Efficient SER measurement method for OFDM receivers with nonlinear distortion

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) system, known to be very practical for multipath environments, suffers from signal waveform with high Peak-to-Average Power Ratio (PAPR). As a consequence nonlinear distortion are introduced by the radio frequency (RF) components, such as amplitude clipping due to the low noise amplifier (LNA) in low cost receivers. This can significantly affect the symbol error rate (SER) performance of such systems. In this article we propose a method to efficiently estimate the SER performance of OFDM receivers in an industrial test environment, where the transmitter is considered ideal and neither fading nor significant noise are observed. To this end importance sampling (IS) techniques are employed. The application of the proposed method shortens the simulation/measurement time up to three orders of magnitude compared to the time needed by intensive Monte Carlo (MC) simulations. We also compare the IS estimated SER with intensive MC simulations and analytically derived results.**

I. Introduction

OFDM modulation has been adopted by many wireless communication standards due to its robustness in multipath environment and quite simple implementation. Two important examples are the widely known WiMAX and 3GPP LTE standard. The main drawback of OFDM is the high PAPR of the transmitted signal. This constitutes a major problem in systems with low cost terminals, since nonlinear distortions result from the clipping effects at the output of the LNA and ADC. This nonlinearity affects the symbol error rate performance of the system. In this work we are interested in developing a new method for fast test and measurement of the SER degradation due to clipping in WiMAX receivers.

In situations where there is a low SER, a direct MC simulation method would require a huge amount of transmitted symbols due to the large number of subcarriers N. In WiMAX the number of subcarriers can be as big as N=1024. We propose here a new method for fast SER measurement when the receiver front-end nonlinearity is the main source of error. The method is based on the IS methods [1], [2]. To make the problem of finding and generating a biased input distribution tractable, a one-parameter characterization of error producing symbols that overcome the large dimensionality of the OFDM symbol is investigated. Contrary to [3] a prior and an off-line classification, pre-selection and saving of millions of OFDM symbols is not needed, but rather they can be generated online, which makes the estimation more accurate. Note that fast measurement or testing is more challenging than fast simulation, since the receiver has to be considered as a black box, thus the biasing of the signal statistics cannot be done at an intermediate point of the system. Additionally, the receiver should be operated under some specified conditions. Through simulations and a new analytically derived SER result, we show that the proposed method is accurate and can shorten the measurement time by a factor up to 1000. Besides, the method is flexible with respect to the specifications of the OFDM receiver and can be adjusted to different standards and properties.

This paper is organized as follows. Section II describes the system model and notational issues. In Section III we derive a new general expression of the SER, then we elaborate on the fast measurement method using the IS technique in Section IV. In Section V, the correctness and usefulness of the method are shown through some simulation scenarios.

II. System model

We are particularly interested here in an industrial test environment of mobile stations. This means that the transmitter is an expensive equipment that has ideal characteristics. We also consider a high SNR received signal, because the transmitter is linked to the receiver by a cable. In that sense, we can assume that the error due to clipping is dominant over the noise and that there is neither frequency nor time selective transmission channel. As a consequence, the clipping occurs only at the receiver and the transmitted signal is considered ideal.

We define $\boldsymbol{s}[\ell] \in C^N$ as the ℓ-th transmitted OFDM symbol, with its elements given by

$$s_n[\ell] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k[\ell] e^{\mathrm{j}\,2\pi nk/N}, \qquad (1)$$

with $c_k[\ell]$ belonging to an M-QAM symbol constellation.

Because the noise power is considered negligible, the received vector is defined as $\boldsymbol{r}[\ell] \approx \boldsymbol{s}[\ell]$.

When the receiver operates under normal conditions, the mean power of the received vector $\boldsymbol{r}[\ell]$ should be constant. We assume here the mean power normalized to unity, i.e. $\mathrm{E}\left[|r_n[\ell]|^2\right] \approx \frac{\sum_0^{N-1}|r_n[\ell]|^2}{N} = 1$.

Let us consider the hard clipping function $g(\bullet)$ [1]. We define the auxiliary variable $a_n[\ell] = |r_n[\ell]|$ and the hard clipping as

$$g\left(r_n[\ell]\right) = \begin{cases} r_n[\ell] & \text{if } a_n[\ell] \leq l, \\ l e^{\mathrm{j}\,\theta_{n,\ell}} & \text{else,} \end{cases} \tag{2}$$

where $\theta_{n,\ell}$ is the phase of the received sample $r_n[\ell]$. In our simulations, the clip level is defined by

$$l = \sqrt{\frac{P_{\text{sat}}}{\mathrm{E}\left[|r_n[\ell]|^2\right]}} = \sqrt{P_{\text{sat}}}, \tag{3}$$

where P_{sat} is the maximum output power of the LNA at the receiver.

III. Analytical SER computation due to clipping

In this section, we deliver a theoretical analysis of the SER caused by clipping in a multicarrier system. We assume by the central limit theorem that the output of the OFDM modulator is almost Gaussian distributed in real and imaginary part. Contrary to [4] a discrete time system model is employed which leads to more accurate results, as observed later from the simulations.

Let $\mathcal{E}$ be the event that a symbol error occurs for a given subcarrier and $\mathcal{C}$ be the event that at least one sample is clipped in one OFDM symbol. Similar to [4], we distinguish two cases. First, when the clip level is set quite low, then many clipping events may occur within one OFDM symbol, so that the additive white Gaussian noise assumption becomes valid. The second case is when the clip level is set high enough such that clipping is a rare event. It means that, very probably, clipping occurs at most once every OFDM symbol. Thus, clipping distortion can be modeled as additive impulsive noise. The validity of the two models are discussed in the simulation section. Herewith, a noise-free testbed channel is assumed, although a generalization to noisy channels is quite straightforward.

A. Additive Gaussian noise model

Under clip level l, the variance of the distortion error normalized by the received signal power can be easily computed as

$$\sigma_c{}^2 = e^{-l^2} - l\sqrt{\pi}\,\mathrm{erfc}\,(l)\,.$$

Then by the additive white noise model, we have [5]

$$\Pr(\mathcal{E}) = 1 - \left(1 - 2\frac{\sqrt{M}-1}{\sqrt{M}}\mathrm{Q}\left(\sqrt{\frac{3}{2(M-1)\sigma_c{}^2}}\right)\right)^2. \tag{4}$$

[1]The proposed approach can also be applied considering soft clipping.

where $\mathrm{Q}(y) = \frac{1}{\sqrt{2\pi}}\int_y^\infty e^{-\frac{u^2}{2}}du$.
Due to its simplicity, this model is commonly used in the literature but may become very inaccurate for high clip levels.

B. Additive impulsive noise model

Now we assume that clipping is a rare event. In this case the probability that exactly one clip event occur is almost the same as the probability that at least one happens within an OFDM symbol, and is obviously given by [2]

$$\Pr(\mathcal{C}) = 1 - (1 - \Pr(a_n^2 > l^2))^N. \tag{5}$$

Then by the law of total probability, we have

$$\Pr(\mathcal{E}) = \Pr(\mathcal{E}|\mathcal{C}) \cdot \Pr(\mathcal{C}) + \Pr(\mathcal{E}|\bar{\mathcal{C}}) \cdot \Pr(\bar{\mathcal{C}}). \tag{6}$$

where $\bar{\mathcal{C}}$ is the complement of $\mathcal{C}$.

Both real and imaginary parts of r_n are normal Gaussian distributed, with mean zero-mean. Additionally, the amplitude $a_n = \sqrt{\Re(r_n)^2 + \Im(r_n)^2}$ follows a Chi-square distribution

$$\mathrm{f}_a(a_n) = 2a_n \cdot e^{-a_n^2},\ a_n > 0. \tag{7}$$

Thus, the following probability density function of the power samples $p_n = a_n^2$ is exponential with unit mean

$$\mathrm{f}_p(p_n) = e^{-p_n},\ p_n > 0. \tag{8}$$

The clipped amplitude portion is $\epsilon_n = a_n - |g(r_n)|$ and follows the probability density function

$$\mathrm{f}_{\epsilon_n}(\epsilon_n) = \begin{cases} 2(\epsilon_n + l)e^{(\epsilon_n+l)^2} & \text{for} \quad \epsilon_n > 0 \\ (1 - e^{-l^2}) \cdot \delta(\epsilon_n) & \text{for} \quad \epsilon_n = 0. \end{cases} \tag{9}$$

Therefore, the conditional probability density function of the clipped amplitude portion given that clipping happens, is

$$\mathrm{f}_{\epsilon_n}(\epsilon_n\,|\epsilon_n > 0\,) = 2(\epsilon_n + l)e^{\epsilon_n(\epsilon_n+2l)}. \tag{10}$$

Afterwards, we evaluate the conditional rate of symbol error due to clipping, which is required to compute the overall symbol error probability.

For this we evaluate the effect of a clipped amplitude, occurring at block position c, on the k-th subcarrier channel. It is given by the DFT of the clipped portion of the amplitude sample ϵ_c

$$\begin{aligned} F_k &= \frac{1}{\sqrt{N}}\epsilon_c\cos(2\pi kc/N) + \mathrm{j}\,\frac{1}{\sqrt{N}}\epsilon_c\sin(2\pi kc/N) \\ &= \frac{\epsilon_c}{\sqrt{N}}e^{\mathrm{j}\,\Theta_k} = \alpha + \mathrm{j}\,\beta, \end{aligned} \tag{11}$$

where $\Theta_k = 2\pi kc/N$ is uniform distributed over $[0, 2\pi]$.

Since a square constellation of M points is used, each real and imaginary component has $\sqrt{M}$ levels, equally spaced and separated by $2d$,

$$d = \sqrt{\frac{3}{2\,(M-1)}}, \tag{12}$$

where the total power of each subcarrier is set to unity.

[2]For the sake of simplicity we will omit the index ℓ and set without loss of generality the received signal power to unity.

Now we assume, for simplicity, that the real and imaginary parts of the F_k are independent (which is not true in reality). The probability that an error occurs in a constellation of M-points is [5]

$$\Pr(\mathcal{E}) = 1 - \left(1 - 2\frac{\sqrt{M}-1}{\sqrt{M}} \cdot \Pr(\sqrt{N}\alpha > \sqrt{N}d)\right)^2. \quad (13)$$

Using the pdf in (10) of the complex clipping noise, we derive the cumulative distribution of the in-phase noise α from (11)

$$\begin{aligned}\Pr(\sqrt{N}\alpha > x) &= \int_x^\infty \int_0^{\arccos(\frac{x}{\alpha})} 2(\alpha + l)e^{-\alpha(\alpha+2l)} \frac{1}{\pi} \, d\theta \, d\alpha \\ &= \int_x^\infty 2(\alpha + l)e^{-\alpha(\alpha+2l)} \frac{\arccos(\frac{x}{\alpha})}{\pi} \, d\alpha. \end{aligned} \quad (14)$$

This integral is not solvable in closed form. However the $\arccos(\cdot)$ function can be approximated in the interval $[0, 1]$ by a linear function, i.e. $\arccos(\frac{x}{\alpha}) \approx \frac{\pi}{2}(1-\frac{x}{\alpha})$. Furthermore, for $l \ll x$, this can be approximated again by $\arccos(\frac{x}{\alpha}) \approx \frac{\pi}{2}(1 - \frac{x}{l+\alpha})$, and so

$$\begin{aligned}\Pr(\sqrt{N}\alpha > x) &\approx \int_x^\infty (1 - \frac{x}{l+\alpha})(\alpha + l)e^{-\alpha(\alpha+2l)} \, d\alpha \\ &\approx \frac{\operatorname{erfc}(l + x)}{2\operatorname{erfc}(l)}, \end{aligned} \quad (15)$$

where we used the approximation $\operatorname{erfc}(x) \approx \frac{e^x}{x\sqrt{\pi}}$ for $x > 1$.

Now, plugging (12) and (15) into (13) yields the final result

$$\Pr(\mathcal{E}) = 1 - \left(1 - 2\frac{\sqrt{M}-1}{\sqrt{M}} \cdot \frac{\operatorname{erfc}(l + \sqrt{\frac{3N}{2(M-1)}})}{2\operatorname{erfc}(l)}\right)^2. \quad (16)$$

IV. Application of Importance Sampling to SER Measurement

A. Importance Sampling (IS)

IS is a simulation technique which aims to reduce a cost function, such as the variance of a given estimator. In our case, to reduce the simulation runs of the SER measurement, we aim to reduce the variance of the SER estimator. The main idea is to simulate those input variables that have the most impact on the estimating parameter. They will be more frequently simulated than the others, so the variance of the estimator can be reduced. Actually, IS consists in finding such a distribution, which encourages these important values [2].

We denote E the set of error producing symbols due to clipping, or simply error region. The estimator of the symbol error probability P_E for a MC simulation over K runs is

$$\mathrm{SER_{MC}} = \frac{1}{K}\sum_{i=1}^{K} 1_E(\boldsymbol{x}_i), \quad (17)$$

where $1_E(\boldsymbol{x})$ is the indicator function of E defined as

$$1_E(\boldsymbol{x}) = \begin{cases} 1, & \text{if } \boldsymbol{x} \in E \\ 0, & \text{if } \boldsymbol{x} \notin E. \end{cases} \quad (18)$$

The biased estimator over $\tilde{K}$ simulation runs applying IS is

$$\mathrm{SER_{IS}} = \frac{1}{\tilde{K}}\sum_{i=1}^{\tilde{K}} 1_E(\boldsymbol{x}_i) \cdot w(\boldsymbol{x}_i), \quad (19)$$

with $w(\boldsymbol{x}_i)$ defined as

$$w(\boldsymbol{x}_i) = \frac{f(\boldsymbol{x}_i)}{f^*(\boldsymbol{x}_i)}, \quad (20)$$

where $f(\bullet)$ is the original distribution and $f^*(\bullet)$ is the biased distribution. The weight function gives the relation between the biased and the original distributions.

The theoretical optimal biased distribution f^*_{opt} can be obtained when the probability of symbol error P_E and the error region E are known a priori, i.e.

$$f^*_{opt}(\boldsymbol{x}) = \frac{1_E(\boldsymbol{x})f(\boldsymbol{x})}{P_E}.$$

Despite this exciting result, it is not practicable since P_E is the parameter to be estimated and $1_E(\boldsymbol{x})$ is unknown [2].

Nevertheless, from $f^*_{opt}(\bullet)$ we can obtain the properties of a practicable biased distribution. $f^*(\bullet)$ is so chosen that

- its support includes the support of the rare event, in our case E.
- the higher probability regions of E are hit more often during the simulation than the lower probability regions of E.

B. Proposed approach

Applying IS means finding and generating a practicable $f^*(\bullet)$ for the system. However, we are limited in this choice. Since we estimated the SER of an OFDM symbol $\boldsymbol{s}$, we are confronted with the high dimensionality of the IS input variable. This leads to a complex computation, generation and evaluation of a high dimensional distribution. Consequently, we first have to find a one dimensional parameter that efficiently characterizes E due to clipping and make therefore the generation of the biased input distribution easier. As follows two parameters are investigated for the characterization of the error region E.

The first parameter previously used in [3] is called Crest Factor (CF) and is defined as

$$\mathrm{CF}(\boldsymbol{s}) = \max_{0 \le n \le N-1}(|s_n|). \quad (21)$$

The second parameter is the variance of the signal power (VP) over a single OFDM symbol about the mean and is given by

$$\begin{aligned}\mathrm{VP}(\boldsymbol{s}) &= \frac{\sum_{n=0}^{N-1}(|s_n|^2 - \mathrm{E}[|s_n|^2)^2}{N} \\ &= \frac{\sum_{n=0}^{N-1}(|s_n|^2 - 1)^2}{N}. \end{aligned} \quad (22)$$

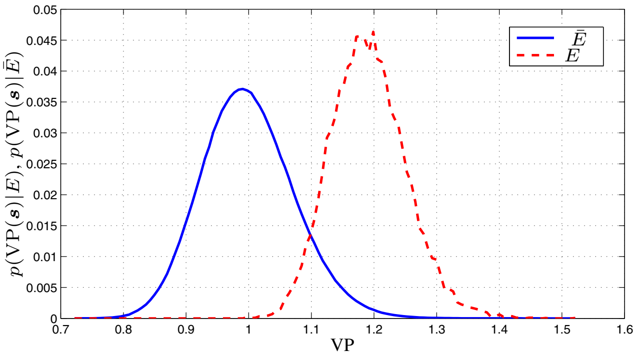

Figure 1. Example of two conditional parameter distributions for the parameter VP, with N=1024, $M = 64$ and $l = 2$.

Both criteria were evaluated on their ability to distinguish between error producing and error free symbols. As an example Fig. 1 depicts the distribution of the parameter VP conditioned on the occurrence or not of an error event.

The Jensen-Shannon Divergence (JSD) [6] between the conditional distributions $p(\mathrm{CF}(\boldsymbol{s})|E)$, $p(\mathrm{CF}(\boldsymbol{s})|\bar{E})$ and between $p(\mathrm{VP}(\boldsymbol{s})|E)$, $p(\mathrm{VP}(\boldsymbol{s})|\bar{E})$ have been calculated to find out which of them distinguished better E and $\bar{E}$. It turned out that the criteria VP maximizes this JSD distance and therefore the search of $f^*(\bullet)$ has been based on that parameter.

We have to notice that the biased distribution $f^*(\bullet)$ must have the properties listed in Section IV-A, i.e. the support of $f^*(\bullet)$ has to include E and to favor the typical error set. Due to the exponential nature of the signal power samples p_n (at least for high N) and in order to increase the number of degrees of freedom, a weighted sum of exponential distributions is proposed as the biased distribution as follows

$$f^*_{exp}(p_n) = ce^{-\frac{p_n}{a}} + de^{-\frac{p_n}{b}}. \tag{23}$$

Besides the fact that $f^*_{exp}(p_n)$ must be a pdf, it also has to possess some properties due to certain measurement specified constraints. For instance, the expected value of the power has to be fixed to unity. On the other hand, the expected value of the variance of the signal power have to be increased to a value that will be denoted as $M_{\mathrm{VP}} > 1$. In order to make errors occur more frequently, this value also represents in fact the stress-factor of the proposed measurement method.

Mathematically we have

$$\int f^*_{exp}(p_n)\,\mathrm{d}p_d = 1,$$
$$\mathrm{E}[(p_n)] = 1,$$
$$\mathrm{E}[\mathrm{E}\left((p_n) - \mathrm{E}[(p_n)]\right)^2] = \mathrm{E}[\mathrm{VP}] = \mathrm{M_{VP}} > 1.$$

From those constraints we obtain the following identities

$$b = \frac{1 + M_{VP} - 2a}{2(1-a)} \tag{24}$$
$$c = \frac{1-b}{a(a-b)} \tag{25}$$
$$d = \frac{1-a}{b(b-a)}. \tag{26}$$

The parameter a is a free variable that can be adjusted to fit the conditional distribution of Frame Error Rate (FER) given VP:

$$\Pr(E\,|\mathrm{VP}(\boldsymbol{s})\,) = \frac{p(\mathrm{VP}(\boldsymbol{s})\,|E\,)\cdot \Pr(E)}{p(\mathrm{VP}(\boldsymbol{s}))}. \tag{27}$$

To generate the symbols according to the chosen distribution we proceed as follows. First, the power samples p_n are generated from $f^*_{exp}(p_n)$. Secondly, the resulting amplitude samples are randomly rotated and demodulated, then transformed with the DFT. Finally, they can be transmitted.

Since it is not possible to find the exact distribution $f^*(\boldsymbol{s})$ which is needed for evaluating (20) and (19), we computed the weight function by means of an estimated distribution of the parameter VP. To this end the relevant parameter range is divided into 100 sections. Then, in order to obtain an estimation of the probability of occurrence of each section respectively for the biased and unbiased distributions, a simulation of 1 million OFDM symbols was carried out. The resulting histograms can be further refined by using curve fitting. Further details are omited due to the lack of space.

V. Simulations Results

In order to evaluate the success of using IS in saving simulation runs, we define the speed-up factor

$$\mathrm{G_{IS,MC}} = \frac{\mathrm{var(SER_{IS})}}{\mathrm{var(SER_{MC})}}. \tag{28}$$

This gain corresponds to the reduction in the number of trials needed to be performed using IS in order to reach the same confidence as the MC simulation.

In the following simulation results we have employed a 64-QAM alphabet and always transmitted 1000 OFDM symbols on each run and a total of 100 runs.

As a first test of the IS method, an OFDM system with $N = 64$ subcarriers was considered. In this case, the number of symbols necessary to get reliable MC simulation results to obtain the real SER is not so high, making it possible to be used as a benchmark.

If we consider a clip level of $l = 2$ we get the following simulated SER

$$\mathrm{SER_{MC}} = 1.703 \cdot 10^{-3} \tag{29}$$
$$\mathrm{SER_{IS}} = 1.696 \cdot 10^{-3}.$$

From the variances of the IS and MC estimators, we can compute the speed-up factor as

$$\mathrm{G_{IS,MC}} = \frac{\mathrm{var(SER_{IS})}}{\mathrm{var(SER_{MC})}} = 4.4564 \cdot 10^{-2} \tag{30}$$

Increasing the clip level to $l = 2.5$ results in

$$\mathrm{SER_{MC}} = 2.125 \cdot 10^{-5} \tag{31}$$
$$\mathrm{SER_{IS}} = 1.0893 \cdot 10^{-5}.$$

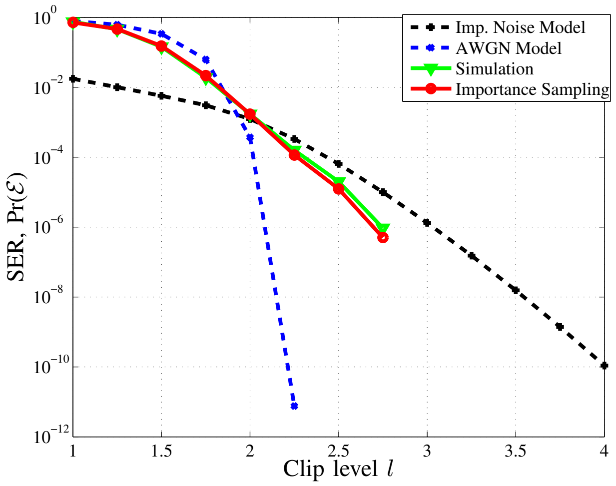

Figure 2. SER and Pr($\mathcal{E}$) using IS, MC and analytical computations for N=64

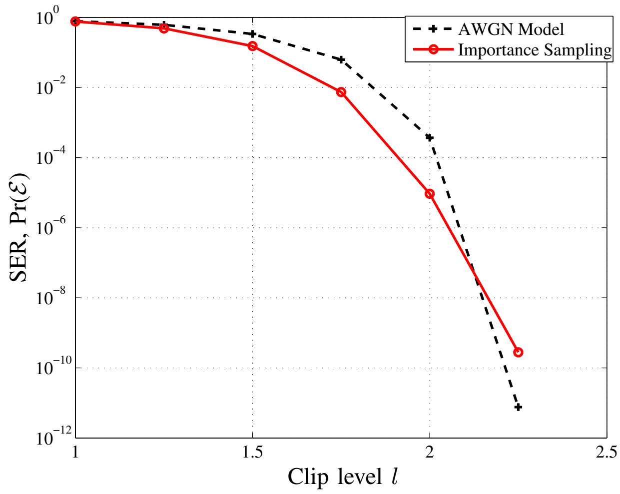

Figure 3. SER and Pr($\mathcal{E}$) using IS and analytical computation for N=1024

The resulting speed-up factor is then

$$G_{\mathrm{IS,MC}} = \frac{\mathrm{var}(\mathrm{SER}_{\mathrm{IS}})}{\mathrm{var}(\mathrm{SER}_{\mathrm{MC}})} = 1.4674 \cdot 10^{-3}.$$

Fig. 2 shows the simulated and analytical results for the first example with various values of clip level. The dashed lines correspond to the analytical methods developed in Section III. For receivers with high levels of nonlinearity, in other words, with a low level of signal clipping, the impulsive noise method sub-estimates the SER, while the AWGN models seems closer to the MC simulation results. This happens because in this case more than one sample of the OFDM symbol are affected by the clipping, making the error similar to a Gaussian process. For a higher level of signal clipping, the roles seem to change and the AWGN model seems too optimistic while the impulsive noise model is closer to the MC simulation results. Besides that, it is important to note the similarity between the MC and the IS simulations.

As a second example we have employed the IS method in an OFDM system with parameters closer to the WiMAX standard. In this case the OFDM symbol had $N = 1024$ samples.

For a clip level $l = 2$ we get

$$\begin{aligned} \mathrm{SER}_{\mathrm{MC}} &= 9.8535 \cdot 10^{-6} \\ \mathrm{SER}_{\mathrm{IS}} &= 9.2974 \cdot 10^{-6}. \end{aligned} \tag{32}$$

The speed up factor is then

$$G_{\mathrm{IS,MC}} = \frac{\mathrm{var}(\mathrm{SER}_{\mathrm{IS}})}{\mathrm{var}(\mathrm{SER}_{\mathrm{MC}})} = 4.8609 \cdot 10^{-2}.$$

Fig. 3 shows the IS simulated result and the analytical SER using the AWGN model for various values of clip level. It can be observed that for a higher number of subcarriers the AWGN model approximates more to the simulated result, because more peaks are clipped and the effect is similar to an additive noise.

VI. Conclusions

A new method based on importance sampling is presented for efficient simulation and/or measurement of OFDM receivers with high number of subcarriers and under nonlinear distortion. With an appropriate biased distribution the runtime speed-up is in the order of 1000. The generation of this distribution was based on an intermediate variable to reduce the dimensional complexity. The proposed method also provides accurate estimates in the low SER regime. Contrary to [3] a prior pre-selection, classification and saving of inputs symbols is no more necessary. The method is also independent of the specifications of the OFDM receiver and can be adjusted to different standards and properties, like the number of subcarriers in OFDM and the modulation scheme, and even can be used for coded transmission.

References

[1] James Antonio Bucklew, *Introduction to Rare Event Simulation*, Springer, 2004.

[2] J.-C. Chen, D. Lu, J.S. Sadowsky, and K. Yao, "On importance sampling in digital communications. i. fundamentals," *Selected Areas in Communications, IEEE Journal on*, vol. 11, no. 3, pp. 289–299, Apr 1993.

[3] G. Vandersteen, P. Wambacq, Y. Rolain, J. Schoukens, S. Donnay, E. Engels, and I. Bolsens, "Efficient bit-error-rate estimation of multicarrier transceivers," in *DATE '01: Proceedings of the conference on Design, automation and test in Europe*, Piscataway, NJ, USA, 2001, pp. 164–168, IEEE Press.

[4] A.R.S. Bahai, M. Singh, A.J. Goldsmith, and B.R. Saltzberg, "A new approach for evaluating clipping distortion in multicarrier systems," *Selected Areas in Communications, IEEE Journal on*, vol. 20, no. 5, pp. 1037–1046, Jun 2002.

[5] Andrea Goldsmith, *Wireless Communications*, Cambridge University Press, 2005.

[6] J. Lin, "Divergence measures based on the shannon entropy," *Information Theory, IEEE Transactions on*, vol. 37, no. 1, pp. 145–151, Jan 1991.