{A} - s_0\boldsymbol{I})^{-1}\boldsymbol{b}\right)$ or $\mathcal{K}_q\left((\boldsymbol{A} - s_0\boldsymbol{I})^{-T}, (\boldsymbol{A} - s_0\boldsymbol{I})^{-T}\boldsymbol{c}^T\right)$ respectively.

For the numerical calculation of $\boldsymbol{V}$, the known Arnoldi algorithm can be used, which returns an orthonormal basis $\boldsymbol{V}$ (with $\boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{I}$) of the required Krylov subspace.

## 4 Structure Preserving Model Order Reduction

Before reducing the original model, a state transformation $\boldsymbol{z} = \boldsymbol{Q}^{1/2}\boldsymbol{x}$, $\boldsymbol{x} = \boldsymbol{Q}^{-1/2}\boldsymbol{z}$ to the system (4) is suggested:

$$\begin{cases} \dot{\boldsymbol{z}} = \overbrace{\boldsymbol{Q}^{1/2}(\boldsymbol{J} - \boldsymbol{R})\boldsymbol{Q}^{1/2}}^{\widetilde{\boldsymbol{A}}}\boldsymbol{z} + \overbrace{\boldsymbol{Q}^{1/2}\boldsymbol{g}}^{\widetilde{\boldsymbol{b}}}u \\ y = \underbrace{\boldsymbol{g}^T\boldsymbol{Q}^{1/2}}_{\widetilde{\boldsymbol{c}}^T}\boldsymbol{z}. \end{cases} \tag{9}$$

In order to match the first $q$ moments of the above model, the orthonormal matrix $\boldsymbol{V}^* \in \mathbb{R}^{n\times q}$ of the projection $\boldsymbol{z} = \boldsymbol{V}^*\boldsymbol{z}_r$ is calculated using the standard input Krylov subspace[1]

$$\begin{aligned} K_{\boldsymbol{V}^*} &= span(\widetilde{\boldsymbol{A}}^{-1}\widetilde{\boldsymbol{b}}, \ldots, \widetilde{\boldsymbol{A}}^{-q}\widetilde{\boldsymbol{b}}) = && (10) \\ &= span\left\{\left(\boldsymbol{Q}^{1/2}(\boldsymbol{J} - \boldsymbol{R})\boldsymbol{Q}^{1/2}\right)^{-1}\boldsymbol{Q}^{1/2}\boldsymbol{g}, \ldots, \left(\boldsymbol{Q}^{1/2}(\boldsymbol{J} - \boldsymbol{R})\boldsymbol{Q}^{1/2}\right)^{-q}\boldsymbol{Q}^{1/2}\boldsymbol{g}\right\}, && (11) \end{aligned}$$

[1]The output subspace can be used instead.

leading to the following state space representation of the reduced model:

$$\begin{cases} \dot{\boldsymbol{z}}_r = \boldsymbol{V}^{*T}\boldsymbol{Q}^{1/2}\left(\boldsymbol{J}-\boldsymbol{R}\right)\boldsymbol{Q}^{1/2}\boldsymbol{V}^{*}\boldsymbol{z}_r + \boldsymbol{V}^{*T}\boldsymbol{Q}^{1/2}\boldsymbol{g}u \\ y = \boldsymbol{g}^T\boldsymbol{Q}^{1/2}\boldsymbol{V}^{*}\boldsymbol{z}_r, \end{cases} \tag{12}$$

which is equivalent to

$$\begin{cases} \dot{\boldsymbol{z}}_r = \left(\boldsymbol{J}_r^{*}-\boldsymbol{R}_r^{*}\right)\boldsymbol{z}_r + \boldsymbol{g}_r^{*}u \\ y = \boldsymbol{g}_r^{*T}\boldsymbol{z}_r. \end{cases} \tag{13}$$

Hence, the matrices of the reduced port-Hamiltonian model are defined as

$$\boldsymbol{J}_r^{*} = \boldsymbol{V}^{*T}\boldsymbol{Q}^{1/2}\boldsymbol{J}\,\boldsymbol{Q}^{1/2}\boldsymbol{V}^{*} \tag{14}$$

$$\boldsymbol{R}_r^{*} = \boldsymbol{V}^{*T}\boldsymbol{Q}^{1/2}\boldsymbol{R}\,\boldsymbol{Q}^{1/2}\boldsymbol{V}^{*} \tag{15}$$

$$\boldsymbol{Q}_r^{*} = \boldsymbol{I} \tag{16}$$

$$\boldsymbol{g}_r^{*} = \boldsymbol{V}^{*T}\boldsymbol{Q}^{1/2}\boldsymbol{g}. \tag{17}$$

It can be seen that the interconnection matrix $\boldsymbol{J}_r^{*}$ of the reduced model is still skew-symmetric, and the dissipation matrix $\boldsymbol{R}_r^{*}$ symmetric and positive semi-definite. Furthermore, the matrix $\boldsymbol{Q}_r^{*} = \boldsymbol{I}$ is clearly symmetric and positive definite. Therefore, the port-Hamiltonian structure of the original model is preserved.

If $\boldsymbol{Q}$ is *diagonal* (which is often true), then the required matrix $\boldsymbol{Q}^{1/2}$ can be easily calculated. In this case, applying the Arnoldi algorithm to (11) is the essential numerical effort in calculating the reduced model (13)-(17).

If, however, $\boldsymbol{Q}$ is *non-diagonal*, then the calculation of $\boldsymbol{Q}^{1/2}$ can be costly and should be avoided. Therefore, the above algorithm will be subsequently modified, resulting in a numerically efficient reduction scheme for models with general matrices $\boldsymbol{Q} > 0$. In a first step a new projection matrix

$$\boldsymbol{V} = \boldsymbol{Q}^{-1/2}\boldsymbol{V}^{*}, \tag{18}$$

is introduced, corresponding to the following Krylov subspace:

$$K_{\boldsymbol{V}} = span\left\{\boldsymbol{Q}^{-1/2}\left[\left(\boldsymbol{Q}^{1/2}(\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q}^{1/2}\right)^{-1}\boldsymbol{Q}^{1/2}\boldsymbol{g}\,,\,\ldots\,,\,\left(\boldsymbol{Q}^{1/2}(\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q}^{1/2}\right)^{-q}\boldsymbol{Q}^{1/2}\boldsymbol{g}\right]\right\} \tag{19}$$

$$= span\left\{\left((\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q}\right)^{-1}\boldsymbol{g}\,,\,\ldots\,,\,\left((\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q}\right)^{-q}\boldsymbol{g}\right\}. \tag{20}$$

By comparing (11) with (19), it can be seen that $\boldsymbol{V}^{*}$ and $\boldsymbol{Q}^{1/2}\boldsymbol{V}$ span the same subspace. This suggests the usage of the projection matrix $\boldsymbol{V}$ (the calculation of which can be done using a standard Arnoldi procedure), thereby avoiding the numerically expensive computation of $\boldsymbol{Q}^{1/2}$. Note that, when calculating the matrix $\boldsymbol{V}^{*}$ by

$$\boldsymbol{V}^{*} = \boldsymbol{Q}^{1/2}\boldsymbol{V}, \tag{21}$$

its orthonormality property is lost, i.e. $\boldsymbol{V}^{*T}\boldsymbol{V}^{*} \neq \boldsymbol{I}$. Therefore, instead of (12) the reduced model becomes

$$\begin{cases} \boldsymbol{V}^{*T}\boldsymbol{V}^{*}\dot{\boldsymbol{z}}_r = \boldsymbol{V}^{*T}\boldsymbol{Q}^{1/2}\left(\boldsymbol{J}-\boldsymbol{R}\right)\boldsymbol{Q}^{1/2}\boldsymbol{V}^{*}\boldsymbol{z}_r + \boldsymbol{V}^{*T}\boldsymbol{Q}^{1/2}\boldsymbol{g}u \\ y = \boldsymbol{g}^T\boldsymbol{Q}^{1/2}\boldsymbol{V}^{*}\boldsymbol{z}_r. \end{cases} \tag{22}$$

Applying the transformation (21) to the system above results in

$$\begin{cases} \boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{V}\dot{\boldsymbol{z}}_r = \boldsymbol{V}^T\boldsymbol{Q}\left(\boldsymbol{J}-\boldsymbol{R}\right)\boldsymbol{Q}\boldsymbol{V}\boldsymbol{z}_r + \boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{g}u \\ y = \boldsymbol{g}^T\boldsymbol{Q}\boldsymbol{V}\boldsymbol{z}_r. \end{cases} \tag{23}$$

With a second state transformation $\boldsymbol{x}_r = \boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{V}\boldsymbol{z}_r$, $\boldsymbol{z}_r = \left(\boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{V}\right)^{-1}\boldsymbol{x}_r$, the reduced model becomes

$$\begin{cases} \dot{\boldsymbol{x}}_r = \boldsymbol{V}^T\boldsymbol{Q}\left(\boldsymbol{J}-\boldsymbol{R}\right)\boldsymbol{Q}\boldsymbol{V}\left(\boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{V}\right)^{-1}\boldsymbol{x}_r + \boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{g}u \\ y = \boldsymbol{g}^T\boldsymbol{Q}\boldsymbol{V}\left(\boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{V}\right)^{-1}\boldsymbol{x}_r. \end{cases} \tag{24}$$

This model still matches the first $q$ moments of the original system, and is again in port-Hamiltonian form, leading to the main result of this paper:

**Theorem 1.** Given a linear port-Hamiltonian system of the form

$$\begin{cases} \dot{\boldsymbol{x}} = \left(\boldsymbol{J}-\boldsymbol{R}\right)\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{g}u \\ y = \boldsymbol{g}^T\boldsymbol{Q}\boldsymbol{x}. \end{cases} \tag{25}$$

Then, applying the projection $\boldsymbol{x} = \boldsymbol{V}\boldsymbol{x}_r$ with the columns of $\boldsymbol{V}$ forming a basis of the Krylov subspace

$$K_{\boldsymbol{V}} \;=\; span\left\{((\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q})^{-1}\boldsymbol{g}\,,\,\ldots\,,\,((\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q})^{-q}\boldsymbol{g}\right\}, \tag{26}$$

results in the reduced port-Hamiltonian system

$$\begin{cases} \dot{\boldsymbol{x}}_r = \left(\boldsymbol{J}_r-\boldsymbol{R}_r\right)\boldsymbol{Q}_r\boldsymbol{x}_r + \boldsymbol{g}_r u \\ y = \boldsymbol{g}_r^T\boldsymbol{Q}_r\boldsymbol{x}_r, \end{cases} \tag{27}$$

which matches the first $q$ moments of the original one, with the matrices

$$\boldsymbol{J}_r \;=\; \boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{J}\boldsymbol{Q}\boldsymbol{V} \tag{28}$$

$$\boldsymbol{R}_r \;=\; \boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{R}\boldsymbol{Q}\boldsymbol{V} \tag{29}$$

$$\boldsymbol{Q}_r \;=\; \left(\boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{V}\right)^{-1} \tag{30}$$

$$\boldsymbol{g}_r \;=\; \boldsymbol{V}^T\boldsymbol{Q}\boldsymbol{g}. \tag{31}$$

*Proof:* The matching of the first $q$ moments is true, as the original system is projected using $\boldsymbol{Q}^{1/2}\boldsymbol{V}$ spanning the same subspace as $\boldsymbol{V}^*$, for which the moment matching property is guaranteed. The resulting system (24) preserves the port-Hamiltonian structure as it is well known that if a square matrix $\boldsymbol{F} \in \mathbb{R}^{n\times n}$ is positive (negative) definite and $\boldsymbol{T} \in \mathbb{R}^{n\times q}$ is of maximum rank, then the matrix $\boldsymbol{T}^T\boldsymbol{F}\boldsymbol{T}$ is positive (negative) definite. In addition, the matrix $\boldsymbol{T}^T\boldsymbol{F}\boldsymbol{T}$ preserves the symmetry (skew-symmetry) of the matrix $\boldsymbol{F}$. ■

**Remark 1.** An alternative proof is to compare the moments of (25) and (27) directly.

**Remark 2.** The presented method can be easily generalized to the multi input, multi output case (MIMO) and to moment matching around $s_0 \neq 0$. For the latter purpose, the matrix $(\boldsymbol{A} - s_0\boldsymbol{I})$ has to be used instead of $\boldsymbol{A}$ in the calculation of the projection matrix, leading to

$$K_{\boldsymbol{V}} \;=\; span\left\{((\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q}-s_0\boldsymbol{I})^{-1}\boldsymbol{g}\,,\,\ldots\,,\,((\boldsymbol{J}-\boldsymbol{R})\boldsymbol{Q}-s_0\boldsymbol{I})^{-q}\boldsymbol{g}\right\}. \tag{32}$$

**Remark 3.** Note that the overall projection matrix can be easily shown to satisfy $\boldsymbol{x}_r = \boldsymbol{V}^T \boldsymbol{Q} \boldsymbol{x}$, which gives a better insight into the new method, by directly connecting the original state $\boldsymbol{x}(t)$ to the reduced one $\boldsymbol{x}_r(t)$.

**Remark 4.** By preserving the port-Hamiltonian structure, the method also preserves *stability*. Thereby, the method offers in fact a solution to the general stability preservation problem in Krylov subspace methods for linear dynamical systems, since every stable linear system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u$ can be written in port-Hamiltonian form $\dot{\boldsymbol{x}} = (\boldsymbol{J} - \boldsymbol{R})\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{g}u$ [5]. The approach is feasible if the numerical effort for this conversion can be accepted. Note that the output $y = \boldsymbol{c}^T \boldsymbol{x}$ does not need to be converted into the form $y = \boldsymbol{g}^T \boldsymbol{Q} \boldsymbol{x}$, since by using the *input* Krylov subspace as suggested in Theorem 1, the definition of the output $y$ does not affect the number of moments matched.

## 5 Conclusion

A new structure-preserving order reduction method for linear port-Hamiltonian systems has been presented. The main advantages of the new approach are that it allows matching the first $q$ moments of the original and reduced systems, preserves the port-Hamiltonian structure (and thus passivity and stability), and is applicable for the reduction of large-scale systems.

Presently, we work on applying the method to the model of a heat transfer problem. The first results are convincing and are intended to be presented soon.

Finally, it would be of interest to evaluate the benefits of the symmetric representation introduced in (9) when combined with Balancing and Truncation, to profit for instance from the error bound already existing.

## Acknowledgments

The authors would like to thank A.J. van der Schaft and K. Schlacher for the fruitful discussions in Oberwolfach and Munich in February and March 2009.

## References

[1] A. C. Antoulas. *Approximation of Large-Scale Dynamical Systems.* SIAM, Philadelphia, 2005.

[2] R. W. Freund. Model reduction methods based on Krylov subspaces. *Acta Numerica*, 12:267–319, 2003.

[3] E. J. Grimme. *Krylov Projection Methods for Model Reduction.* PhD thesis, Dep. of Electrical Eng., Uni. Illinois at Urbana Champaign, 1997.

[4] Ricardo Lopezlena, Jacquelien M. A. Scherpen, and Kenji Fujimoto. Energy-storage balanced reduction of port-hamiltonian systems. In *Proc. IFAC workshop of Lagrangian and Hamiltonian Methods for Nonlinear Control*, pages 79–84, Sevilla, Spain, April 2003.

[5] Stephen Prajna, A.J. van der Schaft, and Gjerrit Meinsma. An LMI approach to stabilization of linear port-controlled hamiltonian systems. *Systems & Control Letters*, 45:371–385, 2002.

[6] A.J. van der Schaft. *$L_2$-Gain and Passivity Techniques in Nonlinear Control.* Springer-Verlag, London, 2000.