We provide the first and second derivatives of (log-) densities and conditional distribution functions of various bivariate copulas. These derivatives are required in order to calculate e.g. the observed Fisher information in multivariate models based on bivariate copulas. In particular, we obtain all derivatives for the bivariate $t$-copula, which have not been available until now. All derivatives are implemented in the R package VineCopula, and we demonstrate the accuracy of our implementation by comparing the Fisher information matrices calculated using our functions with known analytical results where available.