On the Channel Outage Probability of Multi-User CDMA Systems

Benno Zerlin, Johannes Brehmer, and Josef A. Nossek
Institute for Circuit Theory and Signal Processing
Munich University of Technology
{zerlin,brehmer,nossek}@nws.ei.tum.de

Abstract— **Like numerous other applications, novel cross-layer optimization schemes require the determination of the outage probability of fading channels. Relying on the assumption of frequency selective Rayleigh fading channels we develop a closed form expression for the probability density function of the sum of the square channel coefficients at the output of *maximum ratio combining* (MRC) receivers. This allows for the formulation of the channel outage probability directly in terms of the channel energy and thus enables the analytic derivation of the outage probability in multi-user CDMA systems. Moreover, low complexity lower and upper bounds are derived, which provide the means for a well fitting approximation of the exact result directly through the cumulative distribution of the channel energy as obtained at the MRC output.**

I. Introduction

The channel outage probability is an important measure for communications over fading channels. By its means information theoretic concepts like the capacity [1] or the cutoff-rate [2] based modeling of communication systems with some restrictions can be extended to the field of fading channels. The determination of the channel outage probability in single-user settings with frequency flat fading can directly be obtained from the cumulative distribution of the user's channel coefficient. For single-user systems with diversity combining receivers the problem has been solved for CDMA channels by [3] and for arbitrary channel statistics through the numerical inversion of the Laplace transform of the underlying cumulative distribution functions in [4].

For multi-user settings though, the use of the common resource transmit power no longer allows for a user-wise definition of the channel outage probability. Instead, we propose the determination of the channel outage probability in multi-user settings via an K-dimensional integration of the joint distribution of all K user's channel realizations.

We investigate a multi-user system with Rayleigh fading channels and an instantaneous transmit power constraint. Thus, the sum of the transmit powers of all users must never exceed a given maximum $P_{\max}$. The multiple access scheme is *code division multiple access* (CDMA), where the spreading codes due to the use of long scrambling sequences appear time variant. The favorable property of a linear relation between user transmit power and the resulting interference contribution can be employed to model these systems by the resulting *signal to noise and interference ratio* (SNR) using so-called orthogonality factors [5], [6], [7]. Considering the effect of the MRC receivers at the mobile stations, we extend the resulting expression and introduce a stochastic model for the occurring variable 'channel energy', which results as the sum of the squared channel coefficients. As this class of MRC CDMA systems with Rayleigh fading channels appears relevant to us, we demonstrate all following considerations on this background. Nevertheless, the resulting computations of the channel outage probability hold for arbitrary channel distributions, and only require the analytic expression of the corresponding cumulative distributions.

Determining the channel outage probability for such a system, the key issue lies in the mutual dependence of the outage events of different users, due to the common resource transmit power. Thus, a formulation of the channel outage in terms of user SNR is no longer possible. Instead, channel outage has rather to be defined in terms of total transmit power as the case when the available maximum transmit power does not suffice to serve all users with the required SNR. As the determination of the corresponding probability distribution of the total transmit power from the pdf of the channel coefficients is impossible, we propose a formulation of the channel outage directly in terms of the random variable 'channel energy'. The outage probability this way can be determined by a multi-dimensional integral over the set of all infeasible channel realizations in $\mathbb{R}_+^K$. Thus the key issue lies in formulating this set of outage channels and finding a subset and an enclosing set that lead to lower and upper bound expressions, respectively.

To this end, Section II focuses on the assumptions made about the channel and the CDMA system, and introduces a closed formulation for the probability density of the sum of squared Rayleigh channel coefficients. With these stochastic means, the formulation of the outage probability as an integral in the K dimensional space of channel realizations is derived in a generic form. Basing on the SNR expression given in Section II, the investigations in Section III specify the limits of the introduced integration formula and illustrate the proceeding in a two-dimensional example. Finally, Section III-C proves the subset relations between the set of infeasible channel realizations and two alternative proposals, resulting in an tight lower and an upper bound on the channel outage probability. Evaluations in Section IV reveal the tightness of the formulated bounds in different SNR regimes as well as the quality of an approximation that bases on these bounds.

978-3-8007-2909-8/05/$20.00 ©2005 IEEE

II. System Model and Assumptions

We model a system with K non-cooperative receivers. Base and mobile stations are equipped with a single antenna each. Fig. 1 sketches one branch of this multi-user system. The

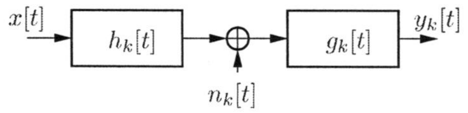

Fig. 1. Channel and receive branch of user k

channels towards the different users are assumed frequency selective and the path delays are integer multiples of the chip duration. Without loss of generality all channels consist of Q channel paths:

$$h_k[t] = \sum_{q=1}^{Q} h_{k,q}\delta[t-q].$$

The fast fading coefficients $h_{k,q}$ of the different channel paths are assumed to be random variables with independent Rayleigh distribution, i.e. real and imaginary part of $h_{k,a}$ are distributed with $\mathcal{N}(0, \sigma_{k,q}^2)$. The channel coefficients are assumed to be mutually independent between paths and users, but are not identically distributed, as $\sigma_{k,q}^2$ might vary drastically due to pathloss and shadowing effects. Employing MRC receivers, the mobile stations processes the receive signal with FIR filters $g[t]$ along:

$$g_k[t] = \sum_{q=1}^{Q} h_{k,q}^{*}\delta[t+q].$$

Through this time diversity scheme, the desired signal component is weighted with the sum of the squared absolute values of all channel coefficients, in the sequel denoted with:

$$r_k = \sum_{q=1}^{Q} |h_{k,q}|^2. \tag{1}$$

It can be shown that $|h_{k,q}|^2$ due to the Rayleigh fading model above is a central chi-square distributed random variable with two degrees of freedom (often called χ_2^2 random variables) with the following distribution:

$$f_{|h_{k,q}|^2}\left(|h_{k,q}|^2\right) = \frac{1}{\sqrt{2\pi|h_{k,q}|^2}\sigma_{k,q}} \exp\left(-\frac{|h_{k,q}|^2}{2\sigma_{k,q}^2}\right).$$

Aiming for a formulation of $f_{r_k}(r_k)$ it has proven advantageous to express r_k as a weighted sum of i.i.d. χ_2^2 random variables $r_{k,q}$ of unit variance, which allows for the representation of r_k as $r_k = \sum_{q=1}^{Q} \lambda_{k,q} r_{k,q}$ with $\lambda_{k,q} = \sigma_{k,q}$. With a proof from [8] for positive definite quadratic forms in random variables, which has been proposed in [9] and has been extended in [10] to indefinite forms, this formulation allows for the derivation of an analytical expression for the probability density function f_{r_k} as:

$$f_{r_k}(r_k) = \sum_{q=0}^{Q} \alpha_{k,q} \exp\left(-\frac{r_k}{2\sigma_{k,q}^2}\right). \tag{2}$$

Within, the coefficients $\alpha_{k,q}$ are independent of r_k and only depend upon the variances $\sigma_{k,q}$ as:

$$\alpha_{k,q} = \frac{\left(\sigma_{k,q}^{Q-2}\right)^2}{2\left(\prod_{\substack{f=1\\f\neq q}}^{Q}\left(\sigma_{k,q}^2 - \sigma_{k,f}^2\right)\right)}. \tag{3}$$

For alternative representations, a detailed proof, and the consideration of cases with $\sigma_{k,q} = \sigma_{k,f}$, $f \neq q$ we refer to [8]. Eq. (2) now provides analytical means to express the pdf of the channel energy (1), which in the course of this paper will allow for the computation of the outage probability π_{out} for the regarded systems.

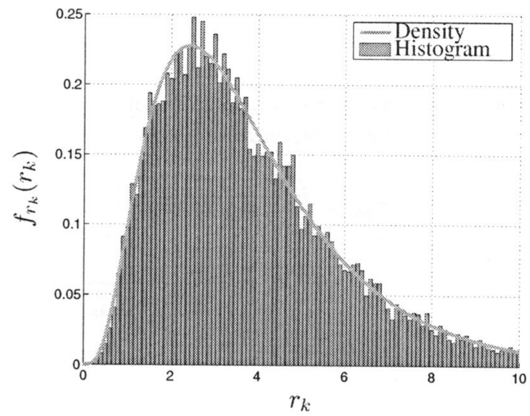

Fig. 2. Validation of Eq. (2)

In single user settings, the channel outage is defined as the probability that the SNR γ_1, cf. (4), falls below the requirement γ_1^{rq}. This definition implicates that all available transmit power P_{max} is used to serve the one user present. Let us employ the expression of the SNR in CDMA systems with spreading factor χ via so-called orthogonality factors ν [7], [5] and extend it to the multi-user case:

$$\gamma_k = \frac{\chi r_k P_k}{\sum_{l=1}^{K} \nu r_k P_l + P_{\text{n}}}. \tag{4}$$

As all users now access the common resource transmit power, a user-wise decoupled mapping of channel energies to SNR is no longer possible. Thus, it becomes advantageous to define the channel outage probability of the complete system directly on the random variables r_k as:

$$\pi_{\text{out}} = \int_{\mathcal{H}} \left(\prod_{k=1}^{K} f_{r_k}(r_k)\mathrm{d}r_k\right). \tag{5}$$

Hence $\mathcal{H}$ is defined as the set of all infeasible channels, i.e. the set of channels in which the SNR requirements $\gamma_1^{\text{rq}}, \ldots, \gamma_K^{\text{rq}}$ can not be fulfilled with non-negative transmit powers $P_k \geq 0$, $k = 1, \ldots, K$ and $\sum_{k=1}^{K} P_k < P_{\text{max}}$. This way, π_{out} is defined as the probability that at least one user will face an outage event.

III. Channel Outage Probability

The considerations in the following subsections will now specify the set $\mathcal{H}$ and the central integration from Eq. (5).

A. Single-User Systems

This introductory paragraph derives the outage probability for the case that the system is loaded with only one user at a time. Note that for this case numerous publications, e.g. [4], have given more general results for the computation of π_{out}.

Exploiting the one-dimensional nature of the setting, Eq. (4) can directly be solved for r_1. As the feasibility of a channel r_1 is directly defined through the SNR requirement $\gamma_1^{(\text{rq})}$, the set $\mathcal{H}$ is defined as:

$$\mathcal{H} = \left\{ r_1 \,\middle|\, r_1 < r_1^{(\text{th})} = \frac{\gamma_1^{(\text{rq})} P_{\text{n}}}{(\chi - \gamma_1^{(\text{rq})}\nu) P_{\max}} \right\}. \tag{6}$$

The outage probability thus can be obtained by integration over the pdf of r_1 introduced in (2) as:

$$\begin{aligned}\pi_{\text{out}} &= \int_0^{r_1^{(\text{th})}} f_{r_1}(r_1)\mathrm{d}r_1 \\ &= F_{r_1}\left(\frac{\gamma_1^{(\text{rq})} P_{\text{n}}}{(\chi - \gamma_1^{(\text{rq})}\nu) P_{\max}} \right).\end{aligned} \tag{7}$$

The cumulative distribution $F_{r_k}(r_k)$ can be obtained analytically by integration of (2) as:

$$F_{r_k}(r_k) = \sum_{q=1}^{Q} 2\sigma_{k,q}^2 \alpha_{k,q} \left(1 - \exp\left(-\frac{r_k}{2\sigma_{k,q}^2} \right) \right), \tag{8}$$

providing the closed solution to the single user case directly. Note that the result in (7) can easily be extended to TDMA multi-user systems by deriving expressions for the distribution of r_k after time-domain scheduling. As these distributions for the prominent scheduling algorithms Round-Robin scheduling, maximum throughput scheduling and proportional fair scheduling can be found analytically, the outage probability of these multi-user systems can be expressed in closed form as well.

B. Multi-User CDMA Systems

Allowing for multiple CDMA users to jointly access the common resource transmit power creates mutual dependencies between the outage events of different users. The key approach to the solution of the CDMA multi-user case is the formulation for the set of channel outage events $\mathcal{H}$. As $\mathcal{H}$ in this case is a set within $\mathbb{R}_+^K$, the hyper-surface enclosing $\mathcal{H}$ is of $K-1$ dimensions, i.e. can be defined by a scalar equation in $\mathbb{R}^K$. With (4) we obtain an implicit description for the set $\mathcal{H}$ of infeasible channel realizations as:

$$\mathcal{H} = \left\{ r_1, \ldots, r_K \,\middle|\, r_K < r_K^{(\text{th})} \right\}, \tag{9}$$

$$r_K^{(\text{th})} = \lim_{\varepsilon \to 0+} \frac{\gamma_K^{(\text{rq})} P_{\text{n}}}{\max\left\{ (\chi - \gamma_K^{(\text{rq})}\nu) P_{\max} - \chi \sum_{i=1}^{K-1} P_i, \varepsilon \right\}}.$$

Let the hyper-surface defined through $r_K^{(\text{th})}$ be denoted as the threshold $\mathcal{T}$. The expression for $r_K^{(\text{th})}$ directly results from (4) when computing the power available to user K by subtracting the powers necessary for serving all other users from $P_{\max}$. These powers result as:

$$P_i = \frac{\gamma_i^{(\text{rq})}}{\chi} \left(\nu P_{\max} + \frac{P_{\text{n}}}{r_i} \right). \tag{10}$$

If the channels $r_1, \ldots r_{K-1}$ already cause an outage, i.e. require more power than $P_{\max}$:

$$P_k = P_{\max} - \sum_{i=1}^{K-1} P_i < \frac{\gamma_k^{(\text{rq})}}{\chi} \left(\nu P_{\max} + \frac{P_{\text{n}}}{r_k} \right), \tag{11}$$

Eq. (9) through $\lim_{\varepsilon \to 0+} \frac{1}{\varepsilon} = \infty$ transforms to the trivial condition $r_K < \infty$. With the definition of $\mathcal{H}$ in Eq. (9) the Kth order integral defined in (5) thus can be computed as:

$$\pi_{\text{out}} = \int_0^{\infty} \cdots \int_0^{\infty} \int_0^{r_K^{(\text{th})}(r_1, \ldots, r_{K-1})} \prod_{i=1}^{K} f_{r_i}(r_i)\mathrm{d}r_i. \tag{12}$$

Fig. 3 visualizes this reasoning in a two-dimensional example, where the threshold $\mathcal{T}$ is the one-dimensional curve $r_2^{(\text{th})} = \text{func}(r_1)$. As obvious from the plot, there exist channel

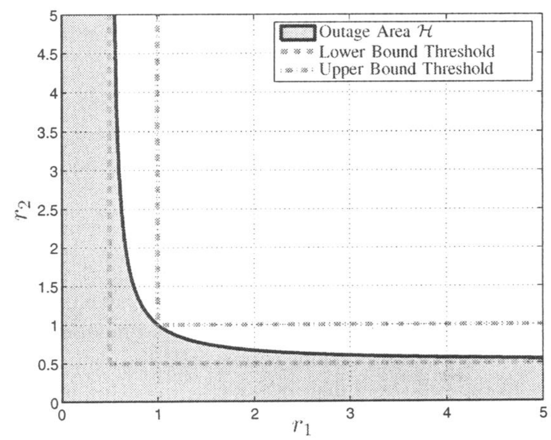

Fig. 3. Graphical representation of (9) for $K = 2$

realizations (here $r_1 < 0.5$) that do not allow for a $\gamma_1^{(\text{rq})}$ compliant service of the first user. In these cases, cf. $\varepsilon > P_{\max} - \sum_{i=1}^{k-1} \left(\gamma_i^{(\text{rq})} P_{\text{n}} / r_i \right)$ in (9), all values $r_2 > 0$ will cause an outage event, the threshold value for r_2 defining the closure of $\mathcal{H}$ thus equals infinity. In all other cases (here $r_1 > 0.5$) the closure of $\mathcal{H}$ is given as:

$$r_2^{(\text{th})} = \frac{\gamma_2^{(\text{rq})} P_{\text{n}}}{(\chi - \gamma_2^{(\text{rq})}\nu) P_{\max} - \gamma_1^{(\text{rq})}(\nu P_{\max} + P_{\text{n}}/r_1)}. \tag{13}$$

Additionally Fig. 3 visualizes the thresholds that are used to derive an upper and a lower bound on π_{out} in the next Section.

C. Bounds on the Outage Probability

The following paragraph aims to find analytical bounds on the multi-user outage probability as the exact expression for large user numbers suffers from the numerical quadrature of the K-dimensional integral in (12).

1) Lower Bound: To this end let the set $\mathcal{H}^{\text{lb}}$ be defined as:

$$\mathcal{H}^{\text{lb}} = \left\{ r_1, \ldots, r_K \,\middle|\, \exists k : \; r_k < r_k^{(\text{lb})} \right\}, \tag{14}$$

$$\text{with} \quad r_k^{(\text{lb})} = \frac{\gamma_k^{(\text{rq})} P_\text{n}}{\left(\chi - \gamma_k^{(\text{rq})}\nu\right) P_\text{max}}.$$

Then $\mathcal{H}^{\text{lb}}$ is a strict subset of $\mathcal{H}$ in Eq. (9), i.e. $\mathcal{H}^{\text{lb}} \subset \mathcal{H}$, and the resulting lower bound $\pi_\text{out}^{\text{lb}} < \pi_\text{out}$:

$$\pi_\text{out}^{\text{lb}} = \int_{\mathcal{H}^{\text{lb}}} \left(\prod_{k \in \mathcal{K}} f_{r_k}(r_k) \right) \mathrm{d}r_k \tag{15}$$

is asymptotically tight for $P_\text{n} \to 0$.

Note that the definition in (14) for all elements of $\mathcal{H}^{\text{lb}}$ requires at least one channel energy r_k, to be smaller than $r_k^{(\text{lb})}$. Without loss of generality we assume $k < K$ in the sequel. Checking these channels for membership in $\mathcal{H}$ yields:

$$\begin{aligned} r_K &\leq \lim_{\varepsilon \to 0+} \frac{\gamma_K^{(\text{rq})} P_\text{n}}{\max\left\{ P_\text{max} - \sum_{i=1}^{K-1} \left(\gamma_i^{(\text{rq})} P_\text{n}/r_i\right), \varepsilon \right\}}, \\ &\leq \lim_{\varepsilon \to 0+} \frac{\gamma_K^{(\text{rq})} P_\text{n}}{\varepsilon}, \\ &\leq \infty, \end{aligned}$$

which obviously is fulfilled. Thus $\mathcal{H}^{\text{lb}}$ is a subset of $\mathcal{H}$ and as $f_{r_k}(r_k)$ is non-negative this also proves the bounding nature of (15). The tightness for $P_\text{n} \to 0$ can be proven through the partial derivatives of (9) with respect to r_k that vanish for $P_\text{n} \to 0$.

For the computation of the bound $\pi_\text{out}^{\text{lb}}$ we introduce the sets $\mathcal{H}_1^{\text{lb}}, \ldots, \mathcal{H}_K^{\text{lb}}$ with $\mathcal{H}_k^{\text{lb}} = \left\{ r_1, \ldots, r_K | r_k < r_k^{(\text{lb})} \right\} \subset \mathcal{H}, \; \forall k$. As the corresponding integrations can be solved mutually decoupled among the users, the computation of the concatenated integral in (5) reduces to the evaluation of $F_{r_k}(r_k)$, $\forall k$. To this end we employ the inclusion-exclusion principle from general set theory to compute the probability measure of $\mathcal{H}^{\text{lb}} = \bigcup_{k=1}^{K} \mathcal{H}_k^{\text{lb}}$ through the cumulative distributions (8) of the channel energies :

$$\begin{aligned} \pi_\text{out}^{\text{lb}} = &\sum_{1 \leq i \leq K} F_{r_i}\left(r_i^{(\text{lb})}\right) \\ &- \sum_{1 \leq i_1 < i_2 \leq K} F_{r_{i_1}}\left(r_{i_1}^{(\text{lb})}\right) F_{r_{i_2}}\left(r_{i_2}^{(\text{lb})}\right) \\ &\vdots \\ &+ (-1)^{K-1} \left(\prod_{i=1}^{K} F_{r_i}(r_i^{\text{lb}}) \right). \end{aligned} \tag{16}$$

2) Upper Bound: Let the set $\mathcal{H}^{\text{ub}}$ be defined as:

$$\mathcal{H}^{\text{ub}} = \left\{ r_1, \ldots, r_K \,\middle|\, \exists k : \; r_k < r_k^{(\text{ub})} \right\}, \tag{17}$$

where $r_k^{(\text{ub})}$ is given as:

$$r_k^{(\text{ub})} = \begin{cases} \dfrac{\gamma_K^{(\text{rq})} P_\text{n}}{\left(\chi - \sum_{i=1}^{K} \nu \gamma_i^{(\text{rq})}\right) P_\text{max} - \sum\limits_{i=1}^{K-1} \frac{P_\text{n}}{r_i^{(\text{lb})}}} & \text{for } k = K, \\ \dfrac{r_i^{(\text{lb})}}{r_K^{(\text{lb})}} r_K^{(\text{ub})} & \text{for } k \neq K. \end{cases}$$

Then $\mathcal{H}$ from Eq. (9) is a strict subset of $\mathcal{H}^{\text{ub}}$ and the probability

$$\pi_\text{out}^{\text{ub}} = \int_{\mathcal{H}^{\text{ub}}} \left(\prod_{k \in \mathcal{K}} f_{r_k}(r_k) \right) \mathrm{d}r_k \tag{18}$$

is an upper bound on π_out.

As $r_K^{(\text{th})}$ in the definition of $\mathcal{H}$ in (9) is strictly monotonically decreasing with all r_k, the bound can be proven by showing, that $[r_1^{(\text{ub})}, \ldots, r_K^{(\text{ub})}]$ lies on the threshold $\mathcal{T}$. As $r_K^{(\text{ub})}$ is determined as the intersection of the one-dimensional subspace containing the origin and $\left[r_1^{(\text{lb})}, \ldots, r_K^{(\text{lb})}\right]$ and $\mathcal{T}$, the proof directly follows from the combination of (17) and (9). The computation of π_out is carried out in complete analogy to (16) once more employing the inclusion-exclusion principle.

3) Approximation: Using the knowledge about the tightness of the introduced lower bound for high SNR, we can introduce an heuristic approximation for the outage probability by combining the presented bounds:

$$\begin{aligned} \pi_\text{out}^{(\text{ax})} &= \beta_\text{ax} \pi_\text{out}^{(\text{ub})} + (1 - \beta_\text{ax}) \pi_\text{out}^{(\text{lb})}, \\ \beta_\text{ax} &= \left(\pi_\text{out}^{(\text{ub})}\right)^{\frac{K}{2}}. \end{aligned} \tag{19}$$

IV. Evaluation

Finally, let us validate the proposed expressions by comparing them with numerical outcome of the corresponding experiments. To this end we evaluated a system with the following properties.

χ	ν	$\sigma_{k,q}^2$	$\gamma_k^{(\text{rq})}$	K	Q
16	0.1	0.5^{q-1}	1	5	4

A. Outage Probability and its Bounds

Visualizing the outage probability of a $K = 5$ user system, Fig. 4 compares the simulation data with the derived bounds and the approximation in (19). The results justify the made considerations, as both bounds hold over the complete SNR range and allow for an extremely well fitting approximation of the outage probability. The plot illustrates that the bounds are asymptotically tight for low and high SNR, respectively. Moreover, both bounds have identical slope for high SNR, which results in a close tunnel for the outage probability, which enables an extremely close approximation of the de facto value of the outage probability through the formula in Eq. (19).

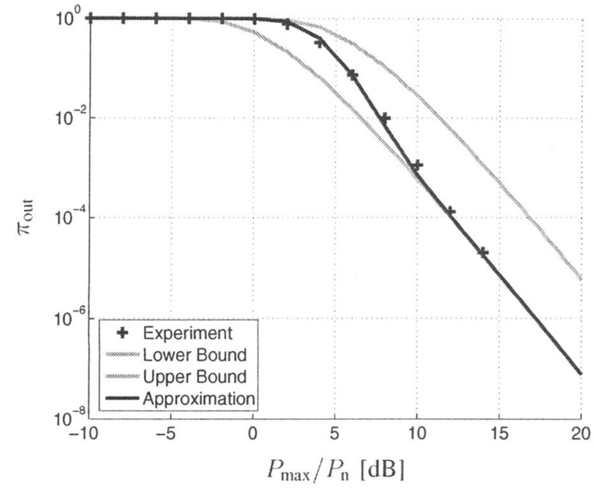

Fig. 4. Visualization of π_{out}

B. Application to XARA

Beside the analytical use, the presented techniques allow for the top-down cross-layer optimization [11], [12] of multi-user systems. Including the relation between SNR demands, channel statistics and outage probability in the mode optimization allows us to predict the packet loss due to channel fading. This is an essential step in top-down cross-layer optimization as it provides the means to derive the optimum mode of operation prior to transmission. With the formulated multi-user relations it becomes possible to derive multi-user scheduling schemes, that simultaneously access the code and time domain (XARA) [13]. As these considerations always are subject to a strict transmit power constraint, the computation of the outage probability in these schemes is of pivotal importance. Fig. 5 demonstrates MAC scheduling in an exemplary HSDPA [14] setting as one possible application of the presented multi-user outage expression.

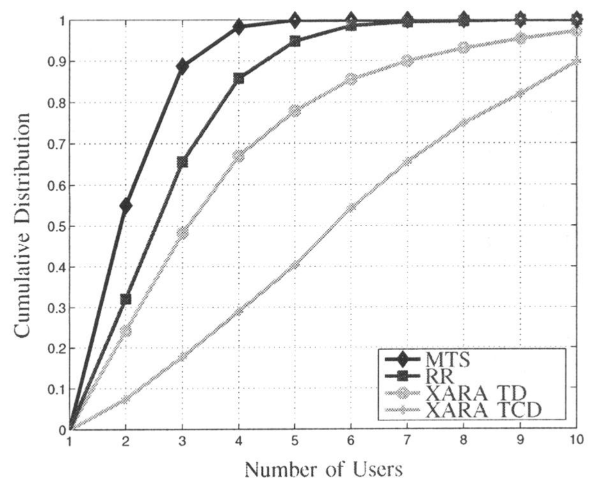

Fig. 5. Capacity Increase through XARA methods

V. Conclusion and Outlook

We have investigated the outage probability of MRC multi-user systems. With an analytical expression for the density function of the resulting channel attenuation after MRC combining, the outage probability can be formulated through multi-dimensional integrals. Formulating outage events as a condition on the realization of the multiple channels allows us to express the integral limits in a closed analytic form, providing explicit expressions for the outage probability even in multi-user systems. Moreover, low complexity upper and lower bounds were introduced that provide a well fitting approximation.

References

[1] Claude Shannon, "A mathematical Theory of Communication," Tech. Rep., Bell Systems Technical Journal, 1948.

[2] R. G. Gallager, *Information Theory and Reliable Communication*. John Wiley and Son, 1968, ISBN: W-471-29048-3.

[3] W. Choi and J. Andrews, "Outage probability for maximal ratio combining receivers in asynchronous CDMA channels," in *Proceedings of the Eighth IEEE International Symposium on Spread Spectrum Techniques and Applications*, September 2004, pp. 779 – 783, Austin, TX, USA.

[4] Y.C. Ko, M.S. Alouini, and M.K. Simon, "Outage Probability of Diversity Systems over Generalized Fading Channels," *IEEE Transactions on Communications*, 2000.

[5] Meelesh B. Mehta, Larry J. Greenstein, Thomas M. Willis, and Zoran Kostic, "Analysis and Results of the Orthogonality Factor in WCDMA downlinks," *IEEE Transactions on Wireless Communications*, vol. 2, no. 6, pp. 1138–1149, November 2003.

[6] Alexander Seeger, Marcin Sikora, and Axel Klein, "Variable Orthogonality Factor: a Simple Interface between Link and System Level Simulation for High Speed Downlink Packet Access," in *Proceedings of the Vehicular Technology Conference, Fall 2003*. IEEE, October 2003, vol. 4, pp. 2531–2534.

[7] Fumiyuki Adachi, "Effects of Orthogonal Spreading and Rake Combining on DS-CDMA forward Link in Mobile Radio," *IEICE Transactions on Communictaions*, vol. E80-B, no. 11, pp. 1703–1712, Nov. 1997.

[8] Serge B. Provost A. M. Mathai, *Quadratic Forms In Random Variables*, vol. 126 of *Statistics: Textbooks and Monographs*, Marcel Dekker, Inc., 1992.

[9] G. E. P. Box, "Some Theorems on Quadratic Forms applied in the Study of Analysis of Variance Problems," *The Annals of Mathematical Statistics*, , no. 25, pp. 290–302, 1954.

[10] J. P. Imhof, "Computing the Distribution of Quadratic Forms in Normal Variables," *Biometrika*, , no. 48, pp. 419–426, 1961.

[11] Benno Zerlin, Michel T. Ivrlač, Wolfgang Utschick, and Josef A. Nossek, " Joint Optimization of Radio Parameters in HSDPA," in *Proceedings of the Vehicular Technology Conference*. IEEE, Spring 2005, Stockholm, Sweden.

[12] Benno Zerlin and Josef A. Nossek, " Cross-Layer QoS Management in Scheduled Multi-User Systems," in *Proceedings of the 2005 Workshop on Signal Processing Advances in Wireless Communications*. IEEE, June 2005, New York City, US.

[13] Benno Zerlin, Michel T. Ivrlač, Josef A. Nossek, and A. Klein, " On Cross-Layer Assisted Resource Allocation in HSDPA," in *Proceedings of the 12th International Conference on Telecommunication*. IEEE, May 2005, Cape Town, South Africa.

[14] Technical Specification Group Radio Access Network, "TS 25.308 - High Speed Downlink Packet Access, Overall Description, Stage 2 (Release 6)," Tech. Rep., Third Generation Partnership Project, Dezember 2004.