Combining Multi-User Diversity with Eigenbeamforming in Correlated Channels

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***Abstract*: In a time varying point-to-multi-point link such as the downlink of a network with a base station and several users, there exists an inherent form of diversity called multi-user diversity. This diversity is harnessed by the use of a proper scheduler such as the proportional fair scheduler. Among other factors, the multi-user diversity gain is determined by the dynamic range of the channel fluctuations. For several scenarios including correlated fading channels, this range can be increased by deploying multiple antennas at the transmitter and using opportunistic beamforming. However, point-to-point link transmitting techniques such as eigenbeamforming exist that have proven to be useful in correlated channels. We present the concept of exploiting multi-user diversity in correlated channels with eigenbeamforming. We show how combining eigenbeamforming with the proportional fair scheduler results in a larger multi-user diversity gain than the one achieved through opportunistic beamforming.**

1. Introduction

An ever increasing demand of bandwidth exists in the downlink of a communications system. For data applications, the transmission between the uplink and the downlink is asymmetric. In such applications as the internet, there is a larger capacity requirement for the downlink than the uplink. Therefore, new wireless systems must be provided with a high-speed downlink shared channel to satisfy this demand. In third generation systems such as CDMA2000 and WCDMA, this need has been addressed by including a high speed shared channel in the downlink through the *High Data Rate* (HDR) [2] mode and the *High Speed Downlink Packet Access* (HSDPA) [8], respectively.

Recent results indicate that the use of multiple antennas at the transmitter or receiver offers a significant increase in the capacity of a wireless communications system over single antenna systems [15]. However, even with the gain produced for example by multiple antennas at the base station, there is still a need for more throughput in the downlink for high rate applications.

Traditionally, fading is mitigated through different forms of link diversity by averaging out the fading, hence making the channel more reliable. In fact, this is done with the purpose to resemble more closely a pure *additive white Gaussian noise* (AWGN) channel [1]. In this sense fading is a non-desired element of the system. Nonetheless, this view pertains to point-to-point links.

Recently another form of diversity has been brought to attention considering another type of link [10]. This diversity is referred to as *multi-user diversity* and is inherent in all multi-user systems, such as the downlink of a system with several users. This is contrary to the link diversity in point-to-point systems, where diversity may or may not be an intrinsic element of the system. To maximize the sum throughput of the users in a multi-user system, a *time division multiple access* (TDMA) scheme must be used, serving at each time instant the user with the largest channel amplitude [10]. In this sense, the system serves the user who undergoes constructive fading, making use of the channel amplitudes from all users as efficiently as possible. If the number of users is high the gain produced is significant as a result of the multi-user diversity. Hence, fading is no longer viewed as a nuisance that has to be overcome, but as a source of randomization that can be exploited through this novel type of diversity. Third generation systems make use of this form of diversity to improve the spectral efficiency. In theory serving users through a TDMA scheme as described before, peak rates of up to 10 Mb/s on the downlink of CDMA/HDR can be achieved [2]. However, multi-user diversity can only be exploited through a proper scheduler.

Instead of using the deterministic *round robin scheduler* (RRS), an opportunistic scheduler that takes into account the channel condition of the users is necessary to harness this multi-user diversity [11]. Nevertheless, by serving the users through this scheduler, the aforementioned scheme is confronted with fairness issues such as an increased delay. The deterministic round robbin scheduler is fair and has no delay issues but achieves a constant total average rate regardless of the number of users. Under the opportunistic scheduler a higher throughput can be achieved as compared to the round robin scheme. However, this gain may come at the expense of fairness. A scheduler that achieves a good tradeoff between fairness and throughput is the *proportional fair scheduler* (PFS) [16].

Furthermore, multi-user diversity depends on several factors of the multi-user system such as the dynamic range of the channel fluctuations. An approach that increases this dynamic range with the use of multiple antennas at the transmitter is called *opportunistic beamforming* [16]. Opportunistic beamforming produces gain in several scenarios but here we focus on correlated channels.

For correlated channels, alternative transmitting schemes for point-to-point links are available that could be combined with multi-user diversity in a point-to-multi-point link. One of this schemes for correlated channels is *eigenbeamforming* [4], [9] . For eigenbeamforming, the transmitter needs to know the principal eigenvector of the channel correlation matrix measured at the receiver. To this end, the receiver feeds back the principal eigenvector to the transmitter and not the whole correlation matrix as stated in [14]. Note that the principal eigenvector is partial channel state information (partial CSI) and not full CSI (instantaneous value of the channel realization). At the same time, the average properties of the channel do not change fast. Then the rate of this feedback is not high. In this paper, we explain how eigenbeamforming can be applied in combination with multi-user diversity. Furthermore, we show that for channels with different degrees of correlation, eigenbeamforming achieves a better performance than opportunistic beamforming when combined with multi-user diversity. This comes at the cost of a slight increase in the overall feedback and complexity in the training phase. However, the increase in feedback due to the principal eigenvector of the channel correlation matrix is practically negligible compared to the necessary feedback required to exploit multi-user diversity. And for the complexity of the training phase, in third generation systems a common pilot channel is present and can be used for the estimation of the channel at the receivers [8].

To this end, the remaining of the paper is structured as follows. Section 2 presents a review of the multi-user diversity concept and the proportional fair scheduler. The opportunistic beamforming scheme is discussed in Section 3. In Section 4, the eigenbeamforming scheme is described. Then, the comparison between eigenbeamforming and opportunistic beamforming with multi-user diversity for correlated fading is shown in Section 5. Finally, in Section 6, we conclude with the results of this comparison.

2. Review of Concepts

2.1 Multi-user Diversity

Traditionally, different types of diversity have been used to mitigate the fading fluctuations of a communications channel in a point-to-point link. Examples of these types of diversity are *time*, *frequency* and *space diversity*. In each of these cases, diversity is used to make the channel resemble more closely an AWGN channel, i.e. a non-fading channel. However, a different approach can be taken in a point-to-multi-point link where there exists intrinsically another type of diversity called multi-user diversity. Multi-user diversity is motivated by the information-theoretic result of Knopp and Humblet [10]. Their result was based on the uplink of a system with several users. They showed that by allowing only the user with the largest channel magnitude to transmit for each time slot, the maximum sum throughput of the uplink can be achieved. This is achieved by exploiting the inherent multi-user diversity in the system. Having more users with independent fading means that the probability of available *stronger channels* is higher. Thus, serving the best user at each time slot among more users results in a larger diversity gain.

Contrary to the point-to-point link type of diversities, to exploit the latent multi-user diversity in a point-to-multi-point link, the base station or transmitter needs the channel magnitude or equivalent SNR from all of the users before it actually schedules a user. This is achieved through a tight feedback of the partial CSI from all the users to the base station, indicating their current channel conditions. The users must be able to track and estimate the channel magnitude through a common downlink pilot. Once having all the partial CSI from all the user, the base station then makes a decision to which user to transmit with constant power. If the base station schedules always the best user this is referred to as the *greedy scheduler* (GS) and it maximizes the sum throughput of the link [10].

The amount of multi-user diversity depends on several factors such as the number of users and the dynamic range and speed of the channel fluctuations [16]. The range of the channel fluctuations is determined by the distribution of the fading. More multi-user diversity can be exploited when the fading distribution of the users have a higher probability of large channel magnitudes as compared to another fading distribution that has a lower probability. The difference of the dynamic range can be observed in Figure 1, where we have plotted the distribution for Rayleigh and Rician fading with a κ-factor $= 10$ (κ stands for the ratio of the line of sight power to the diffused component power). The standard deviation of both types of fading is 1. It can be observed that Rayleigh fading has a higher probability of high peaks or channel magnitudes as compared to Rician fading. This means there is a larger multi-user diversity available for the Rayleigh fading. To view these depen-

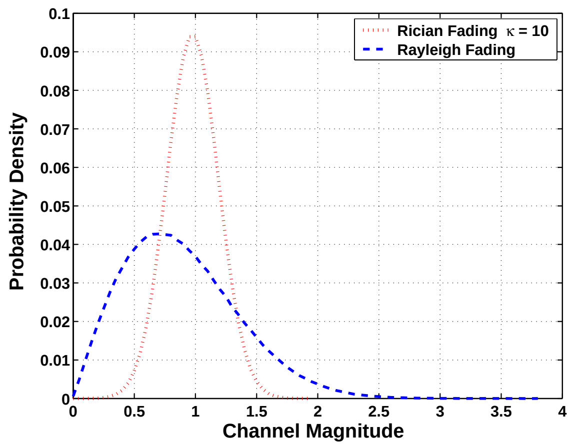

Figure 1: Dynamic range for Rician ($\kappa = 10$) and Rayleigh fading, average SNR $= 0$ dB

dencies, let us consider the downlink of a system with one antenna at the base station and K multiple users with one receiving antenna and let us denote the sum throughput as S. In Figure 2 the average sum throughput S of all the K users in the downlink of a cell is shown as a function of the number of users. The throughput is given by the Shannon equation $C = \log_2(1 + \mathrm{SNR})$ where SNR is the signal to noise ratio at the receiver. The types of fading considered are the Rayleigh and Rician mentioned before and the average SNR at the receiver is 0 dB. Also, included in the plot is the performance over an AWGN channel. It is clearly seen that as the number of users increase, the multi-user diversity increases for both types of fading. At the same time, there is a larger gain for the Rayleigh fading channel since this type of fading has a higher probability of hitting large peaks than in Rician fading as was shown in Figure 1. As for the gain over the AWGN channel, this can be seen in the sense that fading is a source of randomization that can exploit multi-user diversity as noted in [16] through a proper scheduler or that the channel magnitude of an AWGN channel basically has no dynamic range. For a point-to-point link, a non-fading channel is the most reliable and desirable. For a point-to-multi-point link with time varying independent fluctuations a fading channel is better for extracting the multi-user diversity.

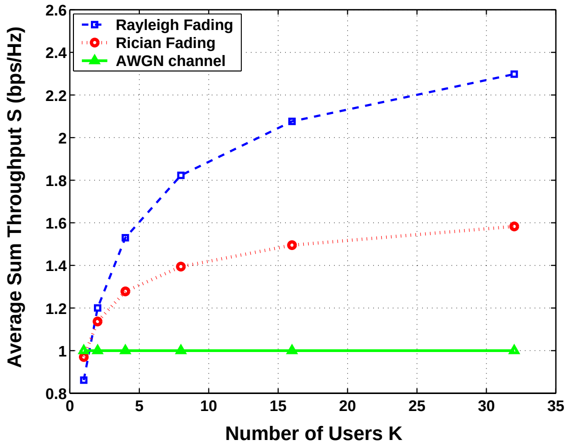

Figure 2: Multi-user diversity gain for Rician ($\kappa = 10$) and Rayleigh fading, average SNR $= 0$ dB

Regarding the speed of the fading, if the channel fades slowly then the possible exploitable diversity is less as we have a time invariant channel for a certain period of time. Hence, the dynamic range is small over a short time period and less multi-user diversity can be extracted. However, there is a limit on how fast the fading can be. If the fading changes very fast due to the speed of the users, then the multi-user diversity decreases as the speed of the users increases [12]. At high speeds, the channel that was measured may no longer be valid when the channel is used. Therefore, the user with the best channel may no longer be the same and this decreases the available multi-user diversity.

2.2 Proportional Fair Scheduler

To extract the inherent multi-user diversity in a network, the transmitter must be able to schedule transmissions among the users after the feedback of their partial CSI. Therefore, this requires an opportunistic scheduler [11]. The maximum sum throughput of a point-to-multi-point link is achieved by transmitting to the users through the greedy scheduler. Furthermore, if the fading statistics among the users are the same, this scheduler achieves the same average throughput for all the users [16], [3]. It is also fair in the sense that the time slots are served equally among the users. Note that fairness here means sharing resources equally but fairness can also be described by other criterions [13]. Meanwhile, when serving users with asymmetric statistics through the greedy scheduler, this conclusion is no longer true and quality of service related issues, such as fairness and delay, must be taken into account because the algorithm is no longer fair. In this case, the statistically stronger user will be given more resources, i.e. time slots, at the expense of the weaker users, who will have to wait long periods of time before being served.

Therefore, to exploit multi-user diversity in a network with asymmetric fading among the users, an opportunistic scheduler is required that exploits this inherent diversity gain while at the same time provides fairness among the users, i.e. serving equal amount of time slots to all. A scheduling algorithm has been developed that addresses these issues and is called the *proportional fair scheduler* (PFS) [7]. For the greedy scheduler, transmission was allocated to the user k who had the best channel or largest requested data rate $R_k(t)$ at time slot t. The requested data rate is a function of the current channel condition or received SNR. When using the proportional fair scheduler, the transmissions are allocated to the user with the largest current rate compared to its own average rate, i.e. the user with the largest ratio:

$$\frac{R_k(t)}{T_k(t)},$$

where $T_k(t)$ is the average throughput of user k. Now a user must no longer wait to have the best channel or largest requested data rate $R_k(t)$ to be served but only a large $R_k(t)$ relative to its own past average throughput $T_k(t)$. The user with the best *relative channel* is served. The average throughput is updated as follows:

$$T_k(t+1) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_k(t) + \frac{1}{t_c} R_k(t) & k = k^*, \\ \left(1 - \frac{1}{t_c}\right) T_k(t) & k \neq k^*, \end{cases} \quad (1)$$

where k^* refers to the user served in time slot t. A user that is served increases its average throughput $T_k(t)$ while the users that are not served decrease their $T_k(t)$, increasing their probability of being served in future slots. The scheduling algorithm serves a user when it is near its own peak within the latency time scale t_c [16]. Moreover, the proportional fair scheduler reduces to the greedy scheduler, with $t_c \to \infty$.

Let us now define the forgetting factor f as the inverse of the time constant t_c ($f = 1/t_c$). The forgetting factor ranges from 0 to 1 and represents the percentage of how much weight does the served data rate for time slot t have on the average throughput $T_k(t)$ for user k. When the forgetting factor is equal to 1 the PFS becomes the RRS and no multi-user diversity can be extracted. Meanwhile, when the forgetting factor is 0, the PFS becomes the GS. As the forgetting factor decreases and approaches 0, the diversity gain increases, but at the expense of delay. The larger the t_c, the higher the $R_k(t)$ needs to be for the user to be served. The user waits longer before hitting a really high peak. Waiting longer for the channel to improve increases $T_k(t)$. Therefore, this produces a higher average sum throughput which is a result of a larger multi-user diversity gain being exploited.

3. Opportunistic Beamforming with Correlated Rayleigh Fading

The degree of multi-user diversity gain that can be extracted from a point to multi-point link depends on the probability of large channel magnitudes and the speed of the fading fluctuations. High probability of large channel magnitudes and fast fading fluctuations result in larger multi-user diversity gain. In a point-to-multi-point link with fading channels that do not have such conditions, multiple antennas can be deployed at the transmitter to induce faster and larger variations through a scheme described as opportunistic beamforming [16]. The larger dynamic range translates into a larger multi-user diversity. Opportunistic beamforming considers multiple antennas at the transmitter and a single antenna at each receiver, although the same concept has been extended to the case when the users have multiple antennas at the receivers [6], [5].

Let us now consider a flat fading downlink of a system, i.e. a multi-user multiple input single output (MU-MISO) system, with N transmit antennas at the base station and one antenna at each receiving user as shown in Figure 3. Let us define $\mathbf{x}(t) \in \mathbb{C}^T$ as the vector of T transmitted symbols for time slot t, $h_{nk}(t) \in \mathbb{C}$ as the complex channel gain from antenna n to the kth user for time slot t, $\mathbf{n}_k(t) \in \mathbb{C}^T$ as the additive white noise at the receiver k for time slot t, and $\mathbf{y}_k(t) \in \mathbb{C}^T$ as the received signal at user k for time slot t.

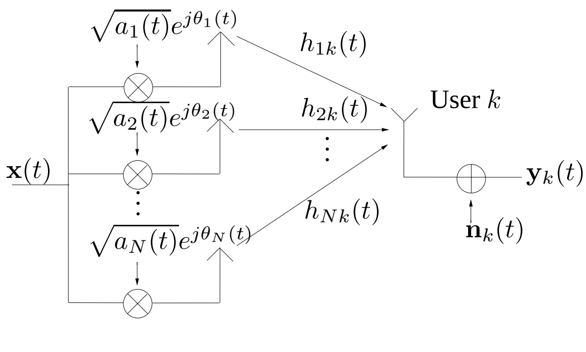

Figure 3: MIMO (MU-MISO) system

On each transmit antenna n a power, $\alpha_n(t) \in [0, 1]$, is allocated with a respective phase shift, $\theta_n(t) \in [0, 2\pi]$, as indicated in Figure 3. The power allocation at the transmitter must satisfy $\sum_{n=1}^{N} \alpha_n(t) = 1$, in order to preserve the total transmit power. The model described is a block fading channel model, where the $h_{nk}(t)$'s remain constant for T symbols. Then, the received signal at user k reads as follows:

$$\mathbf{y}_k(t) = \left(\sum_{n=1}^{N} \sqrt{\alpha_n(t)} \ \mathrm{e}^{\mathrm{j}\theta_n(t)} h_{nk}(t) \right) \mathbf{x}(t) + \mathbf{n}_k(t). \tag{2}$$

Even though there are multiple antennas at the transmitter, the downlink of the system is still a point-to-multi-point link. The links between the base station and user k are represented by an equivalent channel $h_k(t) = \sum_{n=1}^{N} \sqrt{\alpha_n(t)} e^{j\theta_n(t)} h_{nk}(t)$. Then, the use of multiple antennas at the transmitter is transparent to the users. Through this scheme the dynamic range of the $h_k(t)$ can be increased as compared to the original channels $h_{nk}(t)$ and result in a larger multi-user diversity. Just as for the single antenna case, the users must feed back to the base station their received $|h_k(t)|^2$ or their requested data rate $R_k(t)$ resulting from the power $\alpha_n(t)$ and phase $\theta_n(t)$ allocated at the transmitter. If the PFS is used, then the base station decides which is the best relative user and transmits to that user with the given power and phase allocation.

Opportunistic beamforming also produces gain when the variations in the channel are fast and there is correlation among the links. Let us denote all the channels for a user k as $\mathbf{h}^{(k)} = [h_{1k}, h_{2k}, \ldots, h_{Nk}]^{\mathrm{T}}$. To model the correlation among these channels in the array, let us consider that the correlation matrix of the channels, $\mathbf{C}_k = \mathrm{E}[\mathbf{h}^{(k)}\mathbf{h}^{(k),\mathrm{H}}]$, is given by an $N \times N$ matrix as follows:

$$\mathbf{C}_k = \begin{pmatrix} 1 & \rho_k & \rho_k^2 & \cdots & \rho_k^{N-1} \\ \rho_k^* & 1 & \rho_k & \cdots & \rho_k^{N-2} \\ \rho_k^{*,2} & \rho_k^* & 1 & \cdots & \rho_k^{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_k^{*,N-1} & \rho_k^{*,N-2} & \rho_k^{*,N-3} & \cdots & 1 \end{pmatrix}, \tag{3}$$

where $(\bullet)^{\mathrm{H}}$ represents the Hermitian operator, $(\bullet)^*$ represents the conjugate operator and ρ_k denotes the correlation between two adjacent antennas and is complex valued $\rho_k = |\rho_k|\mathrm{e}^{\mathrm{j}\psi_k}$ with ψ_k being a function of the angle of departure of the strongest beam to user k. When $|\rho_k| = 0$ there is no correlation between the links and when $|\rho_k| = 1$ there is full correlation among the links.

When applying opportunistic beamforming for this scenario, the power allocation α_n for $n = 1, \ldots, N$ is varied over 0 to 1, under the constraint that $\sum_{n=1}^{N} \alpha_n(t) = 1$. The allocated phases $\theta_n(t)$ for $n = 1, \ldots, N$ should be given by $\theta_n(t) = (n-1)\theta(t)$, determined by varying one angle of departure $\theta(t)$ [14]. Due to this phase variation, the components of the equivalent channel may add in phase and result in a larger channel magnitude. When we have full correlation, $|\rho_k| = 1$ for $k = 1, \ldots, K$, then the squared magnitude of the equivalent channel for each user ranges from 0 to $N|l_k(t)|^2$, where $l_k(t)$ is Rayleigh distributed. The probability density function of $|h_k(t)|$ for different number of antennas and $|\rho_k| = 1$ is plotted in Figure 4. The probability of large channel magnitudes is higher as the number of antennas N increases, resulting in a larger multi-user diversity.

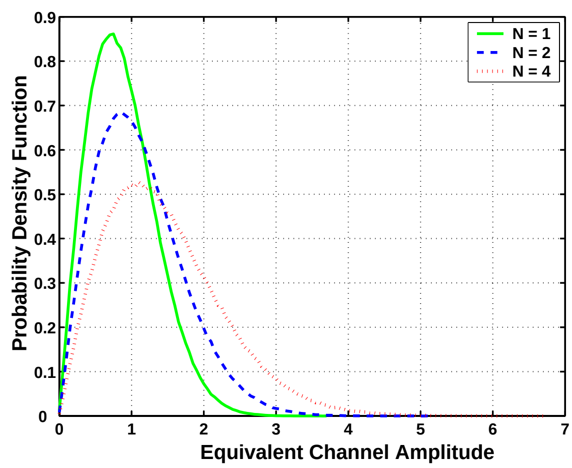

Figure 4: Dynamic range with opportunistic beamforming, correlated Rayleigh fading

4. Eigenbeamforming with Multi-User Diversity

A transmitting scheme that efficiently makes use of the fading correlations in point-to-point links is called eigenbeamforming [4], [9] . To evaluate the performance of this scheme, let us consider correlated Rayleigh fading with the block channel model depicted in Figure 3 as a point-to-point link for each user. The fading for the links between antenna n and the antenna at receiver k are then given by Equation (3). Again let $\mathbf{h}^{(k)} = [h_{1k}, h_{2k}, \ldots, h_{Nk}]^{\mathrm{T}}$ be all the channels for user k. Eigenbeamforming requires the transmitter to have partial CSI of this channel for this user k. In this case, partial CSI means that the transmitter knows in average the principal eigenvector of the spatial correlation matrix of the channel. We have that the sorted eigenvalue decomposition of this correlation matrix is given by:

$$\mathbf{C}_k = \mathrm{E}\left[\mathbf{h}^{(k)}\mathbf{h}^{(k),\mathrm{H}}\right] = \mathbf{V}_k \mathbf{\Lambda}_k \mathbf{V}_k^{\mathrm{H}}, \qquad (4)$$

with $\mathbf{v}_{1,k}$ as the principal eigenvector of $\mathbf{C}_k$, i.e. the eigenvector belonging to the principal eigenvalue of $\mathbf{C}_k$. Let us define the *eigenbeam* $\mathbf{e}_k = \mathbf{v}_{1,k}$. This eigenbeam is then applied as beamforming at the transmitter focusing all the energy on the strongest beam to user k. By applying this power and phase allocation at the transmitter, the data is transmitted over the strongest beam to user k and in average increases the throughput of the point-to-point link under the correlated fading.

Eigenbeamforming can be combined with a proportional fair scheduler to extract multi-user diversity in the following way. We assume that all the users have fed back their eigenbeam $\mathbf{e}_k$ to the base station. For the users to determine this eigenbeam, they need to measure their channels h_{nk}. To this end, the base station must send separate pilot signals on each antenna n for $n = 1, \ldots, N$. With eigenbeamforming the equivalent channel is then $h_k(t) = \mathbf{e}_k^{\mathrm{H}}\ \mathbf{h}^{(k)}$. The users can then calculate what would be their equivalent channel $|h_k|$ if they were served with their respective eigenbeam $\mathbf{e}_k$. The users feedback this $|h_k|$ to the base station. Then, the base station through the PFS decides which would be the relative best user if served with its respective eigenbeam and transmits to that user using its respective $\mathbf{e}_k$. It has been assumed that the base station has knowledge of the eigenbeam $\mathbf{e}_k$ for each user k. Since the average properties of the channel do not change fast, the feedback of $\mathbf{e}_k$ is done at a low rate and is not comparable with the feedback of the SNR from all the users at each time slot to exploit multi-user diversity.

The probability density function of $|h_k|$ is shown in Figure 5 for different number of antennas and for the case of full correlation $|\rho_k| = 1$ for all users. It can be appreciated that as the number of antennas increases there is a higher probability for larger channel magnitude values which results in an increase in the multi-user diversity.

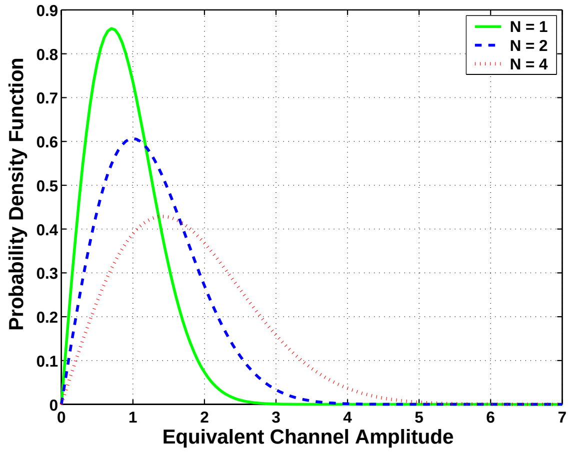

Figure 5: Dynamic range of eigenbeamforming with multi-user diversity, correlated Rayleigh fading

5. Comparison in a Correlated Fading Scenario

Let us now consider the downlink of a cell with a base station with $N = 4$ transmitting antennas with a maximum of $K = 64$ users in a fixed environment with one antenna each. The MISO channel model for each user k is depicted in Figure 3 as a block fading channel. The equivalent system of the downlink then represents a point-to-multi-point link. The channels $h_{nk}(t)$ are given by correlated Rayleigh fading, where the correlation is modeled by a correlation factor ρ_k as expressed in Equation (3) with different ψ_k for each user and with the same magnitude $|\rho_k|$ for all users. The average variance of the fading coefficients $h_{nk}(t)$'s is equal to 1 and the average SNR at each receiver is 0 dB. We consider a training phase where the receiver tracks and estimates the channel, measuring $|h_k|$ for opportunistic beamforming and the $|h_{nk}|$'s for eigenbeamforming, for each user k. The users are served with the proportional fair scheduler with different forgetting factors f.

In Figure 6, the multi-user diversity gain is shown for opportunistic beamforming (OB) and eigenbeamforming (EB) as a function of the number of users. The figure shows this for different magnitudes for the correlation factor $|\rho_k|$. The degree of multi-diversity is represented by the average sum throughput S of the system, where the throughput is given by the Shannon equation $C = \log_2(1 + \mathrm{SNR})$ where $\mathrm{SNR} = |h_k(t)|^2/\sigma_k^2$ with σ_k^2 as the variance of the noise at user k. The results pertain when the users are served through the GS. It can be observed that the multi-user diversity increases as $|\rho_k| \to 1$. This is due to the fact that the dynamic range increases with the degree of correlation for both OB and EB. However, for each value of $|\rho_k|$ eigenbeamforming outperforms opportunistic beamforming. The largest gain is when $|\rho_k| = 1$ even though OB is performing at its optimum for this scenario. This can also be observed from Figures 4 and 5, where for a given large amplitude $|h_k|$, there is a higher probability for this value in the eigenbeamforming case than in the opportunistic beamforming scheme.

The same performance can be observed for all forget-

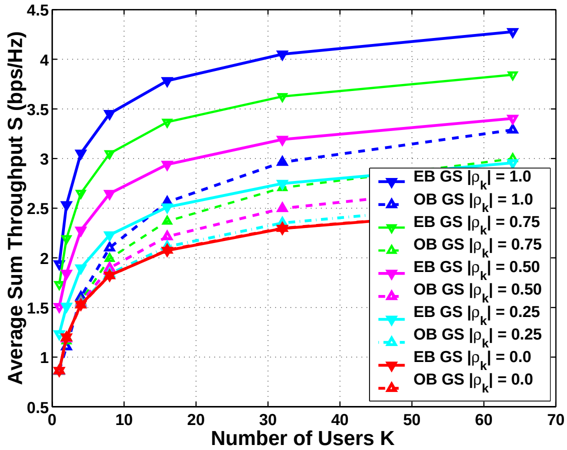

Figure 6: Multi-user diversity gain for OB and EB with different $|\rho_k|$, correlated Rayleigh fading

ting factors in Figure 7, where the average sum throughput S as a function of the forgetting factor has been plotted for a set of $K = 64$ users. As the forgetting factor decreases the multi-user diversity increases, at the expense of delay. We have that when $|\rho_k| = 0$ we have an uncorrelated channel and the performance of eigenbeamforming and opportunistic beamforming is the same. Meanwhile, for a fully correlated channel given by $|\rho_k| = 1$, the performance of eigenbeamforming is in average the same as that of coherent beamforming since the fading is the same on each of the antennas except for a difference in phase. Note again that the gain of EB over OB comes at the expense of a slight increase of the feedback, due to requirement of having the eigenbeam of each user at the transmitter, and with the training phase for estimating the channels $h_{nk}(t)$ for $n = 1, \ldots, N$ for each user.

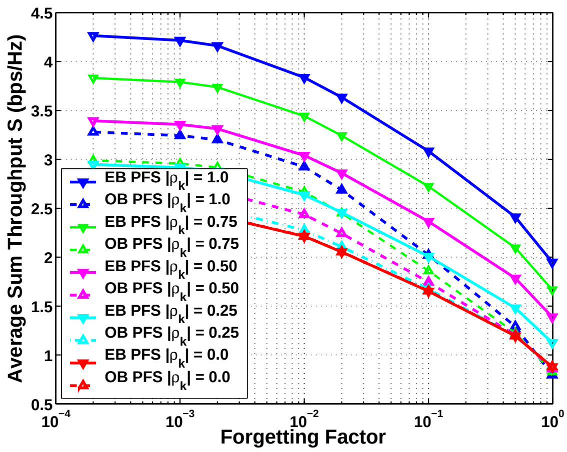

Figure 7: Multi-user diversity for OB and EB for different forgetting factors and $K = 64$, correlated Rayleigh fading

To evaluate this performance of the proportional fair scheduler with different forgetting factors regarding the delay for OB and EB, let us define the *outage delay* D_{out}. The outage delay D_{out} is related to a probability p_{out} as follows:

$$\text{Prob}\{D < D_{\text{out}}\} = 1 - p_{\text{out}}, \tag{5}$$

where p_{out} is the outage probability that a given delay D is larger than D_{out}. The delay D is given in number of time slots each consisting of T symbols. Each forgetting factor translates to an outage delay D_{out} which permits a certain degree of multi-user diversity. To observe this performance, in Figure 8 we plot the average sum throughput S as a function of the outage delay D_{out} for $K = 64$ users served with the PFS. The plot includes results for EB and OB for different $|\rho_k|$ with an outage probability $p = 2\%$. This figure shows the tradeoff between multi-user diversity and delay. As the average sum throughput increases, the outage delay also increases. EB needs a smaller outage delay to achieve a certain sum throughput than compared with OB.

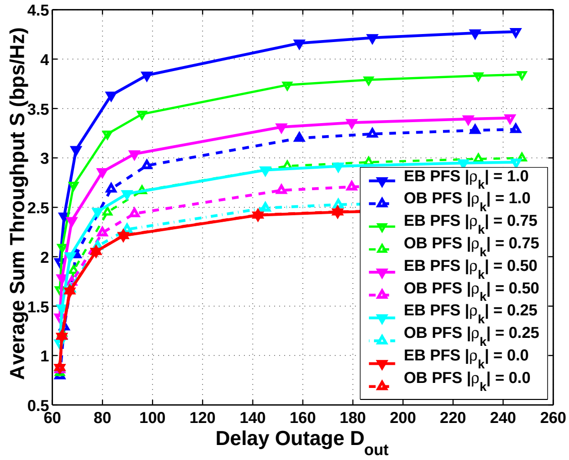

Figure 8: Tradeoff between multi-user diversity and delay, correlated Rayleigh fading

To express the gain of EB over OB as a function of $|\rho_k|$, let us define the relative gain:

$$\eta(|\rho_k|, K) = \frac{S_{\text{EB}}(|\rho_k|, K)}{S_{\text{OB}}(|\rho_k|, K)} \tag{6}$$

where S_{EB} and S_{OB} are the sum throughput achieved with the greedy scheduler as a function of the number of users and of the correlation factor $|\rho_k|$ for eigenbeamforming and opportunistic beamforming, respectively. In Figure 9, we have plotted η as a function of $|\rho_k|$ for different number of users. The value of η converges to 1 for any number of users as $|\rho_k| \to 0$. Meanwhile, for $|\rho_k| = 1$, η is the largest for any number of users. However, as the number of users increase η decreases. This is due to the fact that as there are more users in the cell, the probability of OB directing its opportunistic beam to a user increases. When $K \to \infty$, the performance of EB and OB would be the same for any $|\rho_k|$ and we would have that $\eta = 1$.

6. Conclusions

Opportunistic beamforming is able to increase the multi-user diversity present in a point-to-multi-point link with correlated channels. However, we have shown that by combining eigenbeamforming and the proportional fair scheduler the extracted multi-user diversity gain is larger than compared to opportunistic beamforming for any degree of correlation and for any forgetting factor. When the channels are fully correlated, eigenbeamforming performs in average the same as coherent beamforming as there is only one beam to each user. As the correla-

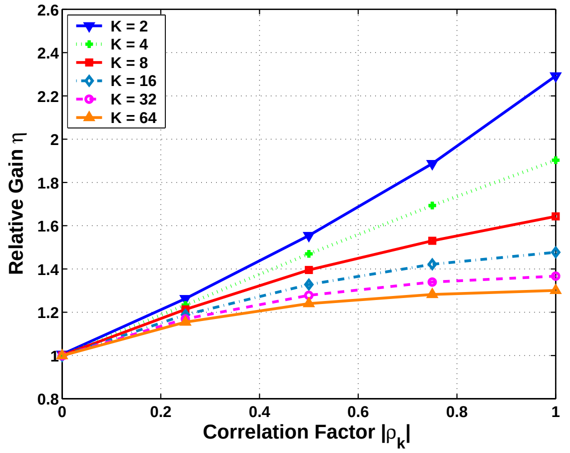

Figure 9: Relative gain of EB over OB as a function of $|\rho_k|$

tion decreases and $|\rho_k| \to 0$, the gain of eigenbeamforming over opportunistic beamforming decreases. When $|\rho_k| = 0$, there is no correlation among the channels, and both schemes perform the same. The performance of eigenbeamforming and opportunistic beamforming also converge as the number of users in the cell increases. Also, the outage delay for eigenbeamforming to achieve a certain performance is less than that for opportunistic beamforming. The gain of eigenbeamforming comes at the cost of a slight increase in feedback and of complexity in estimating the individual channels h_{nk} of the receivers. The partial CSI required for eigenbeamforming, the principal eigenvector of the channel correlation matrix, does not have to be fed back at a high rate. In fact, this partial CSI can be fed back periodically depending on the variation of the channel and at different time instances for each user to avoid a large amount of feedback to the base station at the same time slot in the uplink channel.

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