Throughput Maximizing Transmission Strategy of Energy Harvesting Nodes

Qing Bai, Jingrui Li and Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Technische Universität München, Munich, Germany
Email: bai.qing@nws.ei.tum.de

***Abstract*—We investigate the throughput maximizing data transmission strategy of an energy harvesting node which is able to harvest and store energy for communication. Solar cell and rechargeable battery technologies have made such nodes feasible. In addition to the energy arrival process and the battery capacity limitation, the energy consumption of the circuits of the node also plays an important role in the way how the harvested energy should be utilized. To this end, we assume for the transmitting node an active mode for which a constant circuit power is incurred, and a sleep mode for which no energy is consumed. The criteria that an optimal transmission strategy should satisfy are discussed, and based on them, a construction procedure of the optimal transmission strategy is proposed. Numerical simulations are performed to verify the theoretical results, and the impact of circuit power on the optimal transmission strategy and the maximal achievable throughput is studied.**

I. Introduction

Battery powered communication systems are considerably limited in lifetime. Recharging or replacing the empty batteries may cause notable inconvenience and cost, especially in wireless sensor networks deployed for industrial monitoring and control. Devices with energy harvesting ability offer a possible solution to this problem. Unlike the constant power provided by fixed utilities, the supply to energy harvesting devices is rather variable, unsteady, and depends heavily on environmental conditions or human impact. Therefore, it is of both theoretical and practical importance to investigate energy utilization issues in such systems and evaluate the performance limits [1][2]. On the other hand, improving the energy efficiency of communications, which is often measured by the metric "bit per Joule", is an alternative to expand the lifetime of battery powered systems. This requires a thorough study on the energy consumption of all signal processing blocks in a communication device [3][4]. In this work, we focus on finding the optimal transmission strategy of an energy harvesting transmitter in the sense that the short-term throughput is maximized, where a simple form of the energy consumption model is adopted, *i.e.*, a constant circuit power is assumed when the transmitter is in active mode.

Stemmed from the convexity of the power-rate relation and the timeliness of information delivery, the trade-off between energy efficiency and data rate has been established and many early works have investigated its implications in rate adaptation and scheduling [5]. In [6], energy efficient transmission rate adaptation was considered given the arrival and deadline information on buffered packets in the system. An energy expenditure minimization problem subject to strict deadline constraints has been formulated and a construction method of the optimal service curve is obtained using a calculus approach. The throughput maximization problem at an energy harvesting transmitter, as first proposed by the authors of [2], resembles much mathematical similarity with the energy minimization problem in [6]. We further introduce circuit power of the transmitter into the optimization and pursue a similar calculus approach in tackling the problem. Although the new consideration destroys the convexity of the problem, an optimal solution can be found without much additional effort but is of more practical relevance.

The rest of paper is organized as follows: in Section II we first introduce the data transmission model and the energy arrival/consumption model, and then formulate the throughput maximization problem with constraints. In Section III we analyze the optimal solution of the basic problem which provides us with important insight on how to solve general throughput maximization. Optimality criteria and a construction procedure of the optimal transmission strategy are proposed and proved in Section IV. Simulation results are shown in Section V before we conclude the work in Section VI.

II. System Model and Problem Formulation

We consider an energy harvesting node which transmits data over a single link during the time slot $[0, T]$. A continuous-time model is adopted and it is assumed that the transmit power of the node, denoted by $p_{\mathrm{tx}}(t)$, can be adapted continuously to any desired finite value. Let $f(p_{\mathrm{tx}}(t))$ be the corresponding instantaneous data rate, where $f(\cdot)$ is assumed to be nonnegative, strictly concave, monotonically increasing, and invariant over the time slot $[0, T]$. Obviously, the Shannon formula is a valid choice of $f(\cdot)$ when the transmission link remains unchanged for $t \in [0, T]$.

Besides the energy used for the actual data transmission, *i.e.*, $p_{\mathrm{tx}}(t)$ cumulated over time, there is additional energy consumption within the circuit of the transmitter *e.g.* by active filters, which is relatively independent of transmit power [7]. We denote the circuit power by a constant p_{c} and define the total power usage of the node as

$$p(t) \triangleq \begin{cases} p_{\mathrm{tx}}(t) + p_{\mathrm{c}}, & p_{\mathrm{tx}}(t) > 0, \\ 0, & p_{\mathrm{tx}}(t) = 0. \end{cases} \tag{1}$$

The transmitter is considered as in *active* mode for $p_{\mathrm{tx}}(t) > 0$, in which case the circuit power p_{c} contributes to $p(t)$. For $p_{\mathrm{tx}}(t) = 0$, the transmitter can be turned into a *sleep* mode for which we assume there is no energy consumption. The energy consumption associated with mode switching is also neglected here, but in the design of the optimal transmission strategy we have taken care that the number of mode switches is minimized.

For a simple visualization of the problem and easier application of the theory on the calculus of variations, we utilize a cumulative model to describe the energy input and expenditure of the transmitting node. To this end, we define the following nondecreasing functions for $t \in [0, T]$:

- $W(t)$: total energy expenditure of the node until time t;
- $A(t)$: the total amount of available energy by time t;
- $D(t)$: the minimal amount of energy that has to be consumed by the node by time t.

Due to passivity and causality, $W(t) \le A(t)$ must be satisfied for any $t \in [0, T]$. $D(t)$ on the other hand, is given rise to by physical limitations on the battery. Let $E_{\max}$ be the maximal amount of energy that can be held by the battery and further energy input is lost due to overflow. If $E_{\max}$ is constant, we need to have

$$W(t) \ge D(t) = \max(0, A(t) - E_{\max}), \quad \forall t \in [0, T] \tag{2}$$

in order to avoid battery overflow. Note that in the sequel we do not restrict $D(t)$ to be related to $A(t)$, which gives us the flexibility to allow for any imperfection of the battery.

The throughput maximization problem, which aims to find the function $W(t)$ that maximizes the sum rate over the time interval $[0, T]$, can be formulated as

$$\begin{aligned} \max_{W(t)} \quad & \int_0^T f(p_{\mathrm{tx}}(t))\mathrm{d}t \\ \text{s.t.} \quad & W(t) = \int_0^t p(\tau)\mathrm{d}\tau, && \forall t \in [0, T], \\ & D(t) \le W(t) \le A(t), && \forall t \in [0, T], \\ & W(T) = A(T). \end{aligned} \tag{3}$$

We refer to a function $W(t)$ that fulfills all constraints in (3) as an *admissible curve*, which corresponds to a feasible transmission strategy that can be taken in $[0, T]$. Denote the optimum solution to (3) by $\hat{W}(t)$. The constraints $W(t) \ge D(t)$ and $W(T) = A(T)$ mean that battery overflow should not be allowed and energy should be exhausted by the end of the transmission, respectively. They are included in (3) only because any $W(t)$ that violates them are suboptimal due to the monotonically increasing property of the rate function $f(\cdot)$.

The problem discussed in [2] is a special case of (3) in which $p_{\mathrm{c}} = 0$, $A(t)$, $D(t)$ are discrete and $D(t)$ is $A(t)$ shifted downwards by $E_{\max}$. With $p_{\mathrm{c}} = 0$, Problem (3) is convex and the optimal solution has the nice geometrical interpretation that it is the shortest trajectory in the admissible region [6]. However, the problem becomes nonconvex when $p_{\mathrm{c}} > 0$. We define $D(T^+) \triangleq A(T)$ for the simplicity in explanation. This clearly has no influence on the solution of (3).

III. Analysis of the Basic Problem

We start with analyzing the simplest situation, where the battery has an nonempty initial state A_0 where $A_0 \le E_{\max}$, and there is no energy arrival during $[0, T]$, *i.e.*, $A(t) \equiv A_0$, $D(t) \equiv 0$ for $t \in [0, T]$. Problem (3) under this particular setting will be referred to as the *basic problem*.

It is well-known that for the basic problem with $p_{\mathrm{c}} = 0$, $\hat{W}(t) = \frac{A_0}{T} \cdot t$ (a proof can be found in [6] or [8]), which suggests that using a constant transmit power over the whole time slot leads to the maximal throughput when circuit power is not considered. Now suppose that the transmitter is only active for a period of length u, $u \in (0, T]$, and is asleep for the rest of the time slot. The circuit power causes an energy consumption of $p_{\mathrm{c}} \cdot u$ for the whole active period. Here we assume that $A_0 > p_{\mathrm{c}} \cdot T$, because otherwise the transmitter has to sleep during some part of the time slot which can be dropped from consideration. The rest of the energy, $A_0 - p_{\mathrm{c}} \cdot u$, is optimally used with constant transmit power $p_{\mathrm{tx}} = \frac{A_0}{u} - p_{\mathrm{c}}$. Consequently, an optimization of u can be formulated as

$$\begin{aligned} \max_{u} \quad & g(u) \triangleq u \cdot f\left(\frac{A_0}{u} - p_{\mathrm{c}}\right) \\ \text{s.t.} \quad & 0 < u \le T. \end{aligned} \tag{4}$$

With simple derivations we come to

$$g'(u) = f\left(\frac{A_0}{u} - p_{\mathrm{c}}\right) - \frac{A_0}{u} \cdot f'\left(\frac{A_0}{u} - p_{\mathrm{c}}\right), \tag{5}$$

$$g'(u) \to +\infty, \quad u \to 0 \quad \text{and} \quad g''(u) < 0, \tag{6}$$

which imply that there exists a unique optimal u^* that satisfies

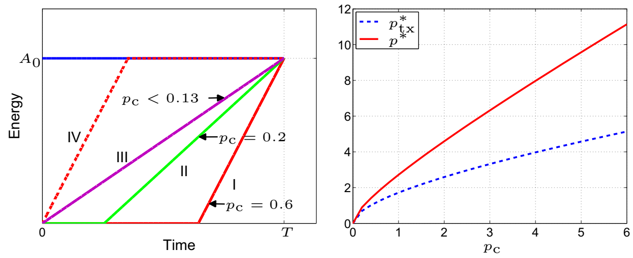
$$\begin{cases} g'(u^*) = 0, & u^* \in (0, T], & \text{if } g'(T) \le 0 \\ g'(u^*) > 0, & u^* = T, & \text{otherwise.} \end{cases} \tag{7}$$

In the first case, noticing that $p_{\mathrm{tx}}^* = \frac{A_0}{u^*} - p_{\mathrm{c}}$ can be plugged into (5), we have

$$f\left(p_{\mathrm{tx}}^*\right) - (p_{\mathrm{tx}}^* + p_{\mathrm{c}}) f'\left(p_{\mathrm{tx}}^*\right) = 0, \tag{8}$$

which means the optimal transmit power p_{tx}^* depends only on p_{c} but not on A_0 or T. This property enables us to easily find the optimal transmission strategy for any basic problem (without the assumption that $A_0 > p_{\mathrm{c}} \cdot T$) given $p^* = p_{\mathrm{tx}}^* + p_{\mathrm{c}}$: if $\frac{A_0}{T} < p^*$, then the transmitter should be turned into sleep mode for a time period of $T - \frac{A_0}{p^*}$ and then transmit with power p_{tx}^*; otherwise the transmitter should transmit over the whole time slot with power $\frac{A_0}{T}$. Note that where and how the sleeping period is located in the time slot does not influence the throughput. The optimal transmission strategy we propose in the next section obeys in general a "sleep first" principle.

We illustrate our analysis on the basic problem in Figure 1. Here and also in later numerical simulations we use the rate function $f(x) = \log(1 + x)$. In Figure 1(a) a basic problem with $A_0 = 10$ and $T = 15$ is depicted. Curves I and II represent $\hat{W}(t)$ for $p_{\mathrm{c}} = 0.6$ and $p_{\mathrm{c}} = 0.2$, respectively. For $p_{\mathrm{c}} < 0.13$ approximately, p^* falls below $10/15$ hence $\hat{W}$ should be the straight line connecting $(0, 0)$ and (T, A_0). The optimal transmit power p_{tx}^* as dependent on p_{c} can be obtained by solving (8) with the bisection method. The results can be stored in a look-up table for quick search. The variations of p_{tx}^* and p^* with increasing p_{c} is shown in Figure 1(b).



(a) Optimal Transmission Strategy (b) p_{tx}^* and p^*

Figure 1. Analysis of the Basic Problem

We will use from here the term *change of mode* to indicate the switch from active mode to sleep mode or vice versa. The term *change of slope* will only refer to the change in transmit power from a positive value to another, but not to the change in transmit power associated with a change of mode. We make the following definition in order to identify an important class of curves in our problem.

Definition 1. *Let $W_1(t)$ and $W_2(t)$ be two admissible curves which differ only in a finite number of subintervals $[a_i, b_i] \subseteq [0, T]$, $i = 1, 2, \ldots, K$. If on these subintervals, $W_1(t)$ and $W_2(t)$ both consist only of horizontal lines and straight lines with slope p^*, then $W_1(t)$ and $W_2(t)$ are equivalent, denoted by $W_1(t) \sim W_2(t)$.*

The Curve I and IV in Figure 1(a) are equivalent. The definition indicates that the transmission strategies represented by equivalent curves are identical in the active periods where only changes of slopes are involved, and differ in the location and length of each sleeping period. Obviously, equivalent curves yield the same throughput.

IV. Optimality Criteria and Construction of the Optimal Admissible Curve

Based on our analysis of the basic problem, we establish the following theorem where we assume that p^* is known.

Theorem 1. *Let $W(t)$ be an admissible curve and $L(t)$, $t \in [a, b]$ be a curve that adjoins $(a, W(a))$ and $(b, W(b))$ where a, b satisfy $0 \leq a < b \leq T$. Denote the slope of the straight line that connects $(a, W(a))$ and $(b, W(b))$ by k.*

1) *If $k < p^*$ and $L(t)$ satisfies*
   - *$L(t)$ consists only of horizontal lines and straight lines with slope p^*,*
   - *$L(t) \nsim W(t)$, $t \in [a, b]$,*
   - *$D(t) \leq L(t) \leq A(t)$, $\forall t \in [a, b]$,*
2) *If $k \geq p^*$ and $L(t)$ satisfies*
   - *$L(t)$ is a straight line segment,*
   - *$L(t) \not\equiv W(t)$, $t \in [a, b]$,*
   - *$D(t) \leq L(t) \leq A(t)$, $\forall t \in [a, b]$,*

then replacing the part of $W(t)$ between $[a, b]$ with $L(t)$ increases the throughput.

Proof: Consider the basic problem between $(a, W(a))$ and $(b, W(b))$ where the upper and lower boundaries are given by $\tilde{A}(t) = W(b)$ and $\tilde{D}(t) = W(a)$, $t \in [a, b]$. The curve $L(t)$ that satisfies the conditions given in Theorem 1 is in fact the optimal solution to this basic problem. Due to the fixed end points $(a, W(a))$ and $(b, W(b))$, $W(t)$ is upper bounded by $\min\left(A(t), \tilde{A}(t)\right)$ and lower bounded by $\max\left(D(t), \tilde{D}(t)\right)$, $t \in [a, b]$, which means Problem (3) localized between $(a, W(a))$ and $(b, W(b))$ has an equal or smaller admissible region than the defined basic problem, and therefore $L(t)$ leads to no less throughput than any other feasible curve on $[a, b]$. As the theorem states that $W(t)$ is not equivalent with $L(t)$ in the first case and not equal to $L(t)$ in the second case, the replacement of $W(t)$, $t \in [a, b]$ with $L(t)$ increases the throughput. ∎

Although Theorem 1 does not give us directly a method to construct $\hat{W}(t)$, it provides the *Optimality Criteria* which can be used to determine whether an admissible curve is optimal: *along $\hat{W}(t)$, $t \in [0, T]$ there do not exist any two points between which the part of $\hat{W}(t)$ can be reconstructed successfully as indicated by Theorem 1.*

From the optimality criteria it is straightforward to observe and prove the following lemmas:

Lemma 1. *The slope at any point of $\hat{W}(t)$ is greater or equal to p^* except for the horizontal part.*

Lemma 2. *Any horizontal part of $\hat{W}(t)$ is at least arrived at or followed by a straight line of slope p^*.*

Lemma 3. *The points at which $\hat{W}(t)$ changes slope are either on $A(t)$ or on $D(t)$. Moreover, the slope change at a point on $D(t)$ is negative, whereas the slope change at a point on $A(t)$ is positive.*

It is clear that the optimal admissible curve is not unique. In fact, there are infinitely many of them which all lead to the same maximal throughput. The theorem below gives us the guarantee of uniqueness in the sense of equivalence.

Theorem 2. *All admissible curves that satisfy the optimality criteria are either equivalent or identical.*

Proof: Suppose that there exist two distinct admissible curves $W_1(t)$ and $W_2(t)$ which do not violate the optimality criterion. As $W_1(0) = W_2(0) = 0$ and $W_1(T) = W_2(T) = A(T)$, the two curves do not differ over the whole time slot of interest. Let (a, b) be an interval over which $W_1(t) \neq W_2(t)$ and $W_1(a) = W_2(a)$, $W_1(b) = W_2(b)$. Without loss of generality, we assume that $W_1(t) > W_2(t)$ which implies $D(t) \leq W_2(t) < W_1(t) \leq A(t)$, $t \in (a, b)$.

The uniqueness of $\hat{W}(t)$ when there is no circuit power can be proved based on Appendix A in [6]. From Lemma 3, we find that there exists a time instance t_0 such that for $t \in [t_0, b]$, $W_1(t)$ is a horizontal line, otherwise the curve $W_2(t)$ with a nonincreasing slope can not meet the curve $W_1(t)$ with a nondecreasing slope at $t = b$, starting from any point $\tilde{t} \in [a, b)$. Then it can be deduced that $W_1(t)$ consists only of straight lines with slope p^* and horizontal lines on $t \in [a, t_0]$. The part of $W_1(t)$ immediately after point $t = a$ is a straight line with slope p^*, otherwise $W_1(t) > W_2(t)$ can not be satisfied, and $W_2(t)$ starting from $t = a$ has to be a horizontal line for its slope has to be smaller than that of $W_1(t)$ but should not exceed p^* according to Lemma 1. It then follows from Lemma 2 that $W_2(t)$ also consists only of straight lines with slope p^* and horizontal lines, which is to say, $W_1(t) \sim W_2(t)$ on $t \in (a, b)$. The argument holds for all intervals on which the two curves differ, which is to say, if $W_1(t)$ is not identical to $W_2(t)$, then it must be equivalent with $W_2(t)$ on $[0, T]$. ∎

Let (t_0, α_0) be in the admissible region, *i.e.*, $D(t_0) \leq \alpha_0 \leq A(t_0)$. Straight lines of nonnegative slopes starting from this point, denoted by $L_{(t_0,\alpha_0)}(t)$, can be distinguished by whether they intersect with $A(t)$ or $D(t)$ first. Note that "intersection" here means, take $D(t)$ as an example, that $L_{(t_0,\alpha_0)}(t_1) = D(t_1)$ for some $t_1 > t_0$ if $D(t)$ is continuous at point t_1, or $L_{(t_0,\alpha_0)}(t) - D(t)$ changes sign at some $t_1 > t_0$ if $D(t)$ is discontinuous at that point. Let $\mathcal{S}_A(t_0, \alpha_0)$ and $\mathcal{S}_D(t_0, \alpha_0)$ denote the sets of slopes which lead $L_{(t_0,\alpha_0)}(t)$ to intersect with $A(t)$ and $D(t)$ first, respectively. Since $A(t) > D(t)$ for all $t \in (0, T)$, it is easy to see that

$$\beta_A > \beta_D, \quad \forall \beta_A \in \mathcal{S}_A(t_0, \alpha_0),\ \beta_D \in \mathcal{S}_D(t_0, \alpha_0),$$

which further leads us to

$$\inf \mathcal{S}_A(t_0, \alpha_0) = \sup \mathcal{S}_D(t_0, \alpha_0) \triangleq \beta(t_0, \alpha_0).$$

Using these notations and definitions we describe the construction of the optimal admissible curve in Algorithm 1.

Algorithm 1 Construction of the Optimal Admissible Curve

1: **initialize** $(t_0, \alpha_0) \leftarrow (0, 0)$, $W(0) \leftarrow 0$;
2: **repeat**
3: $\quad (t_0, \alpha_0) \leftarrow (t_1, \alpha_1)$;
4: $\quad$ **if** $\alpha_0 = A(T)$ **then**
5: $\quad\quad t_1 \leftarrow T$, $W(t) \leftarrow$ horizontal line from (t_0, α_0) to (t_1, α_0), $t \in (t_0, T]$;
6: $\quad$ **end if**
7: $\quad$ compute $\beta(t_0, \alpha_0) = \inf \mathcal{S}_A(t_0, \alpha_0) = \sup \mathcal{S}_D(t_0, \alpha_0)$;
8: $\quad$ **if** $\beta(t_0, \alpha_0) > p^*$ **then**
9: $\quad\quad$ **if** (t_0, α_0) is on $A(t)$ and $\beta(t_0, \alpha_0) = A'(t_0)$ **then**
10: $\quad\quad\quad$ find the largest t_1 such that $\beta(t, A(t)) = A'(t)$, $t \in [t_0, t_1]$;
11: $\quad\quad\quad \alpha_1 \leftarrow A(t_1)$, $W(t) \leftarrow A(t)$, $t \in (t_0, t_1]$;
12: $\quad\quad$ **else if** (t_0, α_0) is on $D(t)$ and $\beta(t_0, \alpha_0) = D'(t_0)$ **then**
13: $\quad\quad\quad$ find the largest t_1 such that $\max(\beta(t, D(t)), p^*) = D'(t)$, $t \in [t_0, t_1]$;
14: $\quad\quad\quad \alpha_1 \leftarrow D(t_1)$, $W(t) \leftarrow D(t)$, $t \in (t_0, t_1]$;
15: $\quad\quad$ **else**
16: $\quad\quad\quad W(t) \leftarrow L_{(t_0,\alpha_0)}(t)$ with slope $\beta(t_0, \alpha_0)$ until the intersection point (t_1, α_1);
17: $\quad\quad$ **end if**
18: $\quad$ **else**
19: $\quad\quad$ find the largest $\tilde{t}_0$ s.t. $\forall t \in [t_0, \tilde{t}_0]$, (t, α_0) is in the admissible region and $\beta(t, \alpha_0) \leq p^*$;
20: $\quad\quad W(t) \leftarrow$ horizontal line from (t_0, α_0) to $(\tilde{t}_0, \alpha_0)$, $t \in (t_0, \tilde{t}_0]$;
21: $\quad\quad W(t) \leftarrow L_{(\tilde{t}_0,\alpha_0)}(t)$ with slope p^* until the intersection point (t_1, α_1);
22: $\quad$ **end if**
23: **until** $t_1 = T$.

We first clarify some points about Algorithm 1 and then prove its optimality. In each iteration we determine how to move from an admissible point (t_0, α_0) by first computing $\beta(t_0, \alpha_0)$ and then comparing it with p^*. If $\beta(t_0, \alpha_0) > p^*$, we move basically in the direction of $\beta(t_0, \alpha_0)$, *i.e.*, the node transmits with power $\beta(t_0, \alpha_0) - p_c$. In order to avoid the ambiguity that when (t_0, α_0) is on $A(t)$ or $D(t)$, moving in the tangent direction of the point does not give any intersection point even with infinitesimal stepsize, we distinguish between the three cases where we move along $A(t)$, $D(t)$, or the straight line with slope $\beta(t_0, \alpha_0)$. At points where $A(t)$ and $D(t)$ are not differentiable, the conditions in Line 11 and 15 are considered as unsatisfied. If $\beta(t_0, \alpha_0) \leq p^*$, we decide for the possibly longest sleeping period and then activate the node to transmit with power $p^* - p_c$.

Proof: From Appendix D in [6] we know that the curve $W(t)$ constructed by Algorithm 1 is admissible and fulfills Lemma 3. It is also clear from the algorithm that the slope at any point of $W(t)$ is no smaller than p^*. Consider the starting point (t_0, α_0) in some iteration with $\beta(t_0, \alpha_0) \leq p^*$. Note that (t_0, α_0) is either on $A(t)$ or on $D(t)$ according to the algorithm. If (t_0, α_0) is on $D(t)$ and $D'(t_0) \neq 0$, no horizontal line is constructed in the current iteration, and the straight line with slope p^* intersects $A(t)$ at $t = t_1$. As a result, if a horizontal line is constructed in the next iteration, it is arrived at with a straight line with slope p^*. If (t_0, α_0) is on $A(t)$, then the constructed horizontal line is always followed by a straight line segment of slope p^*. Therefore, we claim that $W(t)$ satisfies both Lemma 1 and Lemma 2. As a result, around any point on curve $W(t)$, it is impossible to perform a construction as required by Theorem 1 to violate the optimality criteria. The constructed curve $W(t)$ is therefore optimal, as all admissible curves that do not violate the optimality criterion are equivalent and yield the same maximal throughput. ∎

V. Simulation Results

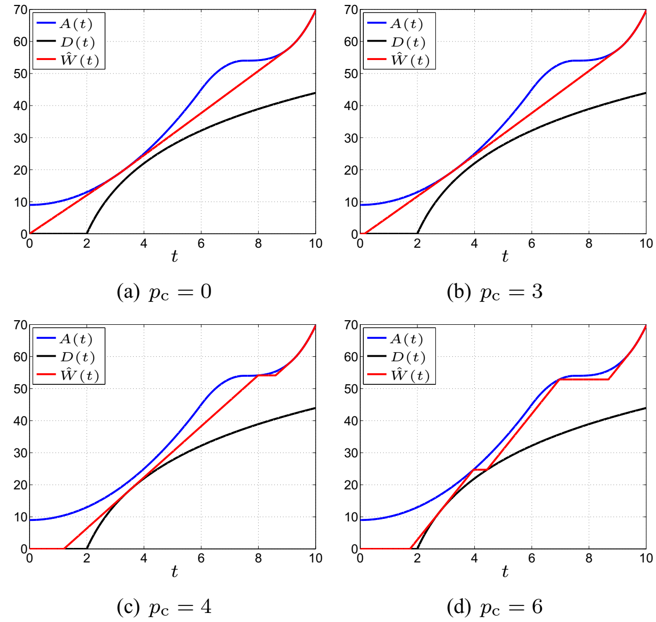
We test our results with both continuous and discrete energy inputs. In the continuous case, we set $T = 10$,

$$A(t) = \begin{cases} t^2 + 9, & t \in [0, 6], \\ 54 - 4(t - 7.5)^2, & t \in (6, 7.5], \\ 54 + (t - 7.5)^3, & t \in (7.5, 10], \end{cases}$$

$$D(t) = \begin{cases} 0, & t \in [0, 2] \\ 20 \log(t - 1), & t \in (2, 10]. \end{cases}$$

The constructed $\hat{W}(t)$ for several p_c values are shown in Figure 2. The optimal curve is composed of 4 segments when

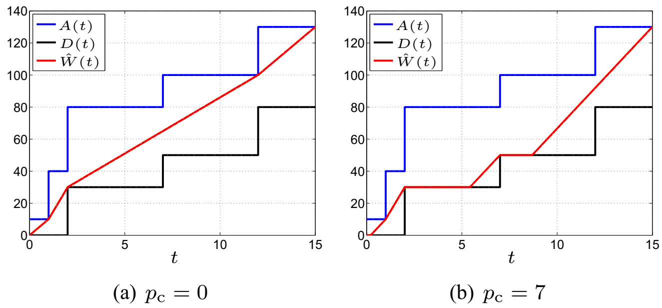
$p_c = 0$: from the origin it moves along the straight line that is tangent to the first part of $A(t)$, then it follows $A(t)$ until the tangent point where the common tangent line between the first and third parts of $A(t)$ is met. After traveling through the common tangent line, $\hat{W}(t)$ follows $A(t)$ again until $t = T$. $D(t)$ is not involved here as it does not obstruct the shortest trajectory from origin to the destination point $(T, A(T))$. The smallest slope of $\hat{W}(t)$ is 6, which is between $p^*(p_c = 2)$ and $p^*(p_c = 3)$. As a result, the optimal admissible curves for $p_c = 0, 1, 2$ are identical. For $p_c \geq 3$, change of mode comes into play to replace the parts of $\hat{W}(t)$ that have smaller slopes than the corresponding p^*. From Figure 2 it can be observed that the larger the circuit power p_c, the more often and for the longer period of time the node is turned into sleep mode.



(a) $p_c = 0$ (b) $p_c = 3$ (c) $p_c = 4$ (d) $p_c = 6$

Figure 2. Continuous Energy Arrivals

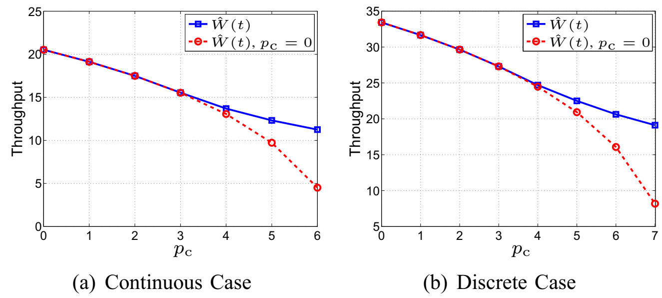
For the discrete energy arrival case, we set $T = 15$ and assume that $D(t)$ is related to $A(t)$ according to (2) where $E_{\max} = 50$. Figure 3 shows $A(t)$, $D(t)$, and two optimal admissible curves. The discrete case requires significantly less computations in the construction of $\hat{W}(t)$ than the continuous case, for only the points of discontinuity on $A(t)$ and $D(t)$ need to be considered.



(a) $p_c = 0$ (b) $p_c = 7$

Figure 3. Discrete Energy Arrivals

The maximal throughput of the two test cases are plotted in Figure 4 with solid lines. The dotted reference curves represent the throughput achieved by using $\hat{W}(t)$ with $p_c = 0$. Note that we need to make sure that $\hat{W}(t)$, $p_c = 0$ is admissible for the current circuit power p_c before we calculate the throughput. Although the maximal throughput seems to decrease approximately linearly with increasing p_c in the plots, the specific decrements still depend mainly on the shape of $A(t)$ and $D(t)$. With relatively large p_c, the gain in throughput by applying the optimal transmission strategy is rather significant.



(a) Continuous Case (b) Discrete Case

Figure 4. Optimal Throughput

VI. Conclusion

We consider data transmission of an energy harvesting node over a single invariant link, where the energy arrival process $A(t)$ and some energy departure constraint $D(t)$ are known. Energy consumption of the circuit of the node is introduced into the model, and a construction procedure which produces the optimal solution to the throughput maximization problem is proposed and proved. Simulation results show that the optimal transmission strategy differs noticeably from the one obtained without the consideration on circuit power, and some gain in throughput can be achieved which is dependent on the specific energy arrivals and the circuit power.

References

[1] Dusit Niyato, Ekram Hossain, Afshin Fallahi, *Sleep and Wakeup strategies in Solar-Powered Wireless Sensor/Mesh Networks: Performance Analysis and Optimization*, IEEE Transactions on Mobile Computing, vol. 6, no. 2, Feb. 2007, pp. 221-236.

[2] Kaya Tutuncuoglu, Aylin Yener, *Short-Term Throughput Maximization for Battery Limited Energy Harvesting Nodes*, IEEE International Conference on Communications 2011, Kyoto, Japan, June 2011.

[3] Shuguang Cui, Andrea J. Goldsmith, *Energy-constrained modulation optimization*, IEEE Transactions on Wireless Communications, Sept. 2005, vol. 4, issue 5, pp. 2349-2360.

[4] A. Mezghani, J. A. Nossek, *Modeling and minimization of transceiver power consumption in wireless networks*, 2011 International ITG Workshop on Smart Antennas (WSA), pp. 1-8, February 2011.

[5] B. Prabhakar, E. Biyikoglu, A. El Gamal, *Energy-efficient transmission over a wireless link via lazy packet scheduling*, IEEE INFOCOM 2001, vol. 1, pp. 386-394, April 2001.

[6] Murtaza Zafer, Eytan Modiano, *A Calculus Approach to Energy Efficient Data Transmission with Quality-of-Service Constraints*, IEEE/ACM Transactions on Networking, vol. 17, pp. 898-911, March 2010.

[7] Guowang Miao, Nageen Himayat, G. Y. Li, *Energy-efficient link adaptation in frequency-selective channels*, IEEE Transactions on Communications, Feb. 2010, vol. 58, issue. 2, pp. 545-554.

[8] Frederick Y. M. Wan, *Introduction to the Calculus of Variations and its Applications*, Chapman & Hall, 1993.