Adaptive Decision Feedback Equalization for Filter Bank Based Multicarrier Systems

Dirk S. Waldhauser, Leonardo G. Baltar and Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Technische Universität München
Arcisstr. 21, 80290 München, Germany
Email: {waldhauser, leo.baltar, josef.a.nossek}@tum.de

Abstract— **In this work we present a per-subchannel nonlinear adaptive equalizer for a class of filter bank based multicarrier (FBMC) systems. We consider the class of exponentially modulated FBMCs with offset quadrature amplitude modulated (OQAM) input symbols. We present a fractionally spaced adaptive decision feedback equalizer (DFE) based on the least-mean-square (LMS) algorithm. The input of each equalizer comprises only the output of each subchannel. In the simulation results we show that, despite its increased computational complexity, the performance and higher bandwidth efficiency of OQAM FBMC systems makes them a competitive alternative to conventional multicarrier systems like cyclic prefix based orthogonal frequency division multiplexing (CP-OFDM).**

I. Introduction

The advantages of using multicarrier (MC) modulation in broadband wired and wireless communication systems is widely known. The idea to divide the frequency spectrum into many narrow subchannels is not new, but only in the last decade a widespread use in practical systems could be observed. There are many classes of MC systems, but the CP-OFDM is certainly the most investigated. It offers the advantage of efficient and simple implementation and trivial channel equalization. Because of the insertion of redundancy (CP), only one tap per subchannel is necessary to compensate the frequency selectivity of the channel. Drawbacks of using CP-OFDM include a loss in spectral efficiency, because some redundancy is inserted, a higher level of out-of-band radiation, since the subcarriers pulse-shaping is trivial, and a higher sensibility to narrowband interferers, because the low attenuation of the sidelobes implies an undesired overlap of the subchannels.

CP-OFDM is based on the general MC concept of transmultiplexers, which are composed of exponentially modulated analysis and synthesis filter banks, also called FBMC systems. Of particular interest are the maximally decimated filter banks. Instead of using a rectangular window for shaping the pulses a finite impulse response (FIR) prototype filter that has longer impulse response than the symbol period, i.e. the number of filter coefficients is higher than the number of subchannels M, is modulated by complex exponentials and employed in each subchannel. As a consequence, the latter can be more concentrated in frequency and only the overlap of immediately adjacent subchannels is significant. But it is known from filter banks and communications theory [1] that the real and imaginary part of the inputs of such a system have to be staggered, resulting in OQAM signals.

The equalization problem in FBMC systems is still an area of active research. We focus here on solutions that depend only on the outputs of each subchannel. In this way per-subchannel equalizers work like single carrier (SC) equalizers for OQAM modulated symbols, but with the difference that ICI is present. Since noise cannot be considered white at the output of a filter with bandwidth smaller than the sampling frequency, we have to consider it in the equalizer design.

From communications theory [2] it is known that the optimal receiver for a frequency selective (band limited) channel with band limited transmit signal and additive white Gaussian noise (AWGN) is composed by a (analog) matched filter (MF), a sampling device (at symbol rate) and a maximum likelihood sequence estimator (MLSE). But the latter is usually impractical in terms of complexity. Two practical sub-optimum solutions with lower computational burden are the linear equalizer (LE) and the DFE that work at symbol rate.

An analog filter matched to the transfer function formed by the transmitter filter and the transmission channel is not easy to be designed (e.g. due to channel variations and the difficult estimation). Alternatively, the receive filter is designed to match the transmitted signal, and is followed by a fractionally spaced equalizer (FSE).

In an FBMC system with OQAM input symbols the equalizer can be inserted before the de-staggering, leading to an FSE working at $2/T$, where $1/T$ is the symbol rate.

In [3] the author has presented a first solution to the equalization problem in an OQAM FBMC systems. The coefficients of the linear MMSE FSE have been calculated iteratively using a steepest descent method, which is more complex than an LMS based solution. In [4] both a LE and DFE for a general class of FBMC have been presented, but they use the output of many subcarriers to feed the equalizer. The authors of [5] have extended the solution of [3] to the DFE case using three feedback filters for each subchannel. The authors of [6] have proposed some solutions for the LE for another class of exponentially modulated FBMC systems. The solution is based on the minimization of the MSE in the frequency domain instead of minimizing the ISI directly. Some analytical solutions have been given, but the choice of the number of coefficients is limited to three. In a recent work [7] we have derived a closed formula for the linear per-subchannel FSE according to the MMSE criterion for a class of OQAM FBMC systems and in [8] for the per-subchannel DFE.

In this paper we have extend the results presented in [9] and derive an LMS based adaptive per-subchannel fractionally spaced DFE for the OQAM based FBMC. By comparison with the analytical solution of [8], the adaptive equalizer has considerably less complexity in the coefficient calculation and is more robust against time variant channel, i.e. it is able to track it.

A. Notation

We employ the following notation throughout this work: The real and imaginary parts of a signal or an impulse response are written as $\mathrm{Re}[(\bullet)] = (\bullet)^{(\mathrm{R})}$ and $\mathrm{Im}[(\bullet)] = (\bullet)^{(\mathrm{I})}$, viz. $(\bullet) = (\bullet)^{(\mathrm{R})} + \mathrm{j}\,(\bullet)^{(\mathrm{I})}$, with $\mathrm{j} = \sqrt{-1}$. Vectors with a time index $n \in \mathbb{Z}$ are always a stack of signal samples, e.g. $\boldsymbol{x}[n] = \left[x[n], x[n-1], \ldots, x[n-N]\right]^{\mathrm{T}} \in \mathbb{C}^{N+1}$, where N is explicitly given each time.

978-1-4244-3828-0/09/$25.00 ©2009 IEEE

II. OQAM FBMC SYSTEM STRUCTURE

A general FBMC system overview is given in Fig. 1. The transmitter is usually called the synthesis filter bank (SFB) and the receiver the analysis filter bank (AFB). We assume that the complex input symbols $d_k[m]$, with $m \in \mathbb{Z}$, are QAM modulated, independent and identically distributed.

In the SFB the real and imaginary parts of each complex input $d_k[m]$ are staggered as shown in Fig. 2 generating OQAM signals $x'_k[n]$, with $n = 2m$ at double the symbol rate. Those signals are upsampled by $M/2$ and fed into the transmitter subfilters, that are defined as

$$h_k[l] = h_0[l] \exp\left(\mathrm{j}\, 2\pi k l/M\right), \quad l = -KM/2, \ldots, KM/2, \quad (1)$$

where $h_0[l]$ is called the prototype filter and has length $KM+1$. It is worth noting that the exponential modulation of the prototype filter can be implemented efficiently using the polyphase decomposition and the fast Fourier transform (FFT), e.g. as shown in [10].

After the subfilters, all the signals are added up and forwarded to the radio frequency (RF) processing. After the RF processing at the receiver the signal is filtered again and downsampled (cf. Fig. 3). The downsampled signal is equalized and then the OQAM samples are de-staggered as shown in Fig. 4.

In Fig. 3 the impulse response $h_{\mathrm{ch}}[l]$ encompasses the whole processing between the SFB and the AFB and the multipath propagation channel.

III. ADAPTIVE PER-SUBCHANNEL DFE

We consider here an FBMC system where only immediate adjacent subchannel filters overlap significantly, i.e. the interference of non-adjacent subchannels is negligible, assured that the prototype filter has a high attenuation level in the stop-band. That is usually true for a polyphase length $K \geq 4$ and roll-off factor $\rho \leq 1$. With those assumptions it is possible to correct the distortions caused by the propagation channel employing just one equalizer working in a $2/T$ sampling rate, i.e. in double symbol rate, for each subchannel k.

The conventional LMS algorithm has to be adapted to the OQAM receive signals. The structure of the LMS based OQAM adaptive DFE is shown in Fig. 5. The outputs $y_k[n]$ of the AFB after the downsampling by $M/2$ are filtered by the feedforward stage $w_k[n]$. From this signal the output of the feedback filter $f_k[n]$ is subtracted. Therefore we get the following estimation of the real and the imaginary part of the transmitted symbol $d_k[m] = a_k[m] + \mathrm{j}\, b_k[m]$

$$\begin{aligned}
\hat{a}_k[m] &= \mathrm{Re}\left[\boldsymbol{w}_k^{\mathrm{H}}[n]\boldsymbol{y}_k[n] - \boldsymbol{f}_k^{\mathrm{H}}[n]\hat{\boldsymbol{x}}_k[n]\right], \\
&= \mathrm{Re}\left[\boldsymbol{u}_k^{\mathrm{H}}[n]\boldsymbol{z}_k[n]\right], \qquad (2)\\
\hat{b}_k[m] &= \mathrm{Im}\left[\boldsymbol{w}_k^{\mathrm{H}}[n-1]\boldsymbol{y}_k[n-1] - \boldsymbol{f}_k^{\mathrm{H}}[n-1], \hat{\boldsymbol{x}}_k[n-1]\right] \\
&= \mathrm{Im}\left[\boldsymbol{u}_k^{\mathrm{H}}[n-1]\boldsymbol{z}_k[n-1]\right], \qquad (3)
\end{aligned}$$

where $\boldsymbol{y}_k[n] \in \mathbb{C}^N$, $\boldsymbol{w}_k[n] \in \mathbb{C}^N$, $\boldsymbol{f}_k[n] \in \mathbb{C}^{B+1}$, $\boldsymbol{u}_k[n] \in \mathbb{C}^{N+B+1}$ is the vertical stacking of both feedforward and feedback filter vectors and $\boldsymbol{z}_k[n] \in \mathbb{C}^{N+B+1}$ is the stacking of their input vectors, viz. $\boldsymbol{u}_k[n] = \left[\boldsymbol{w}_k^{\mathrm{T}}[n], \boldsymbol{f}_k^{\mathrm{T}}[n]\right]^{\mathrm{T}}$ and $\boldsymbol{z}_k[n] = \left[\boldsymbol{y}_k^{\mathrm{T}}[n], \hat{\boldsymbol{x}}_k^{\mathrm{T}}[n]\right]^{\mathrm{T}}$. Moreover, if we consider that all the equalized symbols are detected

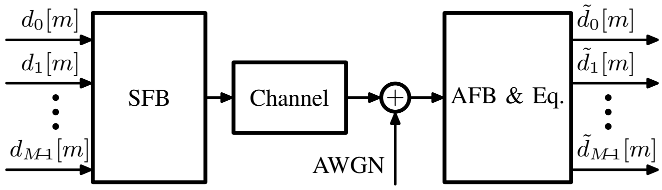

Fig. 1. FBMC System Overview

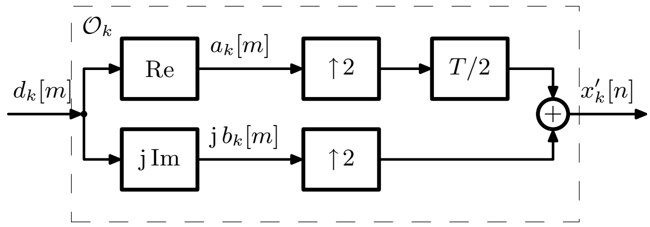

Fig. 2. OQAM Staggering $\mathcal{O}_k$, k even

correctly, the input of the feedback filter $\hat{\boldsymbol{x}}_k[n] \in \mathbb{C}^{B+1}$ is given by

$$\hat{\boldsymbol{x}}_k[n] = \begin{cases} \big[a_k[m-\nu-1],\, \mathrm{j}\, b_k[m-\nu-1],\, a_k[m-\nu-2], \\ \qquad \ldots, \hat{x}_k[n-2\nu-B]\big]^{\mathrm{T}}, & k \text{ even}, \\ \big[\mathrm{j}\, b_k[m-\nu-1],\, a_k[m-\nu-2],\, \mathrm{j}\, b_k[m-\nu-2], \\ \qquad \ldots, \hat{x}_k[n-2\nu-B]\big]^{\mathrm{T}}, & k \text{ odd}, \end{cases} \quad (4)$$

where the last element $\hat{x}_k[n-2\nu-B]$ depends on B and k according to Table I and the parameter ν is the delay inserted by the chain of transmit filter, channel, receive filter and feedforward stage.

TABLE I
LAST ELEMENT OF VECTOR $\hat{\boldsymbol{x}}_k[n]$

$\hat{x}_k[n-2\nu-B]$	B even	B odd
k even	$\mathrm{j}\, b_k[m-\nu-\lfloor B/2\rfloor]$	$a_k[m-\nu-\lfloor B/2\rfloor]$
k odd	$a_k[m-\nu-\lfloor B/2\rfloor]$	$\mathrm{j}\, b_k[m-\nu-\lfloor B/2\rfloor]$

Most of the time the adaptive equalizer operates in the decision directed mode. In this case, the error between the estimated and the demodulated symbol is defined as

$$\epsilon_k[m] = \tilde{d}_k[m] - \hat{d}_k[m] = \mathrm{Q}(\hat{d}_k[m]) - \hat{d}_k[m], \quad (5)$$

where $\mathrm{Q}(\hat{d}_k[m])$ represents the hard decision to the nearest symbol inside the same modulation alphabet used for $d_k[m]$. In the time instants when a training symbol is transmitted, the error is calculated using the transmitted value instead, i.e. $\epsilon_k[m] = d_k[m-\nu] - \hat{d}_k[m]$.

To guarantee the convergence of the LMS algorithm, either a training sequence has to be transmitted before the data symbols, while the equalizer operates in the training mode, or good initial values $\boldsymbol{w}_k[0]$ and $\boldsymbol{f}_k[0]$ have to be provided and the equalizer operates in the decision directed mode. It is worth noting that most current deployments of multicarrier systems envisage the use of scattered pilots in a time and frequency grid. In this case the error signal can be calculated for the pilots like in a training mode.

The filter vector is updated every half symbol duration with either a pure real or a pure imaginary error signal. The error signal $e_k[n]$ for the update process of the $T/2$ spaced filter vector results in

$$e_k[n] = \begin{cases} \epsilon_k^{(\mathrm{R})}[(n-1)/2], & n = 2m+1, \\ \mathrm{j}\, \epsilon_k^{(\mathrm{I})}[n/2], & n = 2m, \end{cases} \quad (6)$$

where the role of real and imaginary parts is interchanged again from subchannel to subchannel.

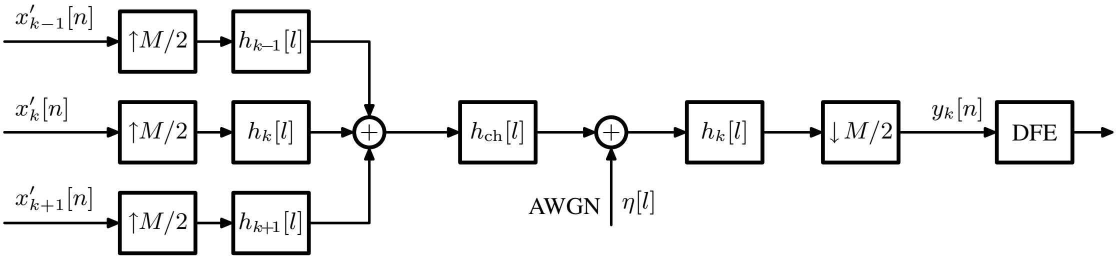

Fig. 3. Subchannel Model for the FBMC System

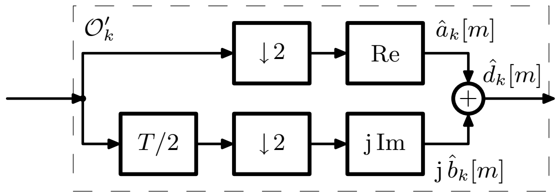

Fig. 4. OQAM De-staggering $\mathcal{O}'_k$, k even

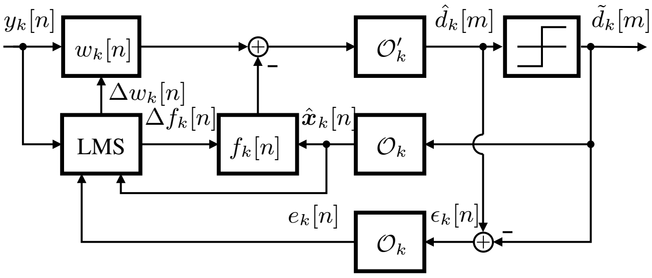

Fig. 5. Per-subchannel decision directed OQAM adaptive DFE

The filter coefficients are updated in the following way

$$\boldsymbol{u}_k[n+1] = \boldsymbol{u}_k[n] + \Delta\boldsymbol{u}_k[n] = \boldsymbol{u}_k[n] + \mu_k \boldsymbol{z}_k[n] e_k^*[n], \tag{7}$$

where $\Delta\boldsymbol{u}_k[n] = \left[\Delta\boldsymbol{w}_k^{\mathrm{T}}[n], -\Delta\boldsymbol{f}_k^{\mathrm{T}}[n]\right]^{\mathrm{T}}$

We consider two possibilities for the choice of the step-size parameter μ_k. The first choice is taken from the traditional LMS algorithm, viz.

$$\mu_k = \frac{2\mathcal{M}}{\mathrm{tr}\,\boldsymbol{R}_{z_k}}, \tag{8}$$

where $\mathcal{M}$ is the dimensionless misadjustment parameter. It is given in % and provides a measure of how close the filter coefficients are from the optimal MMSE solution after the LMS algorithm has converged. In pratice, a misadjustment of 10% gives a satisfactory compromise with convergence speed. The estimated autocorrelation matrix $\boldsymbol{R}_{z_k}$ is calculated using

$$\boldsymbol{R}_{z_k} = \frac{1}{2\mathcal{B}} \sum_{n=0}^{2\mathcal{B}-1} \boldsymbol{z}_k[n]\boldsymbol{z}_k^{\mathrm{H}}[n], \tag{9}$$

where $\mathcal{B}$ is the number of data symbols transmitted in one frame.

The second choice for the step-size parameter is an adaptation of the normalized LMS based on set-membership filtering [11], abbreviated as SM-NLMS. In this algorithm, the step-size is also adaptive and reflects the information provided by the current data input without needing knowledge of the input statistics. In this way the convergence speed of a high step-size LMS and the low steady-state MSE of low step-size LMS can be combined. The step-size for the SM-NLMS is defined as

$$\mu_k[n] = \frac{\nu_k[n]}{\boldsymbol{z}_k^{\mathrm{H}}[n]\boldsymbol{z}_k[n]}, \tag{10}$$

where

$$\nu_k[n] = \begin{cases} 1 - \frac{\gamma}{|e_k[n]|}, & \text{if } |e_k[n]| > \gamma, \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

where $\gamma \geq 0$ is a bound on the error signal. In(10) the step-size is time variant.

IV. Simulation Results

We have simulated the performance of the per-subchannel adaptive DFE in a WiMAX framework. We have considered a transmission bandwidth of 10 MHz and sampling frequency M/T= 11.2 MHz. The input symbols have been modulated using 16-QAM.

We have considered two versions of the subchannel bandwidth, i.e. two different total numbers of subchannels in the FBMC. First, following the WiMAX standard, we have used $M = 1024$ with a subchannel bandwidth of 10.94 kHz, but only 840 subchannels have been occupied with input signals, the others had zero input. Then we have reduced the total number of subchannels to $M = 256$ with a subchannel bandwidth of 43.75 kHz and occupied 210 subchannels. The symbol rate remains the same for both configurations. Since the subchannels are wider for a lower M, the channel equalization is more challenging. For that reason, we have also increased the length of the equalizers to evaluate its effectiveness. The motivation of using wider subchannels in an FBMC system is to benefit from a lower sensitivity to carrier frequency offset (CFO) and to Doppler effects, and a lower peak-to-average power ratio (PAPR).

As prototype filter we have employed a truncated version of a root raised cosine filter with $K=4$ and roll-off factor $\rho=0.5$, since this kind of filter is nearly Nyquist (nearly ISI free) and with that roll-off factor only immediately adjacent subchannels overlap significantly. A lower K would imply a higher interference between non-adjacent subchannels.

As channel model we have chosen the Vehicular A Extended, a variation of the International Telecommunication Union (ITU) model used in WiMAX with a maximum delay spread of 10 μs. We have considered a static and a mobile scenario (30 km/h).

The initial values of the equalizer coefficients at the beginning of each frame have been calculated analytically by using the results from [8].

Fig. 6 depicts the simulations results in terms of uncoded bit error ratio (BER) as a function of E_b/N_0 for a static scenario. For all the curves we have used $N = 3$, $B = 1$ and both LMS and SM-NLMS algorithms. The abbreviation "dec.-dir." stands for decision-directed mode of operation and pilot means that the true transmitted symbols have been used for both error and input of the feedback filter, but the BER has been calculated with the detected symbol after the equalization. We can see that both equalizers perform very similar. The SM-NLMS behaves a little worse for high E_b/N_0 regimes in the case of the dec.-dir. mode, because this algorithm realizes fewer updates of the coefficients than the standard LMS algorithm resulting in lower complexity.

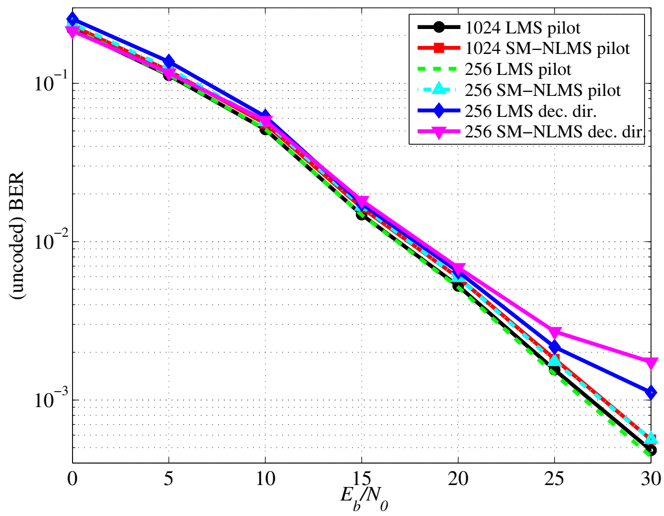

Fig. 6. Comparison between LMS and SM-NLMS for different number of subcarriers in a static channel

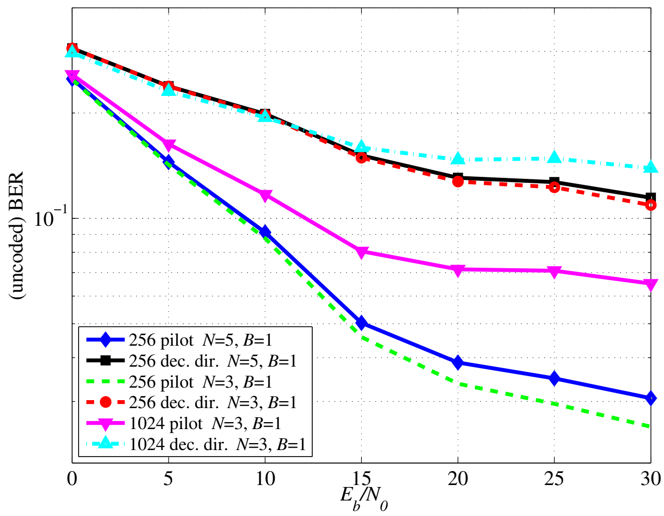

Fig. 7. Comparison between different equalizer lengths using SM-NLMS, with different number of subcarriers and for a speed of 30 km/h

Fig. 7 depicts the simulation results for the SM-NLMS algorithm with different numbers of subcarriers in a mobile scenario. We can see that even if wider subchannels are used, the equalizer is still capable of tracking the channel variations. It is worth noting that a longer equalizer has not shown any advantage in the framework with the considered parameters.

Considering both decision and pilot directed adaptation an upper and a lower limit in the expected performance has been obtained for any scattered pilot structure.

A. Complexity Analysis

In [7] we have derived the increase in computational overhead of the OQAM FBMC system in relation to the CP-OFDM system. We have considered an efficient implementation of the filter banks found in the literature there.

In OFDM the complexity is dominated by the FFT and the equalizer contains only one coefficient per-subcarrier. The complexity is, therefore, obtained by one FFT at the transmitter and by one FFT and M multiplications at the receiver.

Besides one FFT at each receiver and transmitter in an FBMC the polyphase filtering operation has to be considered. Furthermore, the equalizer has a couple of coefficients increasing the receiver complexity. But since only one equalizer per-subcarrier is used, that increase is a linear function of the number of occupied subcarriers. In addition to that, both the FFT and the equalizer operate in a $T/2$ cycle in contrast to CP-OFDM, where the FFT and the one-tap equalizer operate in a T cycle. That is the price paid by the OQAM staggering.

It turns out that an FBMC system has about five times increased multiplications for each received symbol compared to CP-OFDM system. The difference in the case of the DFE is that 1/2 of the feedback filter length has to be added to the feedforward filter length and then used as the total length of the equalizer.

It is important to mention that we have not considered the computation of the equalizer coefficients in the complexity analysis.

V. Conclusion

In this contribution we have investigated the problem of adaptive channel equalization in multicarrier systems. We have introduced a modified version of both LMS and SM-NLMS based DFE, that is adjusted to the requirements of the OQAM FBMC system. The equalizer operates as a fractionally spaced ($T/2$) equalizer in order to compensate for ISI and ICI.

The simulation results have shown that the FBMC system is capable of operating with wider subchannels provided the equalizer is long enough.

Acknowledgment

This work was partially supported by the European Commission under Project PHYDYAS (FP7-ICT-2007-1-211887).

References

[1] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of ofdm/oqam systems based on filterbank theory," *IEEE Trans. Signal Processing*, vol. 50, no. 5, pp. 1170–1183, May 2002.
[2] J. G. Proakis, *Digital Communications*, 4th ed. New York, NY: McGraw-Hill, 2001.
[3] B. Hirosaki, "An analysis of automatic equalizers for orthogonally multiplexed QAM systems," *IEEE Trans. Commun.*, vol. COM-28, no. 1, pp. 73–83, January 1980.
[4] J. Louveaux, L. Vandendorpe, L. Cuvelier, F. Deryck, and O. van de Wiel, "Linear and decision-feedback MIMO equalization for transmultiplexer-based high bit rate transmission over copper wires," *Proc. Int. Zurich Seminar on Broadband Commun.*, pp. 177–184, Feb 1998.
[5] S. Nedic and N. Popovic, "Per-bin DFE for advanced OQAM-based multi-carrier wireless data transmission systems," *Proc. Int. Zurich Seminar on Broadband Commun.*, pp. 38–1–38–6, 2002.
[6] T. Ihalainen, T. H. Stitz, M. Rinne, and M. Renfors, "Channel equalization in filter bank based multicarrier modulation for wireless communications," *EURASIP J. Appl. Signal Process.*, vol. 2007, no. 1, pp. 140–140, 2007.
[7] D. S. Waldhauser, L. G. Baltar, and J. A. Nossek, "MMSE subcarrier equalization for filter bank based multicarrier systems," *Proc. IEEE SPAWC 2008*, pp. 525–529, July 2008.
[8] L. G. Baltar, D. S. Waldhauser, and J. A. Nossek, "MMSE subchannel decision feedback equalization for filter bank based multicarrier systems," *Proc. IEEE ISCAS 2009*.
[9] D. S. Waldhauser, L. G. Baltar, and J. A. Nossek, "Adaptive equalization for filter bank based multicarrier systems," *Proc. IEEE ISCAS 2008*, pp. 3098–3101, May 2008.
[10] T. Karp and N. J. Fliege, "Computationally efficient realization of MDFT filter banks," *Proc. EUSIPCO '96*, vol. 2, pp. 1183–1186, September 1996.
[11] S. Gollamudi, S. Nagaraj, S. Kapoor, and Y.-F. Huang, "Set-membership filtering and a set-membership normalized LMS algorithm with an adaptive step size," *IEEE Signal Processing Lett.*, vol. 5, no. 5, pp. 111–114, May 1998.