Random Beamforming in MIMO Broadcast Channels with Cooperating Receive Antennas

Mario Castañeda, Thomas Lang and Josef A. Nossek
Lehrstuhl für Netzwerktheorie und Signalverarbeitung, Technische Universität München

Abstract

In a Gaussian *multiple-input multiple-output* (MIMO) broadcast channel, the sum rate capacity can be achieved by *dirty paper coding* (DPC). Nevertheless, DPC is computationally intensive and requires that full *channel state information* (CSI) from all users in the system is available at the transmitter. However, with partial CSI at the transmitter, random beamforming [1] can exploit the inherent multiuser diversity in the MIMO broadcast channel with several users. In this work we show that with random beamforming the cooperation of the multiple receive antennas is a better policy than no cooperation at all. To this end, we consider two receive filters: the maximum ratio combiner and the Wiener filter.

1 Introduction

It is well known that the use of multiple antennas in single user links, i.e. *single-user multiple-input multiple-output* (SU-MIMO) systems, provides a remarkable increase in the spectral efficiency [2]. However, when considering a multiuser downlink systems, i.e. the broadcast channel, this spectral efficiency can be further increased by exploiting *multiuser diversity*. Recently, it has been demonstrated that DPC [3] achieves the sum rate capacity of the Guassian MIMO broadcast channel [4] and moreover, also its capacity region [5]. However, DPC in the multiuser context has a high computational burden. Additionally, it relies on the assumption that the channel from all the users is known perfectly at the transmitter. For many scenarios, especially when the number of users or number of transmit antennas is large or the channel coherence time is too short, this is rather not feasible.

Nevertheless, multiuser diversity can also be exploited only with partial CSI at the transmitter. In a *multiuser multiple-input single-output* (MU-MISO) scenario, such as the broadcast channel with multiple transmit antennas and users with a single antenna, Viswanath et. al. have proposed the concept of *opportunistic beamforming* [6]. With a random beam applied at the transmit antennas, each user measures its *signal to interference and noise ratio* (SINR) and feeds this SINR (partial CSI) back to the base station. With the partial CSI from all users, the base station decides which user should be scheduled for transmission with *one stream* on that random beam. However, with multiple antennas at the transmitter, the base station can actually transmit *several streams* simultaneously. In [7], opportunistic beamforming is extended to *multiuser multiple-input multiple-output* (MU-MIMO) scenarios by transmitting on several random beams to one user through *singular value decomposition* (SVD) multiplexing, which is the optimal strategy in the SU-MIMO case. Nonetheless, the base station can assign indeed each random beam to a different user. Sharif and Hassibi have developed the concept of *random beamforming* in [1], where orthonormal random beams, as many as the number of transmit antennas, are generated and assigned to different users.

We consider the problem of supporting multiple users with multiple receive antennas each. In this context we investigate three different receiver strategies:

1. *no cooperation* (NC) between receive antennas of one user, i.e. one user with N antennas acts like N independent single antenna users
2. *maximum ratio combining* (MRC) of the N receive signals from each antenna and
3. *minimum mean square error* (MMSE) processing of the N receive signals, i.e. Wiener filtering.

This paper is organized as follows. In Section 2, the system model and the random beamforming concept is described. In Section 3, we analyze random beamforming with no receive antenna cooperation, with the MRC and with the WF. Afterwards, a comparison among the considered receiver strategies and simulation results are presented in Section 4. Finally, the conclusions are given in Section 5.

2 System Model and Random Beamforming

We consider a Gaussian MU-MIMO broadcast channel with M transmit antennas and K users each equipped with N receive antennas. We assume flat block fading, where the channel matrix remains constant over the coherence time and then changes independently for the next coherence time interval. For ease of notation, in the following we have not included the time index. As in a typical cellular system, we assume that $K \gg M$ and $M \geq N$. Additionally, we assume that the channels from each transmit antenna to each receive antenna for every user are *independent and identically distributed* (i.i.d.) complex Gaussian zero-mean and unit-variance random variables[1]. The channel matrix for user k is

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{h}_{k,1}^{\mathrm{T}} \\ \vdots \\ \mathbf{h}_{k,N}^{\mathrm{T}} \end{bmatrix} \in \mathbb{C}^{N \times M}, \tag{1}$$

where $\mathbf{h}_{k,n} \in \mathbb{C}^M$ is the channel vector from the base station to the n-th antenna of user k. Since the entries of the channel matrix are i.i.d. then

$$\mathrm{E}[\mathbf{h}_{k,n}\mathbf{h}_{k,n}^{\mathrm{H}}] = \mathbf{I}_N,$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix. With random beamforming, M orthonormal random beams $\mathbf{w}_m \in \mathbb{C}^M$, for $m = 1, \ldots, M$, are generated according to an isotropic distribution [8]. Collecting the beamforming vectors into a matrix we have

$$\mathbf{W} = [\mathbf{w}_1, \ldots, \mathbf{w}_M] \in \mathbb{C}^{M \times M}.$$

The equivalent channel $\mathbf{H}_{k,\mathrm{eq}}$ seen by user k after random beamforming (for all the M random beams) is given by

$$\begin{aligned} \mathbf{H}_{k,\mathrm{eq}} &= \mathbf{H}_k\mathbf{W} = \begin{bmatrix} \mathbf{h}_{k,1}^{\mathrm{T}}\mathbf{w}_1 & \ldots & \mathbf{h}_{k,1}^{\mathrm{T}}\mathbf{w}_M \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{k,N}^{\mathrm{T}}\mathbf{w}_1 & \ldots & \mathbf{h}_{k,N}^{\mathrm{T}}\mathbf{w}_M \end{bmatrix} && (2) \\ &= [\mathbf{H}_k\mathbf{w}_1, \mathbf{H}_k\mathbf{w}_2, \ldots, \mathbf{H}_k\mathbf{w}_M] \in \mathbb{C}^{N \times M}, && (3) \end{aligned}$$

where the m-th column of $\mathbf{H}_{k,\mathrm{eq}}$ represents the equivalent channel with the m-th random beam. We use $\chi^2(f)$ to denote a random variable distributed according to a chi-squared distribution with f degrees of freedom. All the entries of the equivalent channel matrix $\mathbf{H}_{k,\mathrm{eq}}$ are complex Gaussians, i.e. $\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_m \sim \chi^2(2)\ \forall\ k, m, n$. This is due to the fact that the entries of the channel vector $\mathbf{h}_{k,n}$ are i.i.d. complex Gaussians and the beamforming vector $\mathbf{w}_m$ has unit norm. Therefore, the inner product $\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_m$ can be expressed as a weighted sum of independent complex Gaussians where the sum is a complex Gaussian random variable with variance equal to the sum of the variances [9]. Additionally, notice that the entries of the equivalent channel matrix $\mathbf{H}_{k,\mathrm{eq}}$ are i.i.d. over the receive antennas (over the columns of $\mathbf{H}_{k,\mathrm{eq}}$), i.e. for receive antennas a and b, $a \neq b$,

$$\begin{aligned} \mathrm{E}[(\mathbf{h}_{k,a}^{\mathrm{T}}\mathbf{w}_m)^{\mathrm{T}}(\mathbf{h}_{k,b}^{\mathrm{T}}\mathbf{w}_m)^*] &= \mathrm{E}_{\mathbf{w}}[\mathbf{w}_m^{\mathrm{T}}\,\mathrm{E}_{\mathbf{h}}[\mathbf{h}_{k,a}\mathbf{h}_{k,b}^{\mathrm{H}}]\mathbf{w}_m^*] \\ &= \mathrm{E}_{\mathbf{w}}[\mathbf{w}_m^{\mathrm{T}} \cdot \mathbf{0} \cdot \mathbf{w}_m^*] = 0, \end{aligned}$$

where $\mathrm{E}_{\mathbf{w}}[\bullet]$ and $\mathrm{E}_{\mathbf{h}}[\bullet]$ denote the expectation over the beams and the channel, respectively, Also, the entries of $\mathbf{H}_{k,\mathrm{eq}}$ are i.i.d. over the beams (over the rows of $\mathbf{H}_{k,\mathrm{eq}}$), i.e. for beams p and q, $p \neq q$,

$$\begin{aligned} \mathrm{E}[(\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_p)^{\mathrm{H}}(\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_q)] &= \mathrm{E}_{\mathbf{w}}[\mathbf{w}_p^{\mathrm{H}}\,\mathrm{E}_{\mathbf{h}}[\mathbf{h}_{k,n}^*\mathbf{h}_{k,n}^{\mathrm{T}}]\mathbf{w}_q] \\ &= \mathrm{E}_{\mathbf{w}}[\mathbf{w}_p^{\mathrm{H}} \cdot \mathbf{I}_M \cdot \mathbf{w}_q] \\ &= \mathrm{E}_{\mathbf{w}}[\mathbf{w}_p^{\mathrm{H}}\mathbf{w}_q] = 0, \end{aligned}$$

due to the assumption of i.i.d. channel coefficients and the fact that beams are orthogonal, i.e. $\mathbf{w}_p^{\mathrm{H}}\mathbf{w}_q = 0$ for $p \neq q$. Let us denote the vector of transmit symbols as $\mathbf{s} = [s_1, \cdots, s_M]^{\mathrm{T}}$, where the m-th entry is the symbol transmitted on the m-th beam. We assume independent symbols with equal power allocation on each beam, i.e. $\mathrm{E}[\mathbf{s}\mathbf{s}^{\mathrm{H}}] = (1/M) \cdot \mathbf{I}_M$, and with transmit power constraint

$$\mathrm{tr}\left(\mathrm{E}[\mathbf{W}\mathbf{s}\mathbf{s}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}]\right) = 1,$$

where tr($\bullet$) is the trace operator. The received signal vector for user k after random beamforming is given by

$$\mathbf{y}_k = \sqrt{\rho}\mathbf{H}_k\mathbf{W}\mathbf{s} + \mathbf{v}_k = \sqrt{\rho}\sum_{m=1}^{M}\mathbf{H}_k\mathbf{w}_m s_m + \mathbf{v}_k, \tag{4}$$

where the n-th entry of $\mathbf{v}_k = [v_{k,1}, \cdots, v_{k,N}]^{\mathrm{T}} \in \mathbb{C}^N$ is the *additive white Gaussian noise* (AWGN) at the n-th antenna. We assume that $v_{k,n}$ are i.i.d complex Gaussian random variables $\forall\ n, k$. Therefore, we have ρ/M as the *signal to noise ratio* (SNR) per beam for each user. The n-th entry of $\mathbf{y}_k$ is the signal received by user k on the n-th antenna

$$y_{k,n} = \sqrt{\rho}\sum_{m=1}^{M}\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_m s_m + v_{k,n}. \tag{5}$$

With the scheme proposed in [1], we assume that each user k knows through training the equivalent channel with each beam, i.e. each user knows $\mathbf{H}_{k,\mathrm{eq}} = \mathbf{H}_k\mathbf{W}$. Assume for now that the users are only equipped with one receive antenna, i.e. $N = 1$. Hence, each user k can compute the SINR with each beam m on the single receive antenna by assuming that s_m is the desired signal and the the other s_i's are interference as follows:

$$\mathrm{SINR}_{k,m,n}^{\mathrm{NC}} = \frac{|\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_m|^2}{M/\rho + \sum_{i \neq m}|\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_i|^2} = \frac{X}{M/\rho + W}, \tag{6}$$

where $\mathrm{SINR}_{k,m,n}^{\mathrm{NC}}$ denotes the SINR per antenna, for the case of no cooperation between the receive antennas. In the following, we drop the index n and express $\mathrm{SINR}_{k,m} = \mathrm{SINR}_{k,m,n}^{\mathrm{NC}}$, since we have assumed $N = 1$. Each user afterwards feeds back its SINR on each beam. The transmitter thus has the partial CSI from each user, SINR with each beam, and assigns s_m with beam $\mathbf{w}_m$ to the user with the highest corresponding SINR with beam

[1]For convenience, in the following we refer to zero-mean and unit-variance complex Gaussian random variables as complex Gaussian random variables.

m, i.e. $\max_{1\leq k\leq K}\mathrm{SINR}_k$. With this scheduling strategy the sum rate is

$$\begin{aligned} R_{\text{sum}} &\approx \mathrm{E}\left[\sum_{m=1}^{M}\log\left(1+\max_{1\leq k\leq K}\mathrm{SINR}_{k,m}\right)\right] \\ &= M\,\mathrm{E}\left[\log\left(1+\max_{1\leq k\leq K}\mathrm{SINR}_{k}\right)\right], \qquad (7) \end{aligned}$$

where the approximate equality is used instead of the strict equality since there is a small probability that user k may be the strongest user for more than one beam. As shown in [1], this is very unlikely for large K and therefore the users do not need to feedback their SINR with each beam. Instead they need to feedback only their maximum SINR over the beams, i.e. $\max_{1\leq m\leq M}\mathrm{SINR}_{k,m}$ for $k=1,\ldots,K$, along with the index m corresponding to the beam in which the SINR is maximized. Therefore, the partial CSI from each user available at the transmitter consists of the maximimum SINR and corresponding beam index per user.

The sum rate in (7), depends on the distribution of the maximum of K i.i.d. SINR's, which in turn depends on the distribution of the SINR. To this end, we study next different receive strategies for $N>1$ and analyze the performance of the distribution of the SINR for each case.

3 Receiver Strategies

3.1 No Cooperation between Antennas (NC)

As suggested in [1], one possibility for the case of multiple receive antennas, i.e. $N>1$, is to treat each receive antenna as an independent user [2]. Instead of considering K users with multiple antennas, we consider actually $N\cdot K$ single antenna receivers. Each user must now feedback the maximum of N SINR's per beam. The transmitter assigns beam m to the antenna of that user with the highest SINR on beam m. The maximization is now done over over $N\cdot K$ i.i.d. SINR's, contrary to only over K i.i.d. SINR's for the single receive antenna case. The highest SINR for a given user and beam results from a maximization over N i.i.d. SINR's. In this sense, this approach can be seen as *antenna selection* since the antenna with the highest SINR could be selected for reception.

From (6), it can be seen that $X\sim\chi^2(2)$ as explained in the previous section with distribution

$$f_X(x)=\mathrm{e}^{-x}, \qquad x>0. \qquad (8)$$

Furthermore, as explained also in the previous section, $\mathbf{h}_{k,n}^{\mathrm{T}}\mathbf{w}_i$ are independent over the beams i for fixed k and n. Hence, we have that the interference in the SINR of (6) $W\sim\chi^2\left(2(M-1)\right)$. With $Y=\frac{M}{\rho}+W$ in (6) and after performing a transformation of variables, we obtain the distribution of Y

$$f_Y(y)=\frac{(y-M/\rho)^{M-2}\mathrm{e}^{-(y-M/\rho)}}{(M-2)!}, \quad y>M/\rho. \quad (9)$$

Denoting $Z=\mathrm{SINR}_{k,m,n}^{NC}$ for the SINR without cooperating receiving antennas of user k at antenna n for beam m, we have that $Z=\frac{X}{Y}$ whose distribution is given by [9]

$$f_Z(z)=\int_{M/\rho}^{\infty} y f_X(yz) f_Y(y)\,\mathrm{d}y. \qquad (10)$$

Plugging (8) and (9) into (10), we can obtain the *probability density function* (pdf) of the SINR for the case of no cooperation between receive antennas

$$\begin{aligned} f_Z(z) &= \frac{\mathrm{e}^{-z/\rho}}{(M-2)!}\sum_{i=0}^{M-2}\binom{M-2}{i}\left(-\frac{1}{\rho}\right)^{M-2-i}\times \\ &\quad \sum_{k=0}^{i+1}\frac{(i+1)!}{(i+1-k)!}\frac{(1/p)^{i+1-k}}{(1+z)^{k+1}} \qquad (11) \\ &= \frac{\mathrm{e}^{-z/\rho}}{(1+z)^M}\left(\frac{1}{\rho}(1+z)+M-1\right) \qquad (12) \end{aligned}$$

and integrating the pdf we obtain the *cumulative distribution function* (cdf)

$$F_Z(z)=1-\frac{\mathrm{e}^{-z/\rho}}{(1+z)^{M-1}}. \qquad (13)$$

As stated before, for a given beam the highest SINR for a given user results from a maximization over N i.i.d. SINR's, distributed according to (12). The cdf of the maximum of N i.i.d. random variables is given by the product of the individual cdfs [9]

$$F_{Z_N}(z_N)=\left(1-\frac{\mathrm{e}^{-z_N/\rho}}{(1+z_N)^{M-1}}\right)^N, \qquad (14)$$

and taking the derivative of the cdf we obtain the pdf

$$\begin{aligned} f_{Z_N}(z_N) &= N\left(1-\frac{\mathrm{e}^{-z_N/\rho}}{(1+z_N)^{M-1}}\right)^{N-1}\times \\ &\quad \left(\frac{\mathrm{e}^{-z_N/\rho}}{(1+z_N)^M}\left(\frac{1}{\rho}(1+z_N)+M-1\right)\right). \qquad (15) \end{aligned}$$

3.2 Maximum Ratio Combining (MRC)

With the assumption that the users know the equivalent channel $\mathbf{H}_{k,\mathrm{eq}}$, another possible strategy with $N>1$, is to perform maximum ratio combining (MRC), in order to detect each stream. Applying a linear filter $\mathbf{G}_k\in\mathbb{C}^{M\times N}$ for each user k to the received signal vector given in (4), we obtain

$$\hat{\mathbf{s}} = \mathbf{G}_k\mathbf{y}_k=\mathbf{G}_k\mathbf{H}_k\mathbf{W}\mathbf{s}+\mathbf{G}_k\mathbf{v}_k \qquad (16)$$

where the m-th entry of the vector $\hat{\mathbf{s}}=[\hat{s}_1,\ldots,\hat{s}_M]^{\mathrm{T}}$ represents the estimate of the symbol transmitted on the m -th

[2] This strategy is denoted by Case 1 in Section VI. in [1].

beam. The filter used to detect the m-th stream corresponds to the m-th row of the matrix $\mathbf{G}_k$, i.e. $\mathbf{e}_m^{\mathrm{T}} \cdot \mathbf{G}_k$, where $\mathbf{e}_m$ is an M dimensional vector where all entries are equal to 0 except for the m-th entry which is set to 1.

For MRC, $\mathbf{G}_k$ is actually a matched filter given by $\mathbf{G}_{k,\mathrm{MRC}} = \mathbf{H}_{k,\mathrm{eq}}^{\mathrm{H}} = \mathbf{W}^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}$. Hence, we have that

$$\mathbb{C}^{M\times M} \ni \mathbf{G}_{k,\mathrm{MRC}}\mathbf{H}_{k,\mathrm{eq}} = \mathbf{G}_{k,\mathrm{MRC}}\mathbf{H}_k\mathbf{W} =$$
$$\begin{bmatrix} \|\mathbf{H}_k\mathbf{w}_1\|_2^2 & \mathbf{w}_1^H\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_2 & \dots & \mathbf{w}_1^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_M \\ \mathbf{w}_2^H\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_1 & \|\mathbf{H}_k\mathbf{w}_2\|_2^2 & \dots & \mathbf{w}_2^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_M \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{w}_M^H\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_1 & \mathbf{w}_M^H\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_2 & \dots & \|\mathbf{H}_k\mathbf{w}_M\|_2^2 \end{bmatrix}.$$

Taking the m-th row of (16) we have that the estimate of the m-th symbol is

$$\hat{s}_m = \|\mathbf{H}_k\mathbf{w}_m\|_2^2 s_m + \sum_{i\neq m} \mathbf{w}_m^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_i s_i + \mathbf{w}_m^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{v}_k$$

After MRC the symbol estimate $\hat{s}_m$ is composed of the desired signal portion for beam m plus the interference from the other beams and the noise. Since the transmitted symbols are independent and the $\mathrm{E}[|s_m|^2] = 1/M\ \forall m$, the SINR for user k for beam m with MRC reads as

$$\begin{aligned} \mathrm{SINR}_{k,m}^{\mathrm{MRC}} &= \frac{\|\mathbf{H}_k\mathbf{w}_m\|_2^4}{\|\mathbf{H}_k\mathbf{w}_m\|_2^2\cdot M/\rho + \sum_{i\neq m}|\mathbf{w}_m^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_i|^2} \\ &= \frac{\|\mathbf{H}_k\mathbf{w}_m\|_2^2}{M/\rho + \sum_{i\neq m}\frac{|\mathbf{w}_m^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_i|^2}{\|\mathbf{H}_k\mathbf{w}_m\|_2^2}} \qquad (17) \\ &= \frac{U}{M/\rho + I} = \frac{U}{T}. \qquad (18) \end{aligned}$$

From the previous expression we can observe that $U = \|\mathbf{H}_k\mathbf{m}_m\|_2^2 \sim \chi^2(2N)$ with pdf

$$f_U(u) = \frac{u^{N-1}\mathrm{e}^{-u}}{(N-1)!}.$$

Let us express I, which represents the interference from the other beams, as

$$I = \sum_{i\neq m}\left|\frac{\mathbf{w}_m^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_i}{\|\mathbf{H}_k\mathbf{w}_m\|_2}\right|^2 = \sum_{i\neq m}\left|\left(\frac{\mathbf{H}_k\mathbf{w}_m}{\|\mathbf{H}_k\mathbf{w}_m\|_2}\right)^{\mathrm{H}}\mathbf{H}_k\mathbf{w}_i\right|^2, \qquad (19)$$

where we can represent as a unit norm vector

$$\frac{\mathbf{H}_k\mathbf{w}_m}{\|\mathbf{H}_k\mathbf{w}_m\|_2} = \left[\alpha_1\,\mathrm{e}^{\mathrm{j}\theta_1}, \alpha_2\,\mathrm{e}^{\mathrm{j}\theta_2}, \dots, \alpha_N\,\mathrm{e}^{\mathrm{j}\theta_N}\right]^{\mathrm{T}} \in \mathbb{C}^N, \qquad (20)$$

with $\sum_{i=n}^{N}\alpha_n^2 = 1$ for $\alpha_n \in \mathbb{R}_+$ and $\theta_n \in [0, 2\pi[$, and also

$$\mathbf{H}_k\mathbf{w}_i = [c_{i,1}, c_{i,2}, \dots, c_{i,N}]^{\mathrm{T}} \in \mathbb{C}^N, \qquad (21)$$

where $c_{i,n} \in \mathbb{C}$ are i.i.d. complex Gaussians $\forall i, n$. Let us recall from Section 2, that $\mathbf{H}_k\mathbf{w}_m$ and $\mathbf{H}_k\mathbf{w}_i$ for $i \neq m$ are independent. Assuming now that $\mathbf{H}_k\mathbf{w}_m$ is given and that $\mathbf{H}_k\mathbf{w}_i$ for $i \neq m$ are random vectors, we can simplify I from (19) by using (20) and (21)

$$\begin{aligned} I &= \sum_{i\neq m}\left|\sum_{n=1}^{N}\alpha_n\,\mathrm{e}^{\mathrm{j}\theta_n}\,c_{i,n}\right|^2 \qquad (22) \\ &= \sum_{i\neq m}\left|\sum_{n=1}^{N}\alpha_n c'_{i,n}\right|^2 \qquad (23) \\ &= \sum_{i\neq m}\left|\left(\sqrt{\sum_{n=1}^{N}\alpha_n^2}\right)\cdot c''_i\right|^2 \qquad (24) \\ &= \sum_{i\neq m}|c''_i|^2 \sim \chi^2(2(M-1)). \qquad (25) \end{aligned}$$

where $c'_{i,n} = \mathrm{e}^{\mathrm{j}\theta_n}c_{i,n}$ and c''_i are complex Gaussian random variables. Equations (23) and (24) follow from the fact that $c_{i,n}$ are circularly symmetric complex Gaussians and (25) follows from $\sum_{i=n}^{N}\alpha_n^2 = 1$. Therefore, it can be observed that $I \sim \chi^2(2(M-1))$. Hence, the interference with one receive antenna, denoted by W in (6), and the interference after the MRC of N signals, denoted by I in (18), have the same distribution.

This result can be intrepreted as follows. The MRC coherently adds the power of the desired signal but does not add coherently the power of the interference. The MRC does not change the distribution of the interference received with one antenna. Note, that the linear filter $\mathbf{G}_{k,\mathrm{MRC}}$ does not take into account the interference.

Retaking it from (18), we see that $T = M/\rho + I$ has the following distribution

$$f_T(t) = \frac{(t - M/\rho)^{M-2}\mathrm{e}^{-t-M/\rho}}{(M-2)!}, \quad t > M/\rho$$

Denoting $d = \mathrm{SINR}_{k,m}^{\mathrm{MRC}} = \frac{U}{T}$ and using (10), the pdf of the SINR with MRC for user k and beam m is given by

$$\begin{aligned} f_D(d) &= \frac{\mathrm{e}^{-d/\rho}\cdot d^{N-1}}{(N-1)!(M-2)!}\sum_{i=0}^{M-2}\binom{M-2}{i}\times \\ &\left(-\frac{1}{\rho}\right)^{M-2-i}\sum_{k=0}^{i+N}\frac{(i+N)!}{(i+N-k)!}\frac{(1/\rho)^{i+N-k}}{(1+d)^{k+1}}. \qquad (26) \end{aligned}$$

3.3 Wiener Filter (WF)

For random beamforming we have assumed in Section 2, that the users know their equivalent channel $\mathbf{H}_{k,\mathrm{eq}}$. Therefore, a better strategy than the MRC for detecting the beams with the N receive antennas is to employ *minimum mean square error* (MMSE) estimation, i.e. Wiener filtering, since it has interference suppression capability. For the following, let us rewrite (4) as

$$\mathbf{y}_k = \underbrace{\sqrt{\rho}\mathbf{H}_k\mathbf{w}_m s_m}_{\text{Desired Signal}} + \underbrace{\sqrt{\rho}\sum_{i\neq m}\mathbf{H}_k\mathbf{w}_i s_i + \mathbf{v}_k}_{\text{Interference + Noise}}. \qquad (27)$$

To detect the m-th stream for user k, we apply the receive filter $\mathbf{g}_{k,m,\text{WF}}^{\text{H}}$, i.e. $\hat{s}_m = \mathbf{g}_{k,m,\text{WF}}^{\text{H}}\mathbf{y}_k$. The $\mathbf{g}_{k,m,\text{WF}}^{\text{H}}$ that minimizes the MSE, $\text{E}[|\mathbf{g}_{k,m,\text{WF}}^{\text{H}}\mathbf{y}_k - s_m|^2]$, results in [10]

$$\mathbf{g}_{k,m,\text{WF}}^{\text{H}} = \frac{\sqrt{\rho}}{M}\mathbf{w}_m^{\text{H}}\mathbf{H}_k^{\text{H}}\left(\frac{\rho}{M}\mathbf{H}_k\mathbf{W}\mathbf{W}^{\text{H}}\mathbf{H}_k^{\text{H}} + \mathbf{I}_N\right)^{-1} \quad (28)$$

However, using the matrix inversion lemma it can be shown that (28) can be expressed as

$$\mathbf{g}_{k,m,\text{WF}}^{\text{H}} = \kappa\frac{\sqrt{\rho}}{M}\mathbf{w}_m^{\text{H}}\mathbf{H}_k^{\text{H}}\left(\frac{\rho}{M}\sum_{i\neq m}\mathbf{H}_k\mathbf{w}_i\mathbf{w}_i^{\text{H}}\mathbf{H}_k^{\text{H}} + \mathbf{I}_N\right)^{-1}, \quad (29)$$

where

$$\kappa = \frac{1}{1 + \mathbf{w}_m^{\text{H}}\mathbf{H}_k^{\text{H}}\left(\sum_{i\neq m}\mathbf{H}_k\mathbf{w}_i\mathbf{w}_i^{\text{H}}\mathbf{H}_k^{\text{H}} + \frac{M}{\rho}\cdot\mathbf{I}_N\right)^{-1}\mathbf{H}_k\mathbf{w}_m}.$$

With the random beam $\mathbf{w}_m$ and the the Wiener filter $\mathbf{g}_{k,m,\text{WF}}^{\text{H}}$ for beam m, (29), the channel for symbol s_m becomes a *single-input single-output* (SISO) link with SINR

$$\text{SINR}_{k,m}^{\text{WF}} = \mathbf{w}_m^{\text{H}}\mathbf{H}_k^{\text{H}}\left(\sum_{i\neq m}\mathbf{H}_k\mathbf{w}_i\mathbf{w}_i^{\text{H}}\mathbf{H}_k^{\text{H}} + \frac{M}{\rho}\mathbf{I}_N\right)^{-1}\mathbf{H}_k\mathbf{w}_m, \quad (30)$$

with capacity

$$C_m^{\text{WF}} = \log_2\left(\left|1 + \text{SINR}_{k,m}^{\text{WF}}\right|\right). \quad (31)$$

Plugging (30) into (31) and using the identity

$$|\mathbf{I}_p + \mathbf{AB}| = |\mathbf{I}_q + \mathbf{BA}|, \quad (32)$$

where $\mathbf{A} \in \mathbb{C}^{p\times q}$, $\mathbf{B} \in \mathbb{C}^{q\times p}$ and $|\mathbf{A}|$ is the determinant of $\mathbf{A}$, one can express $C_m^{\text{WF}} = C_m$, where

$$C_m = \log_2\left(\left|\mathbf{I}_N + \mathbf{H}_k\mathbf{w}_m\mathbf{w}_m^{\text{H}}\mathbf{H}_k^{\text{H}}\left(\sum_{i\neq m}\mathbf{H}_k\mathbf{w}_i\mathbf{w}_i^{\text{H}}\mathbf{H}_k^{\text{H}} + \frac{M}{\rho}\mathbf{I}_N\right)^{-1}\right|\right) \quad (33)$$

is the capacity of the SU-MIMO link given in (27) assuming s_m is our desired signal and the rest of the symbols s_i for $i \neq m$ are treated as interference. Therefore, with the Wiener filter we can achieve the capacity in (33). However, even though that the WF is the strategy which achieves the capacity of the SU-MIMO for one beam, it does not achieve the sum rate capacity with the given random beamforming vectors $\mathbf{W}$. This is due to the fact that we are not optimizing the power allocation over the beams, since we have assumed an equal power allocation over the beams. Since the WF is capacity achieving, we expect to achieve a better performance with the WF than with the MRC. However, computing a closed form for the pdf of the SINR for the Wiener filter given by (30) is not as straightforward as for the previous strategies.

4 Analysis and Simulation Results

The sum rate of the receiver strategies studied in Section 3 depend on the maximum of K i.i.d. SINR's as shown in (7). For large K, the maximum of K i.i.d. SINR's depends on the right tail of the SINR distribution [9]. In **Figure 1**, we depict the right tail of the SINR distribution for the case with no cooperation (NC) between receive antennas (15), with MRC (26) and with the WF. For WF, the distribution of the SINR given by (30) is computed numerically with a histogram. The SINR distributions are shown for different values of M, N and ρ. It can be observed that for every setting and a given high SINR value, there is a higher probability with the WF, while with the NC we have the lowest probability. Therefore, it can be expected that no cooperation between the receive antennas would be the worst policy, while cooperation among the receive antennas would provide a gain.

Hence, one can expect that the performance of the WF is better than the MRC and the NC case with respect to the sum rate. This is shown for different ρ in **Figure 2**, where the sum rate for the considered receiver strategies is depicted as a function of the number of transmit and receive antennas. We have assumed $M = N$ for this case and $K = 50$ users. It can be observed that cooperation between the receive antennas provides a remarkable gain as compared to the case of no cooperation between antennas. This result is in contrast to what is stated in [1], where it has been concluded that the best policy is to have no cooperation between the N receive antennas. Furthermore, it can also be seen that the sum rate increases as $M = N$ increases, but the increase is slower for the NC case, since from Figure 1 it can be observed that the probability of high SINR values is quite small for the NC case as $M = N$ increases.

In **Figure 3** we present the sum rate as a function of the number of receive antennas N for a fixed number of transmit antennas $M = 6$ and $K = 50$. One can appreciate how the sum rate increases for all the strategies as N approaches M. For the NC case, as N increases, the size of the pool from where the maximum SINR is chosen increases and thus the maximum SINR for each beam is higher. However, for the MRC and the WF, increasing N has a larger impact since this translates to increasing the degrees of freedom available for detecting a stream.

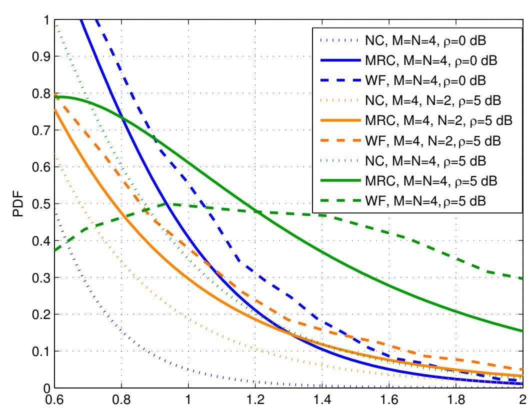

Figure 1: SINR Distributions

Finally in **Figure 4** we show the sum rate as a function of ρ. We assume $M = N = 6$ and recall that the SNR per beam is ρ/M. For small SNR's we have that the MRC and the WF are basically the same as expected. However, as ρ increases the sum rate with the NC and the MRC saturates since the system is interference limited in accordance with [1].

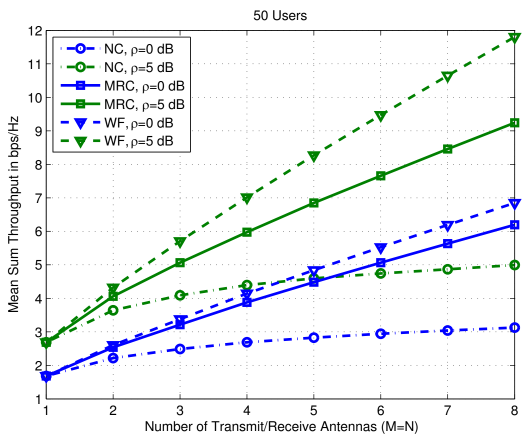

Figure 2: Mean Sum Rate vrs $M = N$

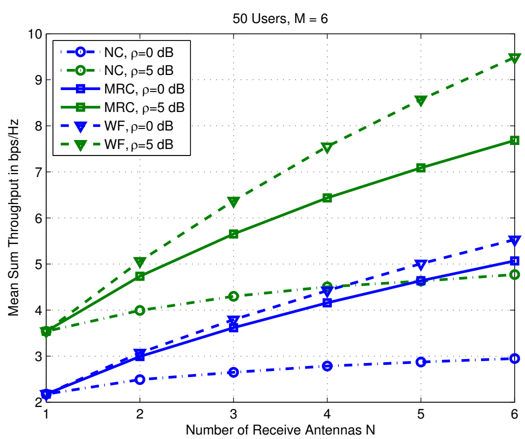

Figure 3: Mean Sum Rate vrs N

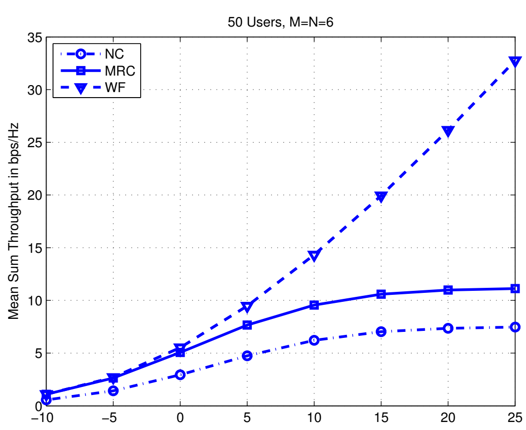

Figure 4: Mean Sum Rate vrs ρ

5 Conclusions

In this work, we have shown that cooperation between N receive antennas with random beamforming at the transmitter is beneficial in comparison to the case of no cooperation between the receive antennas. The gain increases with an increasing N, for both of the considered schemes MRC and WF. It was also shown that the WF was the optimal policy since it achieves the capacity in the SU-MIMO link for one desired stream. In this paper, we have not considered fairness issues and spatial correlations, which could be considerd for future work.

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