

# COVARIANCE BASED LINEAR PRECODING IN THE CASE OF IDENTICAL LONGTERM CHANNEL STATE INFORMATION

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## ABSTRACT

We extend the covariance based linear precoding theory to the spatial division multiple access (SDMA) downlink processing in the case that all the streams belonging to the same user have identical channel covariance matrices, which happens when the antennas of the user are located in close proximity. In this case, the precoders designed for those streams based on covariance channel state information (CSI) will be identical, therefore, they can not separate the user successfully. To overcome this, we modify the covariance matrices by keeping the eigenvectors unchanged and choosing one eigenvalue per stream if the number of streams equals to the rank of the covariance matrix. Otherwise, the eigenvalues will be divided into different groups with the rule making the sums of each group as similar as possible. Then, the precoders are designed based on the modified covariance matrices with the constraint of total transmit power. The proposed method is tested and simulation results show that it works well and can improve the bit error ratio (BER) of the system significantly.

## 1. INTRODUCTION

The design of precoders for multiple-input multiple-output (MIMO) channels, especially for those in the downlink, has drawn tremendous attentions for its capability to support reliable high data rate transmission [1, 2]. The transmitter adjusts parameters such as power levels, constellation sizes, coding schemes, and modulation types according to the instantaneous channel state information (CSI) to optimize power efficiency or maximize data rate. This requires that the CSI should be available at the transmitter. However, to provide the instantaneous CSI at the transmitter, is a demanding task. This is especially true for systems which operate in frequency division duplex (FDD).

More recently, transmitter design based on partial CSI has been proposed and investigated [3–5, 7]. In [3], an adaptive MIMO–OFDM scheme is proposed under the assumption that the transmitter knows only partial CSI. A beamforming ap-

proach for MIMO channels with partial CSI at transmitter is proposed in [4]. In [5], covariance based linear precoding is presented for multi-input single-output (MISO) system downlink transmission [5–8]. In [7] a sub-space based approach is proposed for spatial transmit processing for the multi-user MIMO downlink, when only long-term average CSI is available to the transmitter. Except [7], all of them assume that the channel responses between different transmit and receive antennas are independent, which requires antennas to be spaced sufficiently far apart from each other. Practically, large distance between antennas is hard to realize at the mobile station. That means, the channel response may be highly dependent when the antenna spacing is not large enough in MIMO systems. Trying to employ the covariance based scheme proposed in [5] in MIMO systems, the precoders for different streams turn out to be identical if the receive antennas are close to one another. As a result, the receiver is unable to separate the desired data streams, which results in bad system performance.

The motivation of this work is to single out a solution for precoders designed according to the principle of [5], for the case when streams belonging to the same user exhibit correlated fading. By taking the worst case, we assume the channel covariance matrices of different streams belonging to the same user to be identical and the matrices have more than one dominant eigenvalue. In the proposed approach, new matrices with orthogonal eigenspaces are constructed by keeping the eigenvectors unchanged and partitioning the eigenvalues of the original covariance matrix into groups according to the number of streams per user, such that the sum of eigenvalues per group differ as little as possible. We assume that the rank of the covariance matrices is larger than or equals the number of the streams belonging to that user. This is easy to satisfy, since we can schedule the number of streams to be transmitted to the user according to the rank of its channel covariance matrix. Finally, the precoders are designed based on the modified covariance matrices. Numerical evaluations show that the uncoded bit error ratio (BER) of the system can be improved considerably.

## 2. SYSTEM TRANSMISSION MODEL

We consider the downlink of a multi-user communication system. The system layout is shown in Fig. 1. The transmitter is equipped with  $N_{\text{Tx}}$  transmit antennas. Each user is equipped with  $N_{\text{Rx},k'}$  receive antennas,  $k' \in \{1, \dots, K'\}$ . Herein,  $K'$  is referred to as the number of users.

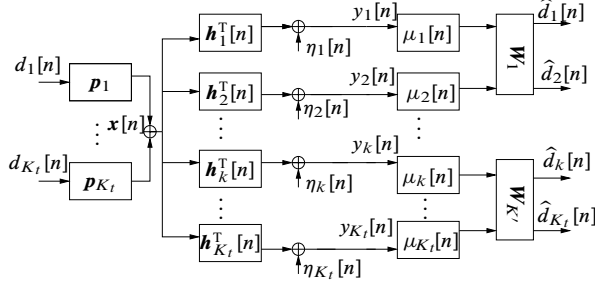


Fig. 1. Multi-user multi-stream system.

Without loss of generality, we assume that each user has the same number of receive antennas,  $N_{\text{Rx}}$ . Let  $k \in \{1, \dots, K_t\}$  denote the index of transmit signal streams and  $K_t$  represent the number of transmit streams at time slot  $t$ . We assume that the number of streams to a user equals the number of its antennas, then,  $K_t = K'N_{\text{Rx}}$ . The transmitted symbol sequence is represented by  $d_k[n]$ . The transmitter has only covariance knowledge of CSI based on restricted reciprocity between up and downlink of an FDD system. The vector  $\mathbf{p}_i \in \mathbb{C}^{N_{\text{Tx}} \times 1}$  is the covariance based precoder for the  $i$ -th data stream. All precoded streams of all users are added up subsequently and sent over the wireless MIMO multi-path channel with  $(Q+1)$  temporal taps. The received signals are processed by receive antenna based matched filters (MF), which have  $(F+1)$  temporal taps. Finally, the obtained signals are stacked and processed by a symbol based equalizer to get the desired signals for each user.

In this paper, we use the following notations:  $(\cdot)^*$ ,  $(\cdot)^{\text{T}}$ , and  $(\cdot)^{\text{H}}$  denote complex conjugation, transposition, complex conjugate transposition, respectively. The terms  $\mathbf{0}_M$ ,  $\mathbf{O}_{M \times N}$  and  $\mathbf{I}_M$  represent the  $M$  dimensional zero vector, the  $M \times N$  zero matrix and the  $M \times M$  identity matrix, respectively. Bold upper and lower case letters denote matrices and column vectors, respectively.

### 2.1. Channel Model

We define the covariance matrix of the  $q$ -th vector-channel coefficient for stream  $k$  as:

$$\mathbf{R}_{k,q} = \text{E} \left[ \mathbf{h}_{k,q} \mathbf{h}_{k,q}^{\text{H}} \right] = \mathbf{U}_{k,q} \mathbf{\Lambda}_{k,q} \mathbf{U}_{k,q}^{\text{H}}, \quad (1)$$

where  $\mathbf{U}_{k,q} = [\mathbf{u}_{k,q,1}, \dots, \mathbf{u}_{k,q,N_{\text{Tx}}}]$  contains the orthonormal eigenvectors of the channel covariance matrix. The diag-

onal matrix  $\mathbf{\Lambda}_{k,q} = \text{diag} \{ \lambda_{k,q,1}, \lambda_{k,q,2}, \dots, \lambda_{k,q,N_{\text{Tx}}} \}$ , contains the corresponding eigenvalues. Let  $\mathbf{R}_{k,q}^{1/2}$  be the square root of the covariance matrix  $\mathbf{R}_{k,q}$ :

$$\mathbf{R}_{k,q}^{1/2} = \mathbf{U}_{k,q} \mathbf{\Lambda}_{k,q}^{1/2} \mathbf{U}_{k,q}^{\text{H}} = \sum_{\xi=1}^{N_{\text{Tx}}} \lambda_{k,q,\xi}^{1/2} \mathbf{u}_{k,q,\xi} \mathbf{u}_{k,q,\xi}^{\text{H}}. \quad (2)$$

Then the channel vectors  $\mathbf{h}_{k,q}$  can be modeled as the realizations of correlated stochastic processes:

$$\begin{aligned} \mathbf{h}_{k,q} &= \sum_{\xi=1}^{N_{\text{Tx}}} \lambda_{k,q,\xi}^{1/2} \mathbf{u}_{k,q,\xi} \mathbf{u}_{k,q,\xi}^{\text{H}} \mathbf{a}_{k,q} \\ &= \sum_{\xi=1}^{N_{\text{Tx}}} \lambda_{k,q,\xi}^{1/2} (\mathbf{u}_{k,q,\xi}^{\text{H}} \mathbf{a}_{k,q}) \mathbf{u}_{k,q,\xi}, \end{aligned} \quad (3)$$

where the random vector  $\mathbf{a}_{k,q} \in \mathbb{C}^{N_{\text{Tx}} \times 1}$  has got the property  $\text{E}[\mathbf{a}_{k,q} \mathbf{a}_{k,q}^{\text{H}}] = \mathbf{I}_{N_{\text{Tx}}}$ . By introducing the channel coefficient  $\rho_{k,q,\xi} = \lambda_{k,q,\xi}^{1/2} \cdot (\mathbf{u}_{k,q,\xi}^{\text{H}} \mathbf{a}_{k,q})$ , the channel for stream  $k$  can be rewritten as

$$\mathbf{h}_k[n] = \sum_{q=0}^Q \sum_{\xi=1}^{N_{\text{Tx}}} \rho_{k,q,\xi} \mathbf{u}_{k,q,\xi} \delta[n-q], \quad (4)$$

with  $\text{E}[|\rho_{k,q,\xi}|^2] = \lambda_{k,q,\xi}$ .

### 2.2. Transceiver Signal Processing

As described before, signals after precoding are added up to form the antenna array transmit signal

$$\mathbf{x}[n] = \sum_{i=1}^{K_t} \mathbf{p}_i d_i[n]. \quad (5)$$

The signal  $\mathbf{x}[n]$  is then fed in to the downlink MIMO channel. At the output of the  $k$ -th channel, we get

$$y_k[n] = \sum_{q=0}^Q \mathbf{h}_{k,q}^{\text{T}} \sum_{i=1}^{K_t} \mathbf{p}_i d_i[n-q] + \eta_k[n], \quad (6)$$

where  $\eta_k[n]$  is zero mean complex Gaussian noise with variance  $\sigma_{\eta}^2$ . The channel coefficients are estimated with the help of the pilot beams  $\mathbf{w}_k$ . Then the matched filter with  $(F+1)$  temporal taps can be written as

$$\begin{aligned} \mu_k[n] &= \sum_{f=0}^F \mathbf{w}_k^{\text{H}} \sum_{\xi=1}^{N_{\text{Tx}}} \rho_{k,f,\xi}^* \mathbf{u}_{k,f,\xi}^* \delta[n+f] \\ &= \sum_{f=0}^F \mu_{k,f} \delta[n+f]. \end{aligned} \quad (7)$$

With this matched filter, the output signal reads

$$\check{d}_k[n] = \sum_{f=0}^F \mu_{k,f} y_k[n+f]. \quad (8)$$

Assuming that stream  $k$  and stream  $k'$  belong to the same user, the received signals are written as

$$\begin{pmatrix} \hat{d}_k[n-D_t] \\ \hat{d}_{k'}[n-D_t] \end{pmatrix} = \mathbf{W}_k^H \begin{pmatrix} \check{d}_k[n] \\ \check{d}_{k'}[n] \end{pmatrix}. \quad (9)$$

Here,  $\check{d}_k[n] \in \mathbb{C}^{M \times 1}$  is a vector obtained by stacking  $M$  samples of  $\hat{d}_k[n]$ , while  $M$  is the length of the receive equalizer and  $D_t$  represents its latency at time-slot  $t$ .

### 2.3. Equivalent covariance based signal

In order to derive an equivalent covariance CSI based signal, the instantaneous channel coefficient  $\rho_{k,q,\xi}$  in the expression of  $\check{d}_k[n]$  should be substituted by a power-equivalent long-term description. Considering all the operations taken are linear,  $\check{d}_k[n]$  can be written as the sum of the  $\check{d}_{k,q,\xi,f}[n]$ , which is the signal traveling through the  $\xi$ -th eigenspace of the transmit antenna array, the  $q$ -th channel tap, and the  $f$ -th matched filter tap, in the way

$$\check{d}_k[n] = \sum_{q=0}^Q \sum_{\xi=1}^{N_{\text{Tx}}} \sum_{f=0}^F \check{d}_{k,q,\xi,f}[n]. \quad (10)$$

With the channel model defined in (4) and the received signal shown in (8), the individual terms in (10) can be further written as

$$\begin{aligned} \check{d}_{k,q,\xi,f}[n] &= \mu_{k,f} \rho_{k,q,\xi} \mathbf{u}_{k,q,\xi}^T \sum_{i=1}^{K_t} \mathbf{p}_i d_i[n-q+f] \\ &\quad + \mu_{k,f} \eta_{k,q,\xi,f}[n+f]. \end{aligned} \quad (11)$$

Note that  $\eta_{k,q,\xi,f}[n]$  are of zero mean with variance  $\sigma_\eta^2$  and are mutually uncorrelated. Taking derivations in [6], we get the instantaneous CSI independent signal as

$$\bar{d}_k[n] = \sum_{q=0}^Q \sum_{\xi=1}^{N_{\text{Tx}}} \sum_{f=0}^F \sum_{i=1}^{K_t} \mathbf{p}_i^T \mathbf{X}_{k,q,\xi,f} \mathbf{d}_i[n] + \tilde{\eta}_{k,q,\xi,f}[n]. \quad (12)$$

Note that the vector  $\mathbf{d}_i[n] \in \mathbb{C}^{Q+F+1}$  is stacked by all the relevant samples of the  $i$ th transmitted signal. The matrix  $\mathbf{X}_{k,q,\xi,f} \in \mathbb{C}^{N_{\text{Tx}} \times (Q+F+1)}$  is defined as

$$\mathbf{X}_{k,q,\xi,f} = \frac{\mathbf{w}_k^H \mathbf{u}_{k,f,\xi}^*}{|\mathbf{w}_k^T \mathbf{u}_{k,f,\xi}|} \sqrt{\sum_{\xi=1}^{N_{\text{Tx}}} \kappa \lambda_{k,q,\xi} \lambda_{k,f,\xi}} \mathbf{v} \otimes \mathbf{u}_{k,q,\xi}, \quad (13)$$

with

$$\kappa = \begin{cases} 2 |\mathbf{w}_k^T \mathbf{u}_{k,f,\xi}|^2 & \text{for } (q=f) \wedge (\xi=\zeta), \\ |\mathbf{w}_k^T \mathbf{u}_{k,f,\xi}|^2 & \text{else.} \end{cases} \quad (14)$$

Herein,  $\mathbf{v} = (\mathbf{0}_{F+q-f}^T, 1, \mathbf{0}_{Q+f-q}^T)$ , and  $\tilde{\eta}_{k,q,\xi,f}[n]$  is defined as the longterm equivalent noise

$$\tilde{\eta}_{k,q,\xi,f}[n] = \sqrt{\frac{1}{N_{\text{Tx}}(Q+1)}} \sum_{\xi=1}^{N_{\text{Tx}}} |\mathbf{w}_k^T \mathbf{u}_{k,f,\xi}| \lambda_{k,f,\xi}^{1/2} \eta_{k,q,\xi,f}[n]. \quad (15)$$

### 3. PRECODER DESIGN WITH MODIFIED CHANNEL COVARIANCE MATRICES

Without loss of generality, let us assume that two streams,  $k$  and  $k'$  belong to the same user and their covariance matrices are identical,  $\mathbf{R}_{k,q} = \mathbf{R}_{k',q}$ . Define  $\mathbf{R}'_{k,q}$  and  $\mathbf{R}'_{k',q}$  as the modified channel covariance matrices for stream  $k$  and  $k'$ , respectively. Let  $\mathbf{A}'_{k,q}$  and  $\mathbf{A}'_{k',q}$  represent the modified matrices containing the corresponding eigenvalues of stream  $k$  and  $k'$ , correspondingly. Then, these matrices can be obtained such that the following equations are satisfied

$$\begin{cases} \mathbf{A}'_{k,q} + \mathbf{A}'_{k',q} = \mathbf{A}_{k,q} \\ \mathbf{A}'_{k,q} \mathbf{A}'_{k',q} = \mathbf{O} \end{cases}, \quad (16)$$

and

$$\{\mathbf{A}'_{k,q}, \mathbf{A}'_{k',q}\} = \operatorname{argmin} \left( \left| \operatorname{tr}(\mathbf{A}'_{k,q}) - \operatorname{tr}(\mathbf{A}'_{k',q}) \right| \right). \quad (17)$$

After having chosen the proper eigenvalue matrices, the new orthogonal channel covariance matrices become

$$\mathbf{R}'_{k,q} = \mathbf{U}_{k,q} \mathbf{A}'_{k,q} \mathbf{U}_{k,q}^H, \quad (18)$$

and

$$\mathbf{R}'_{k',q} = \mathbf{U}_{k,q} \mathbf{A}'_{k',q} \mathbf{U}_{k,q}^H. \quad (19)$$

It is worth mentioning that the eigenvectors are hereby kept unchanged. Note that the newly generated matrices are only used for the precoder design. Considering the fact that the physical channels are unchanged, the precoders for the  $k$ -th and  $k'$ -th stream should be designed based on  $\mathbf{R}'_{k,q}$ ,  $\mathbf{R}'_{k',q}$  and  $\mathbf{R}_{j,q}$ , with  $j \neq k, k'$ , and  $j \in \{1, 2, \dots, K_t\}$ . Then the precoders,  $\mathbf{p}_k \in \mathbb{C}^{N_{\text{Tx}} \times 1}$ , with  $k \in \{1, 2, \dots, K_t\}$ , can be easily designed either to maximize the signal to noise ratio or minimize the MSE of the transmitted signals. Let  $E_{\text{tr}}$  represent the total power available. We define  $\mathbf{X}'_{i,q,\xi,f}$  and

$\pi_{k,q,\zeta,f}$  in the following way:

$$\mathbf{X}'_{i,q,\zeta,f} = \alpha \frac{\mathbf{w}_i^H \mathbf{u}_{i,f,\zeta}^*}{|\mathbf{w}_i^T \mathbf{u}_{i,f,\zeta}|} \mathbf{v} \otimes \mathbf{u}_{i,q,\zeta}, \quad (20)$$

$$\text{with } \alpha = \begin{cases} \sqrt{\sum_{\xi=1}^{N_{\text{Tx}}} \kappa \lambda'_{i,q,\zeta} \lambda'_{i,f,\xi}} & \text{for } (i=k) \vee (i=k') \\ \sqrt{\sum_{\xi=1}^{N_{\text{Tx}}} \kappa \lambda_{i,q,\zeta} \lambda_{i,f,\xi}} & \text{else,} \end{cases}$$

while

$$\pi_{k,q,\zeta,f} = \begin{cases} \sqrt{\sum_{\xi=1}^{N_{\text{Tx}}} \kappa \lambda'_{k,q,\zeta} \lambda'_{k,f,\xi}} & \text{for } q=f \\ 0 & \text{else.} \end{cases} \quad (21)$$

Taking transmit Wiener filter as an example, with which the precoders are design such that the MSE between  $d_k[m]$  and  $\hat{d}_k[m]$  is minimized, the precoder as obtained in [5, 9] can be written as

$$\mathbf{p}_k = \beta \left( \mathbf{X}^* \mathbf{X}^T + \gamma \frac{\sigma_\eta^2}{E_{\text{tr}}} \mathbf{1} \right)^{-1} \mathbf{X}^* \boldsymbol{\pi}_k, \quad (22)$$

$$\beta = \sqrt{\frac{E_{\text{tr}}}{\sum_{k=1}^K \sigma_s^2 \boldsymbol{\pi}_k^H \mathbf{X}^T \left( \mathbf{X}^* \mathbf{X}^T + \gamma \frac{\sigma_\eta^2}{E_{\text{tr}}} \mathbf{1} \right)^{-2} \mathbf{X}^* \boldsymbol{\pi}_k}}.$$

Where  $\mathbf{X}$  is constructed by stacking  $\mathbf{X}'_{k,q,\zeta,f}$  horizontally, while  $\boldsymbol{\pi}_k$  is obtained by stacking  $\pi_{k,q,\zeta,f}$  in one column vector, and  $\gamma$  is a scalar, which is introduced to reduce the notational complexity

$$\gamma = \sum_{k=1}^{K_t} \sum_{f=0}^F \sum_{\xi=1}^{N_{\text{Tx}}} |\mathbf{w}_k^T \mathbf{u}_{k,f,\xi}|^2 \lambda_{k,f,\xi}. \quad (23)$$

Note that only the two precoders out of the obtained set for streams  $k$  and  $k'$  are of use, the rest will be discarded. In this way, the precoder is designed on a user by user basis. The obtained precoder are unified and then scaled by the total transmit power to match the maximum transmit power constraint.

#### 4. NUMERICAL EVALUATIONS AND ANALYSIS

In this section, the proposed approach is evaluated in terms of uncoded bit error ratio (BER), which is averaged over all

**Table 1.** Simulation Parameters

Parameter	Fig. 2	Fig. 3
Case	I	II
$N_{\text{Tx}}$	4	4
$N_{\text{Rx}}$	2	2
Q=F	1	0
Number of $\theta$	2	2
DoD of user 1	$[0^\circ, 30^\circ]$	$[-40^\circ, -15^\circ]$
DoD of user 2	n.a.	$[21^\circ, 48^\circ]$

the streams. The transmitter is equipped with 4 antennas that are equally spaced at a distance of half a wavelength. Waves depart at an angle  $\theta$  to the line perpendicular to the array. The angle  $\theta$  is called direction of departure (DoD). The channel is modeled as Rayleigh fading with  $(Q+1)$  paths, which are separated by one symbol duration. The paths of each user are assumed to have the same DoD-s. The power delay profile (PDP) assumes equally strong paths. Secondary pilot channel (SCPCH) is employed, and the training sequences are transmitted over a fixed grid of 8 beams [10]. Parameters in detail are shown in Table 1. The BER performance is evaluated in two cases: in the first case, there is one user receiving two streams over a channel with two DoD-s which have orthogonal array steering vectors. In the second case, there are two users receiving two streams each, while each user's channel has two DoD-s, while the corresponding steering vectors are not orthogonal.

##### 4.1. Case I: two DoD-s and one user with two streams

Because the covariance matrix is the same for both streams, the same pilot beamforming vector is chosen for both. This leads to the same precoding vector for both streams causing severe inter-stream interference (basically 0dB signal to interference ratio) resulting in poor BER performance. Enforcing different pilot beamforming vectors already leads to different precoding vectors improving the performance. But modifying the covariance matrices according to (16) and (17) leads automatically to different precoding vectors and – in case of orthogonal array steering vectors for the two DoD-s – results in a situation free of spatial interference, such that the joint equalizer only counteracts the temporal interference.

##### 4.2. Case II: two DoD-s, two users, with two streams each

The array steering vectors for the two DoD-s of each user are not orthogonal any more. The covariance matrices per stream are identical for the two streams per user. Using the same pilot beamforming vector for both streams bound for the same user, the performance is again very poor. Enforcing different pilot beamforming vectors helps again. But, as expected, the

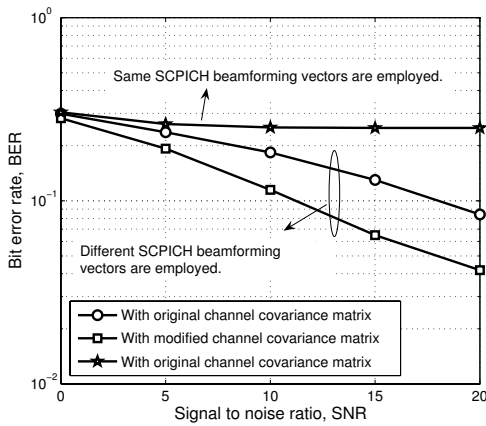


Fig. 2. BER performance of case I over 2 path MIMO channel

modification of the matrices according to (16) and (17), again improves the BER performance considerably. Note that with two users receiving two streams each, the system with four transmit antennas and only covariance CSI at the transmitter is pretty much stressed already. That is why we have considered a channel without temporal inter-symbol interference in this case.

## 5. CONCLUSION

This paper presents an approach for modifying a MISO-based design for precoders such that it becomes usable in a MIMO multi-stream scenario, where the covariance matrices of the channel vectors between the transmit antenna array and each receive antenna of a user are identical for each receive antenna. This approach is based on the generation of modified covariance matrices, which are constructed such that the three formulas shown in (16)-(17) are satisfied. The modified channel covariance matrices are only used for precoder design. First investigations have shown that this approach can significantly improve the uncoded BER performance in multi-streaming applications. Of course, the improvement depends on the channel situation and additional investigations are planned to validate the proposed concept in a broader context.

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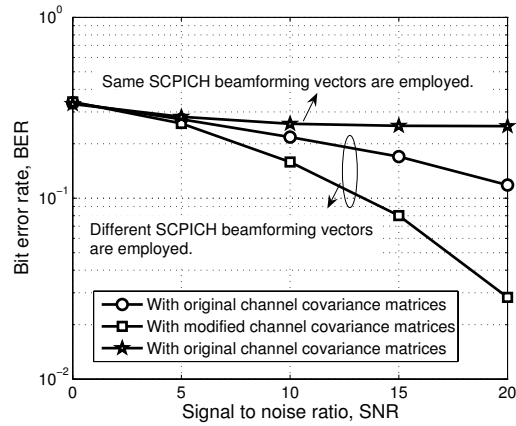


Fig. 3. BER for case II over 1-path MIMO channel

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