We propose a novel class of tempo-spatial Ornstein-Uhlenbeck processes as solutions to Lévy-driven Volterra equations with additive noise and multiplicative drift. After formulating conditions for the existence and uniqueness of solutions, we derive an explicit solution formula and discuss distributional properties such as stationarity, second-order structure and short versus long memory. Furthermore, we analyze in detail the path properties of the solution process. In particular, we introduce different notions of càdlàg paths in space and time and establish conditions for the existence of versions with these regularity properties. The theoretical results are accompanied by illustrative examples.

ambit process, càdlàg in space and time, Lévy basis, long memory, path properties, second-order structure, space–time modeling, stationary solution, stochastic Volterra equation, Volterra-type Ornstein-Uhlenbeck process