

Multicarrier systems and filter banks

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Abstract. Multicarrier systems, especially orthogonal frequency division multiplex (OFDM), are already established in a lot of current communications standards. Moreover, they are very prominent candidates for future systems in mobile communications.

Filter bank based multicarrier systems can be designed to provide a better spectral shaping than OFDM systems. That way, they have some advantages against OFDM, which will be shown in this work and motivate further research in the direction of channel equalizers with low complexity. This leads to optimum bandwidth efficient multicarrier systems without guard interval, i. e. cyclic prefix.

1 Introduction

Optimum adaptability to the time and frequency selectivity of the propagation channel is crucial for mobile communication systems (Kammeyer, 1992). Multicarrier systems provide the possibility to find a trade-off between robustness against frequency selectivity and time selectivity by adjusting the number of subcarriers.

Especially orthogonal frequency division multiplex (OFDM) resp. discrete multitone modulation (DMT) are widely recognized for high data rate communications. They are already established in a lot of current standards like Wireless LAN, HiperLAN, WiMAX, DAB, DVB-T, and xDSL (Akansu et al., 1998). Moreover, they are very prominent candidates for the mobile communication systems of the 4th generation (4G) at least in the downlink, i. e. from the base station to the mobile stations. Even the extension of the system of the 3rd generation (3G+) is likely to be based on multicarrier systems.

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OFDM resp. DMT multicarrier systems used in current standards transmit some kind of redundancy — the cyclic prefix (CP) with a length of at least the equivalent discrete-time channel impulse response — for transforming the time-invariant multipath channel into a periodically time-varying additive white Gaussian noise (AWGN) channel (Ivrlac, 2005). The time-invariance is assumed for the length of at least one OFDM symbol. This represents an elegant way of avoiding channel equalizers, which remove the intersymbol interference (ISI) and adjacent channel interference (ACI), at the expense of reduced bandwidth efficiency up to 25%.

Linear time-domain equalizers are state of the art in OFDM / DMT systems in order to shorten the length of the equivalent discrete time channel impulse response and, therefore, the necessary length of the cyclic prefix. In this way some bandwidth efficiency is saved (Melsa et al., 1996).

Moreover, some research has been done in the field of bandwidth efficient OFDM transmission without any cyclic prefix (Toeltsch and Molisch, 2000; Trautmann and Fliege, 2002; Sinn et al., 2002).

Generally, multicarrier systems can be realized by filter banks, called transmultiplexer filter banks (TMUX) or simply transmultiplexers (Tzannes et al., 1994).

The most important implementation uses one prototype filter $H_0(z)$, from which all other transmit and receive filters $H_k(z)$, $k = 0, 1, \dots, M - 1$, are derived by complex modulation according to Eq. (1), which represents a modulation with the equidistant carrier frequencies (c. f. Fig. 1).

Ideal bandwidth efficiency can be achieved by filters satisfying an overall transfer function according to the first Nyquist criterion. Offset quadrature amplitude modulation (OQAM) (Kammeyer, 1992) or staggered quadrature amplitude modulation (SQAM) (Bingham, 1990) have been proposed in the literature to avoid adjacent channel interference (ACI).

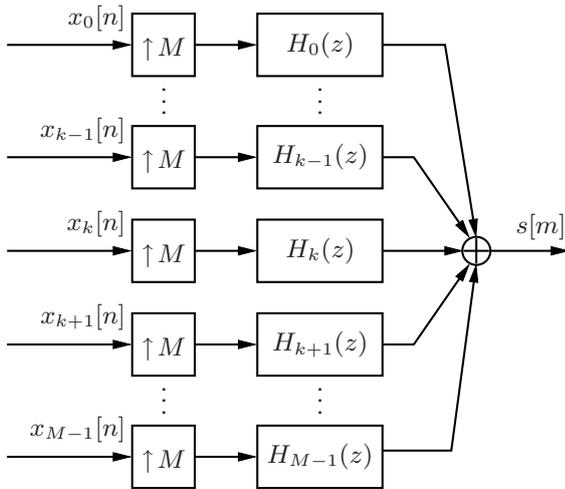


Fig. 1. Synthesis part of a transmultiplexer filter bank.

2 Complexity estimation

The subband filters $H_k(z)$, $k = 0, 1, \dots, M - 1$, of complex modulated transmultiplexer filter banks with M subcarriers are deduced from one prototype filter $H_0(z)$, which determines the spectral shaping of the subbands. Subband frequency responses can be designed which overlap only with directly adjacent subbands and provide separation from other subbands by a high attenuation characteristic. The complex modulation

$$H_k(z) = H_0(W_M^k z), \quad W_M = \exp(-j2\pi/M) \tag{1}$$

in combination with a polyphase decomposition – if the prototype filter is designed as finite impulse response filter (FIR) or infinite impulse response filter (IIR) with special properties – leads to extremely efficient implementations of the transmultiplexers. Figure 2 shows the resulting structure of the synthesis part of the transmultiplexer filter bank. The subband filters are realized as IDFT operation and subsequent polyphase component filters at the low symbol rate. Due to the evolving IDFT operation, which is advantageously calculated by an inverse fast Fourier transformation (IFFT), those transmultiplexer filter banks are denoted as DFT transmultiplexers (Fliege, 1993).

At this point it becomes obvious, that orthogonal frequency division multiplex (OFDM) is a special case of these general DFT-transmultiplexer filter banks with the following prototype filter

$$H_0^{\text{OFDM}}(z) = 1 + z^{-1} + \dots + z^{-(M-1)}. \tag{2}$$

The polyphase components $H_{0,0}(z), H_{0,1}(z), \dots, H_{0,M-1}(z)$ become unity in the case of OFDM.

In the following a square root raised cosine FIR filter (RRC) of length MK is assumed for the the prototype filter $H_0(z)$ in the general transmultiplexer case because of its beneficial properties as square root Nyquist filter. The number of

Table 1. Real multiplications per symbol (synthesis part of FB).

M subcarriers:	128	256	512	1024
OFDM	1792	4096	9216	20480
RRC-TMUX	6912	14336	29696	61440

subcarriers M for the multicarrier system is varied between 128, 256, 512 and 1024.

Both in the OFDM and the general DFT-transmultiplexer case with the RRC prototype filter (RRC-TMUX), the same number of real multiplications per symbol $2M \log_2(M)$ is needed for the IFFT operation, because it only depends on the number of subcarriers. Additionally, the RRC polyphase filters require $4K$ real multiplications per symbol. This is the computational overhead required for the improved spectral shaping.

Because of the duality of the synthesis part and the analysis part of the filter bank, the same observations are sincerely true for the receiver.

One additional issue should be taken into consideration: Especially in OFDM systems with cyclic prefix, a large number of subcarriers M is desired in order to keep the proportion of the cyclic prefix small. If no other reasons, e.g. a very strong frequency diversity, prevent the reduction of the number of subcarriers in the RRC-TMUX case, the number of subcarriers can be reduced compared to OFDM in order to compensate for the additional multiplications of the polyphase filters by saving multiplications of the IFFT operations. Table 1 shows the number of real multiplications per symbol for the OFDM and the RRC-TMUX case for different numbers of subcarriers and an RRC polyphase filter length of $K = 10$. The additional complexity for the RRC polyphase filters can be seen by the difference in each column. The reduction of the number of subcarriers in the RRC-TMUX case by a factor of 4 to $M/4$ leads to an overall complexity even smaller than that of an OFDM system with M subcarriers.

The main difference is that for OFDM with cyclic prefix no additional channel equalizer is needed as long as the cyclic prefix is sufficient in length. A comparison at the basis of the achievable data rates shows that OFDM is able to transmit $M \log_2(\mathcal{M}_O)$ bit per symbol, whereas the transmultiplexer transmits $(M + M_{cp}) \log_2(\mathcal{M}_T)$ bit per symbol, where \mathcal{M}_O resp. \mathcal{M}_T denote the modulation alphabets for OFDM resp. the RRC-TMUX. Moreover, the same modulation alphabets are assumed for all subcarriers. Equal rates are obtained by compensating the cyclic prefix length by the modulation scheme. If 25% cyclic prefix is assumed, i.e. $M_{cp}/M = 1/3$, 256 QAM resp. 16 QAM is necessary for OFDM against 64 QAM resp. 8 QAM for a TMUX without CP. This means that more transmit power is necessary for OFDM in order to get the same (uncoded) bit error rates for the higher modulation schemes.

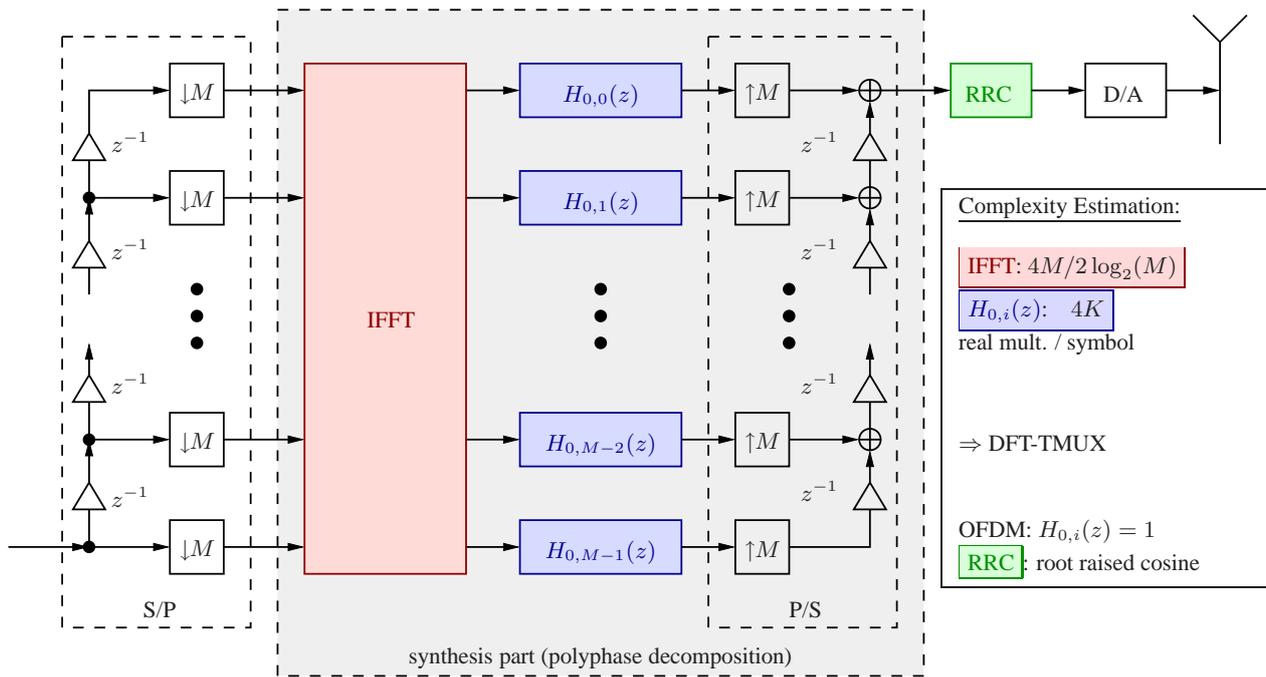


Fig. 2. DFT-transmultiplexer.

3 Power spectral density

The power spectral density of the input symbol vector of both the OFDM and the TMUX system is assumed to be white with unity power, i.e. $E \{ \mathbf{x}[n] \mathbf{x}[n]^H \} = \mathbf{1}_M$. It is easily shown, that both for OFDM and the RRC-TMUX the transmitted symbol vector – disregarding the cyclic prefix for OFDM – keeps the white power spectral density. With OFDM the IDFT operation is a unitary matrix transformation which doesn't change the spectral properties of the vector it is applied to. The RRC filters of the TMUX are approximately square root M -th band filters which means that

$$\sum_{k=0}^{M-1} H^2(zW_M^k) = 1. \tag{3}$$

Therefore, the approximately M -th band spectral power densities of each subcarrier add up to a white power spectral density at the transmitter output.

In practice a certain number of subcarriers at the band edges is omitted for data transmission as frequency guard band in order to simplify the subsequent signal processing tasks concerning the band limitation and D/A conversion. Figures 3a and 3b show the power spectral densities both for an OFDM and an RRC-TMUX multicarrier system. The number of total subcarriers is $M = 32$, and $M_g = 8$ subcarriers are omitted at the band edges. Therefore, in both cases the total power spectral density is not white any more. The power spectral densities of the output signals of each sub-

channel of the TMUX have only overlap with the directly adjacent subchannels and separate the other subchannels by the large attenuation characteristic, whereas the power spectral densities for OFDM show large overlap with all other subchannels.

The subsequent RRC pulse shaping filter – operating at the high, M -fold symbol rate – has to limit the bandwidth to the existing bandwidth restrictions while avoiding intersymbol interference. In Figs. 3a and 3b a rolloff factor $\rho = 0.20$ has been used. The resulting power spectral densities are given in Figs. 3c and 3d. Obviously, the high stopband attenuation of the RRC subchannel filters allows relatively relaxed requirements on the RRC pulse shaping filter. The resulting bandwidth for the OFDM system is about 1.5 times the bandwidth for the TMUX system, when an attenuation threshold of 40 dB is considered.

Therefore, a smaller rolloff factor for the pulse shaping filter is required for the same bandwidth demand in an OFDM system. This increases the complexity for the pulse shaping filter and deteriorates the peak-to-average ratio of the transmit signal.

4 Conclusions

Complex modulated transmultiplexer filter banks with RRC subband filters provide an improved spectral shaping compared to OFDM systems. This means additional costs for the

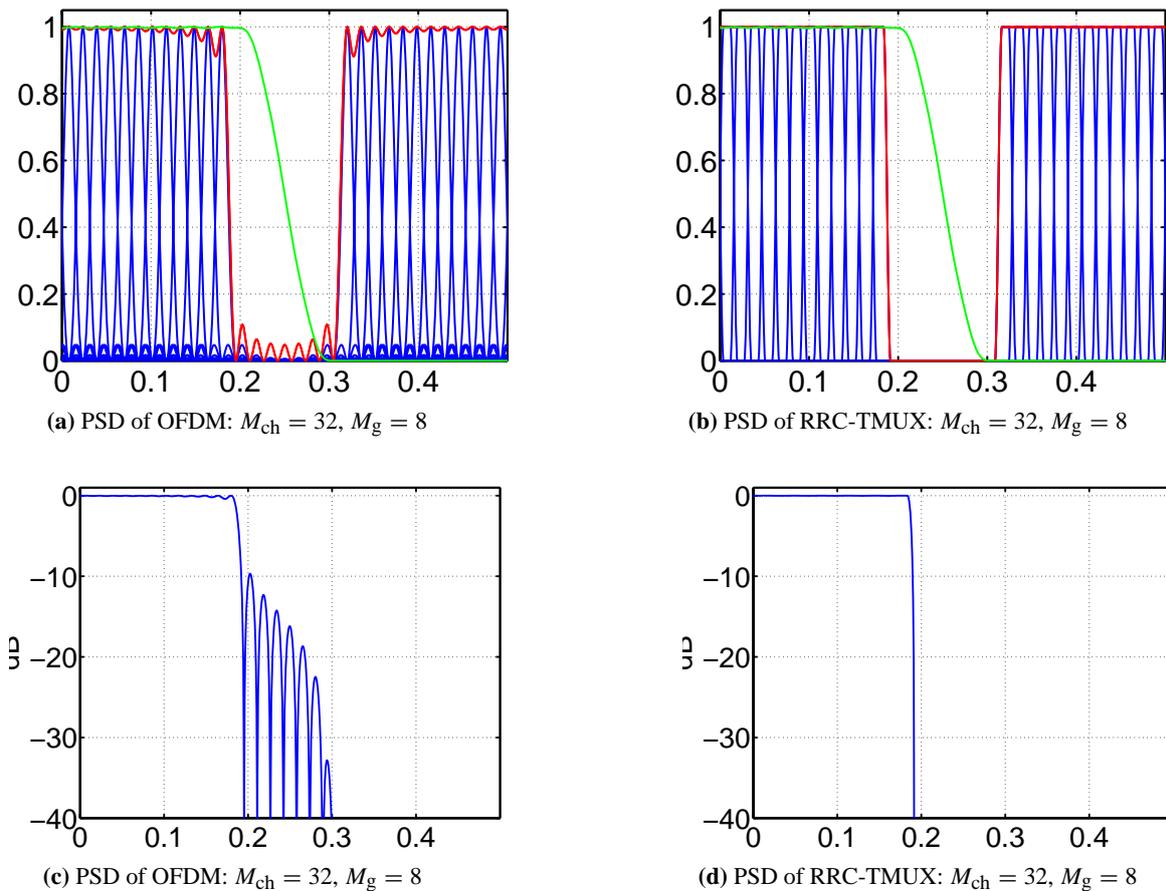


Fig. 3. Comparison: Power spectral densities of OFDM vs. DFT.

pulse shaping filters, but the polyphase decomposition allows for a very efficient implementation.

The power spectral density of the transmit signal with a frequency guard band is nearly band-limited by the subband filters alone, the RRC pulse shaping filter is mainly needed to eliminate periodic repetitions. Therefore, the requirements on the RRC pulse shaping filter are relaxed and a relatively large rolloff factor can be applied. This leads to less complex pulse shaping filters and a lower peak-to-average ratio of the transmit signal because of the faster decaying impulse response of the pulse shaping filter.

The cyclic prefix of OFDM systems is an elegant solution to cope with the distortions of the multipath channel, but it reduces the bandwidth efficiency, because the number of subcarriers cannot be increased arbitrarily due to the time-selectivity of mobile communication channels. Therefore, the modulation alphabet has to be increased to achieve the same data rate as an RRC-TMUX. This requires more transmit power for the same (uncoded) bit error rate. The increase of the transmit power not only boosts the complexity of the transmit amplifier but also enhances the inter-cell interference in cellular systems like mobile communication systems.

The RRC-TMUX necessitates an additional channel

equalizer to cancel intersymbol and adjacent channel interference caused by the frequency selective channel. The alleged complexity of this equalizer has been the main obstacle in the past to the application of DFT-transmultiplexer filter banks with RRC subband filters. The good attenuation properties of the subband filters, which only show overlap with directly adjacent subchannels, should reduce the complexity of the equalization considerably. This will be the subject of future work.

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