# Joint Bit and Power Loading for MIMO OFDM based on Partial Channel Knowledge

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*Abstract*— **Multiple antennas at both transmitter and receiver enable the application of spatial multiplexing as a way of increasing transmission rate. However, this technique suffers from severe performance degradation as soon as fading processes in the multiple-input multiple-output (MIMO) channel exhibit significant correlation. If the transmit correlation matrix of the channel is known at the transmitter, performance can be improved by multiplexing signals across beams pointing along the directions of the eigenvectors of the transmit correlation matrix. These eigenbeams constitute a unitary transformation that has already been shown to be optimum in terms of ergodic capacity in Raleigh-fading channels. However, the effectiveness of this technique decisively depends on bit and power loading across eigenbeams. Here, a method for bit and power loading is presented that based on the notion of *pairwise error probability* (PEP) leads to an optimum distribution of bits and power for a given transmit power and spectral efficiency in an orthogonal frequency-division multiplexing (OFDM) context. Optimality refers to the minimization of a figure of merit that is obtained from adding expected PEPs over single eigenbeams. The effectiveness of this method is verified by means of simulated curves for three channels with different correlation properties.**

## I. INTRODUCTION

Spectrum is a scarce and expensive resource. At the same time there is a demand for wireless mobile systems with ever increasing transmission rates [1]. These two facts make MIMO techniques indispensable due to the high spectral efficiency they offer. Unfortunately, this comes at the expense of more expensive hardware and more complex algorithms.

Multipath propagation in the wireless channel causes temporal inter-symbol interference (ISI), which increases the processing burden required at the receiver to recover the transmitted signals. While narrowband signals are hardly affected by multipath, frequency selectivity poses a serious impediment to the efficient detection of broadband signals. OFDM circumvents this problem by introducing a cyclic prefix between consecutive symbols. If the cyclic prefix is longer than the channel delay profile the frequency selective channel can be effectively decomposed in a number of parallel flat-fading channels, which dramatically eases the detection of transmitted signals [2].

Aiming at the design of broadband wireless systems, the combination of OFDM and MIMO techniques offers a good trade-off between bandwidth efficiency and complexity. Moreover, the flat-fading property of single frequency channels in OFDM makes possible a simple extension of MIMO approaches, mostly conceived using flat-fading channels as a model, to frequency selective channels.

High spectral efficiency requires multiplexing of information over the spatial components offered by a MIMO channel. This technique is called spatial multiplexing and can be readily extended to OFDM on a per subcarrier basis. If no channel knowledge is available at the transmitter, information (bits) and power are uniformly distributed along transmit antennas. Performance will be good as long as the dimensionality offered by the MIMO channel equals the number of transmit antennas. However, a reduced dimensionality, which can originate from spatial correlations or phenomena such as the keyhole effect, can lead to dramatic performance degradation [3]. Partial channel state information (CSI) or knowledge of the transmit correlation matrix of the MIMO channel can help to combat loss of dimensionality due to spatial correlations on the transmitter side, which are the main source of rank reduction in the downlink of a mobile system [4] [5].

While extensive literature exists on the topic MIMO OFDM that considers either perfect, e.g. [6], [7], or no channel knowledge at all, e.g. [2], [8], to date, hardly any publication can be found that considers partial CSI in a multicarrier context as it is understood here, namely as knowledge of the transmit channel correlation matrix.

In the work at hand we consider a MIMO OFDM system with partial CSI at the transmitter. Partial CSI is shown to provide no information about the spectral characteristic of the channel. However, adaptive bit and power loading can still be performed over spatial eigenbeams given by the eigenvectors of the transmit correlation matrix. In [9] the loading algorithm presented in [10] for multitone transmission is straightforwardly applied to distributing bits and power over eigenbeams. Thereby, eigenbeams are treated as if they were orthogonal, i.e. non-intefering, and non-fading. Here, based on

an upper bound of average PEP, a figure of merit is obtained which takes into account the fading nature of eigenbeams and can be optimized over the choice of bit and power loading. This figure of merit includes all significant parameters of the constellations employed over each eigenbeam and does not make use of any approximation for the relationship of average power, bits per symbol and distance between constellation points. The method proposed exhaustively searches for the optimum bit loading over a set of plausible bit allocations, whose cardinality will be low for common numbers of transmit antennas and bits per carrier. For any allocation of bits the optimum power loading is given by a closed form expression.

The remaining of the paper is structured as follows. Section II introduces the system model and some notation. In Section III a figure of merit based on average PEP is introduced that depends on bit and power loading. In Section IV the optimization problem is set up and an algorithm is presented which provides the optimum bit and power loading. Section V shows some simulation results and finally, conclusions are drawn in Section VI.

## II. SYSTEM MODEL

Given a zero-mean complex Gaussian distributed MIMO channel with $M_t$ transmit antennas, $M_r$ receive antennas and $L$ delay paths the relationship between vector $\boldsymbol{y}_k^{(b)} \in \mathbb{C}^{M_r \times 1}$ of receive signals and a vector $\boldsymbol{x}_k^{(b)} \in \mathbb{C}^{N_e \times 1}$ of signals transmitted over subcarrier $k \in \{0, \ldots, N-1\}$ during OFDM block $b \in \{1, \ldots, B\}$ might be written as

$$\boldsymbol{y}_k^{(b)} = \boldsymbol{H}_k \boldsymbol{U}_k \boldsymbol{P}_k^{1/2} \boldsymbol{x}_k^{(b)} + \boldsymbol{n}_k^{(b)} \qquad (1)$$

where $\boldsymbol{n}_k^{(b)} \in \mathbb{C}^{M_r \times 1}$ is a white Gaussian distributed noise vector, $\boldsymbol{H}_k \in \mathbb{C}^{M_r \times M_t}$ is the channel matrix at subcarrier $k$, which is assumed to be approximately constant for the duration of $B$ OFDM blocks, $\boldsymbol{U}_k \in \mathbb{C}^{M_t \times N_e}$ is an arbitrary matrix with orthonormal columns, $\boldsymbol{P}_k^{1/2} \in \mathbb{R}^{N_e \times N_e}$ is a diagonal matrix that assigns a certain transmit power to each signal component and $\mathrm{E}\{|x_{k,n}^{(b)}|^2\} = 1$. $N_e \leq M_t$ is the number of spatial dimensions of the signal to be transmitted over the channel.

The relationship between the channel matrices in the frequency domain and the channel matrices corresponding to each delay path in the time domain is given by [11]

$$\boldsymbol{H}_k = \sum_{\ell=1}^{L} \boldsymbol{H}_\ell^{(t)} e^{j2\pi n_\ell k/N}.$$

where $n_\ell$ is the delay of temporal path $\ell$. If an uncorrelated scatterer assumption holds for the channel, i.e. the fading processes of different delay paths are mutually uncorrelated, then the transmit covariance matrix in the frequency domain is found to be

$$\boldsymbol{R} = \mathrm{E}\left\{\boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{H}_k\right\} = \sum_{\ell=1}^{L} \boldsymbol{R}_\ell^{(t)}, \qquad (2)$$

which turns out to be independent of frequency. Therefore, the conclusion can be drawn that partial knowledge as considered here provides information about the spatial structure of the channel but not about its spectral characteristic. In [4] it has been shown that, assuming complex Gaussian distributed entries in the channel matrices and spatial decorrelation of the fading process at the receiver, for a system model as given by Eq. 1 the optimum matrix $\boldsymbol{U}_k$ has the eigenvectors of the transmit covariance matrix of the channel as columns. This result can be readily extended to channels showing spatial correlation at the receiver. As a consequence, index $k$ is henceforth left out and $\boldsymbol{U}$ will be referred to as eigenbeamforming matrix. Furthermore, due to the fact that no information about the spectral characteristic of the channel is available at the transmitter there is no reason for a frequency dependent allocation of transmit power. Accordingly, a unique power allocation matrix $\boldsymbol{P}$ will be applied to all subcarriers. Taking into account these observations and arranging all $B$ OFDM blocks in a matrix format, our system model can be compactly expressed as

$$\boldsymbol{Y}_k = \boldsymbol{H}_k \boldsymbol{U} \boldsymbol{P}^{1/2} \boldsymbol{X}_k + \boldsymbol{N}_k,$$

where

$$\boldsymbol{Y}_k = [\ \boldsymbol{y}_k^{(1)} \quad \boldsymbol{y}_k^{(2)} \quad \cdots \quad \boldsymbol{y}_k^{(B)} \ ],$$
$$\boldsymbol{X}_k = [\ \boldsymbol{x}_k^{(1)} \quad \boldsymbol{x}_k^{(2)} \quad \cdots \quad \boldsymbol{x}_k^{(B)} \ ],$$
$$\boldsymbol{N}_k = [\ \boldsymbol{n}_k^{(1)} \quad \boldsymbol{n}_k^{(2)} \quad \cdots \quad \boldsymbol{n}_k^{(B)} \ ].$$

The transmitter structure that we consider in this work is depicted in Fig. 1. The *outer code* correlates information transmitted over different space and frequency components and thus leverages diversity. The *adaptive unit* conveniently multiplexes bits over spatial components and assigns them a fraction of the total transmit power. The goal of the present work is a proper design of the *adaptive unit* given a certain raw binary rate at its input and a maximum transmit power as constraints. To this end, a figure of merit $\mathcal{F}$ is required that relates performance to bit and power loadings. In Section III a figure of merit is introduced based on the notion of average PEP.
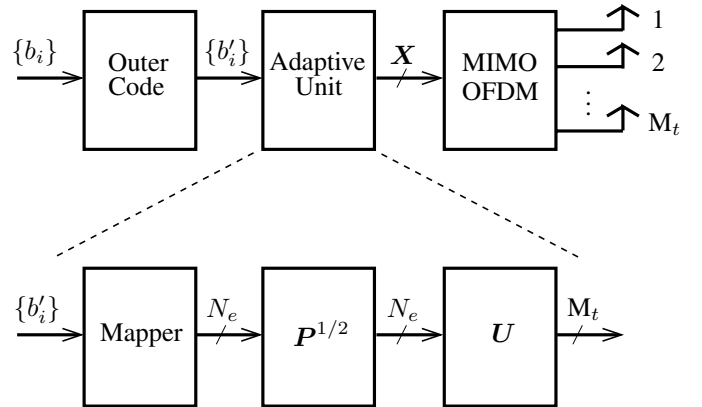


Fig. 1. Transmitter structure for partial CSI.

For purposes of clarity, the following notation is introduced that will be used in the next section. $\tilde{\boldsymbol{A}} = \boldsymbol{I}_L \otimes \boldsymbol{A}$ and

$$\omega_{k,\ell} = e^{-j2\pi k n_\ell/N}.$$

## III. DESIGN CRITERION

Considering all dimensions of a transmitted signal, which can be expressed in matrix form as $\boldsymbol{X} = [\ \boldsymbol{X}_0 \quad \boldsymbol{X}_1 \quad \cdots \quad \boldsymbol{X}_{N-1}\ ]$, and a maximum-likelihood (ML) detector at the receiver, the probability that estimate $\hat{\boldsymbol{X}}$ is equal to $\boldsymbol{X}'$ having transmitted $\boldsymbol{X}$ conditioned on a particular channel realization is called PEP. For our system model this probability can be expressed in closed form using the Gaussian error function [2],

$$P(\boldsymbol{X} \to \boldsymbol{X}'|\boldsymbol{X}, \boldsymbol{H}) = Q\left(\sqrt{\frac{d^2(\boldsymbol{X}, \boldsymbol{X}', \boldsymbol{H})}{2\sigma^2}}\right), \quad (3)$$

where $\sigma^2$ is the variance of the complex-valued noise process, $\boldsymbol{H}$ stands for a particular channel realization and $d(\boldsymbol{X}, \boldsymbol{X}', \boldsymbol{H})$ is the distance of the two transmit signals at the receiver, which is given by

$$d^2(\boldsymbol{X}, \boldsymbol{X}', \boldsymbol{H}) = \sum_{k=0}^{N-1} \left\| \boldsymbol{H}_k \boldsymbol{U} \boldsymbol{P}^{1/2} \boldsymbol{\Delta}_k \right\|^2 , \quad (4)$$

with $\boldsymbol{\Delta}_k = \boldsymbol{X}'_k - \boldsymbol{X}_k$. Since at the transmitter no information about the instantaneous channel realization is available, it is appropriate to calculate the expectation of Eq. 3 over all possible channel realizations, which leads to the average PEP. However, for this expectation no closed solution is available. Fortunately, Eq. 3 can be tightly upperbounded using the Chernoff bound for the Gaussian error function as

$$P(\boldsymbol{X} \to \boldsymbol{X}'|\boldsymbol{X}, \boldsymbol{H}) \le e^{-\frac{d^2(\boldsymbol{X}, \boldsymbol{X}', \boldsymbol{H})}{4\sigma^2}}, \quad (5)$$

whose mean value gives an upper bound of average PEP. This upper bound is computed in [12] and reads,

$$P(\boldsymbol{X} \to \boldsymbol{X}'|\boldsymbol{X}) \le \left(\det\left(\boldsymbol{I} + \frac{\boldsymbol{M}}{4\sigma^2 M_r}\right)\right)^{-M_r}. \quad (6)$$

Thereby,

$$\boldsymbol{M} = \boldsymbol{R}^{(t),H/2} \tilde{\boldsymbol{U}} \tilde{\boldsymbol{P}}^{1/2} \boldsymbol{G}^T \boldsymbol{G}^* \tilde{\boldsymbol{P}}^{1/2} \tilde{\boldsymbol{U}}^H \boldsymbol{R}^{(t),1/2}$$

where

$$\boldsymbol{R}^{(t)} = \text{diag}\{\ \boldsymbol{R}_1^{(t)} \quad \boldsymbol{R}_L^{(t)} \quad \cdots \quad \boldsymbol{R}_L^{(t)}\ \} \in \mathbb{C}^{LM_t \times LM_t}$$

is the block diagonal covariance matrix in the time domain, $\boldsymbol{R}^{(t)} = \boldsymbol{R}^{(t),1/2} \boldsymbol{R}^{(t),H/2}$ and

$$\boldsymbol{G}^T \boldsymbol{G}^* = \sum_{k=1}^{N} \boldsymbol{\Omega}'_k \boldsymbol{\Delta}_k \boldsymbol{\Delta}_k^H \boldsymbol{\Omega}_k'^{,H}$$

with

$$\boldsymbol{\Omega}'_k = [\ \omega_{k,1} \quad \cdots \quad \omega_{k,L}\ ]^T \otimes \boldsymbol{I}_{N_e}.$$

Eq. 6 is a figure of merit that can be computed at the transmitter and depends on the outer code employed, the structure of the mapper and the power loading (cf. Fig. 1). A joint optimization of these parameters would thus be theoretically possible, though practically unfeasible.

Here, we assume that the design of the *adaptive unit* is made independently of the *outer code*. At its input, bits are considered uncorrelated and the mapper is a mere multiplexer of bits over eigenvectors. The *adaptive unit* together with the channel form an equivalent channel for the *outer code*. At the receiver, this channel is equalized, then symbols are detected and mapped to bits. The reliability of these detected raw bits will determine the performance obtained after the decoding process. Thus, properly designing the *adaptive unit* for uncoded transmission, though not optimum, is an effort in the right direction.

Since the available CSI is independent of frequency the design of the *adaptive unit* must also be frequency invariant. Therefore, an upper bound of the average PEP per subcarrier is desirable, which of course will also be frequency invariant for equal pairs $(\boldsymbol{X}'_k, \boldsymbol{X}_k)$. For any subcarrier $k$, setting $\boldsymbol{\Delta}_{k'} = \boldsymbol{0}$ for $k' \ne k$ in Eq. 6, we obtain,

$$P(\boldsymbol{X}_k \to \boldsymbol{X}'_k|\boldsymbol{X}_k) \le \left(1 + \frac{\boldsymbol{\Delta}_k^H \boldsymbol{P} \boldsymbol{\Lambda} \boldsymbol{\Delta}_k}{4\sigma^2 M_r}\right)^{-M_r}, \quad (7)$$

where $\boldsymbol{\Lambda}$ denotes the matrix of eigenvalues of the transmit correlation matrix in the frequency domain and $B = 1$ has been assumed. This assumption is purposeful as we are not interested in coding the signal over several OFDM blocks but in adapting transmission to the available channel knowledge, which happens to be time invariant as long as the channel remains stationary.

Eq. 7 depends very strongly on the particular choice of $\boldsymbol{X}_k$ and $\boldsymbol{X}'_k$. Since we are interested in an optimization of the mapping of bits to symbols, a figure of merit is needed that does not depend on particular symbol pairs but on general constellation parameters. A way to obtain such a figure is by further upperbounding Eq. 7 as follows,

$$P(\boldsymbol{X}_k \to \boldsymbol{X}'_k|\boldsymbol{X}_k) \le \sum_{n=1}^{M_t} \left(1 + \frac{d_n^2 \lambda_n P_n}{4\sigma^2 M_r}\right)^{-M_r}, \quad (8)$$

where $d_n$ denotes the minimum distance between any two points of the constellation employed for transmission over eigenbeam $n$, $\lambda_n$ denotes its associated eigenvalue and $P_n$ its power load. For reasons of generality, $N_e = M_t$ and $d_n = \infty$ if $b_n = 0$, $b_n$ being the bit load of eigenbeam $n$. Each term in Eq. 8 is an upperbound of the maximum average PEP resulting from the transmission over single eigenbeams. Note that cross-talk among eigenbeams is ignored but still the fading nature of eigenbeams is captured.

Now, if we include a factor $\alpha_n$ in each term of Eq. 8 accounting for the average number of closest neighbouring points to any other within a constellation, the following figure of merit is obtained that explicitly depends on power loading and all relevant constellation parameters,

$$\mathcal{F} = \sum_{n=1}^{M_t} \alpha_n \left(1 + \frac{d_n^2 \lambda_n P_n}{4\sigma^2 M_r}\right)^{-M_r}. \quad (9)$$

Each of the terms in Eq. 9 represents an upperbound of the expected symbol error probability (SEP) resulting from

transmission over single eigenbeams. It should be noted that $\mathcal{F}$ is not directly applicable to estimation of SER or BER at the receiver since an important characteristic of the system such as cross-talk among eigenbeams has been lost due to upperbounding (Eq. 8). However, simulations have shown that for common values of SNR and bit rate, $\mathcal{F}$ very faithfully reflects the behaviour of BER curves.

## IV. Bit and Power Loading

The bit and power loading optimization problem reads

$$\min_{b_n, P_n} \mathcal{F}$$

with constraints $\sum_{n=1}^{M_t} P_n = P_T$ and $\sum_{n=1}^{M_t} b_n = R$, where $R$ is the amount of raw bits to be allocated per channel use and $P_T$ the maximum transmit power per subchannel.

Due to the discrete nature of bits the conventional optimization approach based on derivatives is not possible. Two alternatives exist. The first is based on a continuous approximation of $d_n(b_n)$ and $\alpha_n(b_n)$ over an fictitious continuous range of $b_n$. Thereby the conventional approach is applied and the final result is rounded to allowable values of $b_n$ adjusting powers $P_n$ conveniently. The second alternative consists of an exhaustive evaluation of all possible bit allocations with optimized power loading. The former can be implemented very efficiently if certain approximative assumptions are made (cf. [10]) at the cost of accuracy. This is specially advantageous for large numbers of bits and locations. The latter finds the optimum solution but must evaluate all possible bit allocations. However, considering typical values of bits and locations, i.e. eigenbeams, for a system model as described in Section II, we note that the number of meaningful allocations remains relatively low, which renders this second alternative specially atractive.

It is easy to show that for a particular bit allocation

$$P_n = \max\left\{ \frac{M_r}{K_n} \left( (\mu \alpha_n K_n)^{\frac{1}{M_r+1}} - 1 \right), 0 \right\} \qquad (10)$$

minimizes $\mathcal{F}$, where $\mu$ is chosen to satisfy the constraint

$$\sum_{n=1}^{M_t} P_n \leq P_T$$

and

$$K_n = \frac{d_n^2 \lambda_n}{4\sigma^2}.$$

As for the bit loading, solving an elemental combinatorial problem we note that $R$ bits can be allocated over $M_t$ eigenbeams in

$$C(M_t + R - 1, R) = \begin{pmatrix} M_t + R - 1 \\ R \end{pmatrix}$$

different ways. Each of these combinations could be evaluated by an exhaustive search algorithm to chose the allocation yielding the lowest value for $\mathcal{F}$. However, if, without loss of generality, we assume $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{M_t}$, only allocations with the property $b_1 \geq b_2 \geq \ldots \geq b_{M_t}$ seem to make

sense. This is in accordance with the results in [4], where strong eigenbeams receive optimally more power than weak eigenbeams, which implicitly also applies for the bit loading since signals are assumed to be Gaussian.

The number of combinations for which the non-increasing bit loading property applies can be computed as

$$C = 1 + \left\lfloor \frac{R}{2} \right\rfloor + \sum_{b_3 > 0} \left\lfloor \frac{R - 1 - 3(b_3 - 1)}{2} \right\rfloor + \ldots$$

$$\sum_{\substack{b_{M_t} > 0 \\ b_3 \geq \cdots \geq b_{M_t}}} \left\lfloor \frac{R - (M_t - 2) - 3(b_3 - b_4)\ldots - M_t(b_{M_t} - 1)}{2} \right\rfloor,$$

where $\lfloor \cdot \rfloor$ returns the integer part. As an example, for $M_t = 4$ and $R = 6$ a total of 84 bit allocations are possible. If we consider the ordering restriction this number reduces to 9.

In order to find the optimum allocation a search algorithm can be easily implemented using a series of nested loops that evaluate each allocation in a well defined order (cf. Fig. 2).

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for   b_{M_t} = 0 : ⌊R/M_t⌋
   for   b_{M_t-1} = b_{M_t} : ⌊(R - b_{M_t})/(M_t - 1)⌋
      ⋮
      for   b_2 = b_3 : ⌊(R - b_{M_t} - b_{M_t-1} - ... - b_3)/2⌋
         b_1 = R - b_{M_t} - b_{M_t-1} - ... - b_2
         compute   P_n ∀n
         evaluate   F
         if   F < F_min
            P_n^opt = P_n,   b_n^opt = b_n   ∀n
            F_min = F
```

Fig. 2.   Joint power and bit loading search algorithm.

## V. Simulation Results

In order to verify the validity of the proposed design criterion, simulations have been run over frequency selective channels with strong, moderate and weak spatial correlations at the transmitter and weak correlation at the receiver. Transmission is uncoded, $M_t = 4$, $M_r = 4$ and ML detection is considered.

Fig. 3 shows results for a channel with $\Lambda = \text{diag}\{ 14.6 \quad 1.4 \quad 0 \quad 0 \}$ and $R = 4$ bits/subcarrier. For partial CSI, beamforming minimizes $\mathcal{F}$ at all SNR values. For perfect CSI the algorithm in [10] has been used. Due to the high correlation, all channel realizations exhibit a similar spatial structure which partial CSI provides. The difference between partial CSI and perfect CSI is only due to the frequency selectivity of the channel, which is only known by the latter.

In Fig. 4, curves are depicted for a channel with $\Lambda = \text{diag}\{ 5.5 \quad 4.5 \quad 4 \quad 2 \}$ and $R = 4$. Now, the optimum bit loading consists of transmitting two bits over the two strongest eigenbeams. Due to the weak correlation, partial CSI provides very little knowledge about the spatial structure

of concrete channel realizations. This fact increases the gap between perfect and partial CSI. However, partial knowledge is still enough to obtain some gain with respect to the case with no CSI.

Finally, Fig.5 compares the performance of the proposed loading algorithm with the one of the algorithm presented in [9] for a channel with $\Lambda = \text{diag}\{ 9.7 \quad 4.9 \quad 1.2 \quad 0.2 \}$ and $R = 6$. While the algorithm in [9] recommends the use of the three strongest beams with 3, 2 and 1 bits respectively, our algorithm recommends the use of the two strongest beams with 3 bits each at $\text{SNR} = 0$ and 4 and 2 bits at all other values of SNR. The performance gain amounts to more than 1 dB.



Fig. 5. Comparison of the proposed method and the loading method proposed in [9] for a moderately correlated channel.

contrast, a non-trivial loading of bits and power over eigenbeams results in significant performance gains. An algorithm has been presented that relies on the notion of PEP to realize bit and power loading. Simulation results have been shown that corroborate the effectiveness of this method.
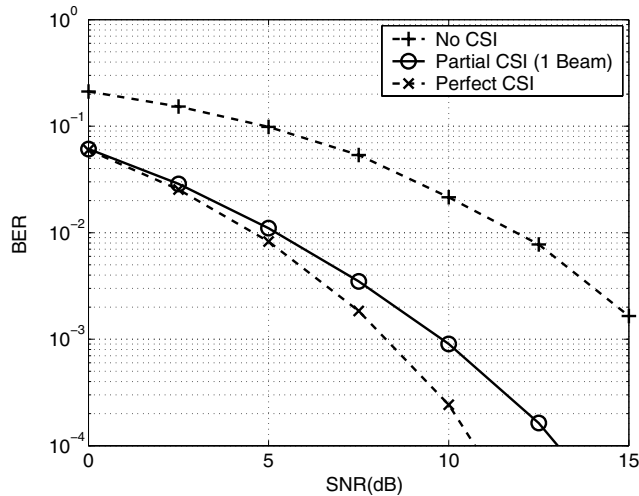


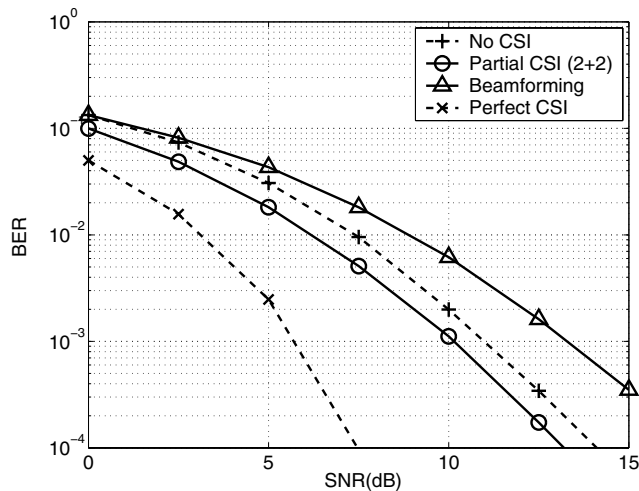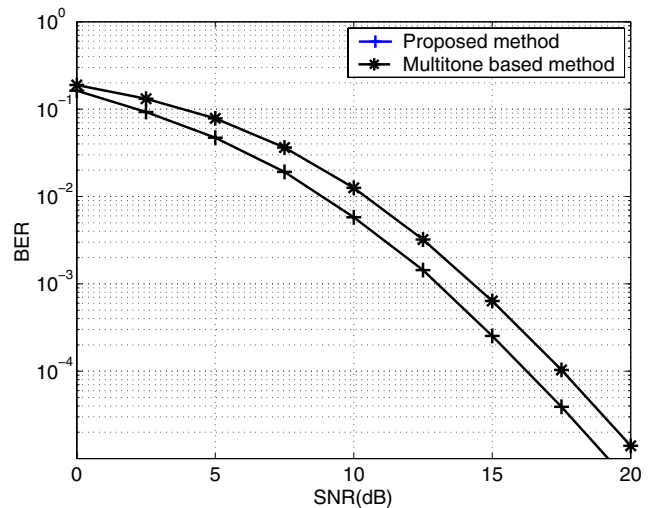Fig. 3. BER curves for strongly correlated channel.



Fig. 4. BER curves for weakly correlated channel.

## VI. CONCLUSIONS

Bit and power loading for a MIMO OFDM system with partial CSI at the transmitter has been discussed. Partial CSI has been shown to be frequency invariant, which leads to a uniform distribution of bits and power over carriers. By

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