Dynamic Cone-Beam Reconstruction Using a Variational Level Set Formulation

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Outline

Motivation :: Clinical

Bringing together

pre-operative 3D imaging (conventional CT, mainly used for rule-out of stenosis)

and

intra-interventional angiography (simultaneous diagnosis and intervention)

by enabling *4D reconstruction from cone-beam projections*.

Image courtesy of Siemens Healthcare

Motivation :: Assumptions

Assumption 1:

Direct tomographic reconstruction not feasible

➔ Perform *symbolic reconstruction* in a first step

(And use recovered motion in subsequent tomographic reconstruction)

Assumption 2:

Separation of shape reconstruction and motion estimation not feasible (“chicken and egg”)

➔ *Simultaneously* estimate shape and motion

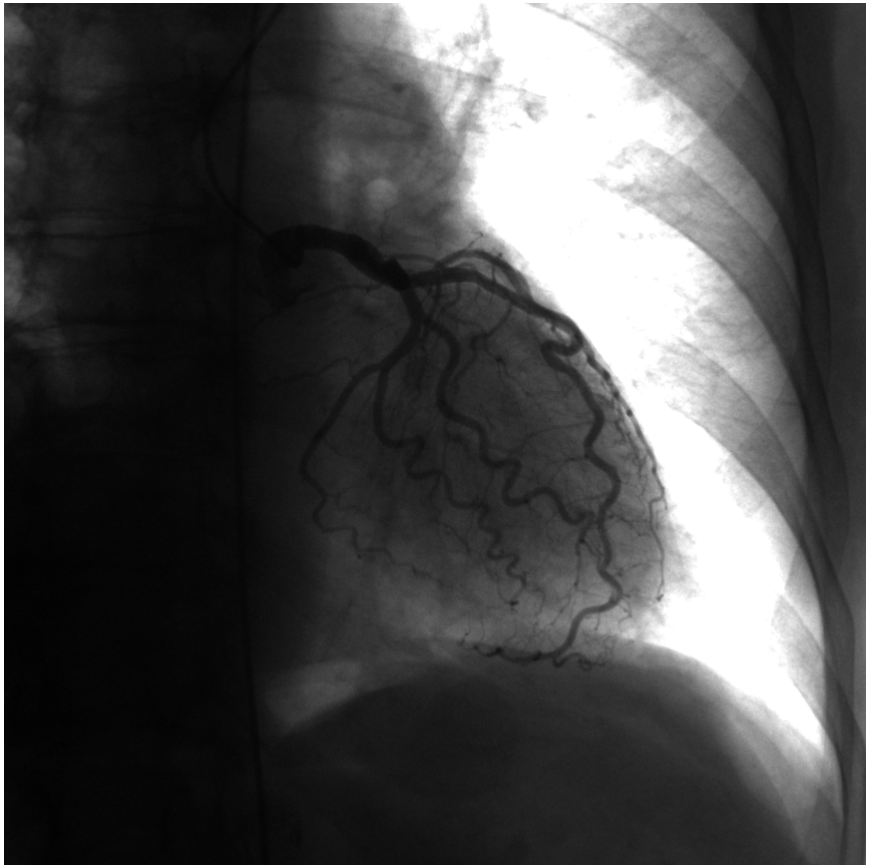
Methods :: Sub-Problems in Our Approach

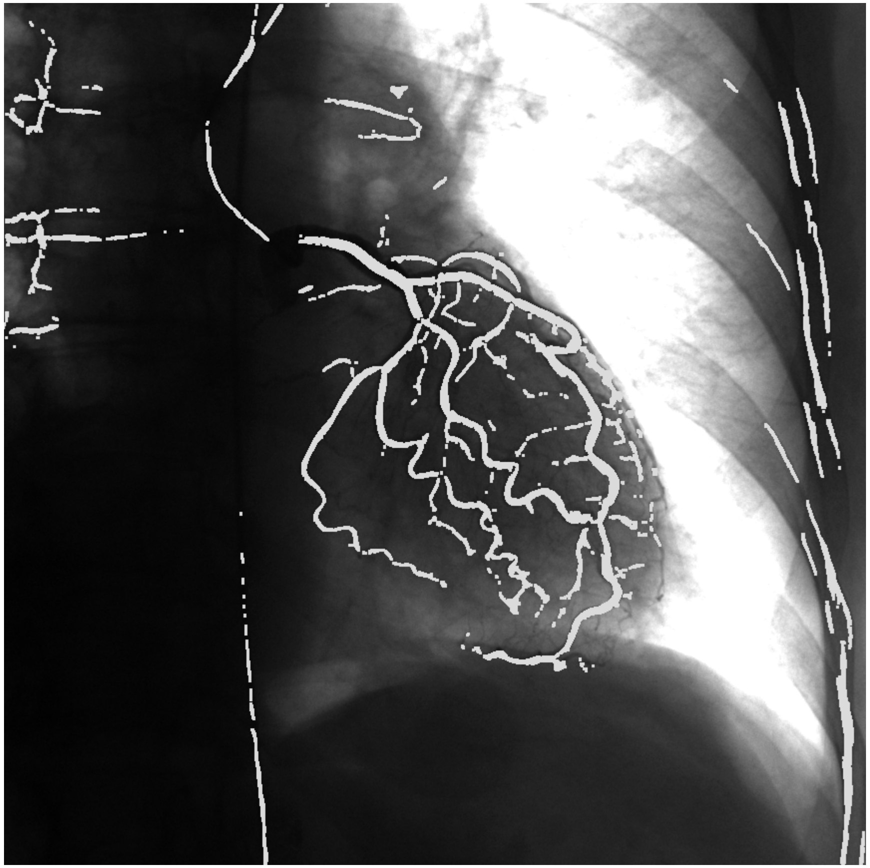
Vessel Enhancement in 2D

Dynamic Shape Models

Optimization Goal / Data Terms

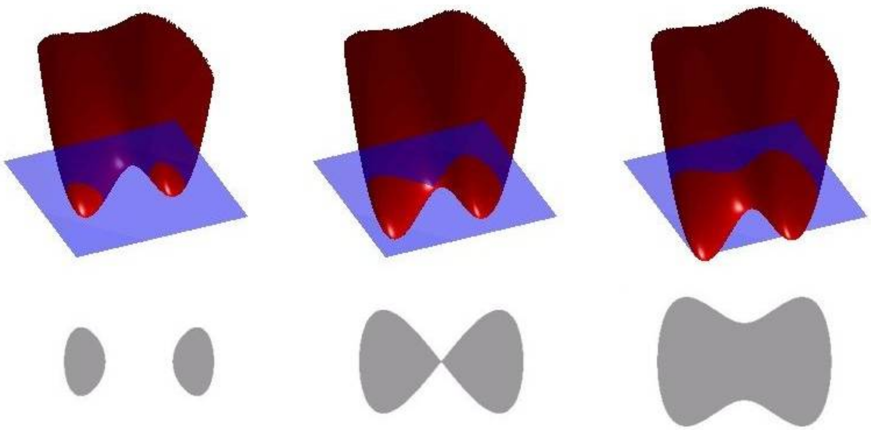
Methods :: Vessel Enhancement

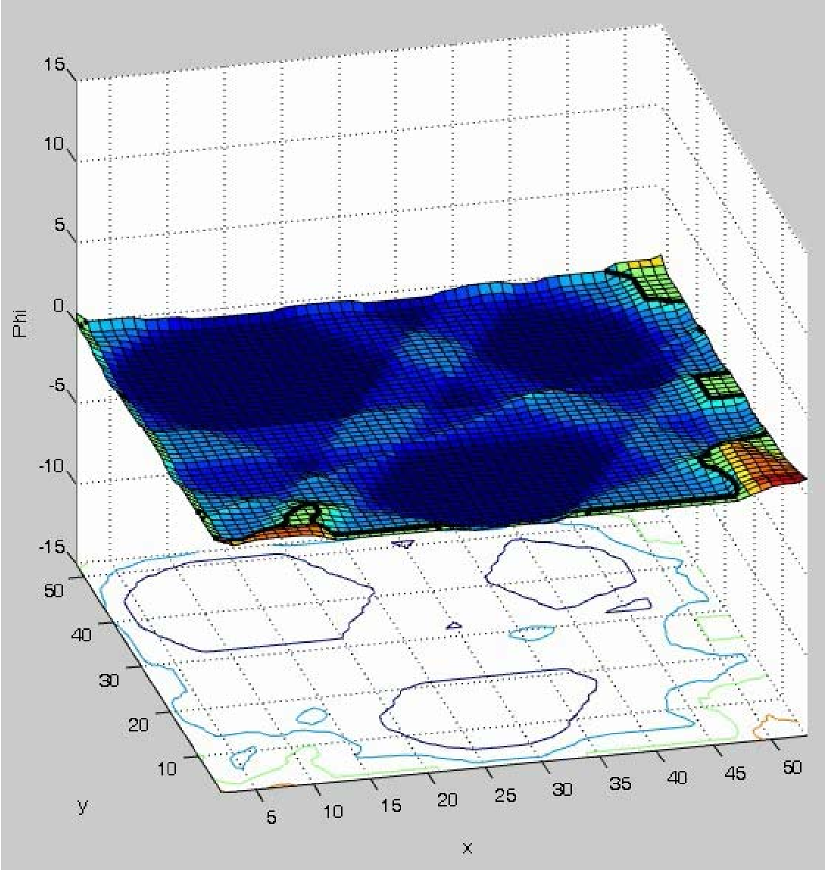




Methods :: Level Sets

- Level sets re-introduced by Osher and Sethian in 1988
- Implicitly represented contour / shape



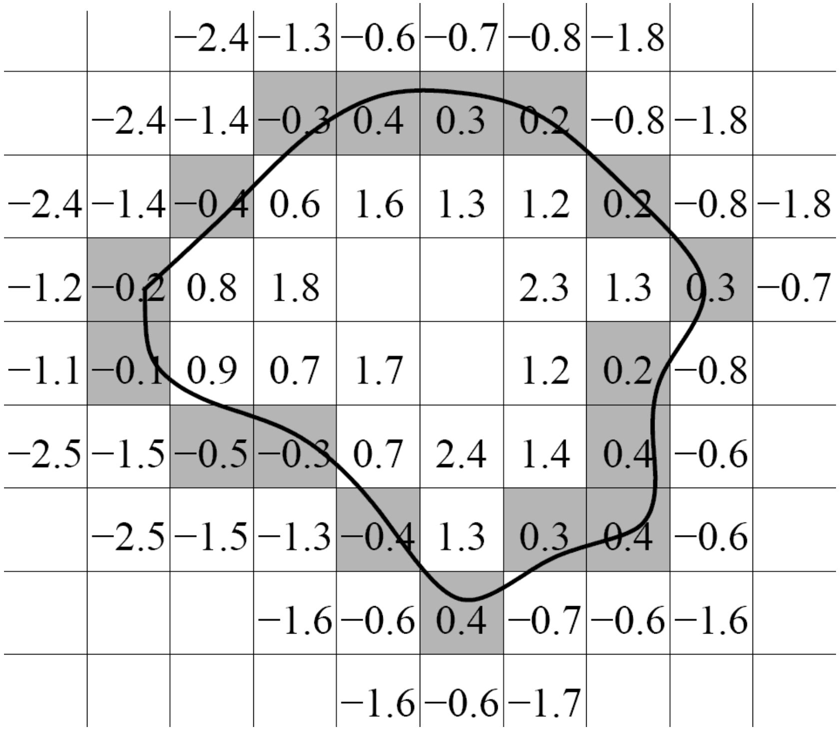


Methods :: Level Sets

$$\Phi : \begin{cases} \Omega \subset \mathbf{R}^3 & \rightarrow & \mathbf{R} \\ \boldsymbol{x} & \mapsto & \Phi(\boldsymbol{x}) \end{cases}$$

$$\Phi(\boldsymbol{x}) \begin{cases} < 0 & : \boldsymbol{x} \text{ is ``inside''} \\ = 0 & : \boldsymbol{x} \text{ is ``on the border''} \\ > 0 & : \boldsymbol{x} \text{ is ``outside''} \end{cases}$$

$$H(\Phi(\boldsymbol{x})) \begin{cases} = \quad 0 & : \boldsymbol{x} \text{ is ``inside''} \\ = +1 & : \boldsymbol{x} \text{ is ``outside''} \end{cases}$$



- Chan and Vese: Active contours without edges. *IEEE Trans. Image Process.*, 10(2):266-277, 2001
- **`http://math.berkeley.edu/~sethian/`**

Methods :: Shape and Motion Models

- Model shape using level set volume

$$
\begin{aligned}
\Phi_0(\boldsymbol{x}_0) < 0 \quad &\Leftrightarrow \quad \boldsymbol{x}_0 \text{ inside vessel} \\
\Phi_0(\boldsymbol{x}_0) = 0 \quad &\Leftrightarrow \quad \boldsymbol{x}_0 \text{ on interface} \\
\Phi_0(\boldsymbol{x}_0) > 0 \quad &\Leftrightarrow \quad \boldsymbol{x}_0 \text{ outside vessel}
\end{aligned}
$$

- Model motion using B-splines

parameters

$$
\boldsymbol{x}_0 = \varphi(\boldsymbol{x}, t, \boldsymbol{\alpha}) = \underbrace{\boldsymbol{R}(t, \boldsymbol{\alpha})\, \boldsymbol{x} + \boldsymbol{T}(t, \boldsymbol{\alpha})}_{\text{rigid dynamic}} + \underbrace{\boldsymbol{u}(\boldsymbol{x}, t, \boldsymbol{\alpha})}_{\text{deformable dynamic}}
$$

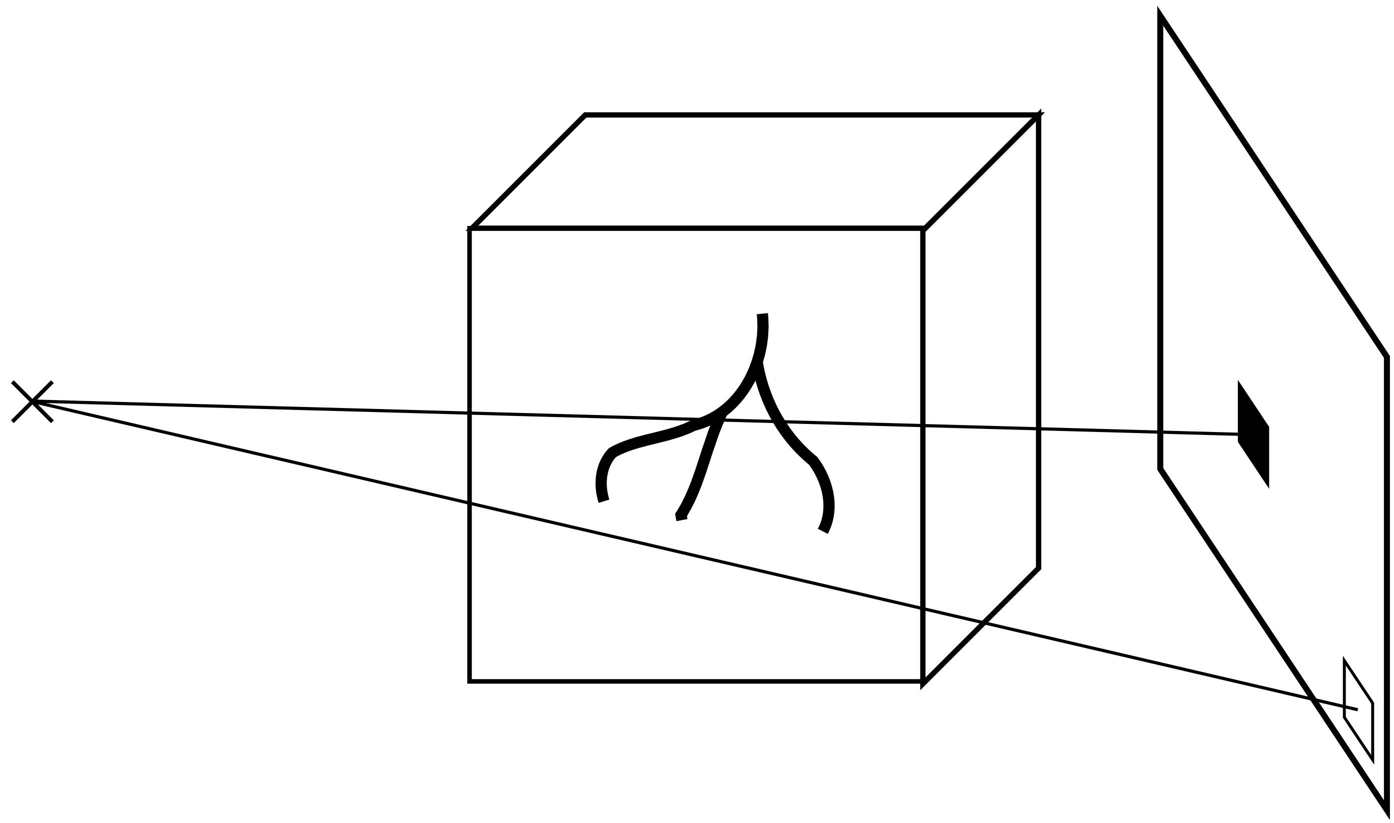
Methods :: Dynamic Level Set

Dynamic shape = static shape & motion model

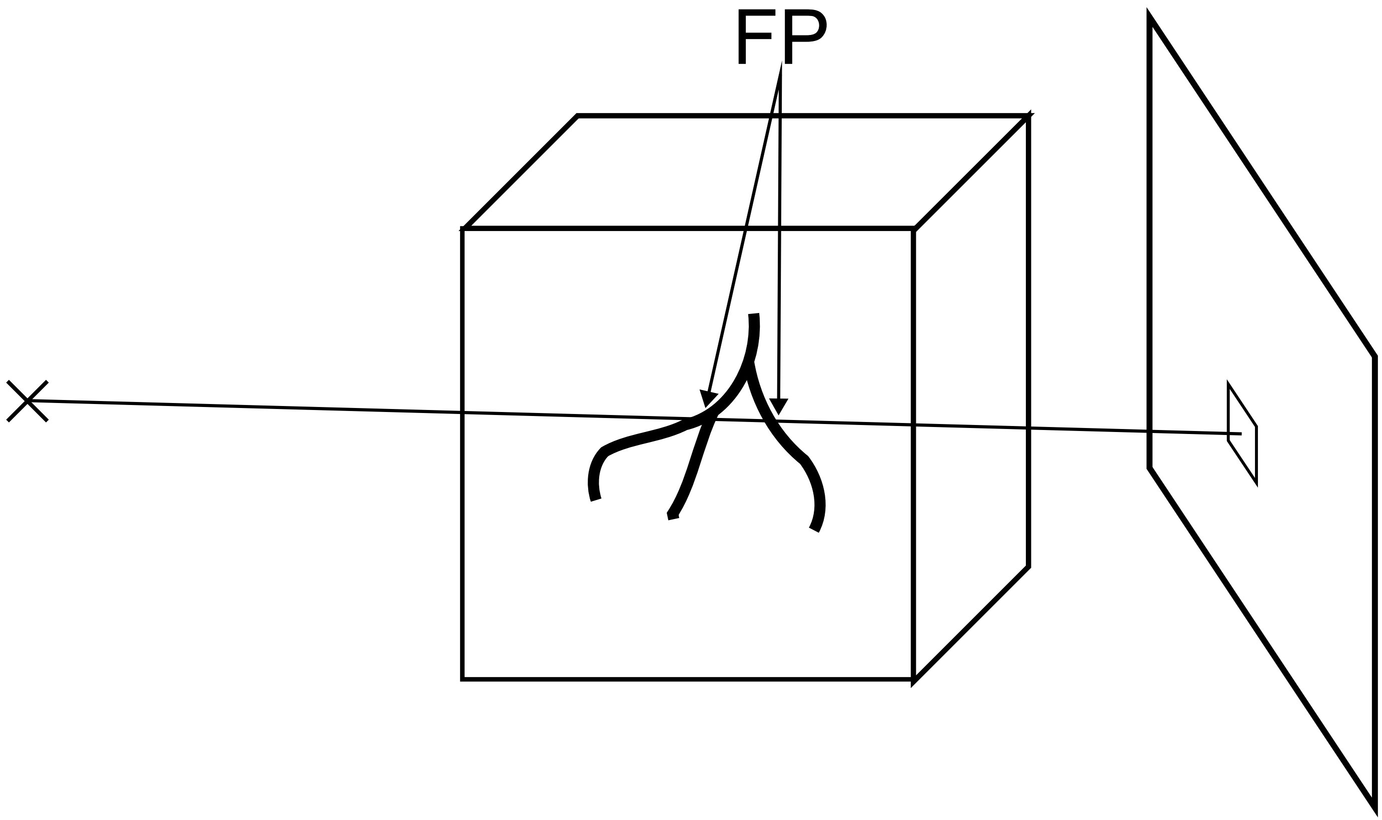
$$\Phi(\boldsymbol{x}, t) = \Phi_0\big(\boldsymbol{\varphi}(\boldsymbol{x}, t, \boldsymbol{\alpha})\big)$$

shape reg. motion reg.

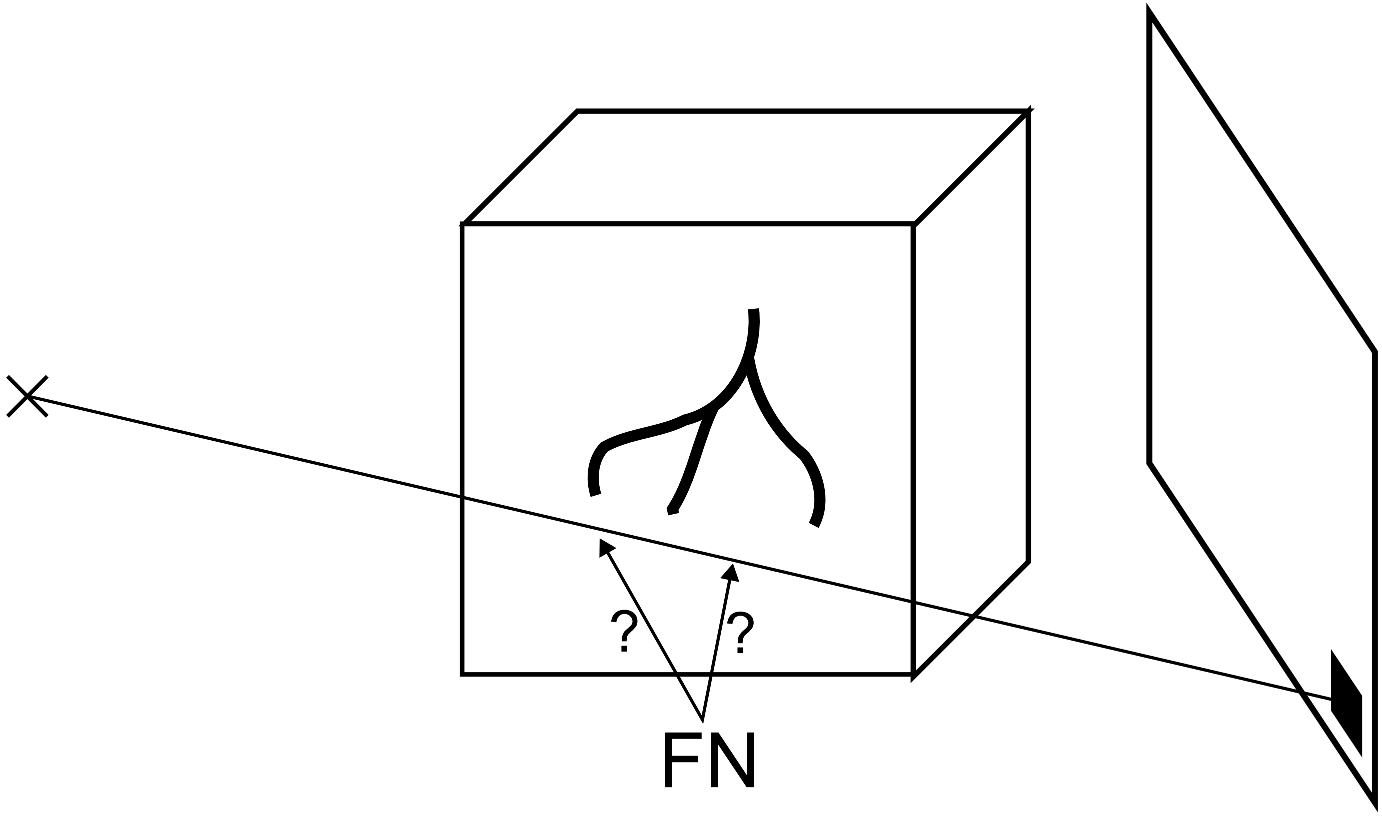
Methods :: Data Terms



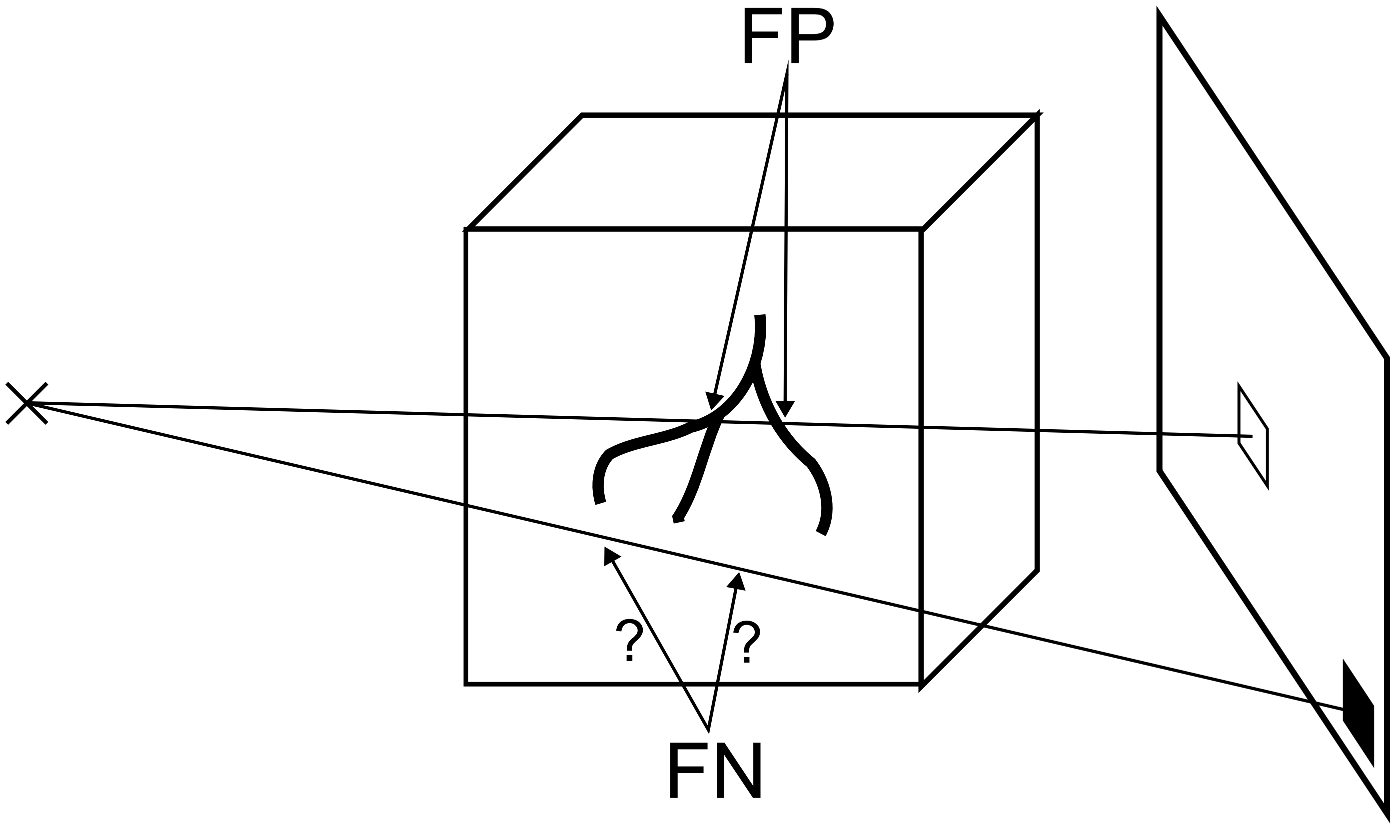
Methods :: Data Terms



Methods :: Data Terms

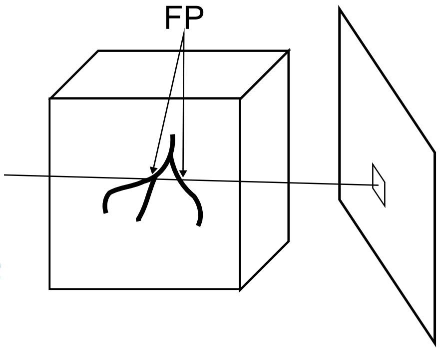


Methods :: Data Terms

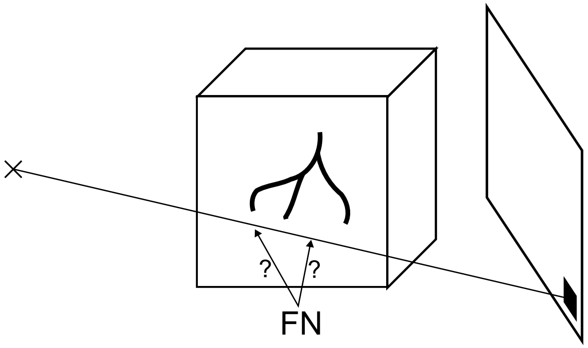


Methods :: Data Terms (cont'd)

$$E_{\mathrm{FP}}(\Phi) = \sum_{l=1}^{L} \int_V \left[1 - H\left(\Phi(\boldsymbol{x}, t_l)\right)\right] \times$$
$$\times S_{\mathrm{FP}}\left(I_l(\boldsymbol{P}_l(\boldsymbol{x}))\right) \times \left[1 - I_l\left(\boldsymbol{P}_l(\boldsymbol{x})\right)\right] \, \mathrm{d}\boldsymbol{x}$$



$$E_{\mathrm{FN}}(\Phi) = \sum_{l=1}^{L} \int_A H\left(\min_{\boldsymbol{x} \in X_l(\boldsymbol{p})} \Phi(\boldsymbol{x}, t_l)\right) \times$$
$$\times S_{\mathrm{FN}}\left(I_l(\boldsymbol{p})\right) \times I_l(\boldsymbol{p}) \, \mathrm{d}\boldsymbol{p}$$

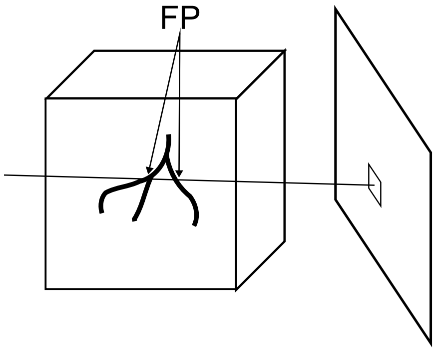


I_l : Image at time t_l
P_l : Projection operator at time t_l
$X_l(\boldsymbol{p})$: Ray for pixel $\boldsymbol{p}$ at time t_l

(un-)reconstructed voxel | contrary pixel indication | weighted penalty

Methods :: Data Terms (cont'd)

$$E_{\mathrm{FP}}(\Phi_0, \boldsymbol{\alpha}) = \sum_{l=1}^{L} \int_V \left[1 - H\left(\Phi_0(\varphi(\boldsymbol{x}, t_l, \boldsymbol{\alpha}))\right)\right] \times \times S_{\mathrm{FP}}\left(I_l(\boldsymbol{P}_l(\boldsymbol{x}))\right) \times \left[1 - I_l(\boldsymbol{P}_l(\boldsymbol{x}))\right] \,\mathrm{d}\boldsymbol{x}$$



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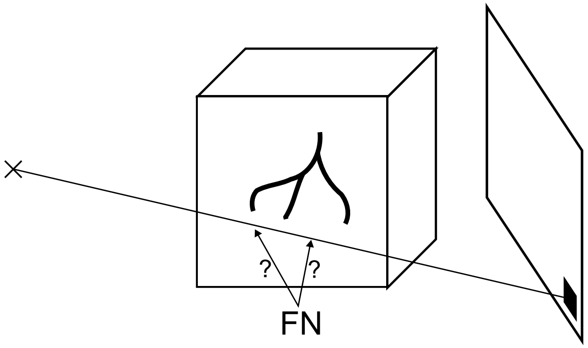

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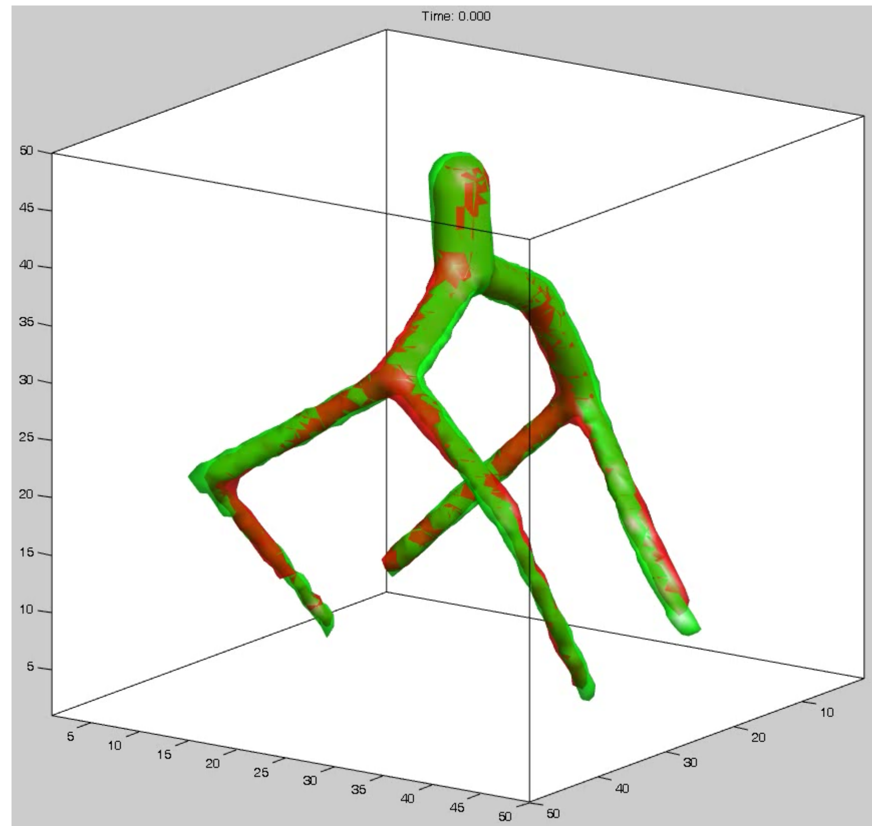
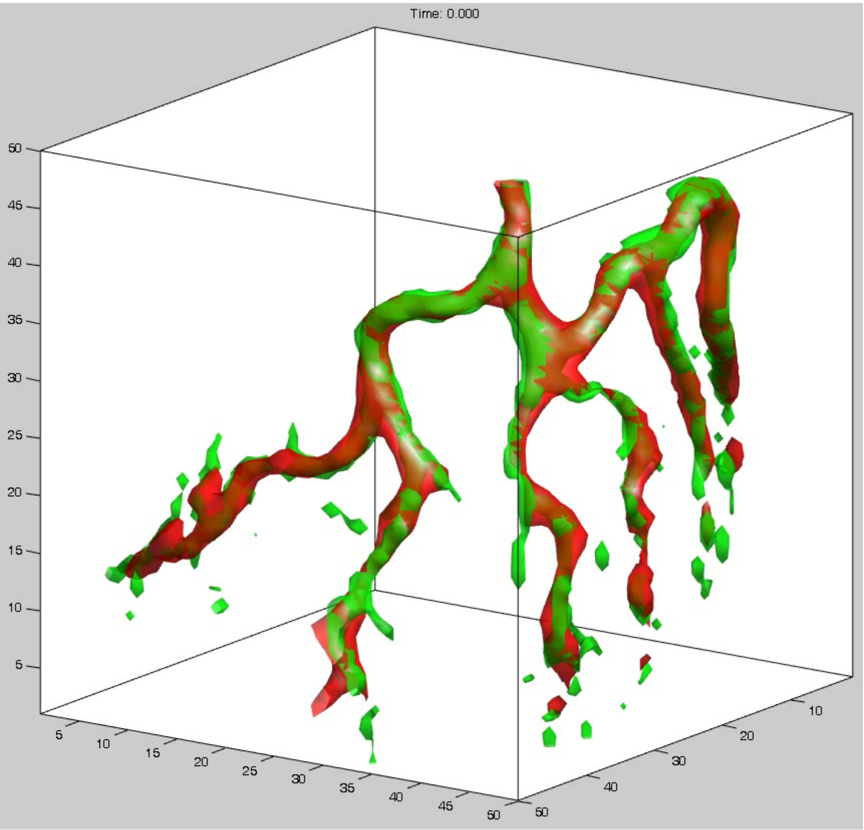
(un-)reconstructed voxel | contrary pixel indication | weighted penalty

Results :: Synthetic Tubular Shapes



Results :: Synthetic Coronaries Phantom



Results :: Quantitative Evaluation

Positional errors (for rigid motions, gaussian noise of 25%, 3mm voxel spacing):

$$\mu \pm \sigma \quad = \quad 0.88\,\mathrm{mm} \pm 0.46\,\mathrm{mm}$$ (sub-voxel accuracy!)

Shape errors (for deformable motions, gaussian noise of 30%, 3mm voxel spacing):

Sensitivity: 74.2% Specificity: 99.6%

Summary

- Contributions:
  - Dynamic level sets for reconstruction
  - Data terms for level set reconstruction
- Future work:
  - Phantom / real data
  - Refined motion models (breathing and non-periodic motions)
- Applicability to real cardiac cone-beam CT:
  - Mainly depending on vessel extraction in 2D
  - Reduction of dependency on vessel extraction by
    - combination with tomographic methods
    - closing loop from reconstruction to segmentation

Acknowledgements

Siemens Healthcare (→ funding)

Moritz Blume (p. 118), Jan Boese, and Martin Brokate (→ discussions)

Tobias Klug and LRR @ TUM (→ multi-core system)

Christopher Rohkohl (→ phantom data)

Fully3D Student Grant