MMSE Block Decision-Feedback Equalizer for Spatial Multiplexing with Reduced Complexity

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***Abstract*— It was shown that enormous capacity advantage could be achieved on a flat MIMO channel compared to single antenna systems. V-BLAST was proposed to obtain such capacity advantage with low complexity. V-BLAST, however, requires multiple matrix (pseudo) inversions that are still computationally intensive for a large number of antennas. Many research activities reducing the complexity have been attracted in the last years. Our contribution is to show that the MMSE block decision-feedback equalizer equivalent to the MMSE V-BLAST can be calculated via Cholesky factorization of the *error covariance matrix* with *symmetric permutation*. Forward and backward filters as well as detection order are jointly optimized with significantly reduced complexity. Simulation results show that the MMSE V-BLAST performance can be achieved by the proposed scheme.**

I. Introduction

It was shown in [1] that enormous capacity increase can be achieved on flat *multiple input multiple output* (MIMO) channels in rich scattering environments. The capacity increase is linear with the number of transmit antennas unless it exceeds the number of receive antennas. To enable reliable communications in such systems, maximum-likelihood detection would be the optimum way, however, as the number of transmit antennas increases, the complexity of the receiver becomes prohibitive.

Vertical Bell Labs layered space-time (V-BLAST) was proposed in [2] as detection scheme with lower complexity. Independent data streams associated with different transmit antennas, called layers, are detected at the receiver by nulling out the interference of other layers in a successive manner. Also suggested is an optimum detection ordering which is of great importance for the successive interference cancellation.

The originally proposed V-BLAST in [2] calculates the nulling vector based on the *zero forcing* (ZF) criterion while in [3], [4] the *minimum mean square error* (MMSE) criterion is adapted to the V-BLAST architecture improving the performance. These detection schemes require calculation of either a pseudo inverse (ZF V-BLAST) or an inverse (MMSE V-BLAST) at every step of the layer detection which is still computationally expensive for a large number of data streams. Many research activities have been attracted in the last years to further reduce the complexity.

For the ZF criterion, computational reduction schemes have been proposed in [5], [6] which are based on QR factorization with suboptimum detection ordering. In [7] a Cholesky factorization is utilized with reordering by unitary transformation leading to the optimum detection ordering.

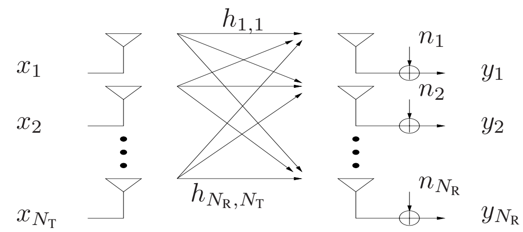

Fig. 1. System model of flat MIMO channel.

Similar contributions for the MMSE criterion based on QR factorization can be found in [8]–[10]. The ordering in [8] is suboptimum while in [9] the authors proposed an additional post-sorting algorithm using unitary transformation to improve the performance. The contribution in [10] also utilizes unitary transformation for reordering. The authors in [11] proposed to apply Cholesky factorization, however, this scheme does not involve their own ordering strategy.

Our contribution is to show that the MMSE block *decision-feedback equalizer* (DFE) equivalent to the optimum MMSE V-BLAST can be calculated via Cholesky factorization with *symmetric permutation* [12] applied to the *error covariance matrix*. Detection ordering, represented by a permutation matrix, is explicitly included into the optimization formulation. Feedforward and backward filters as well as detection ordering are jointly optimized with significantly reduced complexity; lower than the previously proposed schemes [9], [10].

Our system model is introduced in Section II. We review the MMSE V-BLAST algorithm in Section III. In Section IV our proposed algorithm is described and its complexity is analyzed in Section V. Simulation results are presented in Section VI and this paper is summarized in Section VII.

II. System Model

We consider a system equipped with N_T transmit antennas and N_R receive antennas where $N_\mathrm{T} \leq N_\mathrm{R}$. We assume the signals to be narrow band so that a non-dispersive fading channel is present. The discrete time system model in the equivalent complex baseband is illustrated in Fig. 1.

The channel inputs $x_i, i = 1, \ldots, N_\mathrm{T}$, are complex valued baseband signals and are transmitted from N_T antennas simultaneously. The channel tap gain from transmit antenna i

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to receive antenna j is denoted by $h_{j,i}$. These channel taps are assumed to be independent zero mean complex Gaussian variables of equal variance $\mathrm{E}[|h_{j,i}|^2] = 1$. This assumption of independent paths holds if antenna spacing is sufficiently large and the system is surrounded by rich scattering environments.

The received signals can be concisely expressed in matrix form as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \in \mathbb{C}^{N_\mathrm{R}}, \tag{1}$$

where $[\boldsymbol{H}]_{j,i} = h_{j,i}$, $\boldsymbol{y} = [y_1, \ldots, y_{N_\mathrm{R}}]^\mathrm{T}$, $\boldsymbol{x} = [x_1, \ldots, x_{N_\mathrm{T}}]^\mathrm{T}$, $\boldsymbol{n} = [n_1, \ldots, n_{N_\mathrm{R}}]^\mathrm{T}$ is the received noise, and $(\bullet)^\mathrm{T}$ denotes transposition.

III. MMSE V-BLAST

We review the MMSE V-BLAST algorithm in this section. Let us first consider the error signal of a linear filter $\boldsymbol{F}^\mathrm{H}$ applied to the received vector $\boldsymbol{y}$

$$\boldsymbol{\varepsilon} = \boldsymbol{F}^\mathrm{H}\boldsymbol{y} - \boldsymbol{x}, \tag{2}$$

where $(\bullet)^\mathrm{H}$ denotes Hermitian transpose. The linear MMSE filter can be found with the *orthogonality principle*, that is $\mathrm{E}[\boldsymbol{\varepsilon}\boldsymbol{y}^\mathrm{H}] = \boldsymbol{0}$. From (1) and (2), the solution is given by

$$\boldsymbol{F}^\mathrm{H} = \boldsymbol{\Phi}_{xx}\boldsymbol{H}^\mathrm{H}\left(\boldsymbol{H}\boldsymbol{\Phi}_{xx}\boldsymbol{H}^\mathrm{H} + \boldsymbol{\Phi}_{nn}\right)^{-1}, \tag{3}$$

where the covariance matrices of the channel input and the noise are defined as

$$\boldsymbol{\Phi}_{xx} = \mathrm{E}\left[\boldsymbol{x}\boldsymbol{x}^\mathrm{H}\right] \quad \text{and} \quad \boldsymbol{\Phi}_{nn} = \mathrm{E}\left[\boldsymbol{n}\boldsymbol{n}^\mathrm{H}\right], \tag{4}$$

respectively. Assuming that the covariance matrices in (4) are invertible, and with (2) and (3), the error covariance matrix can be expressed as

$$\boldsymbol{\Phi}_{\varepsilon\varepsilon} = \mathrm{E}\left[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^\mathrm{H}\right] = \left(\boldsymbol{\Phi}_{xx}^{-1} + \boldsymbol{H}^\mathrm{H}\boldsymbol{\Phi}_{nn}^{-1}\boldsymbol{H}\right)^{-1}, \tag{5}$$

where we applied the *matrix inversion lemma* [13]. Using this lemma, Equation (3) may be rewritten in alternative form as

$$\boldsymbol{F}^\mathrm{H} = \boldsymbol{\Phi}_{\varepsilon\varepsilon}\boldsymbol{H}^\mathrm{H}\boldsymbol{\Phi}_{nn}^{-1}. \tag{6}$$

Note that the diagonal entries of $\boldsymbol{\Phi}_{\varepsilon\varepsilon}$ represent the MSEs of the respective channel inputs, i.e. $\mathrm{E}[|x_i - \hat{x}_i|^2], i = 1, \ldots, N_\mathrm{T}$. Thus, the channel input having the minimum diagonal entry of $\boldsymbol{\Phi}_{\varepsilon\varepsilon}$ can be seen as the most reliable one in MMSE sense. In the successive manner of interference cancellation, such a most reliable channel input must be detected at the first stage to avoid error propagation. Let $\{k_1, \ldots, k_{N_\mathrm{T}}\}$ be the optimum detection order, then the k_1-th diagonal entry of $\boldsymbol{\Phi}_{\varepsilon\varepsilon}$ must be minimum. The corresponding filter $\boldsymbol{f}_{k_1}^\mathrm{H}$ is the k_1-th row of $\boldsymbol{F}^\mathrm{H}$. The output of $\boldsymbol{f}_{k_1}^\mathrm{H}$ is quantized and decision is made to get $\hat{x}_{k_1}$. Assuming that this decision is correct ($\hat{x}_{k_1} = x_{k_1}$), the contribution of x_{k_1} on the received signal $\boldsymbol{y}$, i.e. x_{k_1} multiplied with the corresponding channel response which is the k_1-th column of $\boldsymbol{H}$, is subtracted. At the second stage, since the k_1-th entry of $\boldsymbol{x}$ has been detected at the first stage, the k_1-th column of the channel matrix $\boldsymbol{H}$ can be neglected; leading to an updated system only with $N_\mathrm{T} - 1$ transmit antennas.

To generalize the procedure, the deflated channel matrix $\boldsymbol{H}^{(i)}$ is introduced for $i = 2, \ldots, N_\mathrm{T}$, where the columns $k_1, \ldots, k_{i-1}$ of $\boldsymbol{H}$ are replaced by zeros and $\boldsymbol{H}^{(1)} \triangleq \boldsymbol{H}$. At the i-th stage, $\boldsymbol{\Phi}_{ee}^{(i)}$ and $\boldsymbol{F}^{(i),\mathrm{H}}$ are calculated from (5) and (6) by replacing $\boldsymbol{H}$ with $\boldsymbol{H}^{(i)}$. Then, the optimum filter calculation with ordering can be described as

$$k_i = \operatorname*{argmin}_{k \notin \{k_1, \ldots, k_{i-1}\}} \boldsymbol{e}_k^\mathrm{T}\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{(i)}\boldsymbol{e}_k, \quad \text{and} \quad \boldsymbol{f}_{k_i}^\mathrm{H} = \boldsymbol{e}_{k_i}^\mathrm{T}\boldsymbol{F}^{(i),\mathrm{H}}, \tag{7}$$

where $\boldsymbol{e}_k$ is the k-th column of the $N_\mathrm{T} \times N_\mathrm{T}$ identity matrix $\boldsymbol{1}_{N_\mathrm{T}}$. The MMSE V-BLAST repeats the procedure in (7) N_T times, thus it requires the matrix inverse calculation in (5) for each channel input. That becomes computationally expensive for large N_T, since we end up with $\mathrm{O}(N_\mathrm{T}^4)$.

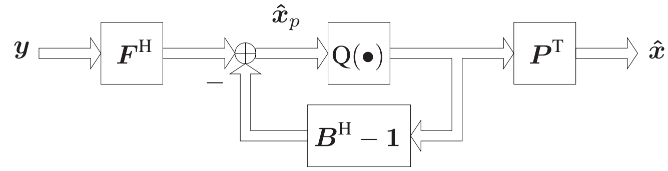

Fig. 2. Our system model for deriving the MMSE block DFE taking into account detection ordering represented by permutation matrix $\boldsymbol{P}$.

IV. Proposed MMSE Ordered Cholesky

As discussed, e.g. in [11], [14], it is useful to describe the successive interference cancellation architecture by a pair of forward and backward block filters with a certain constraint on the backward filter structure. In contrast to the frequently used system model, e.g. in [11], [14], we propose to explicitly include the detection order in our system model as illustrated in Fig. 2. We introduce the permutation matrix

$$\boldsymbol{P} = \sum_{i=1}^{N_\mathrm{T}} \boldsymbol{e}_i\boldsymbol{e}_{k_i}^\mathrm{T} \in \{0,1\}^{N_\mathrm{T}\times N_\mathrm{T}}$$

to express the detection order $\{k_1, \ldots, k_{N_\mathrm{T}}\}$. The matrix $\boldsymbol{B}^\mathrm{H}$ must be unit lower (or upper) triangular[1] so that the outputs of the backward filter $\boldsymbol{B}^\mathrm{H} - \boldsymbol{1}$ are not subtracted from already detected signals. This causality constraint is necessary to describe the successive interference cancellation procedure properly. The optimum as well as a suboptimum solution will be explained in the sequel.

A. Optimum Ordered Cholesky

We optimize the estimated signal $\hat{\boldsymbol{x}}_p$ (cf. Fig. 2) which can be expressed as

$$\hat{\boldsymbol{x}}_p = \boldsymbol{F}^\mathrm{H}\boldsymbol{y} - \left(\boldsymbol{B}^\mathrm{H} - \boldsymbol{1}\right)\boldsymbol{P}\hat{\boldsymbol{x}}, \tag{8}$$

where the subscript 'p' indicates that the variable is permuted by $\boldsymbol{P}$. The desired signal for $\hat{\boldsymbol{x}}_p$ is the channel input $\boldsymbol{x}$ permuted by $\boldsymbol{P}$. Assuming that decisions made prior to every detection stage are correct ($\hat{\boldsymbol{x}} = \boldsymbol{x}$), the error vector reads as

$$\boldsymbol{\varepsilon}_p = \boldsymbol{P}\boldsymbol{x} - \hat{\boldsymbol{x}}_p = \boldsymbol{B}^\mathrm{H}\boldsymbol{P}\boldsymbol{x} - \boldsymbol{F}^\mathrm{H}\boldsymbol{y}. \tag{9}$$

Then, the MSE $\varphi = \mathrm{E}[\|\boldsymbol{P}\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2^2]$ can be calculated as

$$\varphi = \mathrm{E}\left[\boldsymbol{\varepsilon}_p^\mathrm{H}\boldsymbol{\varepsilon}_p\right] = \mathrm{tr}\left(\boldsymbol{\Phi}_{\varepsilon\varepsilon,p}\right). \tag{10}$$

[1] Unit lower (upper) triangular matrices are lower (upper) triangular matrices with ones along the main diagonal.

Here, 'tr' denotes the trace operator and the error covariance matrix $\boldsymbol{\Phi}_{\varepsilon\varepsilon,p} = \mathrm{E}[\boldsymbol{\varepsilon}_p \boldsymbol{\varepsilon}_p^{\mathrm{H}}]$ can be written as

$$\boldsymbol{\Phi}_{\varepsilon\varepsilon,p} = \boldsymbol{B}^{\mathrm{H}}\boldsymbol{P}\boldsymbol{\Phi}_{xx}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B} - 2\mathrm{Re}\left\{\boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Phi}_{xy}^{\mathrm{H}}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}\right\} + \boldsymbol{F}^{\mathrm{H}}\boldsymbol{\Phi}_{yy}\boldsymbol{F}, \tag{11}$$

where the covariance matrices are defined as (cf. Equation 4)

$$\begin{aligned}\boldsymbol{\Phi}_{xy} &= \mathrm{E}\left[\boldsymbol{x}\boldsymbol{y}^{\mathrm{H}}\right] = \boldsymbol{\Phi}_{xx}\boldsymbol{H}^{\mathrm{H}}, \\ \boldsymbol{\Phi}_{yy} &= \mathrm{E}\left[\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}\right] = \boldsymbol{H}\boldsymbol{\Phi}_{xx}\boldsymbol{H}^{\mathrm{H}} + \boldsymbol{\Phi}_{nn}.\end{aligned} \tag{12}$$

Our goal is to jointly optimize the forward and backward filters by minimizing φ in (10). As the backward filter must be triangular, our optimization problem can be stated as

$$\begin{aligned}\{\boldsymbol{F}_{\mathrm{opt}}, \boldsymbol{B}_{\mathrm{opt}}\} &= \underset{\{\boldsymbol{F},\boldsymbol{B}\}}{\mathrm{argmin}}\ \varphi \\ \text{s.t.:}\ \ \boldsymbol{e}_i^{\mathrm{T}}\left(\boldsymbol{B}^{\mathrm{H}} - \boldsymbol{1}_{N_{\mathrm{T}}}\right)\boldsymbol{S}_i^{\mathrm{T}} &= \boldsymbol{0}_i^{\mathrm{T}} \quad \text{for} \quad i = 1, \ldots, N_{\mathrm{T}},\end{aligned} \tag{13}$$

where $\boldsymbol{e}_i$ is the i-th column of $\boldsymbol{1}_{N_{\mathrm{T}}}$, $\boldsymbol{0}_i$ is the zero vector of dimension $N_{\mathrm{T}} - i + 1$, and the selection matrix cuts out the last $N_{\mathrm{T}} - i + 1$ elements of an N_{T}-dimensional vector:

$$\boldsymbol{S}_i = [\boldsymbol{0}_{N_{\mathrm{T}}-i+1\times i-1}, \boldsymbol{1}_{N_{\mathrm{T}}-i+1}] \in \{0,1\}^{N_{\mathrm{T}}-i+1\times N_{\mathrm{T}}}. \tag{14}$$

Note that the constraint in (13) is defined for every row of the backward filter so that its upper triangular part must be zero. Equation (13) can be solved using Lagrangian multipliers and we get the solution for the forward and backward filter:

$$\begin{aligned}\boldsymbol{F}_{\mathrm{opt}}^{\mathrm{H}} &= \sum_{i=1}^{N_{\mathrm{T}}} \boldsymbol{e}_i\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{S}_i^{\mathrm{T}}\left(\boldsymbol{S}_i\boldsymbol{P}\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{S}_i^{\mathrm{T}}\right)^{-1}\boldsymbol{S}_i\boldsymbol{P}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{nn}^{-1}, \\ \boldsymbol{B}_{\mathrm{opt}}^{\mathrm{H}} &= \sum_{i=1}^{N_{\mathrm{T}}} \boldsymbol{e}_i\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{S}_i^{\mathrm{T}}\left(\boldsymbol{S}_i\boldsymbol{P}\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{S}_i^{\mathrm{T}}\right)^{-1}\boldsymbol{S}_i\boldsymbol{P}\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}\boldsymbol{P}^{\mathrm{T}},\end{aligned} \tag{15}$$

respectively. Plugging this result into the MSE φ, yields:

$$\{k_1, \ldots, k_{N_{\mathrm{T}}}\}_{\mathrm{opt}} = \underset{\{k_1,\ldots,k_{N_{\mathrm{T}}}\}}{\mathrm{argmin}} \sum_{i=1}^{N_{\mathrm{T}}} \boldsymbol{e}_{k_i}^{\mathrm{T}}\left(\boldsymbol{\Pi}_i\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}\boldsymbol{\Pi}_i\right)^{+}\boldsymbol{e}_{k_i}, \tag{16}$$

where $\boldsymbol{\Pi}_i = \boldsymbol{1}_{N_{\mathrm{T}}} - \sum_{j=1}^{i-1}\boldsymbol{e}_{k_j}\boldsymbol{e}_{k_j}^{\mathrm{T}}$ and $(\bullet)^{+}$ denotes the pseudo inverse. Note that $\boldsymbol{\Pi}_i$ is independent of $k_i, \ldots, k_{N_{\mathrm{T}}}$ and that we obtain the MMSE V-BLAST of (7), if we minimize each summand separately, e. g. k_i is chosen under the assumption that $k_1, \ldots, k_{i-1}$ are fixed.

As can be observed from (15), the filters are determined row by row, each of which requires one matrix inverse as it is the case for the MMSE V-BLAST (cf. Section III).

Since $\boldsymbol{\Phi}_{\varepsilon\varepsilon}$ is Hermitian and also positive definite, there exist the permutation matrix $\boldsymbol{P}$, the unit lower triangular matrix $\boldsymbol{L}$, and the diagonal matrix $\boldsymbol{D}$ which have the following relation

$$\boldsymbol{P}\boldsymbol{\Phi}_{\varepsilon\varepsilon}\boldsymbol{P}^{\mathrm{T}} = \boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^{\mathrm{H}}. \tag{17}$$

This is called the Cholesky factorization with symmetric permutation [12] of $\boldsymbol{\Phi}_{\varepsilon\varepsilon}$. It plays a central role in this paper. With (17), the forward and backward filters in (15) reduce to

$$\boldsymbol{F}_{\mathrm{opt}}^{\mathrm{H}} = \boldsymbol{D}\boldsymbol{L}^{\mathrm{H}}\boldsymbol{P}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{nn}^{-1} \quad \text{and} \quad \boldsymbol{B}_{\mathrm{opt}}^{\mathrm{H}} = \boldsymbol{L}^{-1}. \tag{18}$$

Due to (18), the error covariance matrix in (11) reads as

$$\boldsymbol{\Phi}_{\varepsilon\varepsilon,p} = \boldsymbol{D} = \mathrm{diag}\left(d_1, \ldots, d_{N_{\mathrm{T}}}\right). \tag{19}$$

TABLE I

CALCULATION OF BLOCK DFE FILTERS WITH DETECTION ORDERING.

```
1 :  Φ_εε = (Φ_xx^{-1} + H^H Φ_nn^{-1} H)^{-1}
     P_o = 1_{N_T}, D = 0_{N_T}
     for  i = 1, ..., N_T
         q = argmin_{q'=i,...,N_T} Φ_εε(q', q')
         P_i = 1_{N_T}  whose i-th and q-th rows are exchanged
         P_o = P_i P_o
         Φ_εε = P_i Φ_εε P_i^T
         D(i, i) = Φ_εε(i, i)
         Φ_εε(i:N_T, i) = Φ_εε(i:N_T, i)/D(i, i)
         Φ_εε(i+1:N_T, i+1:N_T) = Φ_εε(i+1:N_T, i+1:N_T)
               − Φ_εε(i+1:N_T, i) Φ_εε(i+1:N_T, i)^H D(i, i)
12 : L = lower triangular part of Φ_εε
13 : B^H = L^{-1}, F^H = D L^H P_o H^H Φ_nn^{-1}
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This means that the resulting error signal becomes *white* and the ordering optimization in (16) can be rewritten as:

$$\{k_1, \ldots, k_{N_{\mathrm{T}}}\}_{\mathrm{opt}} = \underset{\{k_1,\ldots,k_{N_{\mathrm{T}}}\}}{\mathrm{argmin}} \sum_{i=1}^{N_{\mathrm{T}}} d_i, \tag{20}$$

since $d_i = \boldsymbol{e}_{k_i}^{\mathrm{T}}(\boldsymbol{\Pi}_i\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}\boldsymbol{\Pi}_i)^{+}\boldsymbol{e}_{k_i}$. Remember that minimizing each summand of (16) separately yields the MMSE V-BLAST of (7). Thus, the MMSE V-BLAST in (7) is equivalent to:

$$k_i = \underset{k\notin\{k_1,\ldots,k_{i-1}\}}{\mathrm{argmin}}\ d_i. \tag{21}$$

We can conclude that a successive algorithm computing (17) by minimizing the diagonal entries of $\boldsymbol{D}$ for fixed previous indices $\{k_1, \ldots, k_{i-1}\}$ leads to the optimum MMSE V-BLAST detection ordering as in (7).

In [12], a successive algorithm to compute (17) is presented which finds the *maximum* diagonal entry at each iteration, starting from d_1, and also finds the necessary permutation for positive *semidefinite* systems. Since the diagonal entries in our system represent the MSEs of the ordered channel inputs (d_1 is the MSE of the channel input x_{k_1} detected first), our choice is opposite, i.e. we choose the *minimum* diagonal entry or equivalently, the MMSE at each iteration. As discussed above, this procedure is equal to the MMSE V-BLAST algorithm, but we do not require the multiple matrix inversions. Our proposed algorithm is summarized as a pseudo code in Table I and II for the filter calculation and the detection procedure, respectively.

B. Suboptimum Ordered Cholesky

The proposed optimum ordered Cholesky approach described in Section IV-A needs to calculate the matrix inverse in (5) to determine the error covariance matrix (also cf. line 1 in Table I). To avoid this inversion, we may apply the factorization to $\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1} = \boldsymbol{\Phi}_{xx}^{-1} + \boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{nn}^{-1}\boldsymbol{H}$ which is

$$\boldsymbol{P}'\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}\boldsymbol{P}'^{\mathrm{T}} = \boldsymbol{R}^{\mathrm{H}}\boldsymbol{D}'\boldsymbol{R}, \tag{22}$$

where $\boldsymbol{R}$ is unit upper triangular. If we assume $\boldsymbol{B}^{\mathrm{H}}$ in Fig. 2 to be unit upper triangular instead of lower, then similar to

TABLE II

BLOCK DFE DETECTION USING CALCULATED FILTERS AND ORDERING.

$\hat{\boldsymbol{x}}_p = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{y}, \hat{\boldsymbol{x}} = \boldsymbol{0}_{N_{\mathrm{T}}\times 1}, \boldsymbol{B}^{\mathrm{H}} = \boldsymbol{B}^{\mathrm{H}} - \boldsymbol{1}_{N_{\mathrm{T}}}$
2 : for $i = 1, \ldots, N_{\mathrm{T}}$
$q = \mathrm{find}(\boldsymbol{P}_{\mathrm{o}}(i,:) == 1)$
$\hat{\boldsymbol{x}}(q) = \mathrm{Q}(\hat{\boldsymbol{x}}_p(i))$
$\hat{\boldsymbol{x}}_p = \hat{\boldsymbol{x}}_p - \boldsymbol{B}^{\mathrm{H}}(:,i)\hat{\boldsymbol{x}}(q)$

TABLE III

CALCULATION OF SUBOPTIMUM BLOCK DFE FILTERS WITH DETECTION ORDERING – DIFFERENCE FROM TABLE I.

1 : $\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1} = \boldsymbol{\Phi}_{xx}^{-1} + \boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{nn}^{-1}\boldsymbol{H}$
all appearance of $\boldsymbol{\Phi}_{\varepsilon\varepsilon}$ in Table I is replaced by $\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}$
12 : $\boldsymbol{R}$ = upper triangular part of $\boldsymbol{\Phi}_{\varepsilon\varepsilon}^{-1}$
13 : $\boldsymbol{B}^{\mathrm{H}} = \boldsymbol{R}, \boldsymbol{F}^{\mathrm{H}} = \boldsymbol{D}^{-1}\boldsymbol{R}^{\mathrm{H},-1}\boldsymbol{P}_{\mathrm{o}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{nn}^{-1}$

Section IV-A, the optimum filters can be found as

$$\boldsymbol{F}^{\mathrm{H}} = \boldsymbol{D}'^{-1}\boldsymbol{R}^{\mathrm{H},-1}\boldsymbol{P}'_{\mathrm{o}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{\Phi}_{nn}^{-1} \quad \text{and} \quad \boldsymbol{B}^{\mathrm{H}} = \boldsymbol{R}. \qquad (23)$$

From (11) and (23), the error covariance matrix reads as

$$\boldsymbol{\Phi}_{\varepsilon\varepsilon,p} = \boldsymbol{D}'^{-1} = \mathrm{diag}\left(d_1'^{-1}, \ldots, d_{N_{\mathrm{T}}}'^{-1}\right). \qquad (24)$$

The iterative algorithm determines the diagonal entries starting from d'_1 until $d'_{N_{\mathrm{T}}}$ in (22). However, the upper triangular structure of the feedback filter suggests to detect the ordered channel inputs *from the last*. That means the MSE $d_1'^{-1}$ is calculated first, but the respective channel input is detected last. This undesired reverse direction of optimization also appears in other schemes, e.g. in [8].

Contrary to the optimum case, which *minimizes the MSE starting from the worst* (channel input to be detected first), the suboptimum case *maximizes the MSE starting from the best* (channel input to be detected last) in order to minimize the MSE of the worst because the last channel input does not cause error propagation to others. In Table III, we give the lines to be changed in Table I to end up with the suboptimum filter computation. Due to the detection order opposite to the optimum case, the second line in Table II has to be modified so that the loop starts from N_{T} down to 1.

Consequently, this suboptimum solution does not always lead to the optimum detection ordering. Reordering, e.g. by unitary transformation [9], will be necessary to improve the performance, however, that results in higher complexity than the optimum ordered Cholesky approach.

V. ANALYSIS OF COMPUTATIONAL COMPLEXITY

We compute the number of additions and multiplications required by the proposed algorithms. Since the suboptimum SQRD and the optimum SQRD+PSA in [9] (also similar to [10]) seem to be the most efficient algorithms proposed so far, we also compute their complexity for comparison. The following analysis is performed for white channel input and noise, i.e. $\boldsymbol{\Phi}_{xx} = \sigma_x^2\boldsymbol{1}$ and $\boldsymbol{\Phi}_{nn} = \sigma_n^2\boldsymbol{1}$.

The SQRD performs a modified Gram-Schmidt QR factorization for the extended channel matrix $\boldsymbol{H}_\alpha = [\boldsymbol{H}^{\mathrm{T}}\ \sigma_n\boldsymbol{1}_{N_{\mathrm{T}}}]^{\mathrm{T}}$ of dimension $(N_{\mathrm{T}}+N_{\mathrm{R}})\times N_{\mathrm{T}}$. Its complexity can be computed as $N_{\mathrm{T}}^3 + N_{\mathrm{R}}N_{\mathrm{T}}^2$ each for additions and multiplications [12]. To get the optimum performance the PSA is additionally applied. The PSA performs a Householder QR factorization for the reordered $N_{\mathrm{T}} \times N_{\mathrm{T}}$ matrix $\boldsymbol{Q}_2$ and also updates the $N_{\mathrm{R}} \times N_{\mathrm{T}}$ matrix $\boldsymbol{Q}_1$ (cf. [9]). Its worst case occurs when the detection order is wrong for the first channel input. Then, the full QR factorization is necessary. The complexity of the SQRD+PSA in the worst case can be calculated as $\frac{3}{2}N_{\mathrm{T}}^3 + 2N_{\mathrm{R}}N_{\mathrm{T}}^2$ each for additions and multiplications.

TABLE IV

COMPLEXITY OF SYSTEMS WITH $N_{\mathrm{T}} = N_{\mathrm{R}}$ ANTENNAS FOR A PROCESSOR REQUIRING THE SAME OPERATIONS FOR ADDITION AND MULTIPLICATION.

Nonlinear				Linear
Optimum		Suboptimum		Optimum
Cholesky in Table I	SQRD+PSA worst case [9]	Cholesky in Table III	SQRD [9]	MMSE Equation (6)
$\frac{7}{2}N_{\mathrm{T}}^3$	$7N_{\mathrm{T}}^3$	$\frac{13}{6}N_{\mathrm{T}}^3$	$4N_{\mathrm{T}}^3$	$\frac{13}{6}N_{\mathrm{T}}^3$

To compute the complexity of our algorithms, we can fully make use of the Hermitian structure of the error covariance matrix. In the suboptimum case, we compute the inverse of the error covariance matrix (cf. line 1 in Table III), the Cholesky factorization with symmetric permutation, and $\boldsymbol{R}^{\mathrm{H},-1}$ (cf. Table III). Its complexity can be computed as $\frac{1}{3}N_{\mathrm{T}}^3 + \frac{1}{2}N_{\mathrm{R}}N_{\mathrm{T}}^2$ and $\frac{1}{3}N_{\mathrm{T}}^3 + N_{\mathrm{R}}N_{\mathrm{T}}^2$ for additions and multiplications, respectively. Note that this complexity can be also regarded as that of the linear MMSE filter in (6) because the complexity is due to the calculation of $\boldsymbol{\Phi}_{\varepsilon\varepsilon}$ and it is given as $\boldsymbol{\Phi}_{\varepsilon\varepsilon} = \boldsymbol{R}^{-1}\boldsymbol{D}'^{-1}\boldsymbol{R}^{\mathrm{H},-1}$ from (22) with $\boldsymbol{P}' = \boldsymbol{1}$ (no ordering). In the optimum case, we additionally compute the matrix inversion of the Hermitian matrix to determine the error covariance matrix (cf. Table I). Its complexity can be computed as $N_{\mathrm{T}}^3 + \frac{1}{2}N_{\mathrm{R}}N_{\mathrm{T}}^2$ and $N_{\mathrm{T}}^3 + N_{\mathrm{R}}N_{\mathrm{T}}^2$ for additions and multiplications, respectively.

The complexity is computed separately for additions and multiplications because they might cost differently depending on the processor's architecture in use. As an example, we summarize in Table IV the complexities of a system with $N_{\mathrm{T}} = N_{\mathrm{R}}$ antennas when using a processor which requires the same number of operations for additions and multiplications. From Table IV, it can be seen that our proposed algorithms both in the optimum and suboptimum cases achieve better efficiency than the corresponding SQRD+PSA and SQRD, respectively; *about in a factor of two*. Additionally, our optimum approach requires even less computation than the suboptimum SQRD. The difference in complexity is mainly due to the fact that the SQRD works with the big extended channel matrix $\boldsymbol{H}_\alpha$ (also in [10]) while our approach with the smaller dimension. We also intensively make use of the Hermitian structure of the error covariance matrix. Using the extended channel matrix also results in a memory requirement twice as much as our scheme for $N_{\mathrm{T}} = N_{\mathrm{R}}$ and even more for $N_{\mathrm{R}} \geq N_{\mathrm{T}}$. Finally, it is remarked that our suboptimum approach has the same complexity order of the simple linear MMSE filter.

VI. SIMULATION RESULTS

In the following computer simulations, the channel input and the noise are assumed to be white, i.e. $\boldsymbol{\Phi}_{xx} = \sigma_x^2\boldsymbol{1}$ and

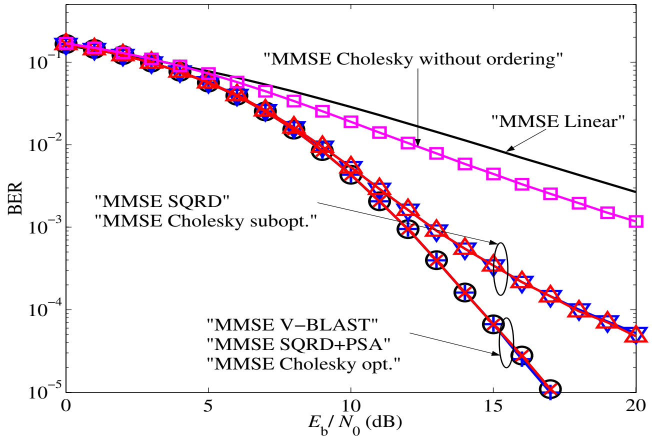

Fig. 3. BER performance comparison for $N_T = N_R = 8$.

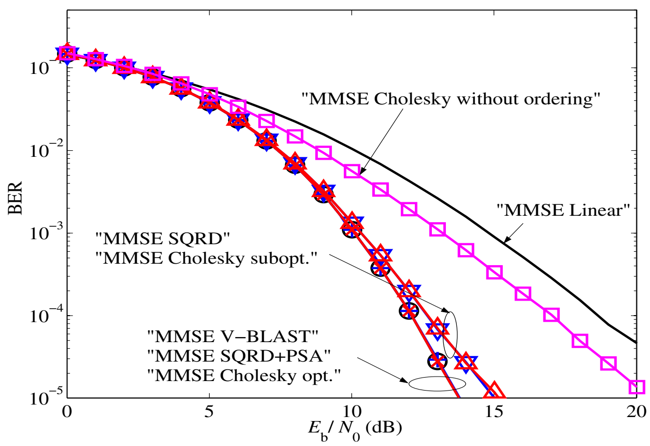

Fig. 4. BER performance comparison for $N_T = 8$ and $N_R = 10$.

$\boldsymbol{\Phi}_{nn} = \sigma_n^2 \mathbf{1}$. For the performance evaluation, *bit error rate* (BER) is computed over $E_b/N_0 = \frac{N_R \sigma_x^2}{M \sigma_\eta^2}$ where E_b, N_0, and M are the average received energy per information bit, noise power spectral density, and the number of information bits per channel input, respectively. In the following, information bits are QPSK modulated ($M = 2$). The channel and the SNR are assumed to be perfectly known at the receiver.

Fig. 3 shows the BER performance of the system with $N_T = N_R = 8$ antennas both at the transmitter and the receiver. It can be seen that our optimum MMSE Cholesky achieves the same performance as the MMSE V-BLAST and the SQRD+PSA, but with significantly lower complexity. The significance of the detection order can be also observed by comparing to the MMSE Cholesky without ordering. Our suboptimum MMSE Cholesky does not approach the optimum performance as it is the case for the SQRD. However, for lower E_b/N_0 values, the performance gap to the optimum one is negligible. Then, the suboptimum Cholesky may be the first choice due to the same complexity order of the simple linear MMSE filter.

Fig. 4 shows the BER performance of the system with $N_T = 8$ and $N_R = 10$ antennas at the transmitter and the receiver, respectively. A tendency similar to the previous example can be observed, but the performance gap between the optimum and the suboptimum cases becomes smaller that makes the suboptimum scheme more attractive.

VII. Summary

We derived a new MMSE block DFE equivalent to the MMSE V-BLAST on flat MIMO channels. Our optimization explicitly includes a detection order represented by a permutation matrix. It was shown that the solution can be simplified drastically by the Cholesky factorization with symmetric permutation applied to the error covariance matrix. The forward and backward filters as well as the detection order were jointly optimized. The proposed iterative algorithm finds the optimum detection order at each iteration in MMSE sense. Our optimum algorithm achieves the same performance as the MMSE V-BLAST, but with significantly lower complexity; lower than the previously proposed schemes. We also proposed a suboptimum scheme with the same complexity order of the linear MMSE filter. The suboptimum scheme becomes a reasonable choice for low SNR or for systems with high numbers of receive antennas.

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