On Energy Efficient Cross-layer Assisted Resource Allocation in Multiuser Multicarrier Systems

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***Abstract*—In this paper the *quality of service* (QoS) constrained radio resource allocation problem at the downlink of a multiuser multicarrier system is investigated. We demonstrate and analyze the trade-off between energy consumption and transmit power within a cross physical and link layer system model, which jointly considers power allocation, adaptive modulation and coding and ARQ/HARQ retransmission protocols. A novel transmit power constrained energy minimization problem is formulated based on the competing nature of the two resources. Due to the combinatorial property of the problem, a suboptimal heuristic resource allocation algorithm is proposed, the accuracy of which is compared with the dual optimal value obtained as a byproduct of the algorithm. Simulation results also provide performance comparisons on ARQ and HARQ protocols, and their different impacts on the choice of the optimal modes of operations.**

I. Introduction

Energy efficiency is conventionally one of the main concerns in the design of mobile devices and sensor networks with limited battery life. More recently, it has drawn a lot of research attention for other types of devices and networks as well, on the purposes to provide better QoS with the available radio resources and to make wireless communications more "green". Cross-layer design has merged as one promising approach to achieve the reduction in energy consumption, by allowing for more information exchange between different layers and a joint adaptation of system parameters functioning across several layers in the protocol stack. In this paper we set energy consumption as the performance metric for the QoS-constrained resource allocation in a multiuser multicarrier system, where the impacts of retransmission protocols on energy and transmit power are studied in a cross-layer fashion.

The trade-off between energy efficiency at link layer and transmit power at physical layer has been touched upon *e.g.* in [1][2], where both authors conclude that power control should benefit from taking energy efficiency at link layer into account. The main contributions of this work are to investigate this trade-off thoroughly by using a cross-layer approach, as well as to mathematically formulate and solve the cross-layer assisted resource allocation problem in multicarrier systems based on it. We are motivated to study this problem through our previous works on energy and transmit power minimization algorithms [9][10], which have been proposed based on ideas and methodologies from [3]-[5]. Some of these algorithms are adopted here to solve certain subproblems in the whole resource allocation procedure.

II. System Model

We consider the downlink scenario of an isolated single-cell with K users, each having one data stream to be served. Resource allocation is done for each *Transmission Time Interval* (TTI), and the consecutive transmissions of data are assumed to be independent from TTI to TTI. Depending on its *throughput* requirement, each data stream may have a number of information bits to transmit at the beginning of a TTI. The other relevant QoS parameter characterizing the data streams, the *latency*, is defined as:

Definition: The latency τ_k of a packet from user k is the delay it experiences until received correctly with an outage probability of no more than the predefined value $\pi^{(\mathrm{out})}$. Let $f_k[m]$ be the probability that it takes exactly m TTI's to transmit a packet error-free, then $\tau_k = (M_k - 1)(\mathrm{RTD} + T) + T$ where RTD represents *round trip delay*, and

$$M_k = \min_M M \quad \text{s.t.} \quad \sum_{m=1}^{M} f_k[m] \geq 1 - \pi^{(\mathrm{out})}.$$

We derive in the following the mathematical descriptions of the regarded system components stemmed from [6], which lay the basis for cross-layer optimization.

A. Channel Model

The downlink broadcast channel is modeled as frequency-selective fading over its whole bandwidth and frequency-flat fading over each *subchannel*, which is consist of N_c adjacent subcarriers. The assignment of any subchannel is exclusive, and *intercarrier interference* is not taken into account. Moreover, we restrict ourselves here to the single-antenna case both at BS and MS. On a particular subchannel n, let $H_{k,n}$ and $\sigma^2_{k,n}$ be the channel coefficient and Gaussian noise variance of user k, and p_n be the amount of power being allocated. When assigned to user k, the *signal-to-noise-ratio* (SNR) is computed as $\gamma_{k,n} = \frac{|H_{k,n}|^2}{\sigma^2_{k,n}} p_n$. For the remaining part of this section we drop the subscripts k and n for simplicity. Assuming that one TTI contains N_s symbols for data transmission, the *minimum allocation unit* (MAU) is defined as an allocation region of one subchannel in the frequency dimension by one TTI in the time dimension, which contains $N_\mathrm{c}N_\mathrm{s}$ symbols.

B. FEC coding and modulation

We assume that modulation and coding across the subchannels are done independently, and with reference to the WiMAX standard 8 modulation and coding schemes (MCS) are chosen as candidates which are listed in Table I.

Table I
MODULATION AND CODING SCHEMES (MCS)

Index	Modulation Type	Alphabet Size A	Code Rate R	$R\log_2 A$
1	BPSK	2	$1/2$	0.5
2	QPSK	4	$1/2$	1
3	QPSK	4	$3/4$	1.5
4	16-QAM	16	$1/2$	2
5	16-QAM	16	$3/4$	3
6	64-QAM	64	$2/3$	4
7	64-QAM	64	$3/4$	4.5
8	64-QAM	64	$5/6$	5

With the absence of intersymbol interference in the system, each subchannel is discrete and memoryless over which the *noisy channel coding theorem* [7] can be applied. Let the modulation alphabet and coding rate on the subchannel under consideration be $\mathcal{A} = \{a_1, \dots, a_A\}$ and R respectively. The *cutoff rate* of the subchannel with SNR γ can be expressed as

$$R_0(\gamma, A) = \log_2 A - \log_2 \left[1 + \frac{2}{A} \sum_{m=1}^{A-1} \sum_{l=m+1}^{A} e^{-\frac{1}{4}|a_l - a_m|^2 \gamma} \right].$$

The noisy channel coding theorem states that there always exists a block code with block length l and binary code rate $R\log_2 A \leq R_0(\gamma, A)$ in bits per subchannel use, such that with maximum likelihood decoding the error probability $\tilde{\pi}$ of a code word satisfies $\tilde{\pi} \leq 2^{-l(R_0(\gamma,A) - R\log_2 A)}$.

In order to apply this upper bound to the extensively used turbo decoded convolutional code, quantitative investigations have been done in [6] and an expression for the *equivalent block length* is derived based on link level simulations as $n_{\text{eq}} = \beta \ln L$, where parameter β is used to adapt this model to the specifics of the employed turbo code, and L is the coded packet length. Consequently, the transmission of L bits is equivalent to the sequential transmission of L/n_{eq} blocks of length n_{eq} and has an error probability of

$$\pi = 1 - (1 - \tilde{\pi})^{\frac{L}{n_{\text{eq}}}} \leq 1 - \left(1 - 2^{-n_{\text{eq}}(R_0(\gamma,A) - R\log_2 A)}\right)^{\frac{L}{n_{\text{eq}}}}.$$

C. Protocol

At the link layer retransmission protocols are studied. The data sequence transmitted in one MAU, *i.e.*, a *packet*, is used as the retransmission unit.

ARQ: The corrupted packets at the receiver are discarded, hence we assume that the *packet error probability* (PEP) of a retransmitted packet is the same as that of its original transmission, *i.e.*, $f[m] = \pi^{m-1}(1-\pi), m \in \mathbb{Z}^+$.

HARQ: The corrupted packets at the receiver are combined and jointly decoded using rate-compatible punctured convolutional codes. For the particular *incremental redundancy* (IR) scheme we employ where the retransmissions contain pure parity bits of the same length as the first transmission, the code rate for the mth transmission can be expressed as $R[m] = \frac{B}{m \cdot L} = \frac{1}{m}R$. Let $\tilde{m}$ denote the maximum number of transmissions determined by the mother code. The equivalent block length n_{eq} is then given by $n_{\text{eq}} = \beta \ln(\tilde{m}L)$. The PEP for the mth transmission can be approximated by

$$\pi[M] = \pi^{(\text{out})}, \quad \pi[m] = 1, m = 1, \dots, M-1$$

when $R_0(\gamma)$ satisfies $\frac{1}{M}R\log_2 A < R_0(\gamma) \leq \frac{1}{M-1}R\log_2 A$.

The system parameters are summarized in Table II.

Table II
SYSTEM PARAMETERS

Total bandwidth		10 MHz
Center frequency	f_{c}	2.5 GHz
FFT size		1024
Number of data subcarriers		720
Number of subchannels	N	30
Number of subcarriers per subchannel	N_{c}	$720/30 = 24$
Transmission Time Interval (TTI)	T	2 ms
Number of data symbols per TTI	N_{s}	16
Round Trip Delay (RTD)	RTD	10 ms
Maximum number of transmissions allowed	$\tilde{m}$	5
Turbo code dependent parameter	β	32
Outage probability	$\pi^{(\text{out})}$	0.01

III. POWER-CONSTRAINED ENERGY MINIMIZATION PROBLEM

For each MAU, the energy consumption for the successful transmission of the B information bits loaded on it is the sum of expected transmit power for each symbol at each transmission, times the number of occupied symbols in one MAU and the duration T_{s} of one symbol. The transmit power required for the current transmission on the other hand, is the transmit power for each symbol times the number of subcarriers occupied. Mathematically, we have

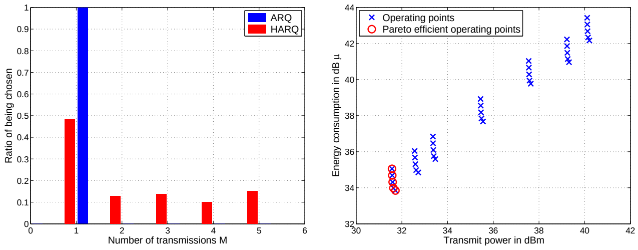
$$\begin{aligned} E &= T_{\text{s}} \cdot \phi_{\text{E}} \cdot \varphi \\ \text{where} \quad \phi_{\text{E}} &= \left\lceil \frac{B}{R\log_2 A} \right\rceil \cdot \gamma(A, R, M), \\ \varphi &= \sum_{m=1}^{M} f[m] \left(\frac{\sigma^2}{|H|^2} + \frac{(m-1)\sigma^2}{|H^{(\text{avg})}|^2} \right), \\ \text{and} \quad P &= \phi_{\text{P}} \cdot \frac{\sigma^2}{|H|^2} \\ \text{where} \quad \phi_{\text{P}} &= \left\lceil \left\lceil \frac{B}{R\log_2 A} \right\rceil / N_s \right\rceil \cdot \gamma(A, R, M). \end{aligned}$$

In the above expressions $\gamma(A, R, M)$ is the SNR required to transmit a packet successfully within M transmissions using MCS (A, R), $|H|^2$ and $|H^{(\text{avg})}|^2$ are the instantaneous and average channel gains, and σ^2 is the noise power on one subcarrier. We refer to the triple (A, R, M) as a *mode of operation*, or equivalently, an *operation mode* from here on. The set of all available modes of operations is denoted by $\mathcal{M}$.

A. Power and Energy: Competing Objectives

From the expressions it is clear that functions ϕ_{E} and ϕ_{P} are channel independent and therefore can be computed offline. The channel dependent part φ is a function only of the number of transmissions M, but not the MCS. What is more, it can easily be shown to be monotonically increasing with M, for both ARQ and HARQ protocols.

As $\gamma(A, R, M)$ monotonically decreases with increasing M when (A, R) is fixed, transmit power P also decreases



(a) Histogram of mode M chosen for a user with $\tau^{(\mathrm{rq})} = 50$ ms

(b) Operating points at $B = 384$

Figure 1. Trade-off between energy and power

with more transmission trials. Yet the monotone of energy consumption E is unclear. In Fig. 1(a) the histogram of the number of transmissions chosen for a user allowing for up to 5 transmissions is shown, which is obtained by simulations on the energy-minimizing scheme [9]. With ARQ protocol, allowing for only one transmission is almost always the best operation mode in terms of energy saving, whereas with HARQ, modes with more transmissions are also chosen yet transmitting with one trial is still the dominating mode. This can be explained roughly as follows: as the ARQ protocol makes no use of the erroneously received packets, it is as expensive a retransmission as the first trial which is energy inefficient. Due to the incremental redundancy obtained, retransmissions are not as expensive with HARQ, and therefore it can be more favorable to go for more than one transmissions when the current channel condition is not good.

To sum up, to allow for more retransmissions saves transmit power for the current TTI but is not energy efficient in general. In fact, transmit power and energy consumption as defined are two conflicting, or competing objectives in the QoS-constrained resource allocation problem. To further illustrate this point, the (E, P) pairs corresponding to all available modes of operations are drawn in Fig. 1(b), where each blue cross represents one (A, R, M) and the Pareto-efficient points are highlighted with red circles. Note that all the Pareto-efficient points are obtained with $(A, R) = (4, 1/2)$.

B. Optimization Formulation

Let the number of information bits intended for user k be b_k, the maximum latency time for the transmission be $\tau_k^{(\mathrm{rq})}$, and the total available transmit power at the BS be P_{tot}. We formulate the energy consumption minimization under transmit power and QoS constraints as

$$\begin{aligned} &\min_{\boldsymbol{B}\in\mathcal{B},\boldsymbol{q}\in\mathcal{M}^N} \quad \sum_{k=1}^{K}\sum_{n=1}^{N} \eta_{k,n}(B_{k,n}, \tau_k^{(\mathrm{rq})}, q_n) \\ &\text{s.t.} \quad \sum_{n=1}^{N} B_{k,n} = b_k, \quad k = 1, \ldots, K, \qquad (1) \\ &\sum_{k=1}^{K}\sum_{n=1}^{N} \xi_{k,n}(B_{k,n}, \tau_k^{(\mathrm{rq})}, q_n) \leq P_{\mathrm{tot}}, \end{aligned}$$

where $\eta_{k,n}(B_{k,n}, \tau_k^{(\mathrm{rq})}, q_n)$ and $\xi_{k,n}(B_{k,n}, \tau_k^{(\mathrm{rq})}, q_n)$ are the energy consumption and transmit power of user k on subchannel n given latency constraint $\tau_k^{(\mathrm{rq})}$ when q_n is chosen as the mode of operation. $\boldsymbol{B} \in \mathbb{Z}_{+,0}^{K\times N}$ represents the bit-loading matrix with its entry $B_{k,n}$ as the number of information bits for the kth user loaded onto the nth subchannel, and q_n is the mode of operation triple (A, R, M) taken on the nth subchannel. As the domain of bit-loading matrix $\boldsymbol{B}$, set $\mathcal{B} \subset \mathbb{Z}_{+,0}^{K\times N}$ represents the set of matrices that have only one nonzero entry in each of their columns and thus implies the FDMA constraint. Explicitly, there are K bit-loading constraints and one transmit power constraint in (1).

Besides the well known combinatorial natured problem of assigning subchannels to users, (1) adds another degree of difficulty by optimizing the modes of operations on the subchannels at the same time, and is obviously impractical to be solved optimally. Instead, simplifications and heuristics have to be employed, which we explain in the following.

C. Simplifications

1) Observations: The simplifications we make are based on observations and analysis on the Pareto-efficient modes of operations on different subchannels loaded with various number of bits, which hold true in most cases.

- There is one best choice of MCS (usually the lowest MCS possible) for given B independent from M, or in other words, Pareto-efficient operating points only differ in M.
- The Pareto-efficient boundary keeps stable with increasing B, until the subchannel is fully loaded and therefore the choice of MCS might change.

The first observation can be explained as when going for a higher MCS, the increment in γ is tremendous and dominating, *i.e.*, there is almost no trade-off between ϕ_{E} and ϕ_{P} when changing (A, R, M). The second observation literally means for B within interval $N_{\mathrm{c}}N_{\mathrm{s}} \cdot (R_1 \log_2 A_1, R_2 \log_2 A_2]$ where (A_2, R_2) is exactly one level higher than (A_1, R_1), the Pareto-efficient modes of operations are the same.

Although there are exceptions with lower MCS, the observations could well be exploited to simplify (1). In Fig. 2 the minimum energy to convey varying B is shown, *i.e.*, the η function with q chosen as the energy minimizing operation mode. Drastic increments of energy can be seen at each transition of optimal MCS, which happens after the subchannel is fully loaded with the current optimal MCS. Besides, energy consumption increases approximately linearly on the interval between two transitions, and the slope of the line segment increases with MCS. As a result, the values of B at transition points can serve as good representatives for all possible B.

2) Simplified Optimization Problem: Let the set of transition points be $\boldsymbol{b}^{(\mathrm{red})}$, and set $\mathcal{B}^{(\mathrm{red})} \subset \mathcal{B}$ have element matrices only taking values from $\boldsymbol{b}^{(\mathrm{red})}$. By restricting $\boldsymbol{B} \in \mathcal{B}^{(\mathrm{red})}$ we

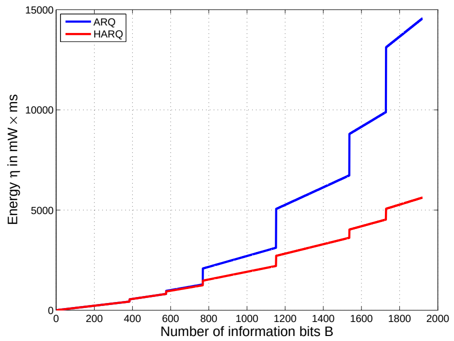


Figure 2. An exemplary η function for ARQ and HARQ protocols

simplify (1) to a tightened version of

$$\begin{aligned} \min_{\boldsymbol{B}\in\mathcal{B}^{(\mathrm{red})},\boldsymbol{q}\in\mathcal{M}^N} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N}\eta_{k,n}(B_{k,n},\tau_k^{(\mathrm{rq})},q_n) \\ \text{s.t.} \quad & \sum_{n=1}^{N} B_{k,n} \geq b_k, \quad k=1,\ldots,K, \\ & \sum_{k=1}^{K}\sum_{n=1}^{N}\xi_{k,n}(B_{k,n},\tau_k^{(\mathrm{rq})},q_n) \leq P_{\mathrm{tot}}, \end{aligned} \tag{2}$$

whose optimal value is an upper bound on that of (1).

IV. Resource Allocation Algorithm

Despite the simplifications we make, to tackle the constrained optimization problem is no easy task. In this section we propose a heuristic algorithm with low complexity which gives us suboptimal solutions to (2).

A. Feasibility Exam

First of all, the feasibility of (2) should be put under test. Let $P_{\min}$ denote the optimal value of the transmit power minimization problem

$$\begin{aligned} \min_{\boldsymbol{B}\in\mathcal{B}^{(\mathrm{red})},\boldsymbol{q}\in\mathcal{M}^N} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N}\xi_{k,n}(B_{k,n},\tau_k^{(\mathrm{rq})},q_n) \\ \text{s.t.} \quad & \sum_{n=1}^{N} B_{k,n} \geq b_k, \quad k=1,\ldots,K. \end{aligned} \tag{3}$$

If $P_{\min} < P_{\mathrm{tot}}$, then (2) is feasible, and solution obtained with (3) provides an upper bound on the optimal energy consumption. Otherwise we determine that (2) is infeasible. Regarding solving (3), two suboptimal algorithms are proposed in [10]. By suboptimally taking $P_{\min}$ we tend to be pessimistic about the system performance, but avoid the complexity to find the optimal solution to (3).

B. Dual Methods

Following the same approach proposed in [5] and adapted in [9], the Lagrange dual problem of (2) can be formulated and solved via Lagrange dual decomposition and the ellipsoid method. At convergence, the dual objective gives the best lower bound on the primal optimal value. The dual optimal solution is however not primal optimal because of the nonzero duality gap. Yet it provides some information about the primal solution as being optimal to a perturbed version of the primal problem [11], and hence we recover the subchannel assignment based on the dual optimal solution.

The main idea of the recovery is to guarantee that each user gets a sufficient number of subchannels to transmit its information bits, *i.e.*, $|\mathcal{S}_k| \geq N_k^{(1)}$ for all k, where $N_k^{(1)}$ is the minimum number of subchannels user k requires. Upon this, the users with insufficient numbers of subchannels indicated by the dual optimal $\boldsymbol{B}$ are sorted by the minimum $\eta\times\xi$ values they have on the spare subchannels, and the assignment is done according to this order until all users have enough subchannels.

C. Determine the bit-loading matrix $\boldsymbol{B}$

Unlike in [9] where frequency band is the only resource the users share, under problem formulation (2) the users are still coupled by the transmit power constraint. Therefore, fixing subchannel assignment does not give us independent optimization problems for individual users. Here we adopt the heuristic method in [10], where for each user, all efficient MCS combinations are enumerated and compared with each other. Again we use the minimum $\eta \times \xi$ value on each subchannel as the comparison criteria.

D. Choose the modes of operations

As for each $B_{k,n} > 0$, there could be a number of Pareto-efficient modes of operations, the problem of which mode of operation to choose needs to be solved with the obtained $\boldsymbol{B}$. Let $\boldsymbol{V}_{\mathrm{E}}, \boldsymbol{V}_{\mathrm{P}} \in \mathbb{R}_+^{\bar{m}\times N}$ be the matrices containing the energy consumption and transmit power of Pareto-efficient operation modes on each of the N subchannels, where $\bar{m}$ is the largest number of efficient modes on one subchannel, and the extra entries for subchannels with less than $\bar{m}$ efficient modes are set to infinity. The optimal selection of operation modes is given by the solution to problem

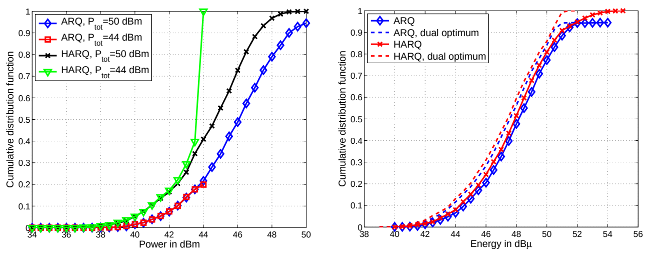
$$\begin{aligned} \min_{\boldsymbol{X}\in[0;1]^{\bar{m}\times N}} \quad & \mathrm{tr}(\boldsymbol{V}_{\mathrm{E}}^T\boldsymbol{X}) \\ \text{s.t.} \quad & \mathrm{tr}(\boldsymbol{V}_{\mathrm{P}}^T\boldsymbol{X}) \leq P_{\mathrm{tot}}, \\ & \sum_{m=1}^{\bar{m}} X_{mn} = 1, \quad n=1,\ldots,N, \end{aligned} \tag{4}$$

where $\boldsymbol{X}$ is the selection matrix. In order to turn (4) into a convex problem, we replace the constraint $X_{mn} \in [0;1]$ with $X_{mn} \in [0,1]$ which actually makes (4) solvable with linear programming. If rounding up the fractional solution is not feasible to (4), adjustments on the selection can be done also based on the minimum $\eta \times \xi$ value on each subchannel.

We have so far solved (2), and the solution obtained is feasible to (1). By cutting down the additionally loaded bits introduced with the coarse granularity of $B_{k,n}$ the solution can be refined. The whole resource allocation procedure is summarized in Algorithm 1.

Algorithm 1 Resource Allocation Algorithm

Solve the power minimization problem (3);
if $P_{\min} < P_{\text{tot}}$ **then**
  With the optimal bit-loading $\boldsymbol{B}_1$, select modes of operations on each subchannel;
else
  Decide that (2) is infeasible and exit;
end if
Solve the dual problem of (2) and recover the subchannel assignment;
Determine the bit-loading matrix $\boldsymbol{B}_2$;
Select modes of operations on each subchannel;
Choose from $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ the one with less energy consumption as the solution to (2);
For each user, cut the extra bits off on the subchannel with the largest line segment slope (see Fig. 2).



(a) CDF of transmit power (b) CDF of energy consumption with $P_{\text{tot}} = 50$ dBm

Figure 3. CDF of transmit power and energy consumption

V. Simulation Results

For simulations, 8 users with QoS requirements as listed in Table III are assumed, where the unit for b_k is bit and the unit for τ_k is ms. Two test scenarios are simulated under 1000 independent channel realizations, where P_{tot} are set to 44 dBm and 50 dBm respectively. All the other simulation parameters remain the same as those taken in [9].

Table III
QoS requirements of 8 users for simulations

User	b_k	τ_k	User	b_k	τ_k
1-4	800	20	5-8	4000	50

In Fig. 3(a) the transmit power values are shown to be in compliance with the respective P_{tot}. At $P_{\text{tot}} = 50$ dBm the ARQ scheme has an outage ratio of 5%, whereas at $P_{\text{tot}} = 44$ dBm the ratio increases to 80%. HARQ scheme on the other hand, is always capable of serving all users with both transmit power constraints. Fig. 3(b) illustrates the cumulative distribution of the minimized energy consumptions and the dual optimal values for both retransmission schemes, where the upper bounds on the suboptimality of the algorithm proposed are shown and the advantage of HARQ over ARQ is again emphasized. Finally, the numbers of transmissions chosen for a user who allows for up to 5 transmissions are drawn in Fig. 4. As compared to Fig. 1(a), the transmit power constraint causes more transmissions especially when being critical to the system. HARQ reveals a more variety of choices than ARQ due to cheaper retransmissions and the less crutial power constraints to it than to ARQ.

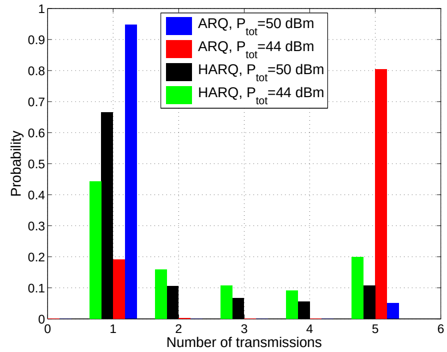


Figure 4. Modes of operations chosen for user 8

VI. Conclusions

At the downlink transmitter with a multicarrier infrastructure, a novel QoS-constrained resource allocation problem is formulated based on studies on the trade-off between energy consumption and transmit power. This trade-off is rooted from optimizing the transmission modes which involve adaptive modulation and coding as well as retransmission protocols, both integrated in a unified cross-layer framework. A suboptimal algorithm is proposed to solve the optimization problem, and its performance has been demonstrated by simulations.

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