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Codes and Bounds for Partially Defective Memory

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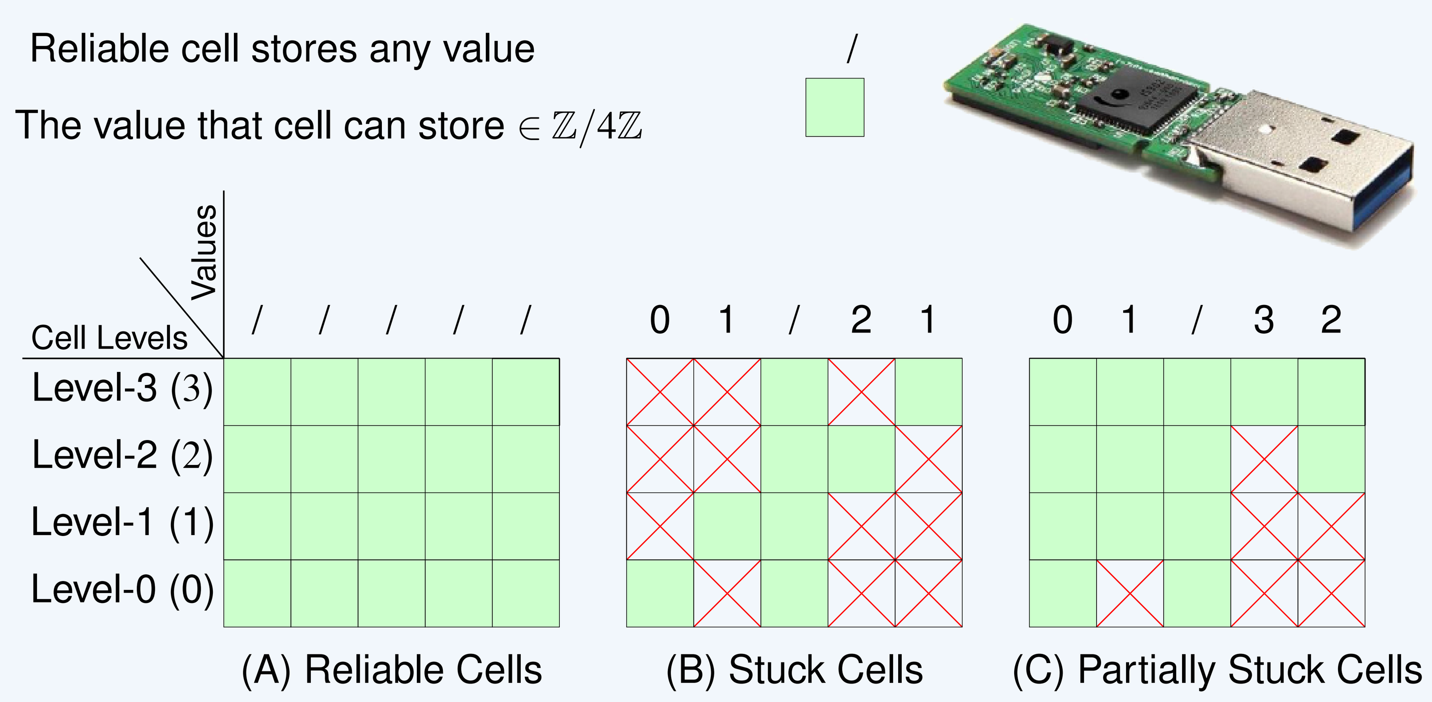
Introduction

The dominance of non-volatile memories and PCMs (phase change memories) as memory solutions for various applications have become evident due to their advantages as permanent storage devices [1].

Problem Description: PCMs may face failures in changing their states; in turn, their cells hold only one phase, becoming **stuck** (**defective**). On the other hand, random errors may occur in these faulty memories.

Solution: A mechanism called **masking** is used to determine a word whose entries coincide with writable levels at the (partially) stuck cells.



Previous works: In [2], the author considered the problem of masking fully stuck cells together with error correction. The error-free case with partially stuck cells has been considered in [3], in which improvements on the redundancy necessary for masking are achieved compared to [2].

Our Contribution: In our code constructions in [4], we consider the problem of combined error correction and masking of partially stuck cells, and we reduce the redundancy necessary for masking, similar to the results in [3], and even reduce further compared to [3, Construction 5].

Focus: This work aims to provide coding schemes and bounds on the memory of partially defective cells that can only store partial information, such as rewritten on these cells without erasing and simultaneously correcting substitution errors.

Definitions

(Σ, t)-PSMC: For $\Sigma \subset \mathbb{F}_q^n$ and non-negative integer t, a q-ary (Σ, t)-*partially-stuck-at-masking code* $\mathcal{C}$ of length n and size M is a coding scheme consisting of a message set $\mathcal{M}$ of size M, an encoder $\mathcal{E}$ and a decoder $\mathcal{D}$. If $\Sigma = \{\mathbf{s} \in \{0,1\}^n \mid wt(\mathbf{s}) \leq u\}$, we say q-ary $(u,1,t)$ PSMC.

Construction 1 [4, Construction 1]

Assume that there is an $[n,k,d]_q$ code $\mathcal{C}$ with a $k \times n$ generator matrix of the form

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{(k-1)\times 1} & \mathbf{I}_{k-1} & \mathbf{P}_{(k-1)\times(n-k)} \\ 1 & \mathbf{1}_{k-1} & \mathbf{1}_{n-k} \end{bmatrix},$$

where $\mathbf{I}_{k-1}$ is the $(k-1)\times(k-1)$ identity matrix, $\mathbf{P} \in \mathbb{F}_q^{(k-1)\times(n-k)}$, and $\mathbf{1}_\ell$ is the all-one vector of length ℓ. From the code $\mathcal{C}$, a PSMC can be obtained, whose encoder and decoder are shown in Algorithm and Algorithm .

Theorem 1 [4, Theorem 1]

The coding scheme in Construction 1 is a $(q-1, 1, \lfloor \frac{d-1}{2} \rfloor)$ PSMC of length n and cardinality q^{k-1}.

Algorithm 1 - Encoder $\mathcal{E}$

Input:

- Message: $\mathbf{m} = (m_0, m_1, \dots, m_{k-2}) \in \mathbb{F}_q^{k-1}$
- Positions of partially stuck-at-1 cells: ϕ such that $|\phi| = u$

1. Compute $\mathbf{w} = (w_1, w_2, \dots, w_{n-1}) = \cdot \mathbf{G}_1$
2. Find $v \in \mathbb{F}_q \setminus \{w_i \mid i \in \phi\}$
3. Compute $\mathbf{c} = \mathbf{w} - v \cdot \mathbf{G}_0$

Output: Codeword $\mathbf{c} \in \mathbb{F}_q^n$

Algorithm 2 - Decoder $\mathcal{D}$

Input: Retrieved

- Retrieve $\mathbf{y} = \mathbf{c} + \mathbf{e}$, $\mathbf{y} \in \mathbb{F}_q^n$

1. $\hat{\mathbf{c}} \leftarrow$ decode $\mathbf{y}$ in $\mathcal{C}$
2. $\hat{v} \leftarrow$ first entry of $\hat{\mathbf{c}}$
3. $\hat{\mathbf{w}} = (\hat{w}_0, \hat{w}_1, \cdots, \hat{w}_{n-1}) \leftarrow (\hat{\mathbf{c}} - \hat{v} \cdot \mathbf{G}_0)$
4. $\hat{\mathbf{m}} \leftarrow (\hat{w}_1, \dots, \hat{w}_{k-1})$

Output: Message vector $\mathbf{m} \in \mathbb{F}_q^{k-1}$

Theorem 2 [4, Theorem 9]: GV-type Bound

Let q be a prime power. Let n,k,t,u be non-negative integers such that

$$\sum_{i=0}^{2(t+\lfloor \frac{u}{q} \rfloor)} \binom{n}{i} (q-1)^i < q^{n-k+1}.$$

There exists a q-ary $(u,1,t)$ PSMC of length n and size q^{k-1}.

Remark [4, Remark 10]

GV-like bound from Theorem 1 is a special case of Theorem 2 for $u \leq q-1$.

Comparison

Comparison of other upper and lower limits to our derived GV-like bound in Theorem 2 taking $n = 114$, $q = 7$, $0 \leq t \leq 56$ and $u \leq q-1$. The dashed-dotted green curve shows the rates for Theorem 1 by Theorem 2 for $u \leq q-1$ in which codes that have the **all-one words** are considered. This curve for several code parameters matches the red line that shows the rates of BCH codes that contain all-one words concerning the designed distances $d \geq 2t+1$.

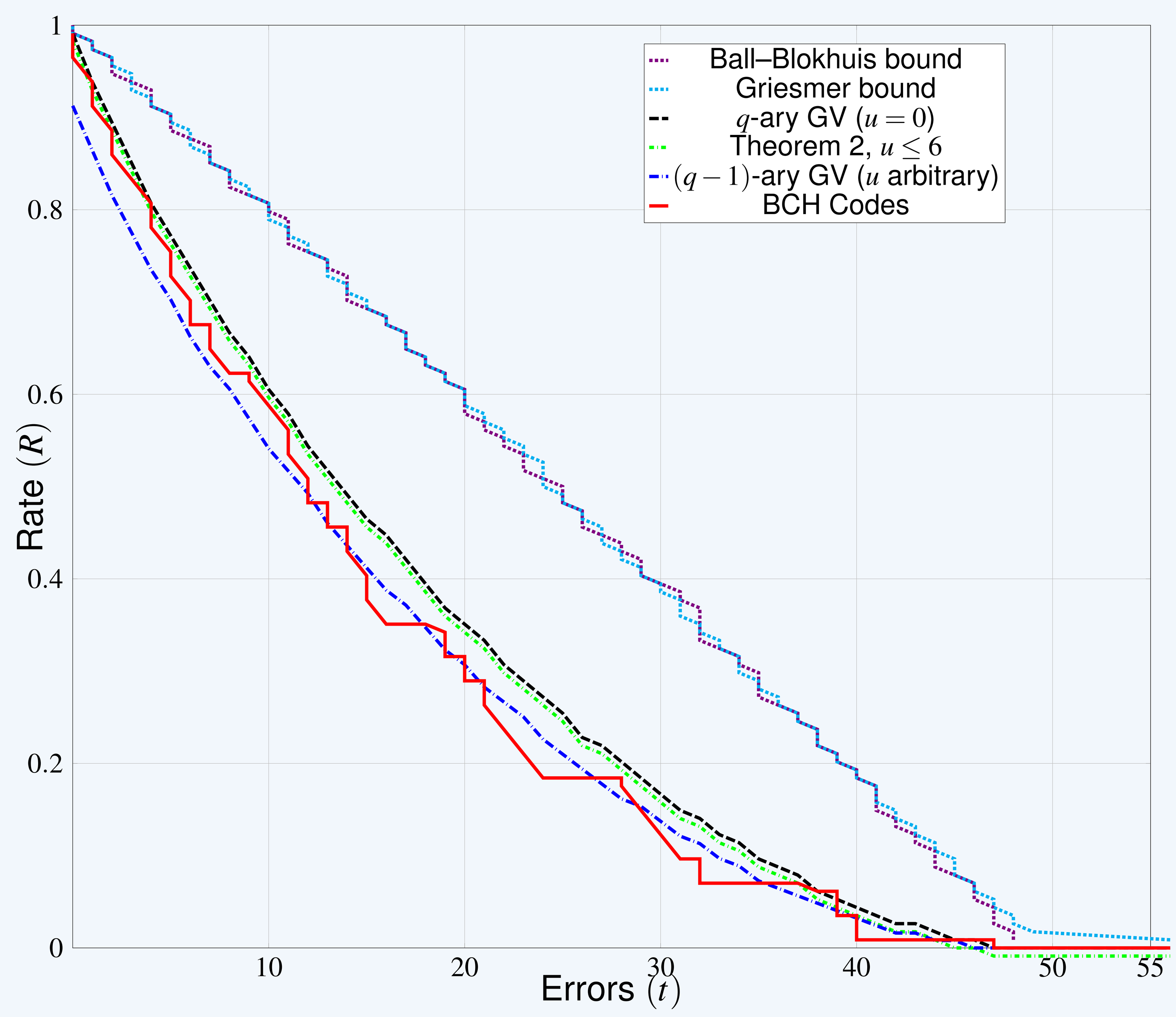


Figure 1. Comparison of other upper and lower limits to our derived GV-like bound in Theorem 2.

Conclusion

Our constructions in [4], including Construction 1, can handle both: partial defects (also called partially stuck cells) and random substitution errors and require fewer redundancy symbols for $u > 1$ and $q > 2$ than the known constructions for stuck cells.

References

[1] G. W. Burr et al., “Phase change memory technology,” J. Vac. Sci. Technol. B, vol. 28, no. 2, pp. 223–262, 2010.

[2] C. Heegard, “Partitioned linear block codes for computer memory with ‘stuck-at’ defects,” IEEE Trans. Inf. Theory, vol. 29, no. 6, pp. 831–842, Nov. 1983.

[3] A.Wachter-Zeh and E. Yaakobi, “Codes for Partially Stuck-At Memory Cells,” IEEE Transactions on Information Theory, vol. 62, no. 2, pp. 639–654, February 2016.

[4] H. A. Kim, S. Puchinger, L. Tolhuizen, A. Wachter-Zeh, ‘Coding and Bounds for Partially Defective Memory Cells’. arXiv, 2022.

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