Primal/Dual Linear Programming and Statistical Atlases for Cartilage Segmentation

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Idea: Automatic Cartilage Segmentation by Constructing a Probabilistic Atlas

Probabilistic Atlas Construction

- $\mathcal{V}_\mathcal{M}$: optimal representative of training set
- $\sigma_\mathcal{M}$: variance map / agreement between atlas and training set
- $p_\mathbf{x}(i)$: pdf defined at each voxel, e.g. Gaussian density

Minimization problem:

$$E(\mathcal{V}_\mathcal{M}, \sigma_\mathcal{M}) = \int_\Omega \sum_{i=1}^{n} \left[\log(\sigma_\mathcal{M}^2(\mathbf{x})) + \frac{(\mathcal{V}_i(\mathbf{x}) - \mathcal{V}_\mathcal{M}(\mathbf{x}))^2}{2\sigma_\mathcal{M}^2(\mathbf{x})} \right] \, d\mathbf{x}$$

Automatic Cartilage Segmentation

- Novel framework for deformable registration based on discrete optimization [1] is used for atlas matching
- Primal/Dual linear programming [2] is used for efficient optimization of the atlas matching problem

Matching criteria:

$$\rho_\mathcal{M}(\mathcal{V}(\mathcal{T}(\mathbf{x}))) = \left[\log(\sigma_\mathcal{M}^2(\mathbf{x})) + \frac{(\mathcal{V}(\mathcal{T}(\mathbf{x})) - \mathcal{V}_\mathcal{M}(\mathbf{x}))^2}{2\sigma_\mathcal{M}^2(\mathbf{x})} \right]$$

Atlas Matching Framework

- Grid-based deformation model (e.g. Free Form Deformation)

$$\mathcal{T}(\mathbf{x}) = \mathbf{x} + \mathcal{D}(\mathbf{x}), \quad \mathcal{D}(\mathbf{x}) = \sum_{p \in \mathcal{G}} \eta(|\mathbf{x} - \mathbf{p}|) \, \mathbf{d}_p$$

- Deformable registration is formulated as a discrete labeling problem

$$\mathcal{L} = \{u^1, ..., u^i\} \qquad \Theta = \{d^1, ..., d^i\} \qquad \mathcal{D}(\mathbf{x}) = \sum_{p \in \mathcal{G}} \eta(|\mathbf{x} - \mathbf{p}|) \, d^{u_p}$$

- Markov Random Field energy formulation of the discrete labeling

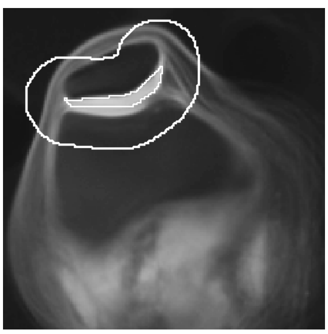
$$E_{\text{total}}(u) = \sum_{p \in \mathcal{G}} V_p(u_p) + \sum_{p,q \in \mathcal{E}(p)} V_{pq}(u_p, u_q).$$

$$E_{\text{data}}(u) = \sum_{p \in \mathcal{G}} \int_\Omega \eta^{-1}(|\mathbf{x} - \mathbf{p}|) \rho_\mathcal{M}(\mathcal{V}(\mathcal{T}(\mathbf{x}))) d\mathbf{x} \approx \sum_{p \in \mathcal{G}} V_p(u_p)$$

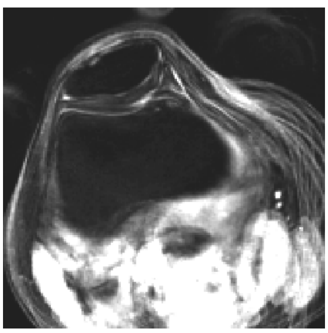
$$E_{\text{smooth}}(u) = \sum_{p,q \in \mathcal{E}(p)} V_{pq}(u_p, u_q), \qquad V_{pq}(u_p, u_q) = \min\left(|d^{u_p} - d^{u_q}|, T\right)$$

Conclusion

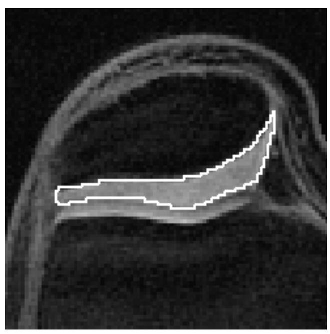
- Automatic segmentation in less than 20 seconds
- Average surface distance of 0.49 (± 0.23) millimeters
- Significant improvement of automatic segmentation initialization for applications with high accuracy constraints
- Flexible and general framework can be used for other anatomy
- Powerful linear programming algorithm allows to incorporate complex priors in future work



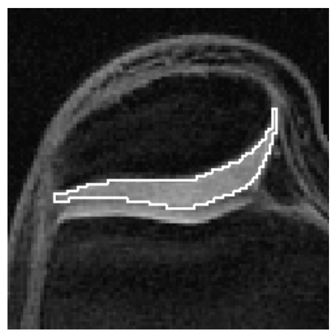
Mean Intensity Image



Variance Map



Automatic Segmentation



Expert Segmentation

Experiments / Results

Method	DSC	Sensitivity	Specificity	Interaction	Cartilage
Grau et al.	0.90 (0.01)	90.03 %	99.87 %	5-10 min	Tibia, Femur, Patella
Dam et al.	0.92 (n/a)	93.00 %	99.99 %	Max 10 min	Tibia, Femur
Cheong et al.	0.64 (0.15)	74.00 %	n/a	0	Medial Tibia
Cheong et al.	0.72 (0.09)	79.00 %	n/a	0	Lateral Tibia
Folkesson et al.	0.80 (0.03)	90.01 %	99.80 %	0	Tibia, Femur
Our Approach	**0.83 (0.06)**	**93.77 %**	**99.94 %**	**0**	**Patella**

Image Data

- 56 data sets of patella cartilage
- 28 data sets used for atlas generation / 28 for evaluation
- Protocol: T1-w 3D FLASH water excitation
- Scanner: Siemens MAGNETOM Symphony 1.5T
- Resolution: 256 x 256 x 20 (.625 x .625 x 3mm)

References
[1] Glocker, B. et al.: Inter and Intra-modal deformable registration: Continuous Deformations meet efficient linear programming. IPMI 2007, Kerkrade, Netherlands.
[2] Komodakis, N. et al.: Fast, approximately optimal solutions for single and dynamic MRFs. CVPR 2007, Minneapolis, Minnesota, USA.

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