SPACE–TIME EQUALIZATION BASED ON V–BLAST AND DFE FOR FREQUENCY SELECTIVE MIMO CHANNELS

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ABSTRACT

In this article, we present different space-time receive processing techniques for frequency selective *multiple input multiple output* (MIMO) channels and evaluate their performance. We present the solutions for linear *zero-forcing* (ZF) and *Wiener filter* (WF) equalization with latency time optimization and incorporate the *Bell Laboratories Layered Space Time* (BLAST) architecture to gain diversity. Furthermore, we present systems based on *decision feedback equalization* (DFE). We also combine this equalization method with the BLAST principle.

1. INTRODUCTION

Considering the uplink of a mobile communication system, receive processing is performed at the *base station* (BS) in order to remove the distortions caused by the channel and the noise. When regarding MIMO systems with frequency selective channels we have to take into consideration the effects of *intersymbol interference* (ISI) and *co-channel interference* (CCI) when designing the receiver structure.

The essential advantage when considering a MIMO system is that higher capacity can be achieved in comparison to *single input single output* (SISO) systems [1]. The problem that arises is to achieve also a low *bit error rate* (BER). To achieve this goal a number of layered space-time (BLAST) architectures have been introduced (e. g. [2], [3]). A member of the BLAST family that has a relative simple structure is the *Vertical-BLAST* (V-BLAST) introduced in [3]–[5]. This architecture is based on a successive interference cancellation [6] and user detection, taking advantage of the rich multipath propagation. Although it has been shown in [7] that there is a different behaviour of various BLAST techniques we focus our attention on the V-BLAST algorithm in this paper.

Most V-BLAST algorithm contributions are restricted to flat fading communication channels, in this way only the CCI is taken under consideration. Many solutions referring to frequency selective channels are related to the principle of MIMO-DFE. The works presented in [8]–[11] derive *minimum mean square error* (MMSE) MIMO-DFEs, with noncausal filters of infinite length and in some cases equal number of transmit and receive antennas. These requirements are dropped in the layered space-time MIMO-DFE systems presented in [12], where MIMO-DFE-BLAST receive pocessing is achieved, as we have successive interference cancellation and user detection. Nevertheless, the design of these systems is not based on a total optimal temporal equalization and in some cases an unneeded enhanced system complexity is involved. In this paper, we present linear and DFE equalization techniques for MIMO frequency selective channels and their supplement with V-BLAST. Throughout this work, we have derived explicit formulas for temporal optimization and have designed systems that are not more complex than the MIMO-DFE system.

This paper is organized as follows. In Section 2, the system model is described. In Section 3, we present different space-time receive processing techniques. The performance of the various receiver structures is evaluated in Section 4 and the paper is concluded by Section 5.

2. SYSTEM MODEL

We consider a discrete-time baseband channel model with symbol-spaced channel taps. The channel coefficients are considered to be complex and Gaussian distributed. We assume perfect carrier recovery, downconversion, channel stationarity between two bursts and channel estimation at the BS [13].

Assuming that we have a channel with L paths, a system with N data streams, and a BS with M antennas the channel impulse response will have the form:

$$\boldsymbol{H}[n] = \sum_{i=0}^{L-1} \boldsymbol{H}_i \delta[n-i], \quad \boldsymbol{H}_i \in \mathbb{C}^{M\times N}.$$

If we define $\boldsymbol{s}[n] = \left[\; s_1[n], s_2[n], \ldots, s_N[n] \;\right]^{\mathrm{T}}$ to be the sent vector, where $(\bullet)^{\mathrm{T}}$ denotes transposition, the received vector $\boldsymbol{x}[n]$ can be expressed as

$$\boldsymbol{x}[n] = \sum_{i=0}^{L-1} \boldsymbol{H}_i \boldsymbol{s}[n-i] + \boldsymbol{\eta}[n], \quad \in \mathbb{C}^{M\times 1},$$

with $\boldsymbol{\eta}[n]$ containing the noise samples. Furthermore, we assume white noise and signal samples that are uncorre-

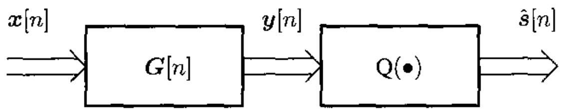

Fig. 1. Linear Receive Processing

lated to each other, resulting in the covariance matrices

$$\mathrm{E}\big[\boldsymbol{s}[n]\boldsymbol{s}^{\mathrm{H}}[n]\big] = \sigma_s^2 \mathbf{1}_N, \quad \mathrm{E}\big[\boldsymbol{\eta}[n]\boldsymbol{\eta}^{\mathrm{H}}[n]\big] = \sigma_\eta^2 \mathbf{1}_M,$$
$$\text{and} \quad \mathrm{E}\big[\boldsymbol{s}[n]\boldsymbol{\eta}^{\mathrm{H}}[n]\big] = \mathbf{0}_{N\times M},$$

where σ_s^2 and σ_η^2 are the average signal and noise power, respectively.

3. SPACE-TIME RECEIVE PROCESSING

3.1. Linear Receive Signal Processing

The system utilized for linear receive processing consists of a *finite-impulse-response* (FIR) filter $\boldsymbol{G}[n]$ followed by a quantizer that detects the filtered vector $\boldsymbol{y}[n]$ (cf. Fig. 1). Assuming that the FIR filter has F taps its impulse response will be

$$\boldsymbol{G}[n] = \sum_{j=0}^{F-1} \boldsymbol{G}_j \delta[n-j], \quad \boldsymbol{G}_j \in \mathbb{C}^{N\times M}.$$

Thus, the vector $\boldsymbol{y}[n]$ can be calculated as

$$\boldsymbol{y}[n] = \boldsymbol{G}\boldsymbol{H}\bar{\boldsymbol{s}}[n] + \boldsymbol{G}\bar{\boldsymbol{\eta}}[n]$$

where

$$\bar{\boldsymbol{s}}[n] = \left[\ \boldsymbol{s}^{\mathrm{T}}[n], \boldsymbol{s}^{\mathrm{T}}[n-1], \ldots, \boldsymbol{s}^{\mathrm{T}}[n-L-F+2]\ \right]^{\mathrm{T}},$$
$$\bar{\boldsymbol{\eta}}[n] = \left[\ \boldsymbol{\eta}^{\mathrm{T}}[n], \boldsymbol{\eta}^{\mathrm{T}}[n-1], \ldots, \boldsymbol{\eta}^{\mathrm{T}}[n-F+1]\ \right]^{\mathrm{T}},$$
$$\boldsymbol{G} = \left[\ \boldsymbol{G}_0, \boldsymbol{G}_1, \ldots, \boldsymbol{G}_{F-1}\ \right] \in \mathbb{C}^{N\times MF},$$

and $\boldsymbol{H} \in \mathbb{C}^{MF\times N(L+F-1)}$ is a block Toeplitz matrix.

1.) In order to achieve ZF equalization we have to solve the following optimization problem (e. g. [6]):

$$\boldsymbol{G}_{\mathrm{ZF}} = \arg\min_{\boldsymbol{G}} \operatorname{tr}\left(\boldsymbol{G}\boldsymbol{R}_\eta\boldsymbol{G}^{\mathrm{H}}\right)$$
$$\text{s.t.: } \boldsymbol{G}\boldsymbol{H} = \boldsymbol{E}_{\nu+1}^{\mathrm{T}}, \qquad (1)$$

with $\operatorname{tr}(\bullet)$ and $(\bullet)^{\mathrm{H}}$ denoting the trace and the Hermitian transpose of a matrix, respectively, $\boldsymbol{R}_\eta = \sigma_\eta^2 \mathbf{1}_{MF}$ and

$$\boldsymbol{E}_{\nu+1}^{\mathrm{T}} = \left[\ \mathbf{0}_{N\times\nu N}, \mathbf{1}_N, \mathbf{0}_{N\times(L+F-\nu-2)N}\ \right],$$

where the identity matrix $\mathbf{1}_N$ is placed at the $(\nu+1)$-th block position, leading to a latency time ν in the decision for the transmitted vector $\boldsymbol{s}[n]$ and to a cancellation of the interference from the post- and precursors of $\boldsymbol{s}[n]$. The solution to this problem is

$$\boldsymbol{G}_{\mathrm{ZF}} = \boldsymbol{E}_{\nu+1}^{\mathrm{T}}(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathrm{H}} = \boldsymbol{E}_{\nu+1}^{\mathrm{T}}\boldsymbol{H}^{+}, \qquad (2)$$

where $(\bullet)^{+}$ denotes the Moore-Penrose pseudoinverse of a matrix. The optimization of the latency time ν is done by substituting the above solution for $\boldsymbol{G}$ into the cost function of Eqn. (1):

$$\nu_{\mathrm{ZF}} = \arg\min_{\nu} \operatorname{tr}\left(\boldsymbol{E}_{\nu+1}^{\mathrm{T}}(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H})^{-1}\boldsymbol{E}_{\nu+1}\right). \qquad (3)$$

2.) Alternatively, we can achieve MMSE equalization by minimizing the error expression

$$\varepsilon_{\mathrm{WF}} = \mathrm{E}\left[\|\boldsymbol{s}[n-\nu] - \boldsymbol{y}[n]\|_2^2\right] \qquad (4)$$

which results in the Wiener-Hopf equation and has the solution

$$\boldsymbol{G}_{\mathrm{WF}} = \boldsymbol{E}_{\nu+1}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{H}}\left(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} + \frac{\sigma_\eta^2}{\sigma_s^2}\mathbf{1}_{MF}\right)^{-1}. \qquad (5)$$

The latency time ν can be again optimized by substituting the above solution for $\boldsymbol{G}$ into the error expression in Eqn. (4) and minimize with respect to ν.

3.2. Linear Receive Signal Processing Combined with V-BLAST

The system combining linear processing with V-BLAST has a structure similar to the one depicted in Fig. 1. In this case, though, the quantizer is replaced by a V-BLAST based detection mechanism [4], achieving successive user detection. First, we choose the latency time ν_{ZF} according to Eqn. (3). Then, we design the linear ZF filter $\boldsymbol{G}[n]$ stepwise like in [4]: 1) Compute $\boldsymbol{G}$ as in Eqn. (2). 2) Use only the row with minimum norm to maximize the SNR. 3) Set the entries in $\boldsymbol{H}$ according to the chosen row to zero and start with 1) again until the whole filter $\boldsymbol{G}[n]$ has been determined. Thus, the ordered transfer function $\boldsymbol{G}\boldsymbol{H}$ will have the form

$$\boldsymbol{G}\boldsymbol{H} = \left[\ \mathbf{0}_{N\times\nu N}, \boldsymbol{L}, \mathbf{0}_{N\times(L+F-\nu-2)N}\ \right],$$

where $\boldsymbol{L}$ is a lower triangular matrix with unit diagonal elements. Hence, by using the principle of the V-BLAST algorithm we can achieve successive user detection based on the structure of the matrix $\boldsymbol{L}$. We can again design our system according to the ZF as well as the MMSE criterion following the derivations of Section 3.1 and [4].

3.3. Linear Receive Signal Processing Combined with DFE

The receive processing system based on the DFE principle [14] has the structure depicted in Fig. 2. Assuming that $\boldsymbol{H}_1$ is the matrix consisting of the first $N(\nu+1)$ columns of $\boldsymbol{H}$ and that

$$\boldsymbol{D}_{\nu+1}^{\mathrm{T}} = \left[\ \mathbf{0}_{N\times\nu N}, \mathbf{1}_N\ \right] \in \{0,1\}^{N\times N(\nu+1)}$$

we can determine the feedforward filter $\boldsymbol{G}[n]$ according to the ZF criterion by solving the optimization of Eqn. (1)

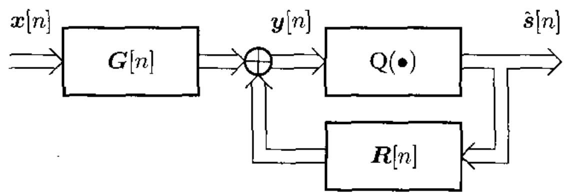

Fig. 2. DFE Receive Processing

but with the reduced constraint that $\boldsymbol{G}\boldsymbol{H}_1 = \boldsymbol{D}_{\nu+1}^{\mathrm{T}}$. This problem will have the solution $\boldsymbol{G}_{\text{ZF-DFE}} = \boldsymbol{D}_{\nu+1}^{\mathrm{T}}\boldsymbol{H}_1^{+}$, analogous to Eqn. (2). By substituting this solution into the minimization expression of Eqn. (1) and solving with respect to ν we can determine the optimal decision delay.

In this case, the transfer function $\boldsymbol{G}\boldsymbol{H}$ will have the form

$$\boldsymbol{G}\boldsymbol{H} = \left[\; \boldsymbol{0}_{N\times\nu N}, \boldsymbol{1}_N, \boldsymbol{A}_{\nu+1}, \dots, \boldsymbol{A}_{L+F-2} \;\right],$$

which means that we have to use DFE to subtract the interference from the precursors. The feedback filter $\boldsymbol{R}[n] = -\sum_{i=1}^{L+F-\nu-2} \boldsymbol{A}_{\nu+i}\delta[n-i]$ removes the interference from the precursors which have been sent prior to $s[n]$ assuming that all previous transmitted symbols have been detected correctly. We note that solutions based on joint feedforward-feedback filter optimizations, like in [12], yield the same result for the feedback filter and have the additional weakness that the latency time can't be optimized explicitly.

The above derivations and the analogous use of Eqn. (5) are sufficient to yield the $G_{\text{WF-DFE}}$ for MMSE processing, the corresponding optimal latency time, and feedback filter.

3.4. Linear Receive Signal Processing Combined with DFE and V-BLAST

The system based on layered space-time DFE processing has a structure similar to the one depicted in Fig. 2, with the additional feature that the user detection is made stepwise based on the V-BLAST algorithm. In this case, the ZF feedforward filter is designed in order to optimize the decision delay and the decision ordering according to V-BLAST, forcing the ordered transfer function $\boldsymbol{G}\boldsymbol{H}$ to be

$$\boldsymbol{G}\boldsymbol{H} = \left[\; \boldsymbol{0}_{N\times\nu N}, \boldsymbol{L}, \boldsymbol{A}_{\nu+1}, \dots, \boldsymbol{A}_{L+F-2} \;\right].$$

Using this expression we apply a V-BLAST based mechanism on the lower triangular matrix $\boldsymbol{L}$ for successive user detection and DFE for the subtraction of the precursors' interference. Following the steps of the previous sections we assert that the extension to MMSE processing is straightforward.

4. PERFORMANCE EVALUATION

In this section, we present simulation results for the MMSE receive processing systems (*Wiener filter*, WF) described

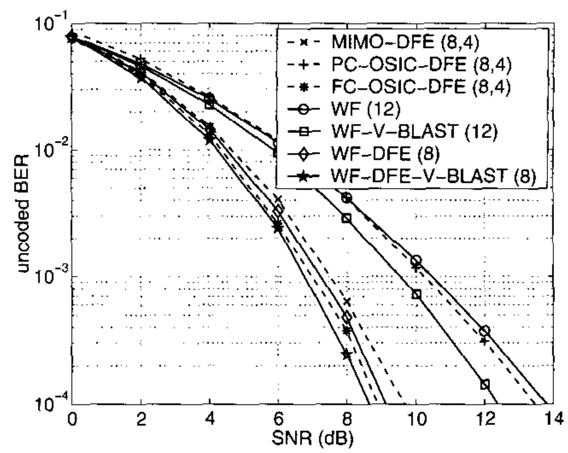

Fig. 3. Uncoded BER versus average SNR for different MMSE–MIMO receivers for QPSK

in Section 3. For this purpose, we have used a system with $M = 4$ receive antennas, $N = 3$ data streams, $L = 5$ channel taps and uncoded transmission. We have used QPSK as well as 16QAM as modulation scheme For the systems described in Section 3.3 and Section 3.4 we have chosen $F = 8$ filter taps whereas for the systems described in Section 3.1 and Section 3.2 we have used $F = 12$ taps since they have no feedback section. In addition, we compare the performance of our systems with the MMSE systems described in [12], namely the MIMO-DFE, the *Partially Connected Ordered Successive Interference Cancellation* (PC-OSIC) DFE, and the *Fully Connected* OSIC-DFE (FC-OSIC-DFE). For the simulation of these systems, we have chosen feedforward filters with 8 taps and feedback filters with 4 taps $(8, 4)$.

From Fig. 3 we assert that the best BER performance is achieved by the WF-DFE-V-BLAST. The performance improvement of the WF and WF-DFE when extending them to layered systems is obvious, as in this case we have an optimal latency time and detection ordering. We also observe that the DFE based systems perform better than the systems without DFE as they evoke more degrees of freedom for the calculation of the feedforward filter. The PC-OSIC-DFE has a performance very close to the one of the WF, a fact that characterizes it as a rather unattractive solution. The MIMO-DFE and the FC-OSIC-DFE yield a satisfactory BER performance but the WF-DFE and the WF-DFE-V-BLAST, respectively, outperform them, despite the fact that the FC-OSIC-DFE requires more complexity than the proposed systems.

Considering Fig. 4 we realize that the performance of the systems changes for 16QAM, when the error propagation due to DFE is severe. In this case, the WF and WF-V-BLAST have the best performance for low SNR values (SNR $\leq$ 10dB) since they are not characterized by error propagation. The WF-DFE and WF-DFE-V-BLAST ex-

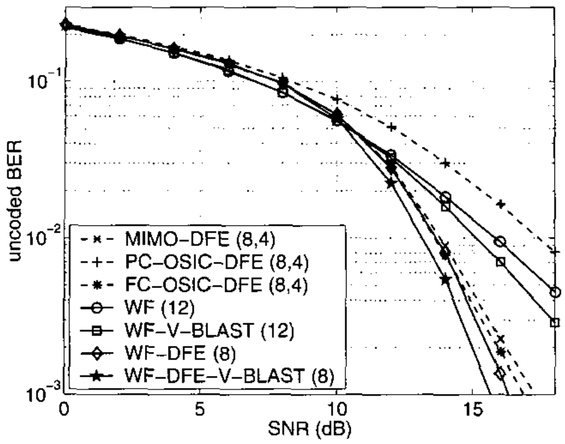

Fig. 4. Uncoded BER versus average SNR for different MMSE–MIMO receivers for 16QAM

hibit bad performance for low SNR values, but their performance improves rapidly with increasing SNR. Thus, for high SNR values the best performance is again achieved by the WF-DFE-V-BLAST. The characteristics of the curves of the DFE based systems are similar to their respectives for QPSK. The error propagation is particularly severe for the the FC-OSIC-DFE as this is confirmed by the fact that its performance is not significantly better than the one of the non-layered MIMO-DFE.

Summarizing, the best performance for low SNR and 16QAM is achieved by the WF-V-BLAST and by the WF-DFE-V-BLAST in all other cases.

5. SUMMARY AND CONCLUSIONS

We have introduced different space-time MIMO receivers for frequency selective channels and optimized them according to the ZF as well as the MMSE criterion. Additionally, we have presented possible optimizations of the latency time and used an optimal detection ordering based on V-BLAST for the layered space-time receiver structures. Simulation results and comparisons with the systems described in [12] show that the layered systems described in this article have a simpler structure and can yield superior performances. The use of the WF-V-BLAST and WF-DFE-V-BLAST assures high user capacity and low BER for both QPSK and 16QAM.

6. REFERENCES

[1] I. E. Telatar, "Capacity of Multi-antenna Gaussian Channels," *Europ. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov. 1999.

[2] G. J. Foschini, "Layered Space-Time Architecture for Wireless Communications in a Fading Environment when Using Multi-Element Antennas," *Bell Labs Tech. J.*, pp. 41–59, 1996.

[3] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniasky, "Simplified Processing for High Spectral Efficiency Wireless Communication Employing Multi-Element Arrays," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 1841–1852, Nov. 1999.

[4] P. W. Wolniasky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates over the Rich-Scattering Wireless Channel," in *Proc. ISSSE-98*, Sep. 1998.

[5] G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Wolniasky, "Detection algorithm and initial laboratory results using v-blast space-time communication architecture," *Electron. Lett.*, Nov. 1998.

[6] S. Verdu, *Multiuser Detection*, Cambridge University Press, 1998.

[7] E. Biglieri, G. Taricco, and A. Tulino, "Decoding space-time codes with BLAST architectures," in *IEEE 7th Symp. on Spread-Spectrum Tech. & Appl.*, Sep. 2002.

[8] A. Duel-Hallen, "Equalizers for Multiple-Input Multiple-Output Channels and PAM Systems with Cyclostationary Input Sequences," *IEEE J. Select. Areas Commun.*, vol. 10, pp. 630–639, Apr. 1992.

[9] A. Lozano and C. Papadias, "A Family of Multiuser Decision-Feedback Detectors for Asynchronous Code-Division Multiple-Access Channels," *IEEE Trans. Commun.*, vol. 43, pp. 421–434, Feb./Mar./Apr. 1995.

[10] S. Roy and D. D. Falconer, "Multi-User Decision-Feedback Space-Time Equalization and Diversity Reception," in *Proc. IEEE Vehicular Technology Conf. (VTC'99 Spring)*, May 1999.

[11] A. Lozano and C. Papadias, "Multi-User Decision-Feedback Space-Time Processing with Partial Cross-Feedback Connectivity," in *Proc. IEEE Vehicular Technology Conf. (VTC'2000 Spring)*, May 2000.

[12] A. Lozano and C. Papadias, "Layered Space-Time Receivers for Frequency-Selective Wireless Channels," *IEEE Trans. Commun.*, vol. 50, pp. 65–73, Jan. 2002.

[13] T. L. Marzetta, "BLAST training: Estimating Channel Characteristics for High-Capacity Space-Time Wireless," in *Proc. 37thAnnu. Allerton Conf. Communications, Control and Computing*, 1999.

[14] J. G. Proakis, *Digital Communications*, McGraw-Hill, Inc., 1995.