Advanced Physical Layer Techniques for Wireless Mesh Networks with Network Coding

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***Abstract*—We consider wireless mesh networks where information is multicasted to multiple terminals in a multi-hop fashion. Due to their strong interdependence, we seek a joint optimization of network and physical layer that are coupled by the per link flow constraint. A common approach is to dualize this constraint and decompose the dual problem into a layered structure; routing at the network layer and rate assignment at the physical layer. For the network layer subproblem, linear network coding is an optimal routing strategy and the solution can be computed by solving a linear or convex program. The physical layer subproblem turns out to be more challenging, due to the nature of the wireless medium and the resulting diminishing effect of multiple access interference. Existing approaches try to avoid interference by full orthogonalization of the channels or building on the concept of conflict graphs. Contrary to these approaches, we are taking into account interference management, for example by exploiting the advanced abilities of multiple antenna systems. Our approach is the factorization of the achievable edge rate region into known rate regions of subgraphs, called Elementary Capacity Graphs (ECGs), which allows for taking into account the half duplex constraint implicitly. The parametrization of the achievable rate region of an ECG depends on the transmission technique used and is in general nonconvex. We demonstrate how the nonconvexity of the physical layer parametrization can be handled within a primal-dual framework without loss of optimality. As our solution is optimal for a given factorization we show by numerically simulations the advances compared to non-optimal schemes.**

I. Introduction and Problem Statement

Communication over a wireless mesh network needs transmission strategies to provide link rates at the physical layer, and a scheme for routing traffic at the network layer. While originally being developed for wired networks with fixed link capacities, network coding, as one possible routing scheme, has recently attracted a lot of attention for being used in wireless networks [1], [2]. At the physical layer, we employ advanced physical layer techniques and utilize the gained flexibility and increased link capacities. This potential gain can only be exploited if network and physical layer are optimized jointly, commonly done via a dual approach, see [1], [3]. Toumpis and Goldsmith [4] give a very general physical layer characterization by scheduling link configurations with fixed link capcities called basic rate matrices. A similar concept is used by Wu et al. [2], who coined the term Elementary Capacity Graphs (ECGs). We adopt this term for our work and extend it to ECGs with variable rates, where each ECG is fully described by its achievable rate region. Having fixed link rates renders the network optimization problem into a linear program, while taking into account variable rates is more challenging and, to the best of our knowledge, only suboptimal solutions are available. Xiao et al. [3] assume that the link rate is only a function of local resources, which implies that links have to be orthogonalized. Cruz and Santhanam [5] assume a linear dependence of the link rate on the SINR which is only true for small SINR values. Whereas Yuan et al. [1] use a convex approximation of how the link rate depends on the SINR. Multiple antenna systems are considered by Liu et al. [6] who construct the network by MIMO-BC systems from each node to its neighbors. The BC systems are orthogonalized by fixed frequency assignment, which is in general suboptimal. In our work we present a major algorithmic framework without loss of optimality, for a given factorization.

We consider a mesh network with graph $\mathcal{G} = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{L}$, $L = |\mathcal{L}|$, is the set of all wireless links in the network. A multicast session is described by its source $s \in \mathcal{N}$ and the set of terminals $\{t_1, \ldots, t_K\} \subset \mathcal{N}$. The decision of a routing scheme at the network layer determines the throughput $r \in \mathbb{R}_+$, and the actual traffic flows on the links $\boldsymbol{f} \in \mathbb{R}_+^L$ that are necessary for obtaining it. Choosing an operating point of the network layer is to select a valid pair of session throughput and traffic assignment $(r, \boldsymbol{f}) \in \mathcal{F}$, where all possible routing decisions are characterized by the routing region $\mathcal{F} \subset \mathbb{R}_+ \times \mathbb{R}_+^L$. For network coding the routing region has a explicit formulation in linear (in)equalities, and therefore $\mathcal{F}$ forms a polyhedron. An extention to multiple multicast sessions that are coded separately is straightforward, see [1].

At the physical layer link rates are assigned to the links in the network by resource allocation, where due to interference and jointly used resources link rates are traded off against each other, described by an achievable edge rate region $\mathcal{R} \subset \mathbb{R}_+^L$. Clearly, the traffic rates established by the network layer are limited to the link rates $\boldsymbol{c} \in \mathcal{R}$ that the physical layer provides, which results in the per link flow constraint $\boldsymbol{f} \leq \boldsymbol{c}$.

In this work we are aiming at the maximization of throughput and the optimization problem can be formulated as

$$\begin{aligned} \max_{r, \boldsymbol{f}, \boldsymbol{c}} \quad & r \\ \text{subject to} \quad & (r, \boldsymbol{f}) \in \mathcal{F} \\ & \boldsymbol{f} \leq \boldsymbol{c} \\ & \boldsymbol{c} \in \mathcal{R}. \end{aligned} \tag{1}$$

A. The Network Layer Characterization

This section is concerned about the constraint $(r, \boldsymbol{f}) \in \mathcal{F}$ of the problem statement (1). A fundamental result of network information theory is that information flows from one source to different terminals do not compete for link capacities, and the maximal throughput is given by the max-flow min-cut theorem and can be achieved by network coding [7]. In other words, a throughput r is achievable if it is achievable for each of the terminals individually. Li et al. [8] prove that optimal throughput can be achieved by linear codes, subsequently Ho et al. [9] show that random linear codes are sufficient. This allows us to exclude code construction in this work, and the optimal routing can be found by a flow allocation problem. For modelling we use additional variables per terminal and link $\boldsymbol{e}^{t_1}, \ldots, \boldsymbol{e}^{t_K} \in \mathbb{R}_+^L$, the so called conceptional flows. The actual traffic flow caused on a link $\ell \in \mathcal{L}$ is the maximum of conceptional flows on the link:

$$\boldsymbol{e}^i \leq \boldsymbol{f}, \ \forall i \in \{t_1, \ldots, t_K\}. \tag{2}$$

A node cannot send more information than it received. Consequently, the "Kirchhoff law" for each node, where $\mathcal{I}(n)$ is the set of incoming links of node n and $\mathcal{O}(n)$ the set of outgoing links, reads

$$\sum_{\ell \in \mathcal{O}(n)} e_\ell^i = \sum_{\ell' \in \mathcal{I}(n)} e_{\ell'}^i, \quad \forall n \in \mathcal{N} \setminus \{s, i\}, \ i \in \{t_1, \ldots, t_K\}.$$

The throughput for a sink is obviously determined by the sum of incoming information flows. By introducing rate incidences $\boldsymbol{a}^i \ i = t_1, \ldots, t_K$ that represent links from the terminals to the sink, we can conveniently express the flow constraints via the incidence matrix $\boldsymbol{A}$ of the network:

$$\begin{bmatrix} \boldsymbol{a}^{t_1} & \boldsymbol{A} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{a}^{t_2} & \boldsymbol{0} & \boldsymbol{A} & \ldots & \boldsymbol{0} \\ \vdots & & & \ddots & \\ \boldsymbol{a}^{t_K} & \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{A} \end{bmatrix} \begin{bmatrix} r \\ \boldsymbol{e}^{t_1} \\ \boldsymbol{e}^{t_2} \\ \vdots \\ \boldsymbol{e}^{t_K} \end{bmatrix} = \boldsymbol{0}. \tag{3}$$

As it is fully characterized by the linear (in)equalities (2) and (3), $\mathcal{F}$ forms a polyhedron.

B. Factorization of the Rate Region into ECGs

As there exists a huge manifold of transmission techniques with complex parametrization, we factorize the achievable edge rate region into known rate regions of subgraphs, called Elementary Capacity Graphs (ECGs). As we will see later this factorization allows to reduce the algorithmic complexity and when established via timesharing, we can take into account the half duplex constraint, which prohibits that a node receives and transmits simultaneously. Formally, an ECG is denoted as $\mathcal{B}_i \subseteq \mathcal{L}$, and the set of ECGs we decide for is given by $\mathcal{B} = \{\mathcal{B}_1, \ldots, \mathcal{B}_B\}$. A transmission schedule alternates between ECGs by assigning the fraction of time t_i, which the ECG $\mathcal{B}_i$ is active. The vector $\boldsymbol{t} = (t_1, \ldots, t_B)^\top \in \mathcal{T}$, where $\mathcal{T} = \{\boldsymbol{t} \geq \boldsymbol{0} : \|\boldsymbol{t}\|_1 = 1\}$, formally describes the transmission schedule. The individual rate regions of the ECGs are given by $\mathcal{R}_1, \ldots, \mathcal{R}_B$ and a parameter set $\boldsymbol{x}_i \in \mathcal{X}_i$ determines a rate point $\boldsymbol{R}_i(\boldsymbol{x}_i)$ from a rate region $\mathcal{R}_i$, i.e. $\mathcal{R}_i = \{\boldsymbol{R}_i(\boldsymbol{x}_i) : \boldsymbol{x}_i \in \mathcal{X}_i\}$. Thus the factorized overall edge rate region $\mathcal{R}$ is obviously defined by the convex hull of all involved rate regions,

$$\begin{aligned} \mathcal{R} &= \{(\boldsymbol{R}_1, \ldots, \boldsymbol{R}_B)\, \boldsymbol{t} : \boldsymbol{R}_1 \in \mathcal{R}_1, \ldots, \boldsymbol{R}_B \in \mathcal{R}_B, \boldsymbol{t} \in \mathcal{T}\} \\ &= \operatorname{co}(\mathcal{R}_1, \ldots, \mathcal{R}_B). \end{aligned} \tag{4}$$

An operating point of the physical layer $\boldsymbol{c}$ is determined by the scheduling vector $\boldsymbol{t}$ and the parameter vector $\boldsymbol{x} = (\boldsymbol{x}_1^\top, \ldots, \boldsymbol{x}_n^\top)^\top \in \mathcal{X} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_B$, i. e.

$$\boldsymbol{c} = (\boldsymbol{R}_1(\boldsymbol{x}_1), \ldots, \boldsymbol{R}_B(\boldsymbol{x}_B))\, \boldsymbol{t}.$$

Dealing with Interference: Physical layer configurations are constructed by timesharing between ECGs that may contain multiple links, which are exposed to destructive interference. Interference is treated as additional noise and no attempt to decode it is made. We consider three basic ways of dealing with interference, which describe a strategy on how the parameters $\boldsymbol{x} = (\boldsymbol{x}_1^\top, \ldots, \boldsymbol{x}_n^\top)^\top \in \mathcal{X} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_B$ are selected.

a) Avoid Interference: The only way to definitely avoid interference within one ECG is to have only a single active link, which is operated at its best transmit strategy.

b) Selfish Transmission: Interference causes a decrease of rate at the nonintended receivers. This strategy simply operates each link assuming there are no other links and takes the decrease in rates into account.

For interference avoidance and selfish transmission we end up with ECGs that correspond to exactly one achievable rate point $\boldsymbol{R}_i$, that might not be optimal. Considering only ECGs that are fixed to a single rate point leads to an physical layer configuration where the edge rate region $\mathcal{R} = \{(\boldsymbol{R}_1, \ldots, \boldsymbol{R}_B)\, \boldsymbol{t} : \boldsymbol{t} \in \mathcal{T}\}$ forms a polytope, and the optimization problem (1) is a linear program. Note that for the special case of one antenna systems where $\boldsymbol{x}$ are the transmit power levels, these rate points correspond to the basic rate matrices in [4].

c) Interference Management: By adjusting the parameter vector $\boldsymbol{x}$ we can trade off the link capacities against each other, which requires cooperation of the transmitters. By opting for interference management we have to include the parameter vector $\boldsymbol{x}$ into the joint optimization of physical and network layer, so this approach demands for major algorithmic solutions.

II. Algorithmic Solutions

Opting for the simple schemes avoid interference and selfish transmission results in an edge rate region given by a polytope where the extreme points are given by the fixed rate vectors of the ECGs, which renders the optimization problem into a linear program. Operating each ECG in an interference management mode requires to handle complex parametrizations that are in general nonlinear and nonconvex. However, the resulting individual rate regions of ECGs can always be made convex by timesharing. Having a convex edge rate region $\mathcal{R}$ and the polyhedral routing region $\mathcal{F}$ the problem (1) is a convex problem and the solution may be found via a dual problem, as for this kind of convex optimization problem the duality gap is zero. A common approach is to dualize the constraint $\boldsymbol{f} \leq \boldsymbol{c}$ and solve the dual problem by a primal-dual algorithm. The algorithm iteratively evaluates the dual function which is a function of the Lagrangian multipliers $\boldsymbol{\lambda}$. For the i-th iteration and the corresponding $\boldsymbol{\lambda}^{(i)}$ we have to solve

$$\begin{aligned} \max_{r,\boldsymbol{f},\boldsymbol{c}} \quad & r - \boldsymbol{\lambda}^{(i),\top}(\boldsymbol{f}-\boldsymbol{c}) \\ \text{subject to} \quad & (r,\boldsymbol{f}) \in \mathcal{F} \\ & \boldsymbol{c} \in \mathcal{R}. \end{aligned} \tag{5}$$

Decomposed into two subproblems, for routing at the network layer we obtain

$$\begin{aligned} \max_{r,\boldsymbol{f}} \quad & r - \boldsymbol{\lambda}^{(i),\top}\boldsymbol{f} \\ \text{subject to} \quad & (r,\boldsymbol{f}) \in \mathcal{F}, \end{aligned} \tag{6}$$

which is a linear program. Rate assignment at the physical layer is

$$\begin{aligned} \max_{\boldsymbol{c}} \quad & \boldsymbol{\lambda}^{(i),\top}\boldsymbol{c} \\ \text{subject to} \quad & \boldsymbol{c} \in \mathcal{R}. \end{aligned} \tag{7}$$

We present a Theorem that allows an elegant reformulation of the physical layer subproblem.

Theorem 1: The optimum solution of the i-th physical layer subproblem (7) is always met by exclusively activating a single ECG.

Proof: Plugging (4) into the physical layer subproblem (7) leads to

$$\max_{\boldsymbol{c}} \quad \boldsymbol{\lambda}^{(i),\top}\boldsymbol{c} \quad \text{subject to } \boldsymbol{c} \in \operatorname{co}\left(\mathcal{R}_1,\ldots,\mathcal{R}_B\right).$$

It is well known that optimizing a linear function over the convex hull of a set can as well be solved over the set itself. Therefore, we can write

$$\begin{aligned} \max_{\boldsymbol{c}} \quad & \boldsymbol{\lambda}^{(i),\top}\boldsymbol{c} \\ \text{subject to} \quad & \boldsymbol{c} \in \bigcup_{i=1,\ldots,B} \mathcal{R}_i. \end{aligned}$$

We now can search for the optimal weighted sum rate point in each of the rate regions $\mathcal{R}_1,\ldots,\mathcal{R}_B$ and select the best point (or one of the best points) as solution to the physical layer subproblem, which corresponds to exclusively activating a single ECG. ■

With this Theorem we can reformulate the physical layer subproblem as

$$\max_{n=1,\ldots,B} \max_{\boldsymbol{c}\in\mathcal{R}_n} \quad {\boldsymbol{\lambda}^{(i)}}^\top \boldsymbol{c}. \tag{8}$$

The new physical layer subproblem has some profound advantages:

- The factorization into smaller problems constituted by ECGs, of which tractable parametrizations and algorithms are available.
- The reformulation provides a clear interface for any type of ECG that has a parametrization of its rate region which allows for the optimization of the weighted sum rate cost function. Weighted sum rate maximization is a well-researched problem and efficient solutions exist for a wide range of physical layer setups.
- The optimum scheduling $\boldsymbol{t}^*$ is found by primal recovery, which avoids an explicit parametrization of the convex hull of the edge rate region of the overall network. The time sharing within the individual ECGs is found via primal recovery as well.

A. Primal-Dual Algorithms and Primal Recovery

A primal-dual algorithm iteratively evaluates the dual function which means to solve an optimization in the primal variables. These optimal primal values are used to update the dual variables, for example by making an adequate step into the direction of a subgradient. For our problem (1) and the chosen dual function (5) the subgradient update rule is given by

$$\boldsymbol{\lambda}^{(i)} = \left[\boldsymbol{\lambda}^{(i)} + v^{(i)}\left(\boldsymbol{f}^{*(i)} - \boldsymbol{c}^{*(i)}\right)\right]^+,$$

where $[\bullet]^+$ denotes $\max(0,\bullet)$, $v^{(i)}$ is determined by a stepsize rule and $\boldsymbol{f}^{*(i)}$ and $\boldsymbol{c}^{*(i)}$ are the solutions to (6) and (7). However, subgradient methods tend to be slow in practice and other update rules for the dual variables should be considered. For our numerical simulations we used a variant of the well known cutting-plane algorithm [10].

Primal-dual algorithms guarantee to find the optimal dual variables $\boldsymbol{\lambda}$, but the primal variables found by evaluating the dual function are in general not feasible to the primal problem. To be explicit, the activation of a single ECG is in general not a feasible physical layer configuration. Feasible primal solutions can be constructed by a convex combination of the solutions found in each iteration. For details on recovering the primal solution of convex optimization problems we refer to [11]. The optimal timesharing of physical layer configurations then equals the convex combining parameters, which are conveniently calculated as a byproduct by the cutting-plane algorithm.

III. An ECG with Two Interfering Links

Having the algorithmic framework at hand, this Section gives an example for advanced physical layer techniques, featuring multiple antenna systems. We consider two types of ECGs, the single link or peer-to-peer connection and the two-user *Interference Channel* (IFC). Using ECGs with a single link effectively represents interference avoidance, whereas in the case of two links we can employ selfish transmission and interference management which requires cooperation of the senders. Figure 1 shows some exemplary ECGs. Other configurations are considered in [12], where ECGs are constituted by *Multicast Channels*, *Broadcast Channels*, and *Multiple Access Channels*.

In this work, without loss of our general conclusion we limit our investigation to multiple-input single-output (MISO) transmission instead of utilizing the enhanced capabilities of the full multiple-input multiple-output (MIMO) channel properties. The MISO two-user interference channel is described by the four channel vectors $\boldsymbol{h}_{ij}$ $i,j = 1,2$. The transmit symbols $\boldsymbol{x}_i \in \mathbb{C}^N$ for the senders $i = 1,2$ are constructed by the scalar data symbol $s_i \in \mathbb{C}$ and the beamforming vector $\boldsymbol{u}_i \in \mathbb{C}^N$ such that $\boldsymbol{x}_i = \boldsymbol{u}_i s_i$. The data symbols

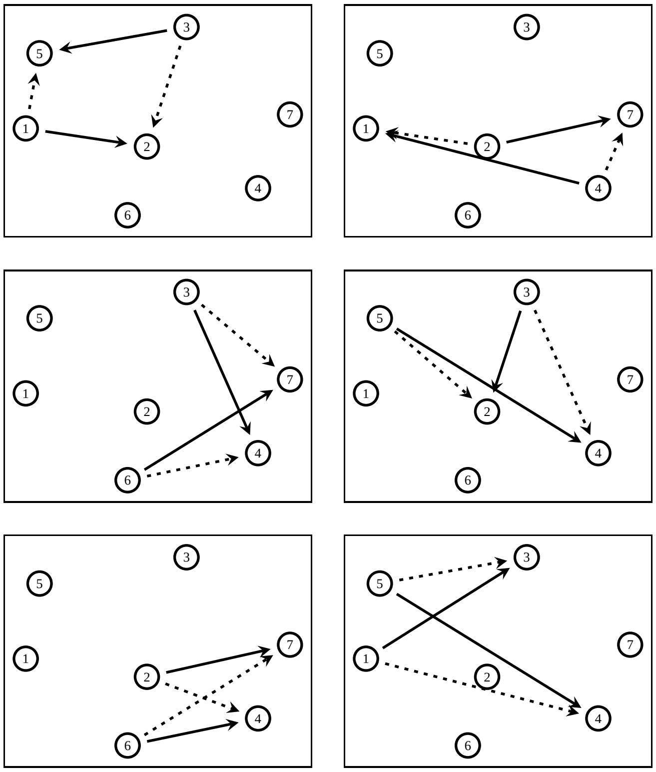

Fig. 1. Exemplary ECGs with the Two User Interference Channel

s_i are circularly symmetric Gaussian with unit variance. The transmitted symbols interfere additively and noise n_i is added at the receiver. The received symbols are

$$\begin{aligned} y_1 &= \boldsymbol{h}_{11}^\top \boldsymbol{x}_1 + \boldsymbol{h}_{12}^\top \boldsymbol{x}_2 + n_1 \\ y_2 &= \boldsymbol{h}_{22}^\top \boldsymbol{x}_2 + \boldsymbol{h}_{21}^\top \boldsymbol{x}_1 + n_2. \end{aligned}$$

Assuming that the receivers treat all interference as additional noise, the achievable rates are given by

$$R_1 = \log\left(1 + \frac{|\boldsymbol{h}_{11}^\top \boldsymbol{u}_1|^2}{\sigma^2 + |\boldsymbol{h}_{12}^\top \boldsymbol{u}_2|^2}\right) \quad (9)$$

$$R_2 = \log\left(1 + \frac{|\boldsymbol{h}_{22}^\top \boldsymbol{u}_2|^2}{\sigma^2 + |\boldsymbol{h}_{21}^\top \boldsymbol{u}_1|^2}\right). \quad (10)$$

The power of the noise $\sigma^2 = \mathrm{E}[|n_1|^2] = \mathrm{E}[|n_2|^2]$ is assumed to be the same at both receivers. The achievable rate region is the union of all beamforming vectors that fulfill a power constraint $||\boldsymbol{u}_1||_2^2, ||\boldsymbol{u}_2||_2^2 \leq P_{\text{max}}$ and can be written as

$$\mathcal{R} = \bigcup_{\substack{\boldsymbol{u}_1, \boldsymbol{u}_2 \\ ||\boldsymbol{u}_1||_2^2 \leq P_{\text{max}} \\ ||\boldsymbol{u}_2||_2^2 \leq P_{\text{max}}}} (R_1(\boldsymbol{u}_1, \boldsymbol{u}_2), R_2(\boldsymbol{u}_1, \boldsymbol{u}_2)).$$

For user i, using $\boldsymbol{u}_i^{\text{MRT}} = \boldsymbol{h}_{ii}^*$ is maximum ratio transmission (MRT) beamforming. Altruistic or zero-forcing (ZF) beamforming, $\boldsymbol{u}_i^{\text{ZF}} = \boldsymbol{h}_{ii}^* - \frac{\boldsymbol{h}_{ji}^\top \boldsymbol{h}_{ii}^*}{||\boldsymbol{h}_{ji}^*||_2^2} \boldsymbol{h}_{ji}^*$, causes no interference to the second user j while the own gain is reduced. Shi et al. [13] and Jorswieck et al. [14] show that the optimal beamforming vector can be written as a combination of $\boldsymbol{u}_i^{\text{MRT}}$ and $\boldsymbol{u}_i^{\text{ZF}}$:

$$\boldsymbol{u}_1(\gamma_1) = P^{\max} \cdot \frac{\gamma_1 \boldsymbol{u}_1^{\text{MRT}} + (1-\gamma_1)\boldsymbol{u}_1^{\text{ZF}}}{||\gamma_1 \boldsymbol{u}_1^{\text{MRT}} + (1-\gamma_1)\boldsymbol{u}_1^{\text{ZF}}||_2} \quad (11)$$

$$\boldsymbol{u}_2(\gamma_2) = P^{\max} \cdot \frac{\gamma_2 \boldsymbol{u}_2^{\text{MRT}} + (1-\gamma_2)\boldsymbol{u}_2^{\text{ZF}}}{||\gamma_2 \boldsymbol{u}_2^{\text{MRT}} + (1-\gamma_2)\boldsymbol{u}_2^{\text{ZF}}||_2}, \quad (12)$$

with $\gamma_1, \gamma_2 \in [0,1]$. By plugging (11)–(12) into (9)–(10) the weighted sum rate problem for the weights λ_1, λ_2 can be formulated as

$$\begin{aligned} \max_{\gamma_1,\gamma_2} \quad & \lambda_1 \cdot \log\left(\sigma^2 + |\boldsymbol{h}_{12}^\top \boldsymbol{u}_2(\gamma_2)|^2 + |\boldsymbol{h}_{11}^\top \boldsymbol{u}_1(\gamma_1)|^2\right) + \\ & \lambda_2 \cdot \log\left(\sigma^2 + |\boldsymbol{h}_{21}^\top \boldsymbol{u}_1(\gamma_1)|^2 + |\boldsymbol{h}_{22}^\top \boldsymbol{u}_2(\gamma_2)|^2\right) - \\ & \lambda_1 \cdot \log\left(\sigma^2 + |\boldsymbol{h}_{12}^\top \boldsymbol{u}_2(\gamma_2)|^2\right) - \\ & \lambda_2 \cdot \log\left(\sigma^2 + |\boldsymbol{h}_{21}^\top \boldsymbol{u}_1(\gamma_1)|^2\right) \\ \text{subject to} \quad & \gamma_1, \gamma_2 \in [0,1]. \end{aligned}$$

The objective can be split into two functions monotonic in γ_1, γ_2 and the problem fits into the framework of optimizing the difference of increasing functions. Having a similar structure, we adopted the approach suggested by Jorswieck and Larsson [15] based on the *Polyblock Algorithm*, which is a global optimization method proposed by Tuy [16]. Selfish transmission coresponds to choose $\gamma_1 = \gamma_2 = 1$ and for interference avoidance by single links the MRT beamformer is chosen.

IV. Results

By numerical simulations we compare the three strategies described in Section I-B: avoid interference, selfish transmission, and interference management. The simulations were made for a fully connected network of seven nodes, which exhibits 42 links. From those we can construct 420 ECGs with two interfering links each as described in Section III, an exemplary selection of these is illustrated in Figure 1. In general the number of ECGs of this type in a network with N nodes is given by:

$$\#\text{ECGs} = 12 \cdot \binom{N}{4}. \quad (13)$$

Each node is equipped with two antennas, and the channel coefficients are complex Gaussian distributed with unit variance. The results are averaged over 500 channel realizations per SNR value. We include simulation results for 4 and 6 terminals, see Figure 2 and Figure 3 respectively. Additionally the solutions of the network optimization problem for one channel realization at 10 dB is given, once for six terminals (flooding), Figure 4, and once for two terminals, Figure 5. For this example the established link capacities are equal to the traffic assigned to it, which is not necessarily always the case. The thickness of the arrows is proportional to the assigned rate.

Interference management by advanced physical layer techniques increases system complexity, so it is a fair question to ask if it is actually worth all the effort. The simulation results give a clear answer by showing a significant increase of the multicast throughput when utilizing the interference

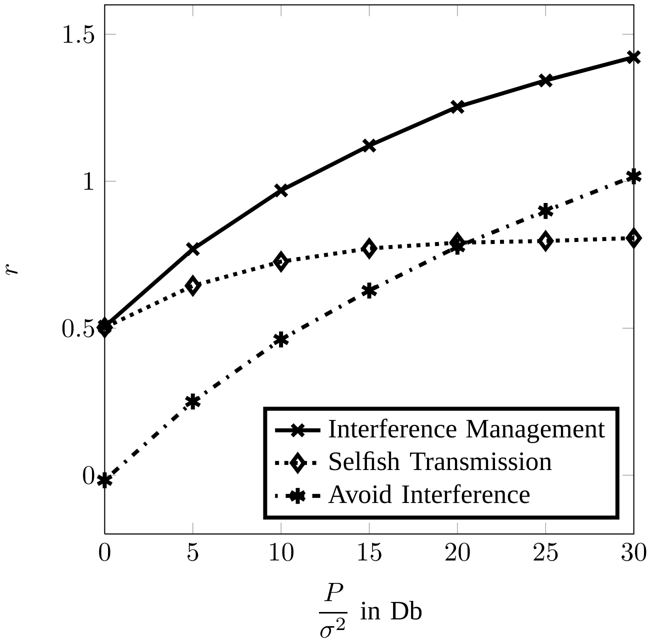

Fig. 2. Multicast Throughput vs. SNR for 4 Terminals

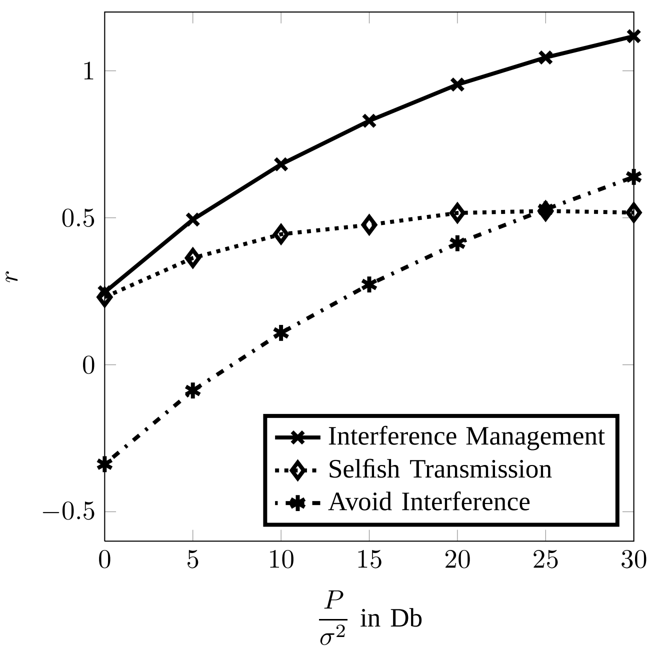

Fig. 3. Multicast Throughput vs. SNR for 6 Terminals

management capabilities in each ECG. For low SNR, selfish transmission is a good strategy, as noise dominates and the impact of interference is almost irrelevant. Obviously, for higher SNR the selfish transmission strategy suffers from being interference limited.

V. Conclusions and Future Work

In this paper, we investigated the potential benefit of advanced physical layer techniques to multicast throughput enhancement in wireless mesh networks. To this end, in a first step we introduced a factorization of the edge capacity region into multiple elementary capacity graphs, each operating in an interference management mode. The proposed optimization approach allows to exploit the dual decomposition framework, although the individual rate regions of the introduced ECGs do not fulfill the required convexity properties. For the physical layer, we used the two user interference channel to illustrate the enhancement of throughput by exploiting the advanced interference management abilities of multiple antenna systems. Although finding the optimal configuration of the two user interference channels requires to run the polyblock algorithm, a global optimization method, a solution to the network optimization problem is found in polynomial time. The ability to decompose the physical layer subproblem into a problem per ECGs keeps the number of variables of the polyblock algorithm constant, while the number of ECGs grows polynomial with the number of nodes, cf. (13). In contrast the naive approach to solve the physical layer subproblem jointly for all ECGs by the polyblock algorithm would result in non-polynomial complexity. By numerical simulations we show a significant gain in throughput compared to systems that do not manage interference. The framework presented in this work is very general with respect to the transmission techniques chosen for the ECGs, as long as the ECG has a parametrization of the rate region and an algorithm to solve the weighted sum rate problem. In ongoing work we will consider the a factorization of the edge rate region in ECGs of various other types. A further direction of future research is on the *Wireless Multicast Advantage* (WMA), which describes the fact that other nodes then the intended receiver might be able to decode the transmitted message, allowing nodes to simultaneously transmit identical data to many receivers. The benefits of considering the WMA haven been shown in [12], where each node has one antenna and ECGs are constructed by BC systems. Motivated by the degradedness of the SISO-BC channel and the superposition coding used, an adequate model for the WMA is derived. However, the WMA is difficult to model in general, and especially for MIMO systems where channels are in general not degraded.

VI. Acknowledgements

We would like to thank Maximilian Riemensberger for providing his expertise and help on implementing the software framework for the numerical simulations.

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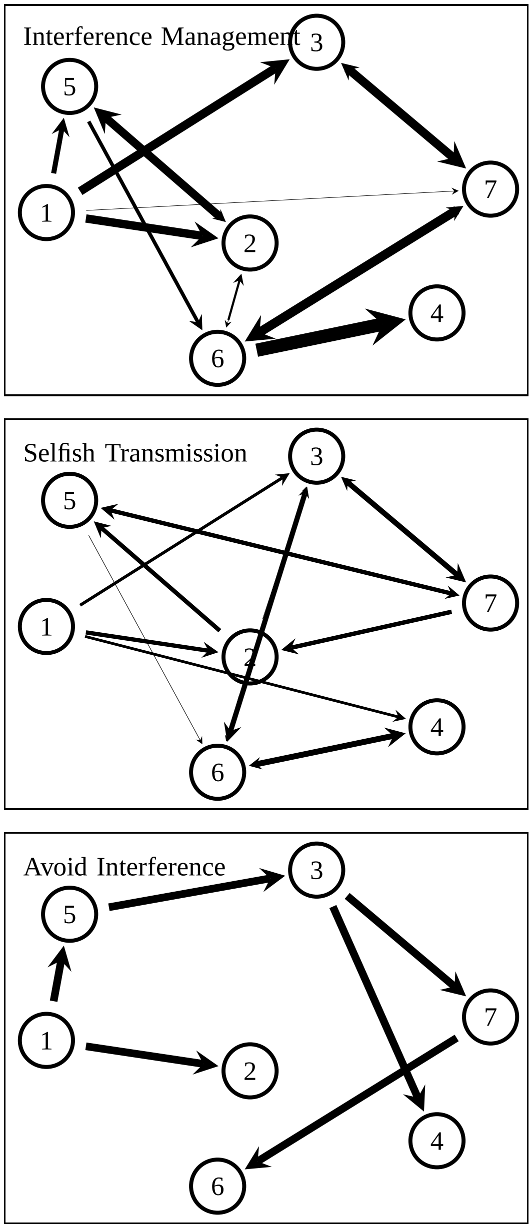

Fig. 4. Solution of the Network Optimization Problem, Source = 1,Terminals = {2,3,4,5,6,7}

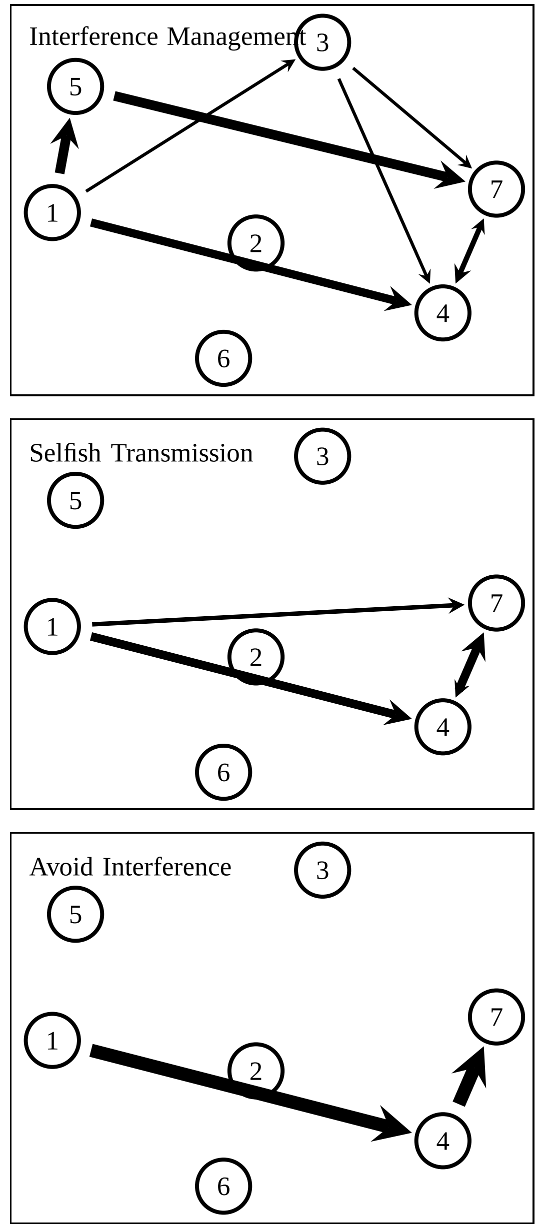

Fig. 5. Solution of the Network Optimization Problem (Source = 1,Terminals = {4,7})

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