Simultaneous In-Plane Motion Estimation and Point Matching Using Geometric Cues Only

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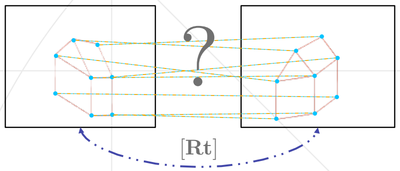
Motivation



Motion Estimation with Unmatched Points

The goal of our method is to estimate an essential matrix $\widetilde{\mathbf{E}} \in \mathbb{E}$ from two sets of unmatched image points ($\{\mathbf{q}_k\} \subset \mathcal{S}$ and $\{\mathbf{p}_l\} \subset \mathcal{T}$).

The is performed by:

1. Discretizing the essential matrix space $\mathbb{E}$
2. A local geometric only matching procedure
3. Minimizing a robust cost

Discretizing the Motion Model

Camera Setup: Camera moving on a plane parameterized by two angles.
$[\theta, \alpha] = \mathbf{u} \in [0, 2\pi]^2$

$$\mathbf{R}(\mathbf{u}) = \begin{bmatrix} \cos(\pi-\theta-\alpha) & 0 & \sin(\pi-\theta-\alpha) \\ 0 & 1 & 0 \\ -\sin(\pi-\theta-\alpha) & 0 & \cos(\pi-\theta-\alpha) \end{bmatrix} \quad \mathbf{t}(\mathbf{u}) = \begin{bmatrix} \sin(\theta) \\ 0 \\ \cos(\theta) \end{bmatrix}$$

It has two useful properties that can be derived from this parameterization:

Property 1 *Overlapping Property*

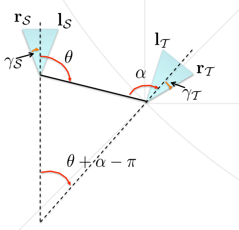
$$\psi(\mathbf{u}) = 0 \quad \Leftrightarrow \quad \pi + \gamma_{\mathcal{S}} + \gamma_{\mathcal{T}} < \theta + \alpha < 3\pi - \gamma_{\mathcal{S}} - \gamma_{\mathcal{T}}$$

Property 2 *Transitivity of the Overlapping Property*

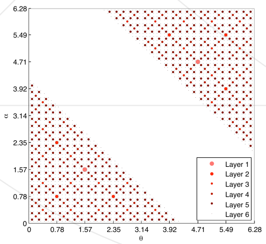
$$\text{Let } \square(\mathbf{a},\mathbf{b}) \subset \square_0 \quad \left(\forall \mathbf{u} \in \mathbf{c}_{\square(\mathbf{a},\mathbf{b})}, \psi(\mathbf{u}) = 0\right) \Rightarrow \left(\forall \mathbf{u} \in \square(\mathbf{a},\mathbf{b}), \psi(\mathbf{u}) = 0\right)$$

We discretized using quadtrees over the domain $[0, 2\pi]^2$.

Camera setup



Motion parameters



Discretized space

Geometric Only Matching

Using a candidate pose $\mathbf{u}$ and unmatched set of points $\{\mathbf{q}_k\}$ and $\{\mathbf{p}_l\}$, we extracted an homogeneous set of matches.

1. **Putative Matches from Geometric Guided Matches [1]**
   We define the hyperbola
   $$\mathbf{C_q} = \mathbf{l}\mathbf{l}^\top - k^2\left(\mathbf{J_u}\Sigma_\mathbf{u}\mathbf{J_u}^\top + \mathbf{J_q}\Sigma_\mathbf{q}\mathbf{J_q}^\top\right)$$
   This gives a first matching criteria
   $$\sigma_{k^2}(\mathbf{q},\mathbf{p},\mathbf{u}) = 1 \quad \Leftrightarrow \quad \mathbf{p} \in \mathbf{C_q} \quad \Leftrightarrow \quad \left(\mathbf{p}^\top\mathbf{C_q}\mathbf{p}\right).\left(\mathbf{p_l}^\top\mathbf{C_q}\mathbf{p_l}\right) > 0$$
2. **Coherent Epipolar Geometry by Spectral Clustering**
   For each match $\langle \mathbf{q}_k, \mathbf{p}_l \rangle$ we estimate an epipolar constraint $\widetilde{\mathbf{u}}_{kl}$ such as
   $$\widetilde{\mathbf{u}}_{kl} = \arg\min_{\widetilde{\mathbf{u}}} d^2_{\mathbb{E}}(\mathbf{u},\widetilde{\mathbf{u}}) \quad \text{s.t.} \quad \mathbf{p}_l^\top \mathbf{F}(\widetilde{\mathbf{u}})\,\mathbf{q}_k = 0$$
   Using the obtained $\widetilde{\mathbf{u}}$ we define a similarity matrix $\mathbf{S}$ that compares two epipolar geometries $\widetilde{\mathbf{u}}_{kl}$ and $\widetilde{\mathbf{u}}_{fg}$ as follows:
   $$\begin{cases} \mathbf{S}(i,j) &= \rho_{4\tau}\begin{pmatrix} & d_l(\mathbf{q}_k, \mathbf{F}(\mathbf{u}_{fg})^\top \mathbf{p}_l) \\ + & d_l(\mathbf{q}_f, \mathbf{F}(\mathbf{u}_{kl})^\top \mathbf{p}_g) \\ + & d_l(\mathbf{p}_l, \mathbf{F}(\mathbf{u}_{fg})\,\mathbf{q}_k) \\ + & d_l(\mathbf{p}_g, \mathbf{F}(\mathbf{u}_{kl})\,\mathbf{q}_f) \end{pmatrix} \\ \mathbf{S}(j,i) &= \mathbf{S}(i,j) \\ \mathbf{S}(i,i) &= 0 \end{cases}$$
   Using the eigen value decomposition of normalized Laplacian of $\mathbf{S}$ we extract most homogeneous set of matches [2].
3. **Enforcing Unicity**
   We built all set of set of matches such that the unicity constraint is satisfied and the set with the largest corresponding $\sum_{i,j} \mathbf{S}_{i,j}$.

Nonlinear Motion Refinement

Using the set of estimated matches we minimize the following cost:

$$q(\mathbf{u}) = \sum_{\langle \mathbf{q},\mathbf{p} \rangle \in \widehat{\mathcal{M}}} \left(\left(1 - \rho_\tau\left(d_l\left(\mathbf{p}, \mathbf{F}(\mathbf{u})\,\mathbf{q}\right)\right)\right)^2 + \left(1 - \rho_\tau\left(d_l\left(\mathbf{q}, \mathbf{F}^\top(\mathbf{u})\,\mathbf{p}\right)\right)\right)^2 \right).$$

References

[1] B. Ochoa and S. Belongie. Covariance Propagation for Guided Matching. Workshop on Statistical Methods in Multi-Image and Video Processing, 2006.
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[3] Z. Zhang. Determining the Epipolar Geometry and its Un- certainty: A Review. IJCV, 1998.
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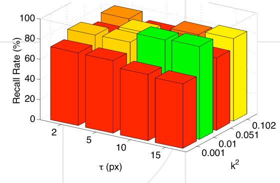
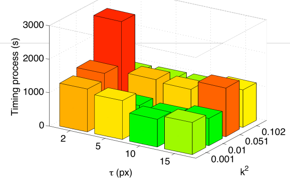
[6] O. Enqvist and F. Kahl. Two View Geometry from Uncertain Correspondences. BMVC, 2009.

Evaluation

Choice of Parameters $\{k^2, \tau\}$

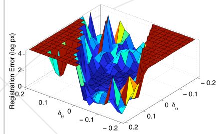


Recall rate of matches

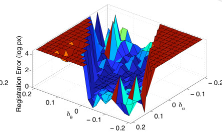


Processing Time

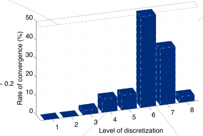
Behavior Analysis $\{k^2, \tau\} = \{0.01, 15\}$



Final Registration Error

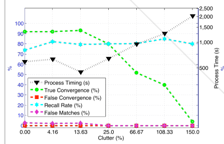


Correct Registration Error

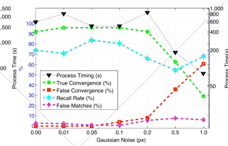


Discretization Level

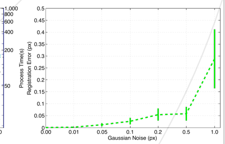
Synthetic Experiments



Increasing Clutter



Increasing Noise



Final Registration Error

Wide Baseline Experiments with Multiple Identical Objects

Future Works

- Uncertainty in the subdivision procedure
- Full five degrees of freedom case

Conclusion

- Motion and matches simultaneously estimated
- Homogeneous matches segmentation
- Pseudo-distance between epipolar geometries