Optimization of Acquisition Geometry for Intra-operative Tomographic Imaging

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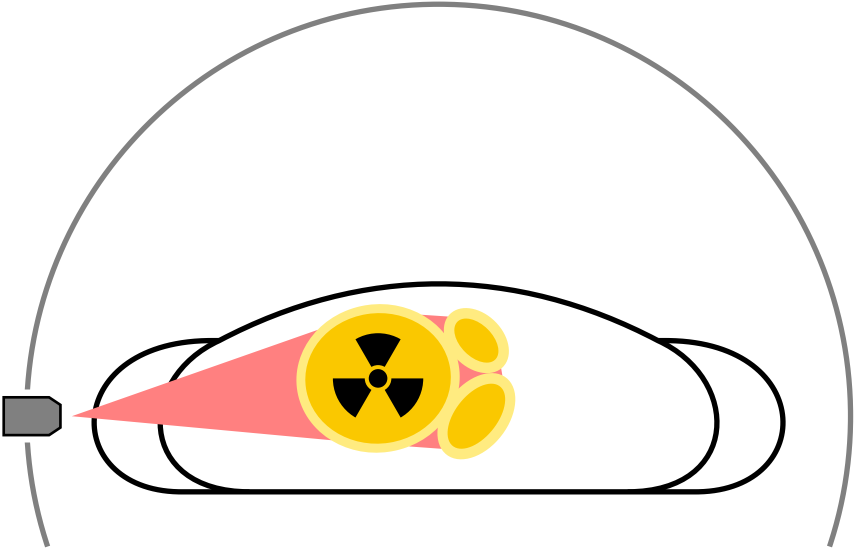
MICCAI
October 4, 2012

HelmholtzZentrum münchen
German Research Center for Environmental Health

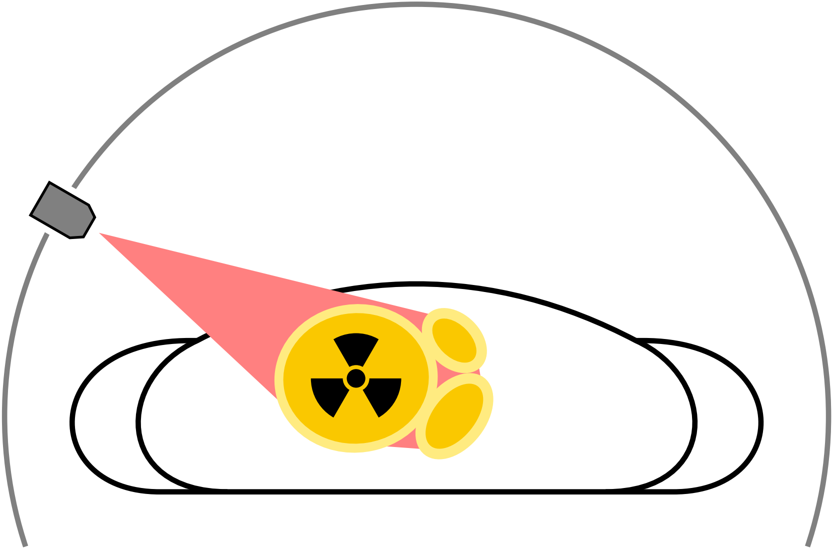
Motivation

- Enable flexible intra-operative functional imaging
- Identify cancer tissue during surgery using radioactive tracer

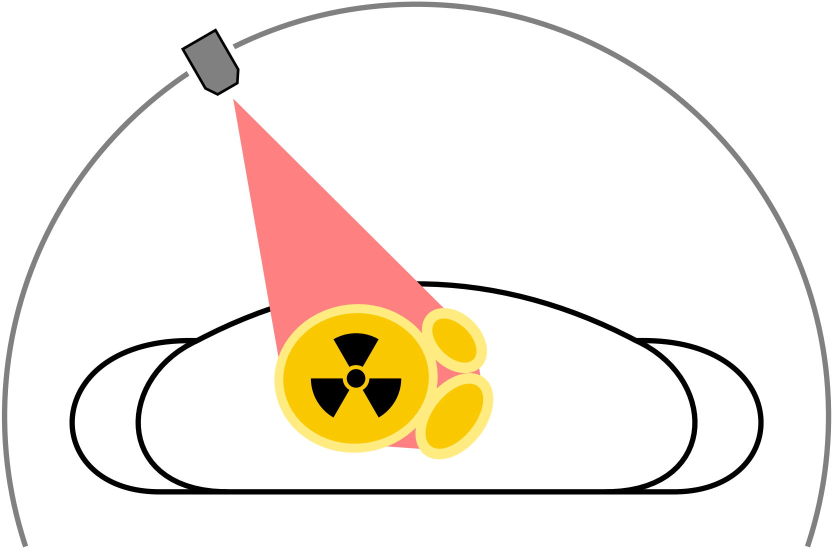
SPECT



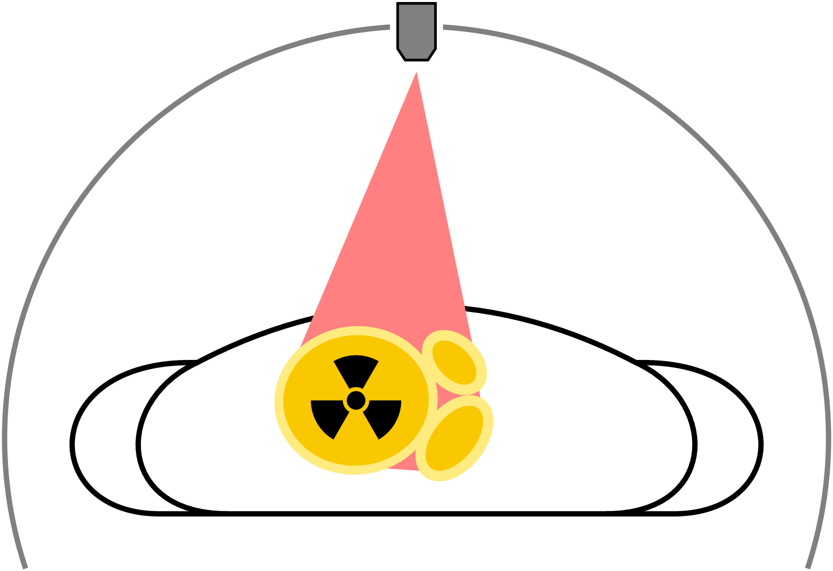
SPECT



SPECT



SPECT



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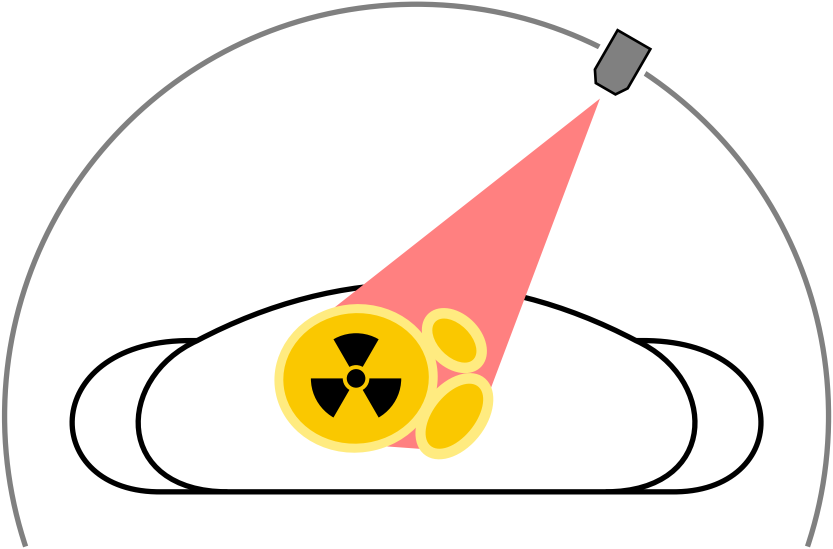

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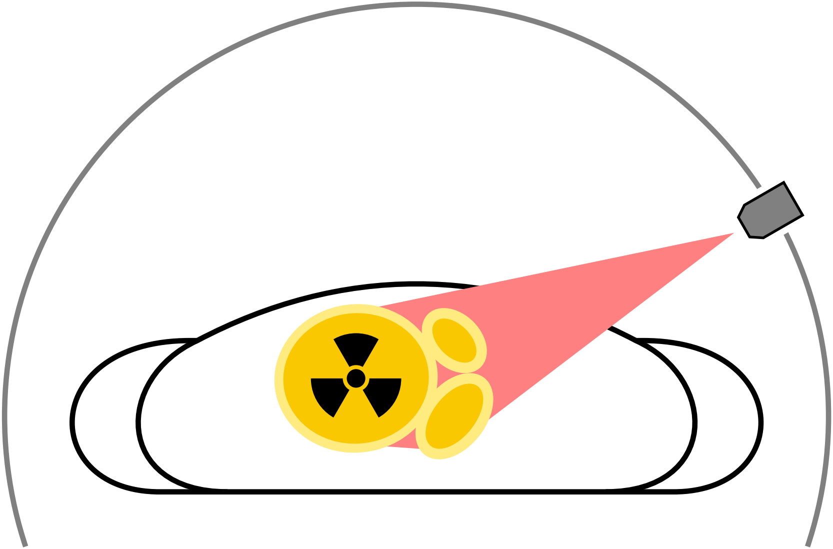
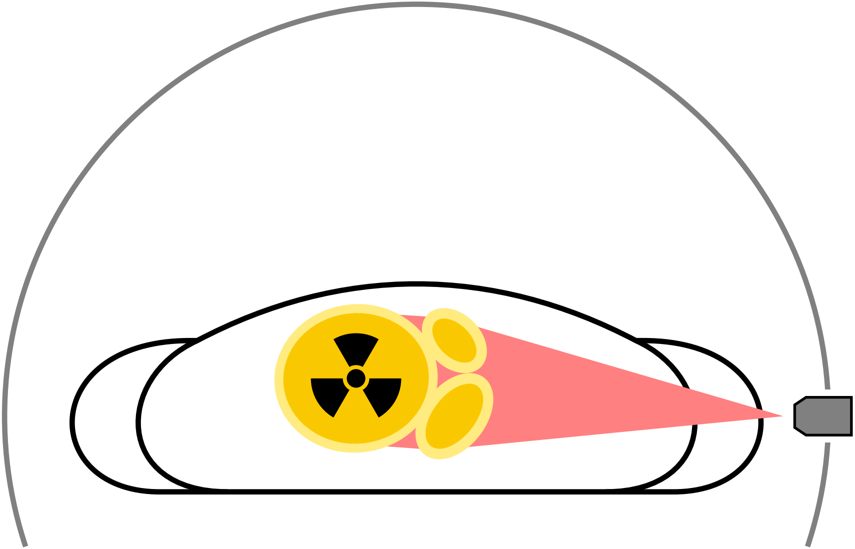


SPECT



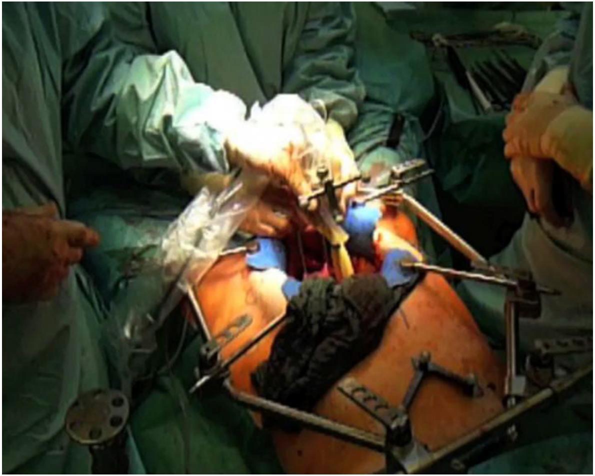
Diagnostic SPECT

Freehand SPECT

SurgicEye Press Picture

T. Wendler et al., Eur J Nucl Med Mol Imaging 37 (8), 2010

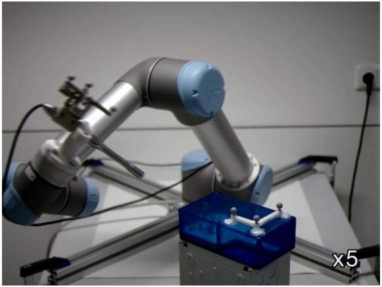
Freehand SPECT



Courtesy of Aslı Okur & Thomas Wendler

T. Wendler et al., Eur J Nucl Med Mol Imaging 37 (8), 2010

Intra-Op Tomographic Imaging



Robotic SPECT

C-Arm CT

Siemens Press Picture

- Problem: Find optimal and reproducible trajectories!

Y. Murayama et al., Neurosurgery 68, 2011

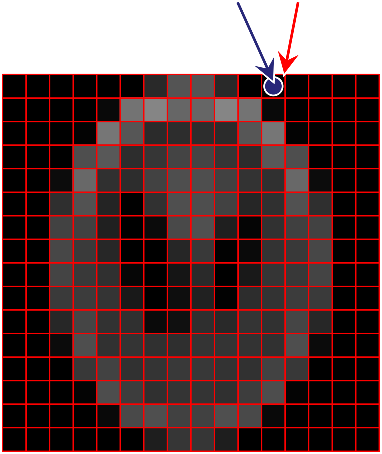
Sneak Peek

- Optimize sensor trajectory for tomographic reconstruction
- Directly use mathematical framework
- Control robotic arm accordingly

Algebraic Reconstruction: Discretized Signal

$f : \Omega \subset \mathbb{R}^3 \to \mathbb{R}$

$f \approx \widehat{f} = \sum_i x_i \cdot b_i$



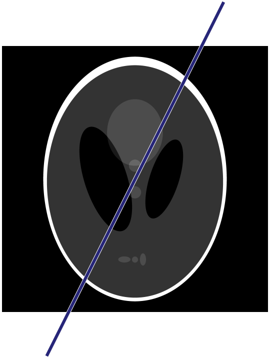
Algebraic Reconstruction: Measurement Model

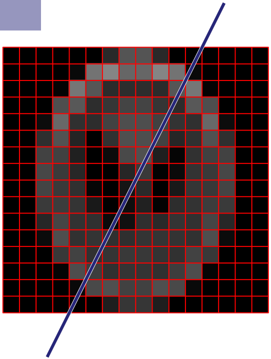
$$\mathcal{M}_j : (\Omega \to \mathbb{R}) \to \mathbb{R}$$

$$m_j = \mathcal{M}_j(f)$$

$$m_j \approx \mathcal{M}_j\left(\sum_i x_i \cdot b_i\right)$$

$$m_j = \int_{L_j} f(\mathbf{x}) d\mathbf{x}$$





Algebraic Reconstruction: Measurement Model

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$$m_j \approx \mathcal{M}_j\left(\sum_i x_i \cdot b_i\right) = \sum_i x_i \cdot \underbrace{\mathcal{M}_j(b_i)}_{=: \, a_{ji}}$$

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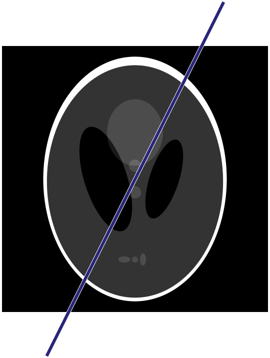
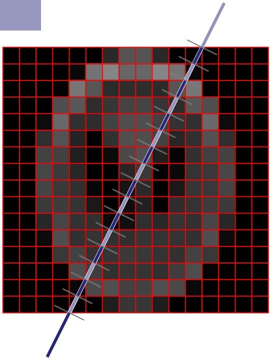
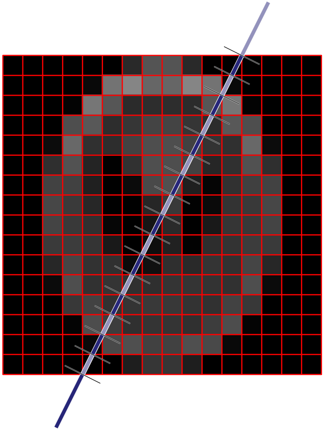




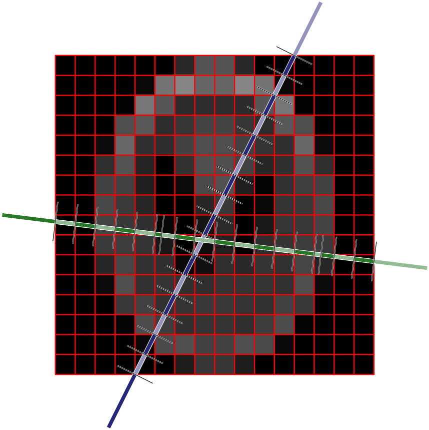
Algebraic Reconstruction: Linear System



$$m_j = \langle \mathbf{a}_j^T, \mathbf{x} \rangle$$

$$\mathbf{m} = \begin{pmatrix} \text{---} \; \mathbf{a}_1 \; \text{---} \\ \\ \\ \end{pmatrix} \mathbf{x}$$

Algebraic Reconstruction: Linear System



$$m_j = \langle \mathbf{a}_j^T, \mathbf{x} \rangle$$

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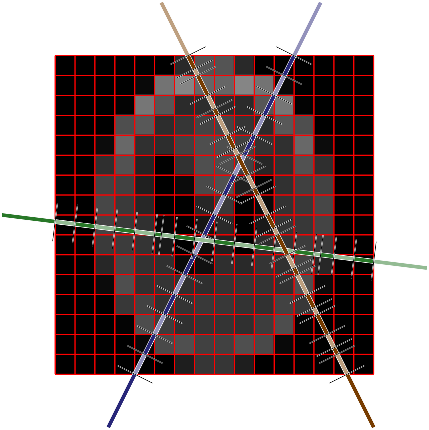
Algebraic Reconstruction: Linear System



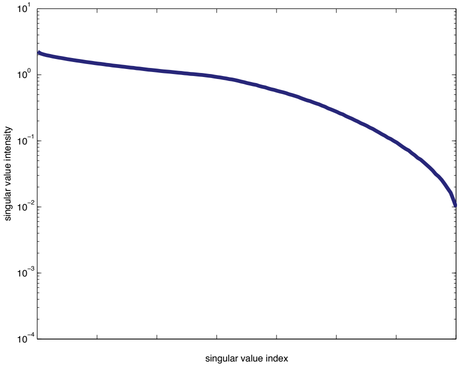
$$m_j = \langle \mathbf{a}_j^T, \mathbf{x} \rangle$$

$$\mathbf{m} = \begin{pmatrix} — \ \mathbf{a}_1 \ — \\ — \ \mathbf{a}_2 \ — \\ — \ \mathbf{a}_3 \ — \\ \vdots \end{pmatrix} \mathbf{x}$$

$$= A \cdot \mathbf{x}$$

- A contains the geometry, **m** the measurements
- Kernel and rank of A are quality indicators

Singular Value Spectrum of a System Matrix



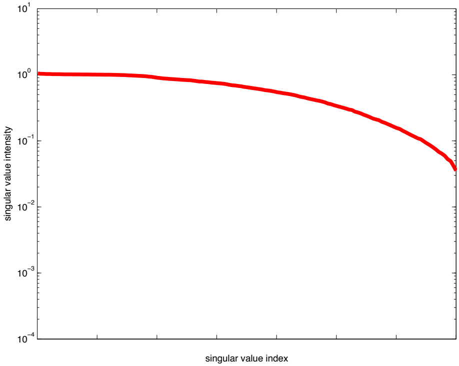
$$A = U \underbrace{\begin{pmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \ddots \end{pmatrix}}_{=:\, S} V^T$$

$$\eta(A) = \sum_i \sigma_i$$

- SVD is slow

T. Lasser et al., Medical Image Analysis 11 (4), 2007

Pivoted QR Decomposition



$$A = Q \underbrace{\begin{pmatrix} r_{11} & r_{12} & \cdots \\ & r_{22} & \cdots \\ & & \ddots \end{pmatrix}}_{=:\ R} P^T$$

$$\eta(A) = \|diag(R)\|_{\ell_1} = \sum_i |r_{ii}|$$

- Pivoted QR is considerably faster

P. C. Hansen et al., Johns Hopkins University Press 2012

Robot Control: Overview

- Find trajectory maximizing cost function η
- Constrain motion to bounding surface

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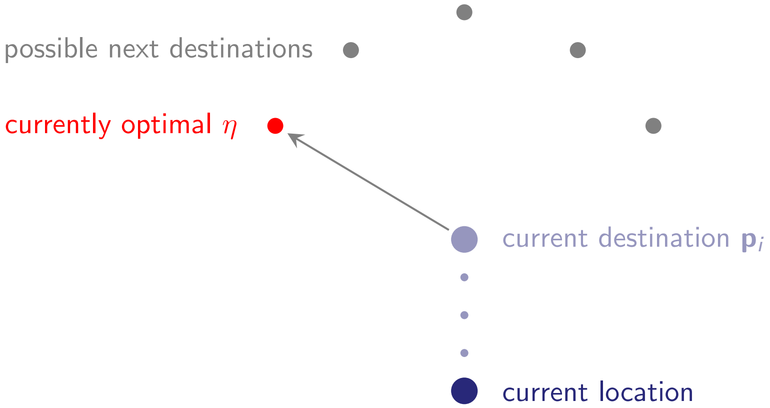

Robot Control: Overview

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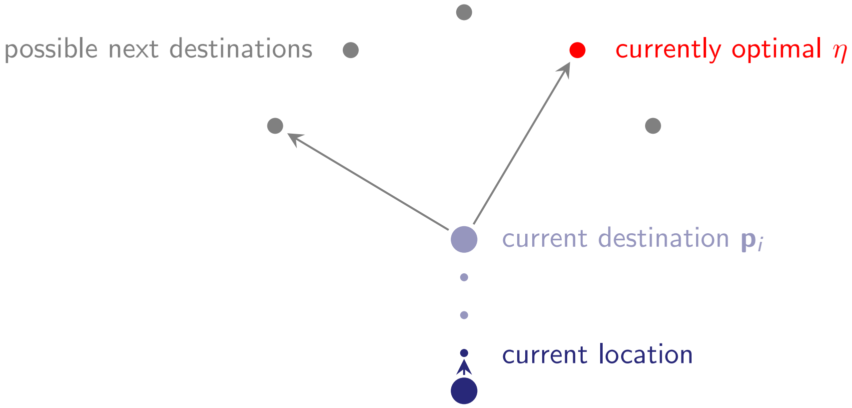

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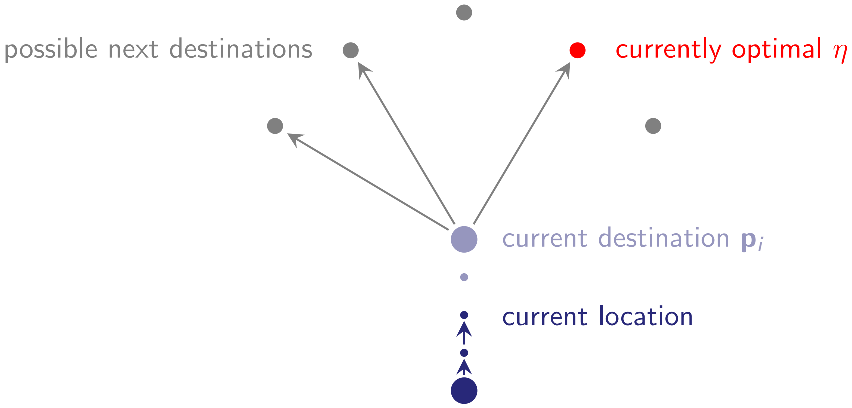

Robot Control: Overview

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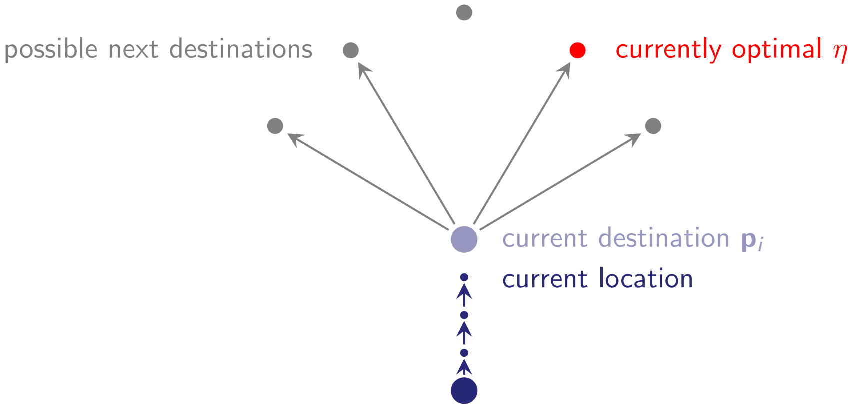

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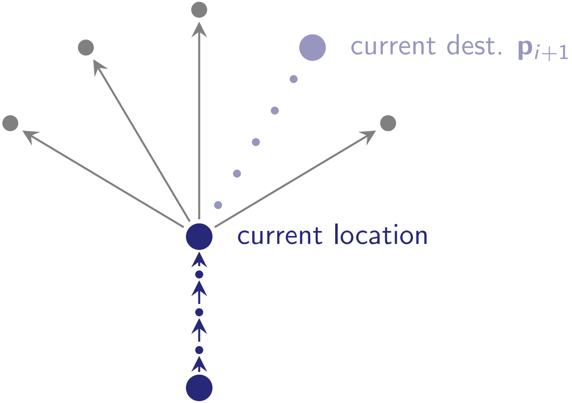
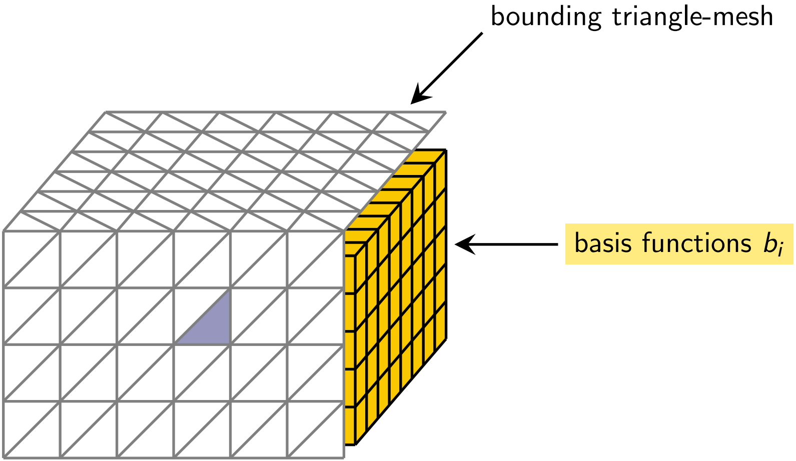
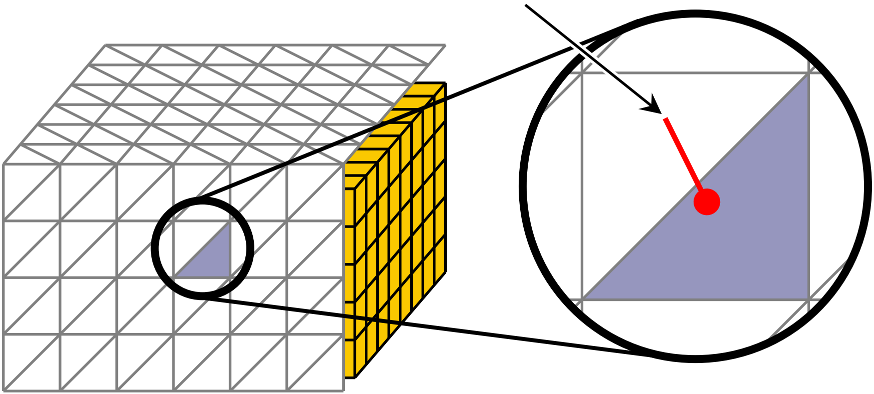


Robot Control: Exploring the Surface



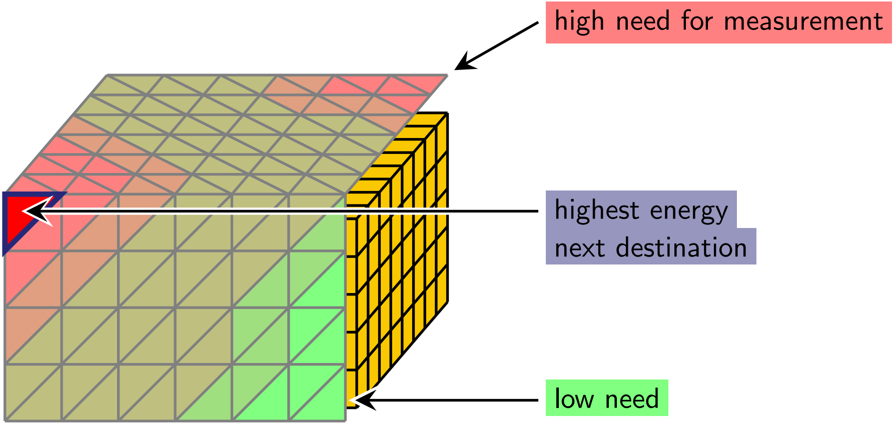
Robot Control: Exploring the Surface

random pose, evaluate $\eta\begin{pmatrix} A_i \\ \text{---}\ \mathbf{a}_* \ \text{---} \end{pmatrix}$



- Predicted matrix A_i after robot reaches *current destination* $\mathbf{p}_i$

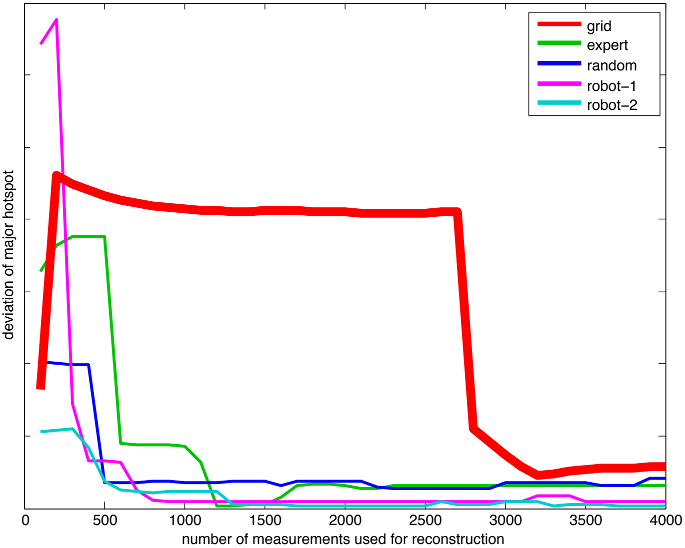
Robot Control: Exploring the Surface



Real-Time Implementation

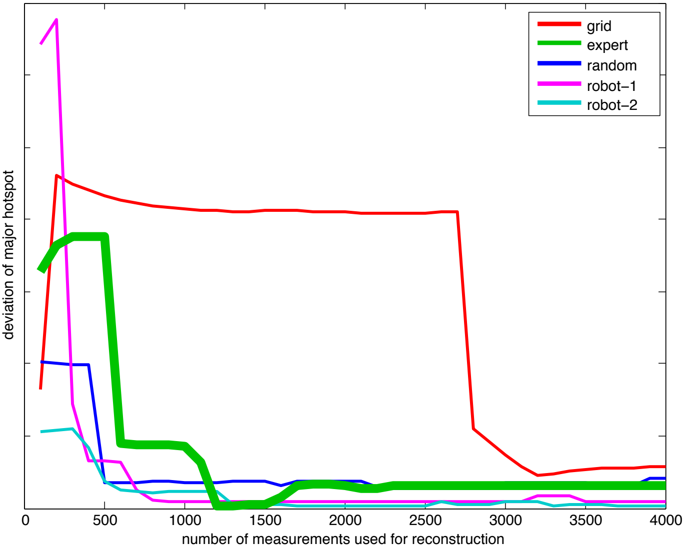
- Small voxel basis ($10 \times 10 \times 7 = 700$ basis functions) for optimization (finer for actual reconstruction)
- Mesh of 200–450 triangles, evaluated in parallel
- In-place decomposition using LAPACK's SGEQP3

Experiments: Simulation



Alexander Hartl contributed.

Experiments: Simulation



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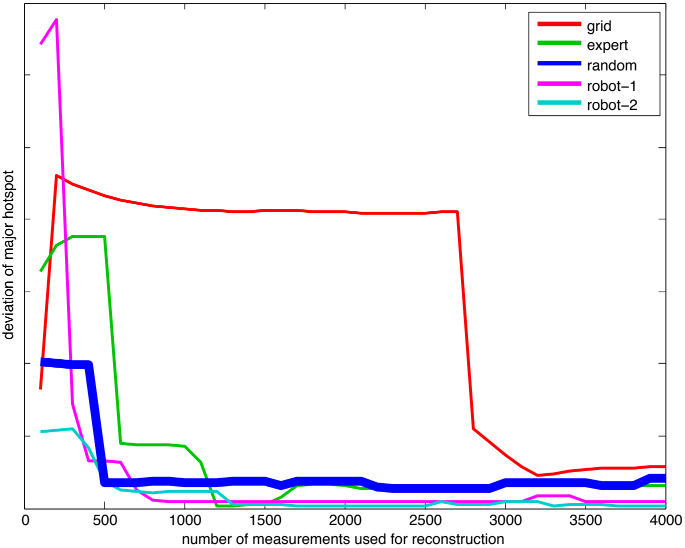

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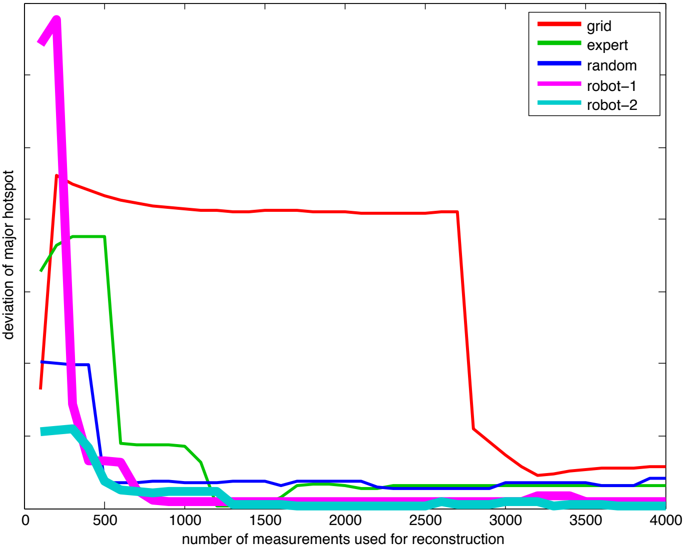
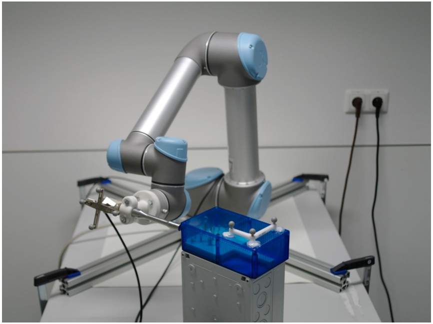
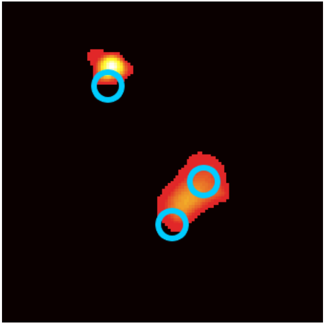


Alexander Hartl contributed.

Experiments: Real World



Setup



Result

Conclusion

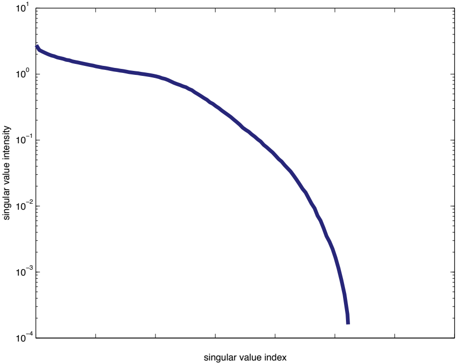
- Sensor trajectory optimization for tomographic reconstruction
- General approach directly using the mathematical framework
- Real-time implementation available

Acknowledgements

- DFG SFB 824
- DFG Cluster of Excellence MAP
- European Union FP7 grant N° 25698

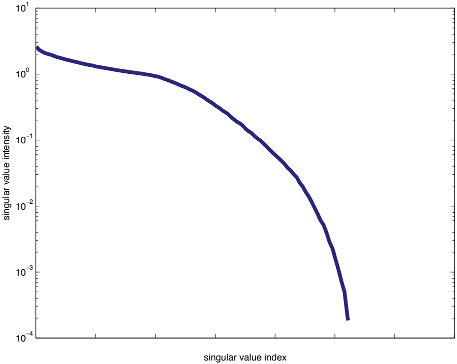
- Looking forward to meet you at our poster (**Th-2-AG-01**), today 3:00 – 4:30 pm

Singular Value Spectrum: Sample Cases



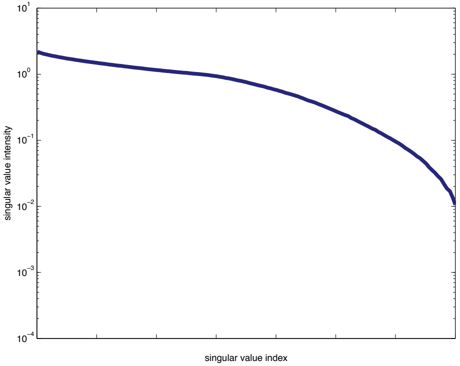
700 basis functions, 522 measurements, case id 94

Singular Value Spectrum: Sample Cases



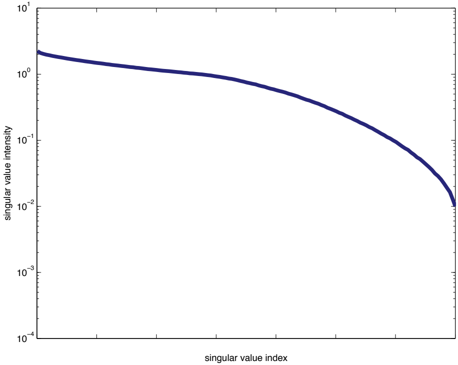
700 basis functions, 522 measurements, case id 171

Singular Value Spectrum: Sample Cases



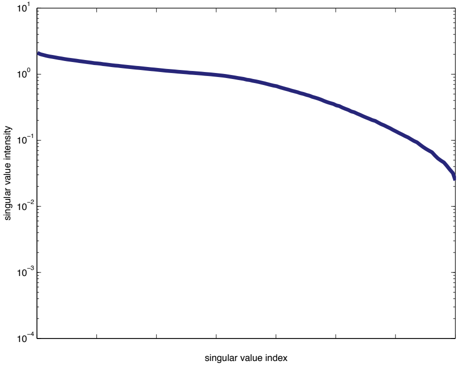
700 basis functions, 853 measurements, case id 190

Singular Value Spectrum: Sample Cases



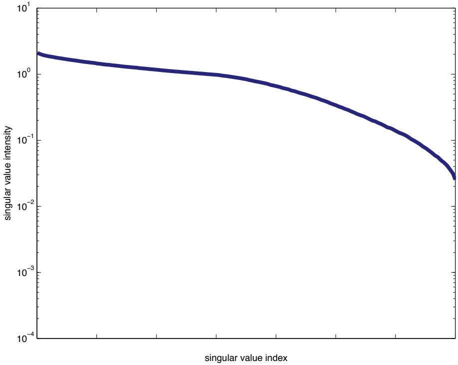
700 basis functions, 853 measurements, case id 249

Singular Value Spectrum: Sample Cases



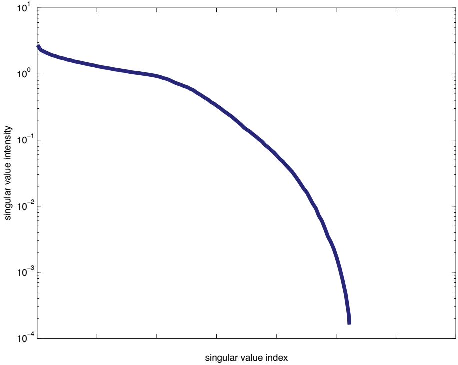
700 basis functions, 939 measurements, case id 167

Singular Value Spectrum: Sample Cases



700 basis functions, 939 measurements, case id 233

SVD vs. QR Spectra: Sample Cases



700 basis functions, 522 measurements, case id 94

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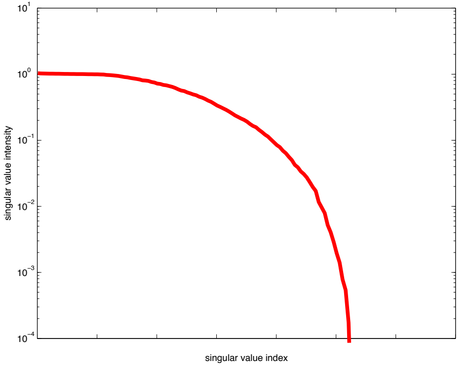

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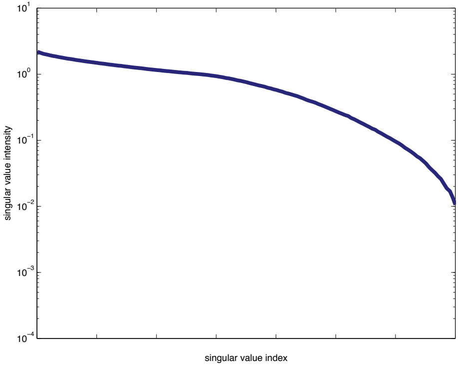

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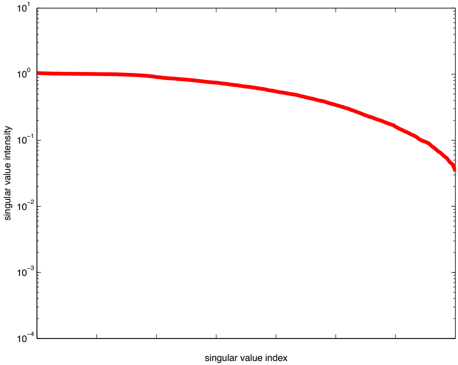

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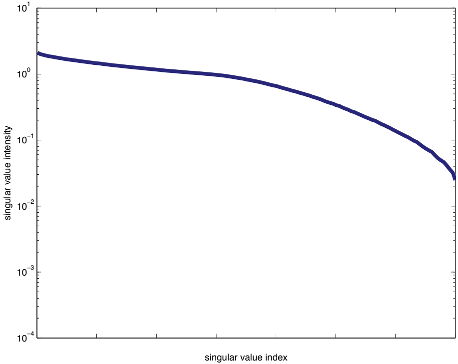

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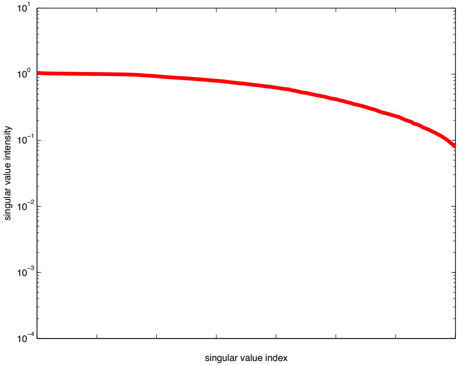

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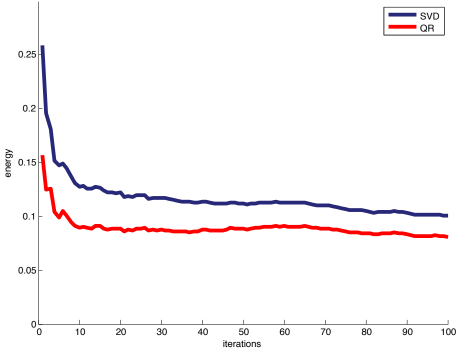
SVD vs. QR Spectra: Random Paths



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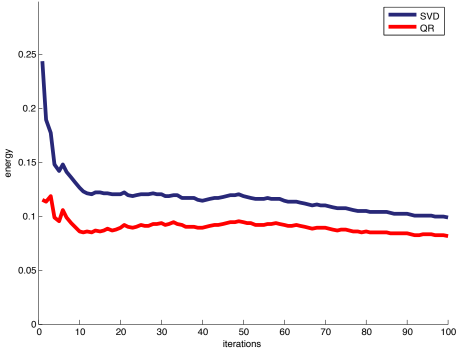

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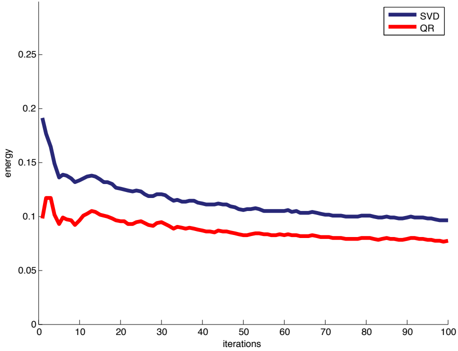

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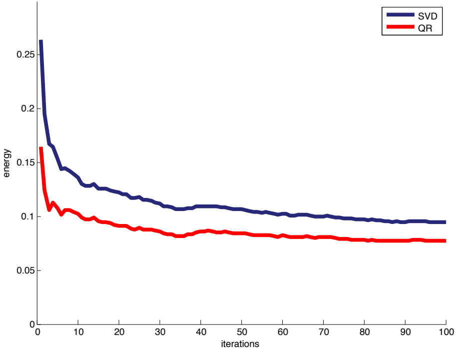
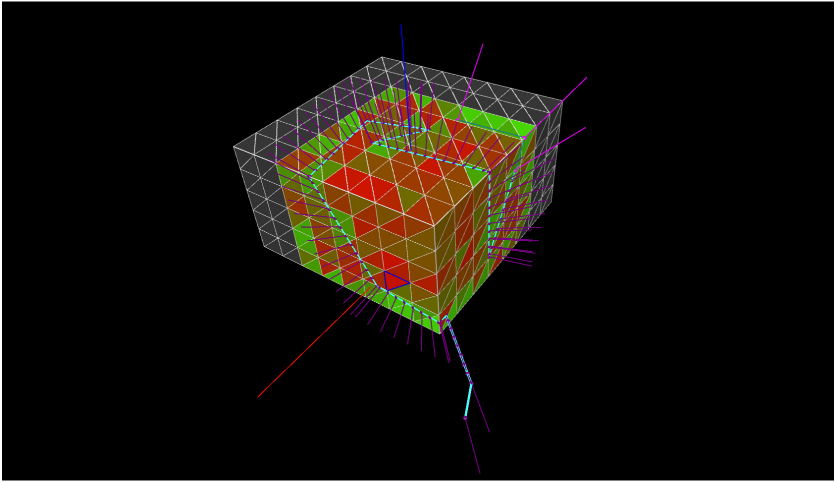


Robot Control: Correlation Path – Energy



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