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Munich University of Technology
Institute for Circuit Theory and Signal Processing
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Michael Joham[1], Ralf Irmer[2], Stefan Berger[1], Gerhard Fettweis[2], and Wolfgang Utschick[1]

1. Institute for Circuit Theory and Signal Processing, Munich University of Technology, 80290 Munich, Germany
Michael.Joham@nws.ei.tum.de

2. Vodafone Chair Mobile Communications Systems, Dresden University of Technology, 01062 Dresden, Germany
irmer@ifn.et.tu-dresden.de

Abstract

We compare different approaches for linear precoding, i. e. processing applied prior to transmission. Our contribution is not only the comparison of the different transmit filters in the same framework, but the design of the transmit filters with FIR structure for a direct sequence code division multiple access system together with an optimization of the latency time. The three basic transmit filter types, namely the transmit matched filter, the transmit zero-forcing filter, and the transmit Wiener filter, are compared to two alternative approaches: the constrained minimum mean square error and the eigenprecoder filter. Simulation results reveal that the eigenprecoder filter is preferable for low signal to noise ratio, whereas the interference suppressing transmit filters are superior otherwise.

1. Introduction

The conventional way to combat the distortions caused by the frequency-selective channel leads to more complex *mobile stations* (MSs) which have to be kept simple, however. Thus, transmit processing is advantageous for the downlink, as the MSs perform an *a priori* known processing, e. g. correlation with the spreading sequence, and the transmitting *base station* (BS) has to adapt to the properties of the channel which can be estimated during uplink reception in a *time division duplex* (TDD) system. The downlink direction is expected to carry most of the traffic for multimedia applications. Examples of TDD systems are the 3GPP TDD mode and the Chinese TD-SCDMA.
The *transmit matched filter* (TxMF, [1, 2]) maximizes the desired signal portion in the estimate, the *transmit zero-forcing filter* (TxZF, [3, 4, 5, 6, 7]) removes the interference, and the *transmit Wiener filter* (TxWF, [8, 9]) minimizes the *modified mean square error* (*modified* MSE). In [7], Noll Barreto et al. proposed the *constrained minimum mean square error transmit filter* (TxCMMSE) which minimizes the MSE together with a transmit power inequality constraint (see also [3, 10]). In [11], a suboptimum TxCMMSE has been examinined which assumes long spreading codes and low system load. If the receive filter is a rake matched to the combination of channel and transmit filter and the transmit filter is designed to maximize the *signal to noise ratio* (SNR) at the receiver, we end up with the *eigenprecoder* [12, 13, 14]. In [15, 16], suboptimum eigenprecoders with a reduced number of rake fingers have been proposed which we do not consider in this paper.

Up to now, no study focused on the comparison of the different transmit filters. Whereas the matched filter types (TxMF and eigenprecoder) were proposed as FIR filters, the interference suppressing filters (TxZF, TxCMMSE, and TxWF) were introduced as block filters which process a whole block at once. However, FIR filters are advantageous for implementation due to their simplicity, therefore, FIR variants of all filters are derived and compared in this paper.

We do not consider the effects of channel mismatches between uplink and downlink and also neglect the outdating of channel estimates obtained during reception in the uplink which deteriorate the results for all transmit filters (especially interference suppressing variants, see e. g. [6, 17]). We also assume perfect knowledge of the noise statistics at the transmitting BS for the TxWF although the MSs have to feed back this information (see the discussion in [8]).

We explain the system model for *direct sequence code division multiple access* (DS-CDMA) transmit processing in Section 2 and in Section 3, we review the transmit filters. The simulation results are presented in Section 4 and conclusions are drawn in Section 5.

1.1. Notation

Vectors and matrices are denoted by lower case bold and capital bold letters, respectively. We use $\mathrm{E}[\bullet]$, '$*$', '$\otimes$', $(\bullet)^*$, $(\bullet)^{\mathrm{T}}$, and $(\bullet)^{\mathrm{H}}$ for expectation, convolution, the Kronecker product, complex conjugation, transposition, and conjugate transposition, respectively. The pseudo inverse of a matrix is denoted by $(\bullet)^+$. All random processes are assumed to be zero-mean and stationary. The covariance matrix of the vector process $\boldsymbol{x}[n]$ is denoted by $\boldsymbol{R}_{\boldsymbol{x}} = \mathrm{E}\left[\boldsymbol{x}[n]\boldsymbol{x}^{\mathrm{H}}[n]\right]$, whereas the variance of the scalar process $y[n]$ is denoted by $\sigma_y^2 = \mathrm{E}\left[|y[n]|^2\right]$. The floor operator is denoted by $\lfloor \bullet \rfloor$ which gives the integer number smaller than or equal to the argument. For the time index of symbols and chips we use $(\bullet)^{[m]}$ and $[n]$, respectively. The $N \times M$ zero matrix is $\mathbf{0}_{N\times M}$ and the $N \times N$ identity matrix is $\mathbf{1}_N$, whose n-th column is $\boldsymbol{e}_n$. Throughout the paper, we use the selection matrix $\boldsymbol{S}_{(q,M,N)} = [\mathbf{0}_{M\times q}, \mathbf{1}_M, \mathbf{0}_{M\times N-q}] \in \{0,1\}^{M\times M+N}$.

2. System Model

The downlink from the BS with N_{a} antenna elements to the U MSs each having one antenna element is shown in Fig. 1. The transmit signals $\boldsymbol{s}_u[n] = \boldsymbol{p}_u[n] * d_u[n] \in \mathbb{C}^{N_{\mathrm{a}}}, u = 1, \ldots, U$, are summed up by the BS and propagate over the u-th channel $\boldsymbol{h}_u = \sum_{q=0}^{Q} \boldsymbol{h}_{u,q}\delta[n-q] \in \mathbb{C}^{N_{\mathrm{a}}}$ to the u-th MS. The output of the receive filter at the u-th MS reads as

$$\hat{d}_u[n] = c_u^*[-n] * g_u[n] * \left(\boldsymbol{h}_u^{\mathrm{T}}[n] * \sum_{k=1}^{U} \boldsymbol{s}_k[n] + \eta_u[n]\right),$$

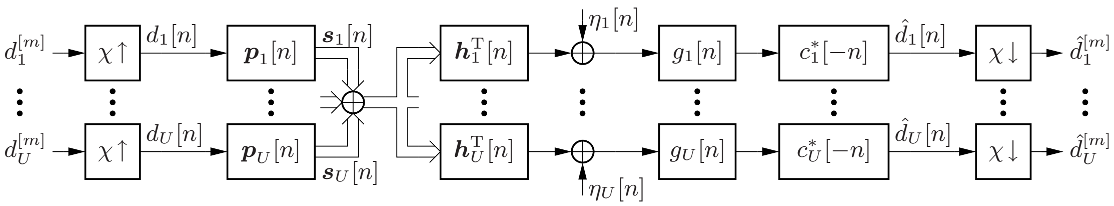

Figure 1: Downlink System Model with U Mobiles

where $\eta_u[n]$ denotes the complex Gaussian noise. All investigated transmit filters assume a code matched filter $c_u^*[-n]$ at the u-th MS, whereas for all transmit filters except the eigenprecoder, the filter $g_u[n]$ is inactive, that is $g_u[n] = \delta[n]$.

Since the interpolator can be described as follows

$$d_u[n] = \begin{cases} d_u^{[m]} & n = \chi m,\ m \in \mathbb{Z}, \\ 0 & \text{otherwise}, \end{cases}$$

and the operation of the decimator is $\hat{d}_u^{[m]} = d_u[\chi m + \nu]$, we get for the estimate of the u-th MS (for $g_u[n] = \delta[n]$):

$$\hat{d}_u^{[m]} = \sum_{k=1}^{U} \boldsymbol{p}_k^{\mathrm{T}} \boldsymbol{A}_u(\nu) \boldsymbol{d}_k^{[m]} + \boldsymbol{c}_u^{\mathrm{H}} \boldsymbol{\eta}_u^{[m]}, \qquad (1)$$

where we used χ for the spreading factor, collected the $L+1$ coefficients of the transmit filter $\boldsymbol{p}_u[n] = \sum_{\ell=0}^{L} \boldsymbol{p}_{u,\ell}\delta[n-\ell]$ in the vector

$$\boldsymbol{p}_u = \left[\boldsymbol{p}_{0,u}^{\mathrm{T}}, \ldots, \boldsymbol{p}_{L,u}^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{N_{\mathrm{a}}(L+1)}$$

and introduced the latency time $\nu \in \{-\chi+1, \ldots, L+Q\}$. The chips of the code $c_u[n] = \sum_{i=0}^{\chi-1} c_{u,i}\delta[n-i]$ and the noise samples effecting the estimate $\hat{d}_u^{[m]}$ are comprised in

$$\boldsymbol{c}_u = \left[c_{u,\chi-1}, \ldots, c_{u,0}\right]^{\mathrm{T}} \in \mathbb{C}^{\chi} \quad \text{and}$$
$$\boldsymbol{\eta}_u^{[m]} = \left[\eta_u[\chi(m+1)+\nu-1], \ldots, \eta_u[\chi m + \nu]\right]^{\mathrm{T}} \in \mathbb{C}^{\chi},$$

respectively. The operations of the channel $\boldsymbol{h}_u[n]$ and the correlator $c_u^*[-n]$ are described by the system matrix

$$\boldsymbol{A}_u(\nu) = \boldsymbol{H}_u \boldsymbol{C}_u^{\mathrm{H}} \boldsymbol{Y}(\nu) \in \mathbb{C}^{N_{\mathrm{a}}(L+1)\times 2\mu+1},$$

where

$$\boldsymbol{H}_u = \sum_{q=0}^{Q} \boldsymbol{h}_{u,q} \otimes \boldsymbol{S}_{(q,L+1,Q)} \in \mathbb{C}^{N_{\mathrm{a}}(L+1)\times L+Q+1} \quad \text{and}$$
$$\boldsymbol{C}_u = \sum_{i=0}^{\chi-1} c_{u,i} \boldsymbol{S}_{(\chi-i-1,L+Q+1,\chi-1)}^{\mathrm{T}} \in \mathbb{C}^{L+Q+\chi\times L+Q+1}.$$

The matrix $\boldsymbol{Y}(\nu) = \boldsymbol{\Phi}(\nu)\boldsymbol{\Psi} \in \{0,1\}^{L+Q+\chi\times\chi(2\mu+1)}$ can be divided into

$$\boldsymbol{\Phi}(\nu) = \boldsymbol{S}_{(\mu\chi-\nu,L+Q+\chi,(2\mu+1)\chi-L-Q-1)}, \quad \text{and}$$
$$\boldsymbol{\Psi} = \begin{bmatrix} \mathbf{0}_{\chi-1\times 2\mu+1} \\ \mathbf{1}_{2\mu+1} \otimes \boldsymbol{e}_1 \end{bmatrix} \in \{0,1\}^{(2\mu+2)\chi-1\times 2\mu+1}$$

representing the decimator and the interpolator, respectively. Here, $\boldsymbol{e}_1 \in \{0,1\}^{\chi}$ and we used the abbreviation

$$\mu = \left\lfloor \frac{L+Q+\chi-1}{\chi} \right\rfloor$$

for the number of symbols which potentially influence the estimate $\hat{d}_u^{[m]}$ and which are put into

$$\boldsymbol{d}_k^{[m]} = \left[d_k^{[m+\mu]}, \ldots, d_k^{[m]}, \ldots, d_k^{[m-\mu]}\right]^{\mathrm{T}} \in \mathbb{C}^{2\mu+1}.$$

Note that $d_u^{[m]} = \boldsymbol{e}_{\mu+1}^{\mathrm{T}} \boldsymbol{d}_u^{[m]}$ is the estimate's $\hat{d}_u^{[m]}$ desired value and $\boldsymbol{e}_{\mu+1} \in \{0,1\}^{2\mu+1}$.

The receive filters $g_u[n] = \sum_{b=0}^{L+Q-\chi+1} g_{u,b}\delta[n-b]$ are only different from $\delta[n]$ for the eigenprecoder which maximizes the SNR at the decimator output:

$$\gamma_u = \frac{\left|\mathrm{E}\left[d_u^{[m],*}\hat{d}_u^{[m]}\right]\right|^2}{\sigma_d^2\ \mathrm{E}\left[|c_u^*[-n] * g_u[n] * \eta_u[n]|^2\right]\big|_{n=\chi m+\nu}}.$$

For the eigenprecoder, the transmit filter has the special form

$$\boldsymbol{p}_u[n] = \tilde{\boldsymbol{p}}_u[n] * c_u[n] \in \mathbb{C}^{N_{\mathrm{a}}},$$

where $\tilde{\boldsymbol{p}}_u[n] = \sum_{\ell=0}^{L-\chi+1} \tilde{\boldsymbol{p}}_{u,\ell}\delta[n-\ell]$. Under the assumption of white symbols and noise, we get for $\nu = L+Q-\chi+1$ and $\sigma_d^2 = \sigma_{d_1}^2 = \cdots = \sigma_{d_U}^2$:

$$\gamma_u = \frac{\sigma_d^2 \left|\boldsymbol{g}_u^{\mathrm{T}} \tilde{\boldsymbol{C}}_u^{\mathrm{H}} \tilde{\boldsymbol{C}}_u \tilde{\boldsymbol{H}}_u \tilde{\boldsymbol{p}}_u\right|^2}{\sigma_{\eta_u}^2 \boldsymbol{g}_u^{\mathrm{T}} \tilde{\boldsymbol{C}}_u^{\mathrm{H}} \tilde{\boldsymbol{C}}_u \boldsymbol{g}_u^*}. \qquad (2)$$

Here, $\boldsymbol{g}_u = [g_{u,0}, \ldots, g_{u,L+Q-\chi+1}]^{\mathrm{T}} \in \mathbb{C}^{L+Q-\chi+2}$ and $\tilde{\boldsymbol{p}}_u \in \mathbb{C}^{N_{\mathrm{a}}(L-\chi+2)}$ is defined similar to $\boldsymbol{p}_u$. The channel matrix $\tilde{\boldsymbol{H}}_u$ and the code matrix $\tilde{\boldsymbol{C}}_u$ are equal to $\boldsymbol{H}_u^{\mathrm{T}}$ and $\boldsymbol{C}_u$, respectively, when L is replaced by $L-\chi+1$.

3. Transmit Filters

The TxZF, TxMF, TxWF, and TxCMMSE share the form:

$$\boldsymbol{p}_u^{\mathrm{T}}(\nu) = \beta(\nu)\boldsymbol{w}_u^{\mathrm{T}} \left(\zeta \boldsymbol{A}^{\mathrm{H}}(\nu)\boldsymbol{A}(\nu) + \xi \mathbf{1}_{U(2\mu+1)}\right)^{-1} \boldsymbol{A}^{\mathrm{H}}(\nu), \qquad (3)$$

where $\boldsymbol{A}(\nu) = [\boldsymbol{A}_1(\nu), \ldots, \boldsymbol{A}_U(\nu)] \in \mathbb{C}^{N_{\mathrm{a}}(L+1)\times U(2\mu+1)}$ is the total system matrix and $\boldsymbol{w}_u = \boldsymbol{e}_u \otimes \boldsymbol{e}_{\mu+1} \in \{0,1\}^{U(2\mu+1)}$ with $\boldsymbol{e}_u \in \{0,1\}^U$. The scaling $\beta(\nu) \in \mathbb{R}_+$ follows from the transmit power constraint, i. e. $\sum_{u=1}^{U} \mathrm{E}[\|\boldsymbol{s}_u[n]\|_2^2] = E_{\mathrm{tr}}$ and $\mathrm{E}[\|\boldsymbol{s}_u[n]\|_2^2] = \sigma_d^2\|\boldsymbol{p}_u\|_2^2$. In the following, we utilize the abbreviation $\boldsymbol{p} = [\boldsymbol{p}_1^{\mathrm{T}}, \ldots, \boldsymbol{p}_U^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{N_{\mathrm{a}}U(L+1)}$.

3.1. Transmit Zero-Forcing Filter – TxZF

The TxZF removes interference, uses the whole available transmit power E_{tr}, and minimizes the *modified* MSE (e. g. [8]):

$$\{\boldsymbol{p}_{\mathrm{ZF}}^{\mathrm{T}}, \beta_{\mathrm{ZF}}\} = \arg\min_{\{\boldsymbol{p}^{\mathrm{T}},\beta\}} \sum_{u=1}^{U} \mathrm{E}\left[\left|d_u^{[m]} - \beta^{-1}\hat{d}_u^{[m]}\right|^2\right]$$
$$\text{s. t.: } \mathrm{E}[\hat{d}_u^{[m]}] = \beta d_u^{[m]} \quad \text{and} \quad \sum_{u=1}^{U} \mathrm{E}[\|\boldsymbol{s}_u[n]\|_2^2] = E_{\mathrm{tr}}. \qquad (4)$$

The TxZF can be found with $\zeta_{\text{ZF}} = 1$ and $\xi_{\text{ZF}} = 0$ or equivalently $\boldsymbol{p}_{\text{ZF},u}^{\text{T}}(\nu) = \beta_{\text{ZF}}(\nu)\boldsymbol{w}_u^{\text{T}}\boldsymbol{A}^{+}(\nu)$. The optimum latency time further maximizes the gain $\beta_{\text{ZF}}(\nu)$ (see [18]):

$$\nu_{\text{ZF}} = \arg\min_{\nu} \sum_{u=1}^{U} \boldsymbol{w}_u^{\text{T}}\boldsymbol{A}^{+}(\nu)\boldsymbol{A}^{+,\text{H}}(\nu)\boldsymbol{w}_u.$$

Therefore, the TxZF latency time optimization in DS-CDMA systems is very comlex compared to the one in TDMA systems (cf. [18]), since the pseudo inverse has to be computed for every value of $\nu \in \{-\chi+1, \ldots, L+Q\}$. However, we observed that setting the latency time to the fixed value $\nu_{\text{fix}} = Q$ leads to near optimum results.

3.2. Transmit Matched Filter – TxMF

The desired signal portion $d_u^{[m]}$ in the estimate $\hat{d}_u^{[m]}$ is maximized by the TxMF [2, 8]:

$$\begin{aligned} \boldsymbol{p}_{\text{MF}}^{\text{T}} &= \arg\max_{\boldsymbol{p}^{\text{T}}} \left| \sum_{u=1}^{U} \mathrm{E}\left[d_u^{[m],*}\hat{d}_u^{[m]} \right] \right|^2 \\ \text{s.t.:} & \sum_{u=1}^{U} \mathrm{E}[\|\boldsymbol{s}_u[n]\|_2^2] = E_{\text{tr}}, \end{aligned} \tag{5}$$

which is obtained with $\zeta_{\text{MF}} = 0$ and $\xi_{\text{MF}} = 1$ and can be expressed as $\boldsymbol{p}_{\text{MF},u}^{\text{T}}(\nu) = \beta_{\text{MF}}(\nu)\boldsymbol{e}_{\mu+1}^{\text{T}}\boldsymbol{A}_u^{\text{H}}(\nu)$. The optimum latency time ν_{MF} is simply Q.

3.3. Transmit Wiener Filter – TxWF

The *modified* MSE is minimized by the TxWF which uses the whole available transmit power (see [8]):

$$\begin{aligned} \{\boldsymbol{p}_{\text{WF}}^{\text{T}}, \beta_{\text{WF}}\} &= \arg\min_{\{\boldsymbol{p}^{\text{T}},\beta\}} \sum_{u=1}^{U} \mathrm{E}\left[\left| d_u^{[m]} - \beta^{-1}\hat{d}_u^{[m]} \right|^2 \right] \\ \text{s.t.:} & \sum_{u=1}^{U} \mathrm{E}[\|\boldsymbol{s}_u[n]\|_2^2] = E_{\text{tr}}. \end{aligned} \tag{6}$$

The parameters of the TxWF can be written as $\zeta_{\text{WF}} = 1$ and $\xi_{\text{WF}} = \sum_{u=1}^{U}\sigma_{\eta_u}^2\|\boldsymbol{c}_u\|_2^2/E_{\text{tr}}$. The optimum latency time further minimizes the *modified* MSE (cf. [18]):

$$\nu_{\text{WF}} = \arg\min_{\nu} \sum_{u=1}^{U} \boldsymbol{w}_u^{\text{T}} \left(\boldsymbol{A}^{\text{H}}(\nu)\boldsymbol{A}(\nu) + \xi_{\text{WF}}\mathbf{1} \right)^{-1} \boldsymbol{w}_u.$$

As the inverse has to be computed for all latency time values, the TxWF latency time optimization for DS-CDMA systems is much more complex than for TDMA systems [18], but setting the latency time to the fixed value $\nu_{\text{fix}} = Q$ is very close to optimum.

3.4. Constrained MMSE Filter – TxCMMSE

The TxCMMSE minimizes the MSE together with an inequality transmit power constraint, where the noise power $\sigma_{\eta_u}^2$ can be assumed to be unknown at the transmitter [7]:

$$\begin{aligned} \boldsymbol{p}_{\text{CMMSE}}^{\text{T}} &= \arg\min_{\boldsymbol{p}^{\text{T}}} \mathrm{E}\left[\left| d_u^{[m]} - \hat{d}_u^{[m]} \right|^2 \right] \\ \text{s.t.:} & \sum_{u=1}^{U} \mathrm{E}[\|\boldsymbol{s}_u[n]\|_2^2] \leq E_{\text{tr}}. \end{aligned} \tag{7}$$

The weight β_{CMMSE} (cf. Eqn. 3) of the TxCMMSE is set to one, since it is not used to meet the transmit power constraint contrary to the TxZF, TxMF, and TxWF. Additionally, $\zeta_{\text{CMMSE}} = 1$ and $\xi_{\text{CMMSE}}(\nu)$ is the Lagrange multiplier for the transmit power inequality constraint. Consequently, $\xi_{\text{CMMSE}}(\nu) \geq 0$ and $\xi_{\text{CMMSE}}(\nu)$ is only zero, if $\boldsymbol{w}_u^{\text{T}}\boldsymbol{A}^{+}(\nu), u = 1, \ldots, U$, already fulfills the constraint in Eqn. (7). Otherwise, the constraint is an equality and $\xi_{\text{CMMSE}}(\nu)$ is the only positive root of the resulting polynomial (see [10] for more details). The optimum latency time for the TxCMMSE can be found by further minimizing the MSE. This optimization is even more difficult than the latency time optimizations for the TxZF and the TxWF, because $\xi_{\text{CMMSE}}(\nu)$ depends on the latency time ν.

3.5. TxEigenfilter and RxMF – Eigenprecoder

The eigenprecoder arises from the maximization of the SNR in Eqn. (2). The SNR is first maximized by the choice of the receive filter $\boldsymbol{g}_u^{\text{T}}$ under the assumption of an already determined transmit filter $\tilde{\boldsymbol{p}}_u$. Consequently, $\boldsymbol{g}_u^{\text{T}} = \tilde{\boldsymbol{p}}_u^{\text{H}}\tilde{\boldsymbol{H}}_u^{\text{H}}$ due to Cauchy-Schwarz inequality (e. g. [19]) and the SNR simplifies to:

$$\gamma_u = \tilde{\boldsymbol{p}}_u^{\text{H}}\tilde{\boldsymbol{H}}_u^{\text{H}}\tilde{\boldsymbol{C}}_u^{\text{H}}\tilde{\boldsymbol{C}}_u\tilde{\boldsymbol{H}}_u\tilde{\boldsymbol{p}}_u\sigma_d^2/\sigma_{\eta_u}^2.$$

Hence, the eigenprecoder $\tilde{\boldsymbol{p}}_u$ is the eigenvector corresponding to the principal eigenvalue of $\tilde{\boldsymbol{H}}_u^{\text{H}}\tilde{\boldsymbol{C}}_u^{\text{H}}\tilde{\boldsymbol{C}}_u\tilde{\boldsymbol{H}}_u$. The optimum latency time for the eigenprecoder is simply $\nu = L + Q - \chi + 1$ which has been used for Eqn. (2). Note that our eigenprecoder solution is more general than the one given in [12, 13] which results from the assumption that $\tilde{\boldsymbol{C}}_u^{\text{H}}\tilde{\boldsymbol{C}}_u = \mathbf{1}_{L+Q-\chi+2}$. Obviously, the eigenprecoder is the same for the single and multi user case, since the SNR is the optimization criterion which does not include interference.

4. Simulation Results

We applied the transmit filters discussed in the previous section to the downlink of a DS-CDMA system with $N_{\text{a}} = 2$ antenna elements deployed at the BS to be able to perform linear zero-forcing also for fully loaded systems ($U = \chi$). The channels to the U MSs are constant during the transmission of one block and have $Q + 1 = 5$ uncorrelated Rayleigh fading paths with exponential *power delay profile* (PDP), that is $\mathrm{E}[\|\boldsymbol{h}_q\|_2^2] = \exp(-q)/\sum_{q'=0}^{Q}\exp(-q'), q = 0, \ldots, Q$. For simplicity, we set the latency times of the TxZF, TxWF, and TxCMMSE to $\nu_{\text{fix}} = Q$, but the TxMF and the eigenprecoder use the optimum latency times. The order of the transmit filter $\boldsymbol{p}_u[n]$ is $L = Q + \chi - 1$ to end up with a fair comparison with the TxMF. Therefore, the eigenprecoder receive filter $g_u[n]$ has the order $2Q$. We set the transmit power to $E_{\text{tr}} = 1$ and the SNR is defined as the transmitted energy per symbol divided by the chip noise power. We show the *uncoded bit error ratio* (uncoded BER) results which are the mean of 10000 channel realizations where a block of 1000 QPSK symbols was transmitted for each channel realization.

Fig. 2 depicts the results for a TDMA system ($U = \chi = 1$). We can observe that the TxWF is the lower bound of the TxMF, TxZF, and TxCMMSE. The TxWF converges to the TxMF for low and to the TxZF for high SNR. The BER of the TxMF saturates for high SNR, since the TxMF does not suppress interference as can be followed from the optimization in Eqn. (5). On the other hand, the TxZF does not saturate for high SNR, because it completely eliminates the interference, but is bad for low SNR, as noise and not interference is the limiting source in this SNR region. The TxCMMSE behaves like a suboptimum TxWF with ξ_{CMMSE} independent of the noise power at the MSs.

For one point, it coincides with the TxWF, but has a worse performance for higher and lower SNRs. We can also see in Fig. 2 that the slop of the BER for the TxCMMSE becomes smaller with increasing SNR. We can expect that the TxCMMSE saturates for very high SNR as was shown in [10], but at a much lower level than the TxMF, however. This behaviour follows from the prementioned property of the TxCMMSE that it is independent of the actual SNR: for some channel realizations ξ_{CMMSE} is very large and hence, the TxCMMSE is like a TxMF.

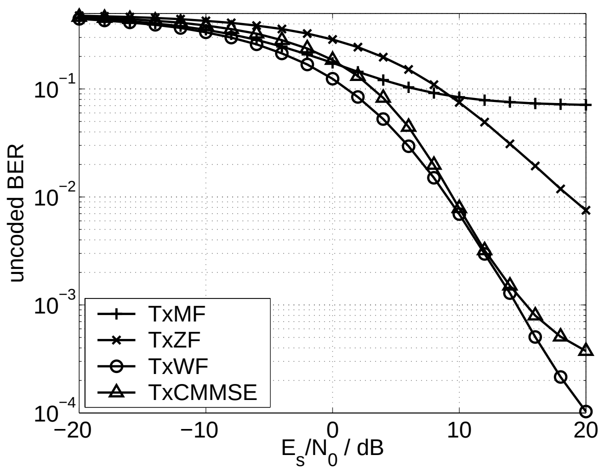

Figure 2: System with $U = 1$ user and spreading factor $\chi = 1$

In Fig. 3, we see that we get similar results for a fully loaded DS-CDMA system ($U = \chi = 2$). However, the results of all transmit filters are improved compared to Fig. 2 which can be explained with *multi-user diversity* — two bad channels are less likely than one bad channel. We also included the results of a weighted version of the TxCMMSE with $\beta_{\text{CMMSE}} > 1$, if $\xi_{\text{CMMSE}} = 0$, i. e. it always uses the whole available transmit power. Both versions of the TxCMMSE behave like the TxZF for low SNR and touch the TxWF at an SNR of approximately $18\,\text{dB}$. From Fig. 2 and the results presented in [10], we can conclude that also the TxCMMSEs in Fig. 3 will exhibit a BER saturation at high SNR.

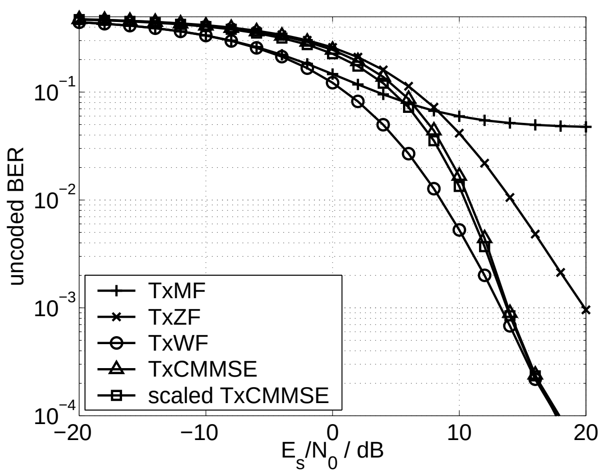

Figure 3: System with $U = 2$ users and spreading factor $\chi = 2$

In Fig. 4, we present the results for a partially loaded system with $U = 2$ and $\chi = 4$. Due to the additional degrees of freedom, the TxWF and the TxZF show dramatically improved BER results compared to Fig. 3 where $U = \chi = 2$. Interestingly, the TxZF seems to gain more from the additional degrees of freedom than the TxWF. The TxMF saturates at a lower BER level for high SNR, since the higher spreading factor $\chi = 4$ (instead of $\chi = 2$) for the same number of MSs leads to lower amounts of *intersymbol interference* (ISI) and *multiple access interference* (MAI). The eigenprecoder outperforms the other filters for the noise-dominated region due to its SNR maximizing property and the additional receive filter, but it fails in interference-dominated regions, where it shows even a higher BER saturation than the TxMF. We can follow that the eigenprecoder is the best choice, if an uncoded BER of 10^{-1} or above is satisfactory and the additional complexity at the MSs due to the receive filters $g_u[n], u = 1, \ldots, U$, and due to the necessary channel estimation at the MSs is possible in this case.

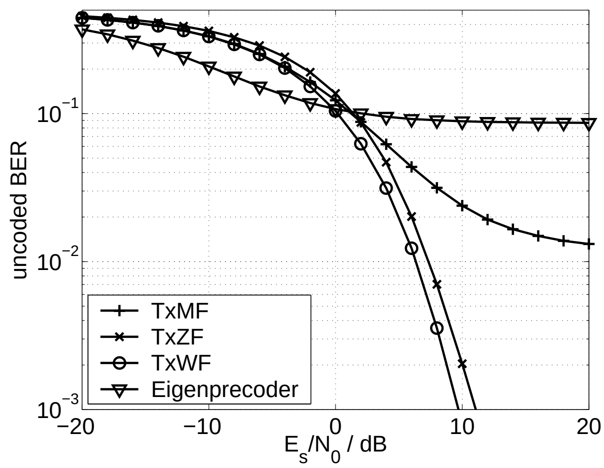

Figure 4: System with $U = 2$ users and spreading factor $\chi = 4$

Fig. 5 again depicts the results for a fully loaded system with $U = \chi = 4$. The TxMF, TxZF, and TxWF exhibit further improved BER results compared to Figs. 2 and 3 due to *multi-user diversity*. However, the less degrees of freedom because of the increased number of MSs ($U = 4$ instead of $U = 2$) leads to worse results than in Fig. 4. The BER of the eigenprecoder also deteriorates, because a higher number of MSs leads to more MAI and the BER saturates at a higher level for high SNR. Therefore, the eigenprecoder is only an alternative to the TxMF and TxWF, if the necessary uncoded BER lies above $15\,\%$.

5. Conclusions

We presented the system model necessary for the design of FIR transmit filters for DS-CDMA systems which also incorporates a latency time. We compared the TxMF, TxZF, TxWF, TxCMMSE, and eigenprecoder for different system loads. The TxCMMSE and also a weighted variant of the TxCMMSE behave like a suboptimum TxWF and are thus no alternatives for the TxWF, since they are independent of the noise power at the receiver. However, the simulation results revealed that the eigenprecoder outperforms all other transmit filters for low SNR due to the additional receive filter and is therefore an alternative for the TxWF, if the necessary uncoded BER is high enough.

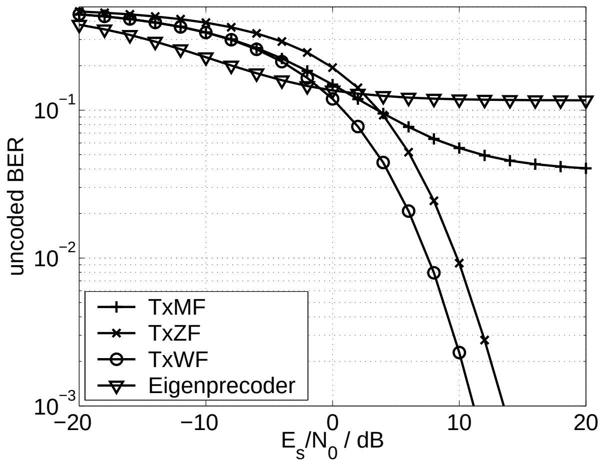

Figure 5: System with $U = 4$ users and spreading factor $\chi = 4$

6. References

[1] R. E. McIntosh and S. E. El-Khamy, "Optimum Pulse Transmission Through a Plasma Medium," *IEEE Transactions on Antennas and Propagation*, vol. AP-18, no. 5, pp. 666–671, September 1970.

[2] R. Esmailzadeh and M. Nakagawa, "Pre-RAKE Diversity Combination for Direct Sequence Spread Spectrum Mobile Communications Systems," *IEICE Transactions on Communications*, vol. E76-B, no. 8, pp. 1008–1015, August 1993.

[3] B. R. Vojčić and W. M. Jang, "Transmitter Precoding in Synchronous Multiuser Communications," *IEEE Transactions on Communications*, vol. 46, no. 10, pp. 1346–1355, October 1998.

[4] G. Montalbano, I. Ghauri, and D. T. M. Slock, "Spatio-Temporal Array Processing for CDMA/SDMA Downlink Transmission," in *Proc. Asilomar Conference on Signals, Systems, and Computers*, November 1998, vol. 2, pp. 1337–1341.

[5] M. Meurer, P. W. Baier, Y. Lu, A. Papathanassiou, and T. Weber, "TD-CDMA Downlink: Optimum Transmit Signal Design Reduces Receiver Complexity and Enhances System Performance," in *Proc. ICT 2000*, May 2000, pp. 300–305.

[6] F. Kowalewski and P. Mangold, "Joint Predistortion and Transmit Diversity," in *Proc. Globecom '00*, November 2000, vol. 1, pp. 245–249.

[7] A. Noll Barreto and G. Fettweis, "Joint Signal Precoding in the Downlink of Spread Spectrum Systems," *IEEE Transactions on Wireless Communications*, vol. 2, no. 3, pp. 511–518, May 2003.

[8] M. Joham, K. Kusume, M. H. Gzara, W. Utschick, and J. A. Nossek, "Transmit Wiener Filter for the Downlink of TDD DS-CDMA Systems," in *Proc. ISSSTA 2002*, September 2002, vol. 1, pp. 9–13.

[9] R. L. Choi and R. D. Murch, "Transmit MMSE Pre-Rake Pre-processing with Simplified Receivers for the Downlink of MISO TDD-CDMA Systems," in *Proc. Globecom 2002*, November 2002.

[10] M. Joham, "On the Constrained Minimum Mean Square Error Transmit Filter and Its Relation to the Transmit Wiener Filter," Tech. Rep. TUM-LNS-TR-03-01, Munich University of Technology, March 2003.

[11] S. L. Georgoulis and D. G. M. Cruickshank, "Transmitter Based Inverse Filters for MAI and ISI Mitigation in a TDD/CDMA downlink," in *Proc. VTC Fall 2002*, September 2002, vol. 2, pp. 681–685.

[12] J. Wang, M. Zhao, S. Zhou, and Y. Yao, "A Novel Multipath Transmission Diversity Scheme in TDD-CDMA Systems," *IEICE Transactions on Communications*, vol. E82-B, no. 10, pp. 1706–1709, October 1999.

[13] R. Irmer, A. Noll Barreto, and G. Fettweis, "Transmitter Precoding for Spread-Spectrum Signals in Frequency-Selective Fading Channels," in *Proc. 3G Wireless*, May 2001, pp. 939–944.

[14] R. Irmer and G. Fettweis, "Combined Transmitter and Receiver Optimization for Multiple Antenna Frequency-Selective Channels," in *Proc. WPMC 2002*, October 2002, pp. 412–416.

[15] R. Irmer and G. Fettweis, "MISO Concepts for frequency-selective channels," in *Proc. 2002 International Zurich Seminar on Broadband Communications*, February 2002, pp. 40–1–40–6.

[16] J.-K. Han, M.-W. Lee, and H.-K. Park, "Principal Ratio Combining for Pre/Post-Rake Diversity," *IEEE Communications Letters*, vol. 6, no. 6, pp. 234–236, June 2002.

[17] F. Dietrich, R. Hunger, M. Joham, and W. Utschick, "Linear Precoding over Time-Varying Channels in TDD Systems," in *Proc. ICASSP 2003*, April 2003, vol. V, pp. 117–120.

[18] M. Joham, W. Utschick, and J. A. Nossek, "Latency Time Optimization for FIR and Block Transmit Filters," in *Proc. ISSPA 2003*, July 2002.

[19] R. N. McDonough and A. D. Whalen, *Detection of Signals in Noise*, Academic Press, 1995.