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Design of Single-Group Multicasting-Beamformers

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***Abstract*— For the single-group multicast scenario, where K users are served with the same data by a base station equipped with N antennas, we present two beamforming algorithms which outperform state-of-the-art multicast filters and feature a drastically reduced complexity at the same time. For the power minimization problem, where QoS constraints need to be satisfied, we introduce a successive beamforming filter computation approach aiming at satisfying at least one additional SNR constraint per *orthogonal* filter update. As long as the number of users K is smaller than the number N of transmit antennas, this procedure delivers excellent results. Our second approach is an iterative update algorithm for the *max-min* problem subject to a limitation of the transmit power. Given a low-complexity initialization beamformer, we search within the local vicinity of this filter vector for a filter-update preserving the transmit power and achieving a larger minimum SNR. To this end, we improve the weakest user's SNR during each iteration and keep on applying this procedure as long as the updates increase the smallest SNR. Otherwise, we adapt the step-size and continue investigating the local vicinity. It turns out that this novel approach is superior to existing state-of-the-art multicast beamformers for an arbitrary number of users.**

I. Introduction

In contrast to the broadcast channel scenario with different messages, where K different users are served with K different data streams by a base station equipped with possibly several antennas, multicasting is characterized by the existence of at least one group whose members are requesting the *same* data. Thus, there is no intra-group interference, only inter-group interference exists. Multi-group multicasting has for example been investigated in [1], [2], [3], whereas single-group multicasting, which we focus on, can be found in [4], [5], [6]. Given *Quality-of-Service* (QoS) constraints for the receive SNR of each user, a common problem formulation is the minimization of the transmit power constrained on a minimum level of these SNR values. In [4], it was shown that this problem is NP-hard in general. Sun et. al. tackle the problem by means of *sequential quadratic programming* (SQP) to obtain approximate solutions in [5]. A widely spread procedure to cope with nonconvex problems is to drop all nonconvex constraints, which is then termed as *semidefinite relaxation* (SDR) [7]. Obviously, leaving out constraints yields an improved cost function which may not be achievable, giving rise to an additional effort in order to end up with a solution that is based on the argument achieving the infeasible bound but fulfills also the nonconvex constraints. In [4], [6], [2], [1], and [3], the authors make use of the SDR of the NP-hard problem and apply *randomization* techniques to end up with a rank-one beamforming covariance matrix.

The drawback of these SDR approaches follows from the fact that *interior point* methods have to be applied, which may not be available on the specific hardware platform and can be slow. We present two fast algorithms which do not rely on any semidefinite programming tool to compute a beamforming vector either minimizing the required transmit power under minimum SNR constraints or maximizing the minimum SNR, subject to a limitation of the transmitted power. In Section II, we discuss the fundamental differences between the broadcast channel with *different* and *common* messages. After reviewing existing beamforming strategies for the multicast scenario in Section III, we introduce a beamformer computation strategy satisfying the constraints of a selection of two out of the K users in Section IV. This will later serve as an initialization beamformer for the iterative SNR increasing algorithm in Section VI. A successive version for the filter computation in the power minimization problem is discussed in Section V, simulation results in Section VII conclude the paper.

Notation: Vectors and matrices are denoted by bold-faced lower and upper case italic letters, respectively. The operators $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, and $(\cdot)^*$ stand for transposition, Hermitian transposition, and complex conjugate, respectively. $\mathrm{tr}(\cdot)$ denotes the trace of a matrix, $\angle(\cdot)$ the argument of a complex scalar, and $\Re\{\cdot\}$ the real part, respectively.

II. Broadcast with Separate/Common Messages

In this section, we highlight key differences between the broadcast channel with *separate* and *common* messages. We assume a centralized multi-antenna base station serving decentralized single-antenna receivers.

A. Broadcast Channel with Separate Messages

The broadcast channel with *separate* messages is characterized by the transmission of *different* data streams to the individual users resulting in the reception of unwanted data at each user, *i.e.*, in interference. Fig. II-A depicts the transmission of K different data streams $s_1, \ldots, s_K$ over the frequency flat channel to the K receivers. The propagation from all N transmit antennas to user k is described by the channel vector $\boldsymbol{h}_k \in \mathbb{C}^N$. Each data stream s_k, $k \in \{1, \ldots, K\}$ is spatially filtered by a vector $\boldsymbol{p}_k$ constituting to the transmit signal

$$\boldsymbol{y}_{\mathrm{BC}} = \sum\nolimits_{k=1}^{K} \boldsymbol{p}_k s_k \in \mathbb{C}^N. \tag{1}$$

At the receivers, zero-mean noise $\eta_1, \ldots, \eta_K$ with variance σ_η^2 is added. Thus, the desired signal portion at user k reads as $\boldsymbol{h}_k^{\mathrm{T}} \boldsymbol{p}_k s_k$, whereas the unwanted interference can be expressed

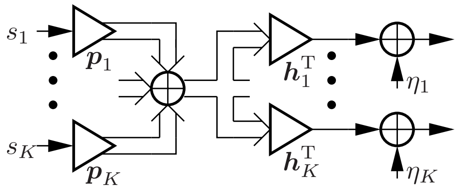

Fig. 1. Broadcast downlink

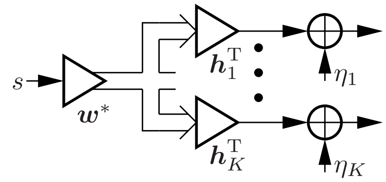

Fig. 2. Multicast downlink

as $\boldsymbol{h}_k^{\mathrm{T}} \sum_{j \neq k} \boldsymbol{p}_j s_j$. The noise amount and the interference give rise to the SINR (*signal to interference plus noise ratio*) definition of user k (e.g. [8], [9], [10]):

$$\mathrm{SINR}_k(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_K) = \frac{|\boldsymbol{h}_k^{\mathrm{T}} \boldsymbol{p}_k|^2}{\sum_{j \neq k} |\boldsymbol{h}_k^{\mathrm{T}} \boldsymbol{p}_j|^2 + \sigma_\eta^2}, \tag{2}$$

where we assumed uncorrelated unit variance symbols $s_k \ \forall k$. A frequently arising problem formulation is the minimization of the transmit power $\sum_{\ell=1}^{K} \|\boldsymbol{p}_\ell\|_2^2$ constrained on minimum levels γ_k for each SINR_k:

$$\min_{\{\boldsymbol{p}_1, \ldots, \boldsymbol{p}_K\}} \sum_{\ell=1}^{K} \|\boldsymbol{p}_\ell\|_2^2 \quad \mathrm{s.t.:} \ \mathrm{SINR}_k(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_K) \geq \gamma_k \ \forall k. \tag{3}$$

B. Broadcast Channel with Common Messages: Multicast

In contrast, the multicast scenario consists of at least one group of receivers within which *all* users request *the same* data, *i.e.*, $s_k = s \ \forall k$. Hence, there is no intra-group interference [3], [2], so it can be seen as a broadcast channel with common messages. Since we focus on the single-group multicasting case (see also [5], [4], [6]), interference is not present within this system configuration. The transmit signal $\boldsymbol{y}_{\mathrm{MC}}$ thus reduces to

$$\boldsymbol{y}_{\mathrm{MC}} = \sum\nolimits_{k=1}^{K} \boldsymbol{p}_k s_k = s \sum\nolimits_{k=1}^{K} \boldsymbol{p}_k =: \boldsymbol{w}^* s, \tag{4}$$

where $\boldsymbol{w} \in \mathbb{C}^N$. Consequently, the precoder structure in Fig. II-B slightly changes compared to Fig. II-A. By virtue of the lack of interference, the SINR of user k is the same as its SNR (*signal to noise ratio*) and reads as

$$\mathrm{SNR}_k(\boldsymbol{w}) = \sigma_\eta^{-2} |\boldsymbol{w}^{\mathrm{H}} \boldsymbol{h}_k|^2. \tag{5}$$

Again, the power minimization problem can be expressed as (cf. [4], [5], [6], [2])

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|_2^2 \quad \mathrm{s.t.:} \ |\boldsymbol{w}^{\mathrm{H}} \boldsymbol{h}_k|^2 \geq \gamma_k \sigma_\eta^2 \ \forall k. \tag{6}$$

(6) is a *quadratically constrained quadratic program* (QCQP) and obviously, the constraints in (6) are nonconvex, leading to the fact that no efficient solutions are available. Note that (6) is always feasible as long as $K < \infty$ and that the cost function and the constraints are invariant w.r.t. a unit norm complex scalar. In [4], it is shown that (6) is NP-hard in general.

Besides the lack of interference, another decisive difference between the BC and the MC scenario concerns the constraints in (3) and (6) for the optimum precoder(s): in the BC case, it was shown in [8], [9], [10] that *all* constraints are fulfilled with *equality*. By contrast, the optimum $\boldsymbol{w}$ need *not* fulfill all inequalities with equality [4], but *at least* one inequality must be fulfilled with equality, otherwise $\boldsymbol{w}$ could be scaled to reduce the transmit power. Up to K equalities may occur.

Another frequently occurring problem we will focus on later is the maximization of the minimum SNR constrained on a maximum power consumption:

$$\max_{\boldsymbol{w}} \min_{k} |\boldsymbol{w}^{\mathrm{H}} \boldsymbol{h}_k|^2 \quad \mathrm{s.t.:} \ \|\boldsymbol{w}\|_2^2 \leq P_{\mathrm{tr}}. \tag{7}$$

In [4], problem (7) was shown to be equivalent up to a scalar to problem (6), and thus, is NP-hard.

III. Review of Existing Multicast Beamforming Strategies

Sun et al. apply *sequential quadratic programming* (SQP) to find approximate solutions to the max-min problem (7), see [5]. There, the constraint in (7) is linearized and the cost function is modeled by a quadratic one at each iterate, and then, a new search direction is obtained on the basis of subproblems, whose objectives are approximations of the Lagrangian functions [11]. In general, SQP is a class of *nonlinear program* (NLP) solvers, such as *interior point* (IP) methods also represent an NLP solver class. A disadvantage of SQP methods is the need for a suitable initialization point and a proper stopping criterion. The initialization point drastically influences the local optimum in which the SQP algorithm will become trapped. Another problem of SQP is the fact, that the Hessian of the Lagrangian may become indefinite if the starting point is not close to a minimum.

Another popular approach [3], [4], [6], [2] is to solve the *semidefinite relaxed* (SDR) version of (6) by dropping all *nonconvex* constraints of the reformulated version of (6):

$$\begin{aligned} \min_{\boldsymbol{X}} \mathrm{tr}(\boldsymbol{X}) \quad \mathrm{s.t.:} \ & \mathrm{tr}(\boldsymbol{X}\boldsymbol{Q}_k) \geq \sigma_\eta^2 \gamma_k \ \forall k, \quad \boldsymbol{X} \succeq \boldsymbol{0}, \\ & \mathrm{rank}(\boldsymbol{X}) = 1, \end{aligned} \tag{8}$$

where $\boldsymbol{X} := \boldsymbol{w}\boldsymbol{w}^{\mathrm{H}}$ and $\boldsymbol{Q}_k := \boldsymbol{h}_k \boldsymbol{h}_k^{\mathrm{H}}$. Note that the set of matrices $\boldsymbol{X}$ fulfilling the first constraint is convex and the set of positive definite matrices is a convex cone [7]. The cost function is also convex in $\boldsymbol{X}$. However, the third constraint does *not* describe a convex set, as the convex sum of two rank-one matrices is in general of rank two. Dropping this constraint, the SDR version of (6) achieves a lower bound on the required transmit power $\|\boldsymbol{w}\|_2^2$. Interestingly, the SDR problem corresponds to the *Lagrange dual problem* [4], which is convex by nature and always yields a bound on the optimum value of the cost function [7].

Now, the convex SDR problem is solved by means of any interior point algorithm, such as *Sedumi* [12]. If the resulting optimum solution $\boldsymbol{X}_{\rm opt}$ has rank one, the global optimum has been found. If not, *randomization* is applied: Given the EVD (*eigenvalue decomposition*) $\boldsymbol{X}_{\rm opt} =: \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\rm H}$ of $\boldsymbol{X}_{\rm opt}$, a weighted sum of the columns $\boldsymbol{u}_k$ in $\boldsymbol{U}$ constitutes the beamforming vector $\boldsymbol{w}$: E.g., the *randA* method in [4] applies

$$\boldsymbol{w} = \alpha_m \boldsymbol{U}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{v}_m, \tag{9}$$

with $\boldsymbol{v}_\ell$ denoting the realization of a complex random vector, where each entry in $\boldsymbol{v}_\ell$ has norm one. Thus, $\|\boldsymbol{U}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{v}_m\|_2^2 = \operatorname{tr}(\boldsymbol{X}_{\rm opt})$, but obviously, not all SNR constraints $|\boldsymbol{w}^{\rm H}\boldsymbol{h}_k|^2 \geq \gamma_k\sigma_\eta^2$ can be fulfilled with $\boldsymbol{w} = \boldsymbol{U}\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{v}_m$. Therefore, a rescaling is necessary with $\alpha_\ell > 1$, such that all constraints are met. The resulting transmit power then reads as $\alpha_\ell^2 \operatorname{tr}(\boldsymbol{X}_{\rm opt})$. Having generated $\ell = 1, \ldots, L_{\max}$ realizations $\boldsymbol{v}_\ell$, one selects $\boldsymbol{v}_m$ with the minimum transmit power, where $1 < \alpha_m \leq \alpha_\ell\ \forall \ell$. For some optimizations like the MAX-CUT problem and certain randomization techniques, there exist limits on how far away the bound lies from the randomized solution [13]. In [2], the computational cost is rated 'negligible' compared to the solution of the semidefinite relaxed problem via interior point methods. We will later show simulation results on the rank of the optimum SDR matrix $\boldsymbol{X}_{\rm opt}$.

IV. Two-User-Combination Filter Approach

Let us assume for a moment that only $K = 2$ users shall be served. Then, the power minimization problem (6) reads as

$$\min_{\boldsymbol{w}} \|\boldsymbol{w}\|_2^2 \text{ s.t.}: |\boldsymbol{w}^{\rm H}\boldsymbol{h}_1|^2 \geq c_1, \text{ and } |\boldsymbol{w}^{\rm H}\boldsymbol{h}_2|^2 \geq c_2, \tag{10}$$

where $c_i = \gamma_i\sigma_\eta^2$. If the base station consists of only a single antenna, *i.e.*, $N = 1$, the obvious solution of (10) is

$$|w| = \max_{i\in\{1,2\}} \frac{\sqrt{c_i}}{|h_i|} \tag{11}$$

with arbitrary phase angle. As soon as the base station is equipped with at least $N \geq 2$ antennas, three relevant cases[1] need to be distinguished for the optimum precoding filter $\boldsymbol{w}$:

1) The first constraint in (10) is met with equality, whereas the second one is over-satisfied. $\boldsymbol{w}$ must then be collinear to $\boldsymbol{h}_1$ to achieve a minimum transmit power $\|\boldsymbol{w}\|_2^2$:
$$\boldsymbol{w} = \frac{\sqrt{c_1}}{\|\boldsymbol{h}_1\|_2^2}\boldsymbol{h}_1, \tag{12}$$
with $\|\boldsymbol{w}\|_2^2 = c_1\|\boldsymbol{h}_1\|_2^{-2}$. If the second constraint shall be over-satisfied, the following must hold:
$$|\boldsymbol{h}_1^{\rm H}\boldsymbol{h}_2|^2 \geq \frac{c_2}{c_1}\|\boldsymbol{h}_1\|_2^4. \tag{13}$$
2) The second constraint in (10) is met with equality, whereas the first one is over-satisfied. $\boldsymbol{w}$ then must be collinear to $\boldsymbol{h}_2$ to achieve the minimum transmit power:
$$\boldsymbol{w} = \frac{\sqrt{c_2}}{\|\boldsymbol{h}_2\|_2^2}\boldsymbol{h}_2, \tag{14}$$
with $\|\boldsymbol{w}\|_2^2 = c_2\|\boldsymbol{h}_2\|_2^{-2}$. If the first constraint shall be over-satisfied, the following must hold:
$$|\boldsymbol{h}_1^{\rm H}\boldsymbol{h}_2|^2 \geq \frac{c_1}{c_2}\|\boldsymbol{h}_2\|_2^4. \tag{15}$$
Inequalities (13) and (15) are mutually exclusive[2].
3) Both constraints in (10) are fulfilled with equality:
$$\boldsymbol{H}\boldsymbol{w} = \begin{bmatrix} \sqrt{c_1}{\rm e}^{{\rm j}\varphi_1} \\ \sqrt{c_2}{\rm e}^{{\rm j}\varphi_2} \end{bmatrix} \tag{16}$$
with $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2]^{\rm H} \in \mathbb{C}^{2\times N}$ and arbitrary, real-valued φ_1 and φ_2. Since we already treated the case $N = 1$, (16) can be solved:
$$\boldsymbol{w} = \boldsymbol{H}^{\rm H}(\boldsymbol{H}\boldsymbol{H}^{\rm H})^{-1}\begin{bmatrix} \sqrt{c_1}{\rm e}^{{\rm j}\varphi_1} \\ \sqrt{c_2}{\rm e}^{{\rm j}\varphi_2} \end{bmatrix} = y_1\boldsymbol{h}_1 + y_2\boldsymbol{h}_2. \tag{17}$$
Then, the transmit power $\|\boldsymbol{w}\|_2^2$ can be computed as
$$\|\boldsymbol{w}\|_2^2 = c_1 z_1 + c_2 z_2 + 2\sqrt{c_1 c_2}\Re\{{\rm e}^{{\rm j}(\varphi_2-\varphi_1)}z_3\}, \tag{18}$$
with z_1, z_2, and z_3 being the upper left, lower right, and upper right element of the 2×2 matrix $(\boldsymbol{H}\boldsymbol{H}^{\rm H})^{-1}$. Since the cost function in (10) is invariant with respect to a phase rotation, we can choose $\varphi_1 = 0$ w.l.o.g. Minimizing $\|\boldsymbol{w}\|_2$ is therefore equivalent to minimizing $\Re\{{\rm e}^{{\rm j}\varphi_2}z_3\}$ w.r.t. φ_2. The optimum solution reads as
$$\varphi_2 = \pi - \angle z_3, \tag{19}$$
leading to $\Re\{{\rm e}^{{\rm j}\varphi_2}z_3\} = -|z_3|$. Inserting (19) into (18) and explicitly computing the 2×2 inverse, we obtain
$$\|\boldsymbol{w}\|_2^2 = \frac{c_1\|\boldsymbol{h}_1\|_2^2 + c_2\|\boldsymbol{h}_2\|_2^2 - 2\sqrt{c_1c_2}|\boldsymbol{h}_1^{\rm H}\boldsymbol{h}_2|}{\|\boldsymbol{h}_1\|_2^2\|\boldsymbol{h}_2\|_2^2 - |\boldsymbol{h}_1^{\rm H}\boldsymbol{h}_2|^2}. \tag{20}$$

The number of *floating point operations* (FLOPS) to find the optimum precoder in the two user case is $7N + \mathcal{O}(1)$, if case 1 or 2 is optimal,[3] or $9N + \mathcal{O}(1)$, if case 3 is optimal [14].

A. Full Featured Combine-2 (FF-C2)

Having selected two out of the K users, this algorithm computes the lowest norm precoding vector $\boldsymbol{w}_{\rm C2}^{i,j}$ that meets the SNR-constraints of these two users i and j, *i.e.*, either (12), (14), or (17) with indices i, j instead of $1, 2$. Based on this precoder, the SNRs of the remaining $K-2$ users are computed, and $\boldsymbol{w}_{\rm C2}^{i,j}$ is scaled by $\alpha^{i,j} \geq 1$ such that *all* SNR constraints are fulfilled. There are $K(K-1)/2$ possibilities to choose two users out of K. Finally, the *full featured Combine-2* filter vector reads as $\alpha^{\check{i},\check{j}}\boldsymbol{w}_{\rm C2}^{\check{i},\check{j}}$ with $\{\check{i},\check{j}\} = \operatorname{argmin}_{i,j,i\neq j} \|\alpha^{i,j}\boldsymbol{w}_{\rm C2}^{i,j}\|_2^2$, *i.e.*, the precoder out of the $K(K-1)/2$ with the smallest transmit power is chosen. A smart implementation takes about $K^2(N + 10) + KN + \mathcal{O}(1)$ FLOPs.

B. Reduced Complexity Combine-2 (RC-C2)

Based on the heuristic that the user with the smallest metric $c_k^{-1}\|\boldsymbol{h}_k\|_2^2$ is likely to be part of the two-user set $\{\check{i},\check{j}\}$, a complexity reduction of above full featured algorithm can be achieved, as only $K-1$ sets with cardinality 2 exist under this assumption. However, this approach may be inferior to the one in Section IV-A in some cases, but it will serve as an excellent initialization for the iterative algorithm in Section VI. A smart implementation takes about $4NK + 21K - 2N + \mathcal{O}(1)$ FLOPs.

[1] The case when both inequalities in (10) are not met with equality need not be taken into account since then, $\boldsymbol{w}$ could be reduced in length to save transmit power.

[2] Multiply (13) by (15) and apply the *Cauchy-Schwarz-inequality*.

[3] This can be determined by comparing (20) with either $c_1^{-1}\|\boldsymbol{h}_1\|_2^{-2}$ or $c_2^{-1}\|\boldsymbol{h}_2\|_2^{-2}$.

V. Successive Beamforming-Filter Computation

A more sophisticated approach to minimizing the transmit power while satisfying all SINR constraints is the following *successive* filter computation:

Starting with an initial precoder $\boldsymbol{w}^{(1)} = \alpha^{(1)}\boldsymbol{h}_{\ell^{(1)}}$, we update $\boldsymbol{w}^{(n)}$ in step n, such that at least one additional SNR constraint is fulfilled. In order not to destroy previously *with equality* fulfilled constraints, the update direction in step n is *orthogonal* to the channel vectors of all users $\ell^{(1)}, \ldots, \ell^{(n-1)}$, whose SNR constraints have already been met with equality, thus leaving their respective SNRs unchanged. A detailed description is shown in Algorithm 1:

Algorithm 1 Successive Beamforming Filter Computation

1: $\boldsymbol{h}_k^{(\mathrm{orig})} = \boldsymbol{h}_k \ \forall k$ *save channels*
2: $\ell^{(1)} \leftarrow \operatorname{argmin}_k c_k^{-1}\|\boldsymbol{h}_k\|_2^2$ *select 'weakest' user*
3: $\boldsymbol{w}^{(1)} \leftarrow \frac{\sqrt{c_{\ell^{(1)}}}}{\|\boldsymbol{h}_{\ell^{(1)}}\|_2^2}\boldsymbol{h}_{\ell^{(1)}}$ *set filter*
4: $\mathbb{U} \leftarrow \{1,\ldots,K\}\backslash\ell^{(1)}$
5: **for** $n = 2$ to N **do**
6: $\quad \ell^{(n)} \leftarrow \operatorname{argmin}_{k\in\mathbb{U}} \frac{\mathrm{SNR}_k(\boldsymbol{w}^{(n-1)})}{\gamma_k}$ *most violated constr.*
7: $\quad$ **if** $\frac{\mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n-1)})}{\gamma_{\ell^{(n)}}} > 1 \rightarrow$ **EXIT** *exit if all constr. met*
8: $\quad$ Orthogonalization $\forall k \in \mathbb{U}$: *see (21)*
$\quad \boldsymbol{h}_k \leftarrow \boldsymbol{h}_k$ orth. w.r.t. $\boldsymbol{h}_{\ell^{(n-1)}}$
9: $\quad \boldsymbol{w}^{(n)} \leftarrow \boldsymbol{w}^{(n-1)} + \alpha^{(n)}\boldsymbol{h}_{\ell^{(n)}}$ *update via (27) and (28)*
10: $\quad \mathbb{U} \leftarrow \mathbb{U}\backslash\ell^{(n)}$
11: **end for**

First of all, all channel vectors are saved into $\boldsymbol{h}^{\mathrm{orig}}$ as they will be reassigned during the procedure. In Line 2, we select the 'weakest' user $\ell^{(1)}$ whose SNR-constraint satisfaction would take most energy. This choice is motivated by the fact that if only a single constraint in (6) was fulfilled with equality and all other constraints were over-satisfied, the filter vector $\boldsymbol{w}$ would be a scaled version of the channel vector $\boldsymbol{h}_{\ell^{(1)}}$ of user $\ell^{(1)}$. Line 3 sets the filter vector as the scaled channel of user $\ell^{(1)}$. The set $\mathbb{U}$ stores the indices of all users, whose SNR constraint have not yet been fulfilled *with equality*, see Line 4. The successive filter update is done in Lines 6 to 10, which run at most $N-1$ times, namely then, if all SNR constraints are met with *equality* in the end. In Line 6, we search again for the SNR constraint that is farthest away from being fulfilled among the set of users $\mathbb{U}$.[4] The algorithm aims at satisfying the constraint of this user $\ell^{(n)}$ with *equality* during step n. Within this update procedure, other SNR constraints might become over-satisfied. If *all* constraints have been met after a filter update, the algorithm terminates in Line 7. The computation of the SNR values $\mathrm{SNR}_k(\boldsymbol{w}^{(n-1)})$ is achieved by computing $|\boldsymbol{w}^{(n-1)\,\mathrm{H}}\boldsymbol{h}_k^{\mathrm{orig}}|^2\sigma_n^{-2} \ \forall k \in \mathbb{U}$, the required number of FLOPs to do so reads as $(2N+1)(K-n+1)$.

In order not to destroy the constraints that are met *with equality*, we allow only for update directions $\boldsymbol{h}_{\ell^{(n-1)}}$ that are orthogonal to all previous update directions. To this end, we rewrite the $\boldsymbol{h}_k$ in Line 8 by orthogonalizing them w.r.t. to the previous update direction. Thus, the resulting update direction $\boldsymbol{h}_{\ell^{(n)}}$ is perpendicular to all $\boldsymbol{h}_{\ell^{(m)}}^{\mathrm{orig}}$ with $m = 1,\ldots,n-1$ due to the manifold projection:

$$\boldsymbol{h}_k \leftarrow \boldsymbol{h}_k - \frac{\boldsymbol{h}_{\ell^{(n-1)}}^{\mathrm{H}}\boldsymbol{h}_k}{\|\boldsymbol{h}_{\ell^{(n-1)}}\|_2^2}\boldsymbol{h}_{\ell^{(n-1)}} \ \forall k \in \mathbb{U}. \tag{21}$$

Having found the update direction $\boldsymbol{h}_{\ell^{(n)}}$, it remains to find the step size $\alpha^{(n)}$ ensuring equality of user $\ell^{(n)}$'s SNR constraint. The filter update rule in Line 9

$$\boldsymbol{w}^{(n)} = \boldsymbol{w}^{(n-1)} + \alpha^{(n)}\boldsymbol{h}_{\ell^{(n)}} \tag{22}$$

leads to the norm

$$\|\boldsymbol{w}^{(n)}\|_2^2 = \|\boldsymbol{w}^{(n-1)}\|_2^2 + |\alpha^{(n)}|^2\|\boldsymbol{h}_{\ell^{(n)}}\|_2^2, \tag{23}$$

since $\boldsymbol{h}_{\ell^{(n)}}$ is also orthogonal to $\boldsymbol{w}^{(n-1)}$. This follows from the manifold orthogonalization of the channel vectors in (21), leading to the fact that the update $\alpha^{(n)}\boldsymbol{h}_{\ell^{(n)}}$ in (22) is orthogonal to all previous updates. From (23), we observe that a small $|\alpha^{(n)}|$ is desirable. Furthermore, the SNR constraint

$$|\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}}|^2 \stackrel{!}{=} c_{\ell^{(n)}} \tag{24}$$

of the weakest user $\ell^{(n)}$ must hold. Inserting (22) into (24), we obtain

$$\begin{aligned} c_{\ell^{(n)}} \stackrel{!}{=} & 2|\alpha^{(n)}|\Re\{\mathrm{e}^{\mathrm{j}\angle\alpha^{(n)}}\boldsymbol{w}^{(n-1)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}\} \\ & + |\alpha^{(n)}|^2|\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}}|^2 + \mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n-1)})\sigma_\eta^2, \end{aligned} \tag{25}$$

cf. (5). Because the constraint of user $\ell^{(n)}$ is not fulfilled, $c_{\ell^{(n)}} - \mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n-1)})\sigma_\eta^2 > 0$, and thus, the sum of the first two expressions in (25) that depend on $|\alpha^{(n)}|$ is positive. Moreover, $\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}} = \boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}$, and we find:

$$\begin{aligned} c_{\ell^{(n)}} - \mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n-1)})\sigma_\eta^2 \stackrel{!}{=} & |\alpha^{(n)}|\|\boldsymbol{h}_{\ell^{(n)}}\|_2^2 \cdot \\ & \left[|\alpha^{(n)}|\|\boldsymbol{h}_{\ell^{(n)}}\|_2^2 + 2\Re\{\mathrm{e}^{\mathrm{j}\angle\alpha^{(n)}}\boldsymbol{w}^{(n-1)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}}\}\right]. \end{aligned} \tag{26}$$

Minimizing $|\alpha^{(n)}|$ subject to (26) yields

$$\angle\alpha^{(n)} = -\angle\boldsymbol{w}^{(n-1)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}}, \tag{27}$$

$$|\alpha^{(n)}| = \frac{-|z| + \sqrt{|z|^2 + c_{\ell^{(n)}} - \mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n-1)})\sigma_\eta^2}}{\|\boldsymbol{h}_{\ell^{(n)}}\|_2^2}, \tag{28}$$

with $|z| = |\boldsymbol{w}^{(n-1)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{orig}}|$. Finally, user $\ell^{(n)}$ is removed from the set $\mathbb{U}$ in Line 10. Alg. 1 is always applicable, when $K \leq N$, for $K > N$, not all SNR constraints might be fulfilled after $n = N$. Then, the beamforming vector $\boldsymbol{w}^{(N_{\mathrm{Tx}})}$ can for example be rescaled such that all constraints are fulfilled.

The worst case complexity of Algorithm 1 approximately computes to $K^2(6N+2.5) + K(4N+7.5) - 7N + \mathcal{O}(1)$ with $K \leq N$ assumed, cf. [14], hence it is quadratic in K.

VI. Iterative SNR-Increasing Update Algorithm

The successive algorithm presented in Section V turns out to perform very good as long as the number of users K is smaller than the number of transmit antennas N. For $N < K$, above algorithm needs to be modified, since after N steps, not all SNR constraints might be fulfilled. However, simply scaling the final beamformer $\boldsymbol{w}^{(N)}$ after N iterations to meet

[4] In case $\mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n-1)}) \geq \gamma_k$, the constraint is fulfilled.

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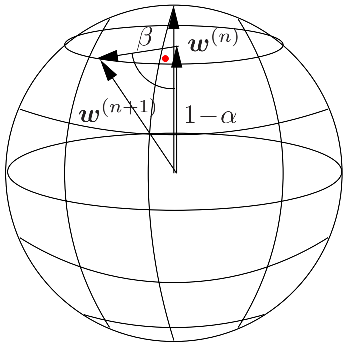

Fig. 3. Illustration of the filter update.

all constraints leads to poor performance results. Hence, we seek for an algorithm capable of handling this case.

In the following, we present an iterative update procedure which is applicable to *any* initialized beamformer. It targets at increasing the *minimum* SNR, hence focusing on the *max-min* problem (7), but as mentioned earlier, the two problems (7) and (6) are equivalent up to a scalar. The real-valued 'step size' $\alpha \in (0,1)$ is adapted during the iterations, but is not chosen optimally, since otherwise the algorithm would terminate after one iteration, and being suboptimal during each step even turns out to allow for better long-term properties (cf. introduction of an *over-relaxation* parameter during Gauss-Seidel iterations to improve convergence [15]).

Given a beamformer $\boldsymbol{w}^{(n)}$, the update procedure for step $n+1$ reads as

$$\boldsymbol{w}^{(n+1)} = (1-\alpha)\boldsymbol{w}^{(n)} + \beta \boldsymbol{w}^{(n)\perp}, \tag{29}$$

with $\boldsymbol{w}^{(n)\perp}$ being orthogonal to $\boldsymbol{w}^{(n)}$, *i.e.*,

$$\boldsymbol{w}^{(n)\,\mathrm{H}} \boldsymbol{w}^{(n)\perp} = 0, \tag{30}$$

$\|\boldsymbol{w}^{(n)\perp}\|_2 = 1$, and $\beta \in \mathbb{C}$. Geometrically speaking, the filter vector $\boldsymbol{w}^{(n)}$ is scaled in length by a factor of $(1-\alpha)$. Then, an orthogonal vector $\beta \boldsymbol{w}^{(n)\perp}$ is added, the length $|\beta|$ of which is chosen such that the updated vector $\boldsymbol{w}^{(n+1)}$ has the same norm (power) P_{tr} as $\boldsymbol{w}^{(n)}$ (cf. Fig. 3):

$$\|\boldsymbol{w}^{(n+1)}\|_2^2 \overset{!}{=} \|\boldsymbol{w}^{(n)}\|_2^2 = P_{\mathrm{tr}}. \tag{31}$$

Plugging (29) into (31), we find

$$|\beta| = \sqrt{P_{\mathrm{tr}}(2\alpha - \alpha^2)}. \tag{32}$$

The update procedure in (29) is

$$\boldsymbol{w}^{(n+1)} = (1-\alpha)\boldsymbol{w}^{(n)} + \boldsymbol{w}^{(n)\perp} \sqrt{P_{\mathrm{tr}}(2\alpha - \alpha^2)}\,\mathrm{e}^{\mathrm{j}\angle\beta}. \tag{33}$$

Following the idea of improving the SNR of the user

$$\ell^{(n)} = \underset{k}{\operatorname{argmin}}\, \mathrm{SNR}_k(\boldsymbol{w}^{(n)})/\gamma_k \tag{34}$$

with the weakest ratio $\mathrm{SNR}_k(\boldsymbol{w}^{(n)})/\gamma_k$ as in Section V, we choose the direction $\boldsymbol{w}^{(n)\perp}$ of the update such that the observed gain to user $\ell^{(n)}$ is as high as possible with the constraint that $\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{w}^{(n)\perp} = 0$. To this end, we express the SNR of user $\ell^{(n)}$ in terms of α, $\boldsymbol{w}^{(n)\perp}$, and $\angle\beta$, by means of (5), (29), and (32):

$$\begin{aligned}\frac{\mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n+1)})}{\sigma_\eta^{-2}} = {} & \underbrace{(1-\alpha)^2 \mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n)})\sigma_\eta^2}_{:=s_1} \\ & + \underbrace{P_{\mathrm{tr}}(2\alpha-\alpha^2)|\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp}|^2}_{:=s_2} \\ & + \underbrace{2(1-\alpha)\sqrt{P_{\mathrm{tr}}(2\alpha-\alpha^2)}}_{:=s_3} \\ & \cdot \underbrace{\Re\{\mathrm{e}^{\mathrm{j}\angle\beta}\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp}\}}_{:=s_4}.\end{aligned} \tag{35}$$

Now, $\angle\beta$ and $\boldsymbol{w}^{(n)\perp}$ are chosen to maximize (35) with α kept fixed in round n. The first term s_1 in (35) is proportional to the $\mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n)}) = \sigma_\eta^{-2}|\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}|^2$ obtained by the previous beamformer $\boldsymbol{w}^{(n)}$ and is constant for fixed α, as well as s_3. The only term depending on $\angle\beta$ is s_4: It is maximized for

$$\angle\beta = -\angle\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}} - \angle\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp}, \tag{36}$$

leading to the expression

$$s_4 = |\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}||\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp}|. \tag{37}$$

Now, we see that $\boldsymbol{w}^{(n)\perp}$ only occurs within the norm of the inner product $\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp}$: Both s_2 and s_4 from (37) depend on $|\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp}|$. Hence, (35) is maximized by choosing $\boldsymbol{w}^{(n)\perp}$ as 'collinear' as possible to $\boldsymbol{h}_{\ell^{(n)}}$, subject to (30). To this end, we project $\boldsymbol{h}_{\ell^{(n)}}$ into the null space of $\boldsymbol{w}^{(n)}$:

$$\boldsymbol{h}_{\ell^{(n)}}^{\perp} = \boldsymbol{h}_{\ell^{(n)}} - \frac{\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}}{P_{\mathrm{tr}}}\boldsymbol{w}^{(n)}. \tag{38}$$

Since $\boldsymbol{w}^{(n)\perp}$ is assumed to have unit norm, we must choose

$$\boldsymbol{w}^{(n)\perp} = \frac{\boldsymbol{h}_{\ell^{(n)}}^{\perp}}{\|\boldsymbol{h}_{\ell^{(n)}}^{\perp}\|_2}. \tag{39}$$

Making use of (38) and (39), the expression $\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp}$ in s_4 and s_2 boils down to the positive real-valued expression

$$\boldsymbol{h}_{\ell^{(n)}}^{\mathrm{H}}\boldsymbol{w}^{(n)\perp} = \boldsymbol{h}_{\ell^{(n)}}^{\perp\mathrm{H}}\boldsymbol{w}^{(n)\perp} = \|\boldsymbol{h}_{\ell^{(n)}}^{\perp}\|_2 \in \mathbb{R}_+. \tag{40}$$

From (36), we find by means of (40):

$$\angle\beta = -\angle\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}. \tag{41}$$

By means of (32), (41), (38), and (39), we have determined all relevant parameters to perform the update in (29).

We ensured the increase of the SNR of user $\ell^{(n)}$, but updating $\boldsymbol{w}^{(n)}$ to $\boldsymbol{w}^{(n+1)}$ might decrease the quotient $\mathrm{SNR}_k(\boldsymbol{w}^{(n+1)})/\gamma_k$ of an arbitrary user k below the *smallest* quotient $\mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n)})/\gamma_{\ell^{(n)}}$ of the previous step n. Then, the update step was not successful, and we decrease the step-size by a factor of 2, and β is computed again to check whether $\min_k \mathrm{SNR}_k(\boldsymbol{w}^{(n+1)})/\gamma_k > \mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n)})/\gamma_{\ell^{(n)}}$ is true. If not, α is again decreased, otherwise, the update was successful, we apply (29) and again *double* the step-size α as long as it is not already as large as the maximum step-size $\alpha_{\max}$. If the number of iterations reaches the maximum number of steps $n_{\max}$, the update procedure terminates, hopefully having found a beamforming vector $\boldsymbol{w}^{(n_{\max}+1)}$ which achieves a larger ratio $\min_k \mathrm{SNR}_k(\boldsymbol{w}^{(n_{\max}+1)})/\gamma_k$ than the initial one $\min_k \mathrm{SNR}_k(\boldsymbol{w}^{(1)})/\gamma_k$. Alg. 2 shows the detailed pseudo-code

Algorithm 2 Iterative SNR increasing update algorithm

1: $\alpha_{\max} \leftarrow 0.2,\ \alpha \leftarrow \alpha_{\max},\ n_{\max} \leftarrow 100$ *initialize variables*
2: $new\_\boldsymbol{w} \leftarrow 1$ *semaphore for change of* $\boldsymbol{w}^{(n)}$
3: **for** $n = 1$ to $n_{\max}$ **do**
4: **if** $new\_\boldsymbol{w} == 0$ **then**
5: $\ell^{(n)} \leftarrow \operatorname{argmin}_k \frac{\mathrm{SNR}_k(\boldsymbol{w}^{(n)})}{\gamma_k}$ *search 'weakest' constr.*
6: $m_{\min} \leftarrow \frac{\mathrm{SNR}_{\ell^{(n)}}(\boldsymbol{w}^{(n)})}{\gamma_{\ell^{(n)}}}$ *store minimum metric*
7: $\boldsymbol{h}^{\perp}_{\ell^{(n)}} \leftarrow \boldsymbol{h}_{\ell^{(n)}} - \frac{\boldsymbol{w}^{(n)\,\mathrm{H}}\boldsymbol{h}_{\ell^{(n)}}}{P_{\mathrm{tr}}}\boldsymbol{w}^{(n)}$ *orthogonalize, (38)*
8: $\boldsymbol{w}^{(n)\perp} \leftarrow \frac{\boldsymbol{h}^{\perp}_{\ell^{(n)}}}{\|\boldsymbol{h}^{\perp}_{\ell^{(n)}}\|_2}$ *norm-1 update direction, (39)*
9: $d\boldsymbol{w} \leftarrow \boldsymbol{w}^{(n)\perp}$ *save update direction*
10: **end if**
11: $|\beta| \leftarrow \sqrt{P_{\mathrm{tr}}(2\alpha - \alpha^2)}$ *compute magnitude of* β*, (32)*
12: $\boldsymbol{w}_{\mathrm{temp}} \leftarrow (1-\alpha)\boldsymbol{w}^{(n)} + \beta d\boldsymbol{w}$ *tryout update, (29)*
13: **if** $\min_k \frac{\mathrm{SNR}_k(\boldsymbol{w}_{\mathrm{temp}})}{\gamma_k} > m_{\min}$ **then**
14: $\boldsymbol{w}^{(n+1)} \leftarrow \boldsymbol{w}_{\mathrm{temp}}$ *perform update*
15: $new\_\boldsymbol{w} \leftarrow 1$ *indicate change of* $\boldsymbol{w}$
16: **if** $\alpha \leq \alpha_{\max}/2$ **then** $\alpha \leftarrow 2\alpha$ *increase step-size again*
17: **else**
18: $\boldsymbol{w}^{(n+1)} \leftarrow \boldsymbol{w}^{(n)}$ *no beamformer change*
19: $new\_\boldsymbol{w} \leftarrow 0$ *indicate NO change of* $\boldsymbol{w}$
20: $\alpha \leftarrow \alpha/2$ *reduce step-size*
21: **end if**
22: **end for**

of the approach. Line 4 reduces the computational load when the step-size α has been reduced, *i.e.*, $\boldsymbol{w}$ has not been updated. Then, Lines 5 to 9 need not be executed.

VII. Simulation Results

In Fig. 4, we see the required transmit power in dB versus the number of users K to be served by $N = 6$ transmit antennas. Here, we averaged over 1000 channel realizations of an i.i.d. channel. The solid curve with the square marker [Bound] represents the bound from the semidefinite relaxed problem in (8) without the rank-1 constraint. Note that this bound is not achievable, since the optimum $\boldsymbol{X}_{\mathrm{opt}}$ will in general turn out to have a rank larger than one. Then, randomization is applied, see Section III. We check $L_{\max} = 100$ random vectors $\boldsymbol{v}_m$ in (9) and chose the best one. The resulting beamformer has the star marker [Sedumi + Rand] and deviates approximately 2 dB from the infeasible bound for $K = 8$ users.

The two combine-2 approaches derived in Subsections IV-A and IV-B are shown as solid lines with empty circle markers [FF-C2] for the *full featured combine-2* version, and dashed lines with circle markers [RC-C2] for the reduced complexity version, respectively. They show a strong increase in the required transmit power when increasing the number of users from two to three, since an additional constraint needs to be fulfilled, but only two additional cardinality-two sets can be taken out of the $K = 3$ users. For $K = 5$, 213 FLOPs occur for the [RC-C2] algorithm.

Simulation results of the successive beamforming filter computation from Section V are plotted with triangle down

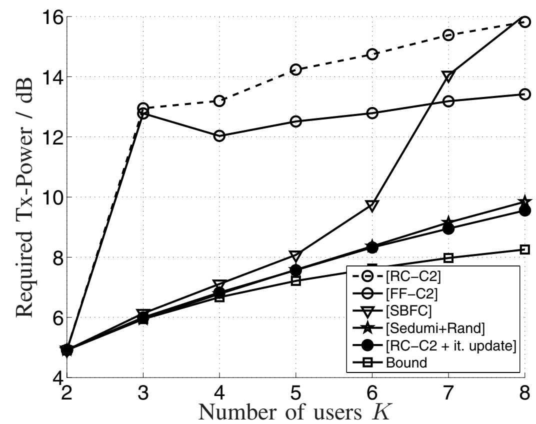

Fig. 4. Required Tx-power/dB vs. number of users K for $N = 6$ antennas.

markers [SBFC]. Up to $K = 5$ users, the curve is only slightly above [Sedumi+Rand], but at the same time, the complexity of [SBFC] is much lower: Applying the complexity formula at the end of Section V we obtain for $K = 5$ and $N = 6$ only 1078 FLOPs. In contrast, computing all SNRs during the $L_{\max}$ randomization operations already needs $K(2N+1)L_{\max} = 6500$ FLOPS for $L_{\max} = 100$. But in order to obtain the solution of [Sedumi+Rand], we also need to compute the EVD for (9) and set up $L_{\max}$ possible filter vectors $\boldsymbol{w}$ in (9). Moreover, the solution of the SDR problem is rated to entail a much larger complexity (see [2]) than the randomization process. Hence, [SBFC] can be regarded as an excellent low complexity solution as long as $K < N$. For $K \geq N$, [SBFC] faces severe degradations.

Applying the iterative SNR increasing algorithm from Section VI, initialized with the low-complexity solution [RC-C2] from Subsection IV-B, yields the solid curve with the filled circle marker [RC-C2+it. update]. Note that we chose $n_{\max} = 100$ iterations, *i.e.*, as many as for the randomization-based Sedumi technique. Interestingly, we outperform the randomization based Sedumi version [Sedumi+Rand] over the complete user range. Both the randomization and the iterative algorithm approximately share the same amount of FLOPs, since in every round, all SNRs for a given precoder need to be computed in both algorithms. However, the solution of the SDR problem is rated to entail a much larger complexity than the randomization as stated in [2], whereas the computational load to compute the low complexity initialization based on the *reduced complexity combine-2* approach (see Subsection IV-B) can completely be neglected compared to the iterative update of the beamformer. Summing up, our proposed filter solution outperforms existing ones with drastically less complexity.

Fig. 5 shows the dependency of the minimum SNR on the number K of users with $N = 4$ transmit antennas again averaged over 1000 channel realizations of an i.i.d. channel. For $K = 20$ users, the combination of randomization and Sedumi yields an SNR that has dropped approx. 2.5 dB below the infeasible bound. It can be observed, that for $K \leq 10$ users, there is a loss of approx. 2–3 dB for the full featured combine-

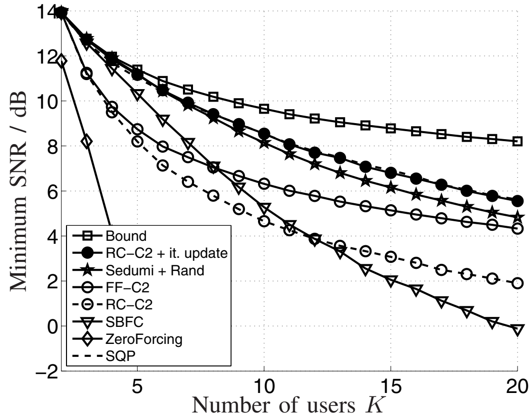

Fig. 5. Minimum SNR/dB vs. number of users K for $N = 4$ transmit antennas.

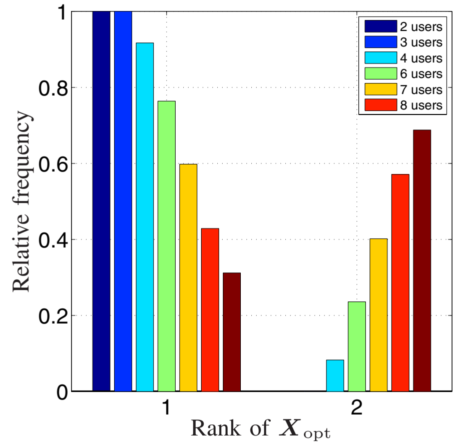

Fig. 6. Relative frequency vs. Rank for a different number of users K.

2 filter [FF-C2], and 3–4 dB for the reduced complexity version [RC-C2] compared to [Sedumi+Rand]. However, the [FF-C2] filter almost reaches the Sedumi solution based on randomization for $K = 20$ users. For $K \leq N = 4$, the results of the successive beamforming filter [SBFC] are very good, but for $K > N = 4$, simple rescaling of the filter vector leads to poor results. For example, [SBFC] is outperformed by the combine-2 approach [FF-C2] for $K > 8$ users. As long as $K \leq N = 4$, simple zero-forcing techniques $\boldsymbol{w} = c\boldsymbol{H}^{\mathrm{H}}(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})^{-1}\mathbf{1}$, with c chosen to fulfill the power constraint, known from the broadcast scenario with different messages [16], can also be applied, see the diamond marker curve [ZeroForcing]. However, the results are very poor, as the multicast structure (identical data symbols, no interference) is not exploited. For the simulation results of the SQP (dashed line, [SQP]), we utilized a free solver named *SOLNP* [17]. We observe that the SQP solutions almost exactly coincide with the results obtained by our novel iterative algorithm. However, as mentioned earlier, several drawbacks have to be faced when using the sequential quadratic programming: A good initialization point need to be chosen (we took [RC-C2]), and the Hessian of the Lagrangian turned out to become not positive definite in some cases, we terminated the SQP and chose the [RC-C2] solution then. We observe that our iterative SNR increasing approach [RC-C2+it.update] (filled circle marker) outperforms the semidefinite relaxed problem with randomization *over the complete user range*, despite the reduced complexity.

Finally, we investigated the rank of the optimum solution $\boldsymbol{X}_{\mathrm{opt}}$ of the SDR in (8) *without* the rank-one constraint for a different number of users K ranging from two up to eight. Here, we again have $N = 6$, *i.e.*, the rank must lie between one and $N = 6$. Eigenvalues smaller than 10^{-4} times the largest one have been set to zero. From Fig. 6 we see, that the maximum occurring rank for the i.i.d. channel is two and becomes more frequent, when the number of users increases. Simulation results with 20 antennas revealed, that the rank may become larger than two.

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