Simultaneous In-Plane Motion Estimation and Point Matching Using Geometric Cues Only
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Appendix

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Abstract

This appendix introduces the proofs of Property 1 and 2 related to the discretization scheme; and a new compact kernel that we use throughout our method.

Notation. Matrix are in upper case bold (*e.g.* $\mathbf{A}$) and vector in lower case bold (*e.g.* $\mathbf{a}$). We consider two cameras: a source $\mathcal{S}$ and a target $\mathcal{T}$. $\square(\mathbf{a},\mathbf{b}) \subset \mathbb{R}^2$ is the quad formed by the two points $\mathbf{a} \in \mathbb{R}^2$ and $\mathbf{b} \in \mathbb{R}^2$. $\mathbf{o}_{\square(\mathbf{a},\mathbf{b})} \in \mathbb{R}^2$ is the center of the quad $\square(\mathbf{a},\mathbf{b})$ and $\mathbf{c}_{\square(\mathbf{a},\mathbf{b})} \in \mathbb{R}^{2\times 4}$ its four corners. $\mathbb{E}$ is the set of Essential Matrices [2]; it is a variety of dimension N ($N \leq 5$) in $\mathbb{R}^9$ (the set of 3×3 matrices).

A. Overlapping properties

As a reminder we redefine the relative rotation $\mathbf{R}(\mathbf{u})$ and translation $\mathbf{t}(\mathbf{u})$

$$\begin{aligned}\mathbf{R}(\mathbf{u}) &= \begin{bmatrix} \cos(\pi-\theta-\alpha) & 0 & \sin(\pi-\theta-\alpha) \\ 0 & 1 & 0 \\ -\sin(\pi-\theta-\alpha) & 0 & \cos(\pi-\theta-\alpha) \end{bmatrix}, \\ \mathbf{t}(\mathbf{u}) &= \begin{bmatrix} \sin(\theta) \\ 0 \\ \cos(\theta) \end{bmatrix}. \end{aligned} \tag{1}$$

Property 1 *Overlapping Property – See supplementary material for Proof*

$$\psi(\mathbf{u}) = 0 \quad \Leftrightarrow \quad \pi + \gamma_{\mathcal{S}} + \gamma_{\mathcal{T}} < \theta + \alpha < 3\pi - \gamma_{\mathcal{S}} - \gamma_{\mathcal{T}}$$

with $\gamma_{\mathcal{S}}$ (resp. $\gamma_{\mathcal{T}}$) half the field of view of the source camera (resp. target). $\psi(\mathbf{u}) = 0$ means no overlap.

Proof of Property 1 – Overlapping Property

In order to have no overlap two criteria have to be matched as the same time $\mathbf{r}_{\mathcal{S}} < \mathbf{l}_{\mathcal{T}}$ *and* $\mathbf{r}_{\mathcal{T}} < \mathbf{l}_{\mathcal{S}}$ (see figure 1 for definition of the variables). These condition lead to theses conditions:

$$\begin{aligned} \mathbf{r}_{\mathcal{S}} &< \mathbf{l}_{\mathcal{T}} \\ \gamma_{\mathcal{S}} &< \theta + \alpha - \pi - \gamma_{\mathcal{T}} \\ \pi + \gamma_{\mathcal{S}} + \gamma_{\mathcal{T}} &< \theta + \alpha \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{\mathcal{T}} &< \mathbf{l}_{\mathcal{S}} \\ \theta + \alpha - \pi - \gamma_{\mathcal{T}} &< 2\pi - \gamma_{\mathcal{S}} \\ \theta + \alpha &< 3\pi - \gamma_{\mathcal{S}} - \gamma_{\mathcal{T}} \end{aligned}$$

$$\pi + \gamma_{\mathcal{S}} + \gamma_{\mathcal{T}} < \theta + \alpha < 3\pi - \gamma_{\mathcal{S}} - \gamma_{\mathcal{T}}.$$

Property 2 *Transitivity of Overlapping Property – See supplementary material for Proof*

$$\textit{Let } \square(\mathbf{a},\mathbf{b}) \subset \square_0 \quad \left(\forall \mathbf{u} \in \mathbf{c}_{\square(\mathbf{a},\mathbf{b})}, \psi(\mathbf{u}) = 0\right) \Rightarrow \quad \left(\forall \mathbf{u} \in \square(\mathbf{a},\mathbf{b}), \psi(\mathbf{u}) = 0\right)$$

Proof of Property 2 Transitivity of Overlapping Criteria

$$\begin{aligned} \mathbf{c}_1 &= [\theta_1, \alpha_1]^\top, \text{with } \psi(\mathbf{c}_1) = 0, \\ \mathbf{c}_2 &= [\theta_2, \alpha_2]^\top, \text{with } \psi(\mathbf{c}_2) = 0, \\ \mathbf{a} &= [\theta_{\mathbf{a}}, \alpha_{\mathbf{a}}]^\top \in \square_{\mathbf{c}_1,\mathbf{c}_2}, \\ &\Rightarrow \theta_1 < \theta_{\mathbf{a}} < \theta_2, \quad \alpha_1 < \alpha_{\mathbf{a}} < \alpha_2 \\ &\Rightarrow \theta_1 + \alpha_1 < \theta_{\mathbf{a}} + \alpha_{\mathbf{a}} < \theta_2 + \alpha_2 \\ &\Rightarrow \pi + \gamma_{\mathcal{S}} + \gamma_{\mathcal{T}} < \theta_{\mathbf{a}} + \alpha_{\mathbf{a}} < 3\pi - \gamma_{\mathcal{S}} - \gamma_{\mathcal{T}} \\ &\Rightarrow \psi(\mathbf{c}_{\mathbf{a}}) = 0. \end{aligned}$$

B. Compact Kernel

We experimented with different kernels that we modify to obtain a compact support (Cootes [1] , wendland [3]). But they performed poorly with our robust cost as can be seen in figure 4 where we optimized a pose using our robust cost and matching points. They seldomly converge to

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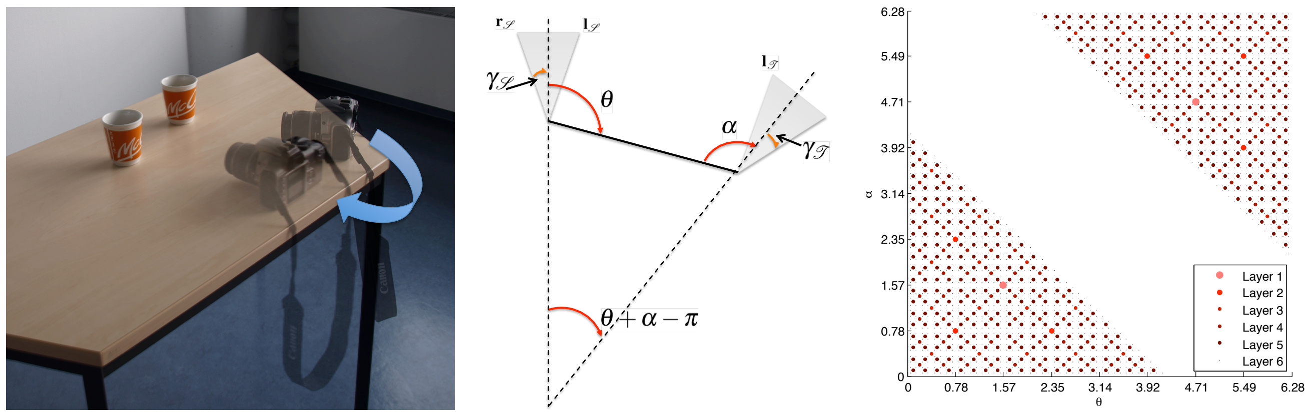

Figure 1. (left) Exemplary in-plane motion (middle) Top view of the 2-parameter camera setup we consider with $\mathbf{u} = [\theta, \alpha]^\top$. The translation only depends on the angle θ while the rotation uses both angles θ and α. (right) A quad-tree subdivision to 6 layers of the essential matrix space $\mathbb{E}$ for the camera setup we consider. The empty diagonal comes from the non-overlapping criterion.

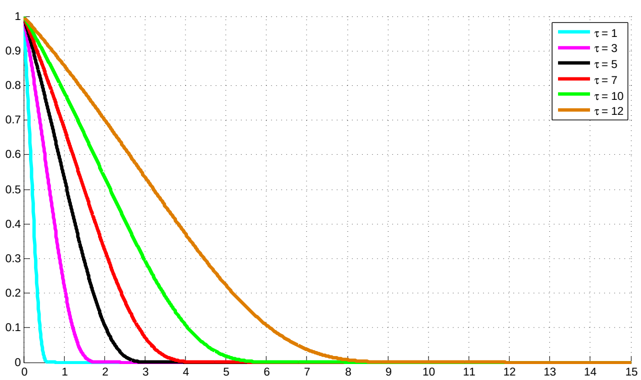

Figure 2. Behavior of our kernel function with varying τ.

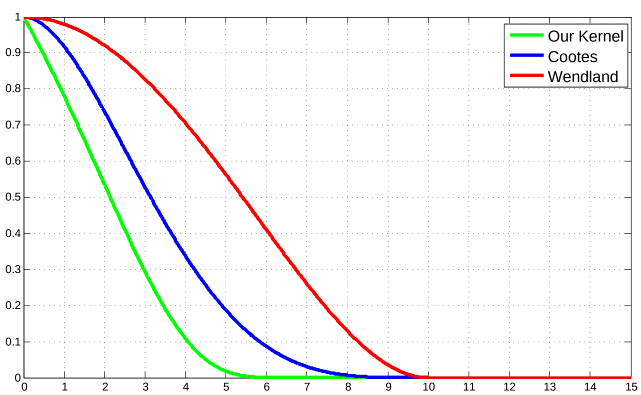

Figure 3. Our kernel with $\tau = 10$ compared to Cootes' and Wendland's.

the 'true' solution even if they are, they lack precision. The missing precision would have a huge impact on the algorithm. Therefore we introduce the following kernel:

$$\rho_\sigma(r) = \begin{cases} e^{\frac{-2\sigma r}{(\sigma - r)^2}} & \text{if } |r| < \sigma \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

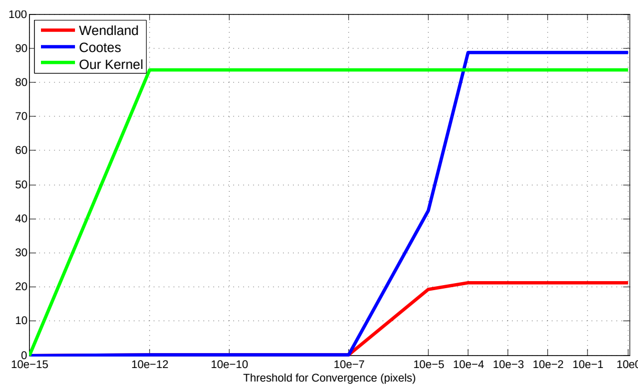

Figure 4. Convergence rate of the robust cost function with different type of kernel.

The behavior of our kernel can be pictured in figure 2 and 3. As it can be seen our kernel give precise results therefore we will be able easily to decide weather we converged in to the global minimum based on the value at convergence.

References

[1] T. Cootes, C. Twining, and C. Taylor. Diffeomorphic Statistical Shape Models. In *BMVC*, 2004.

[2] T. Huang and O. Faugeras. Some properties of the E matrix in Two-View Motion Estimation. *IEEE PAMI*, 1989.

[3] H. Wendland. Piecewise Polynomial, Positive Definite and Compactly Supported Radial Functions of Minimal Degree. *Advances in Computational Mathematics*, 1995.