Removing Inconsistencies of Reduced Bases in Parametric Model Order Reduction by Matrix Interpolation

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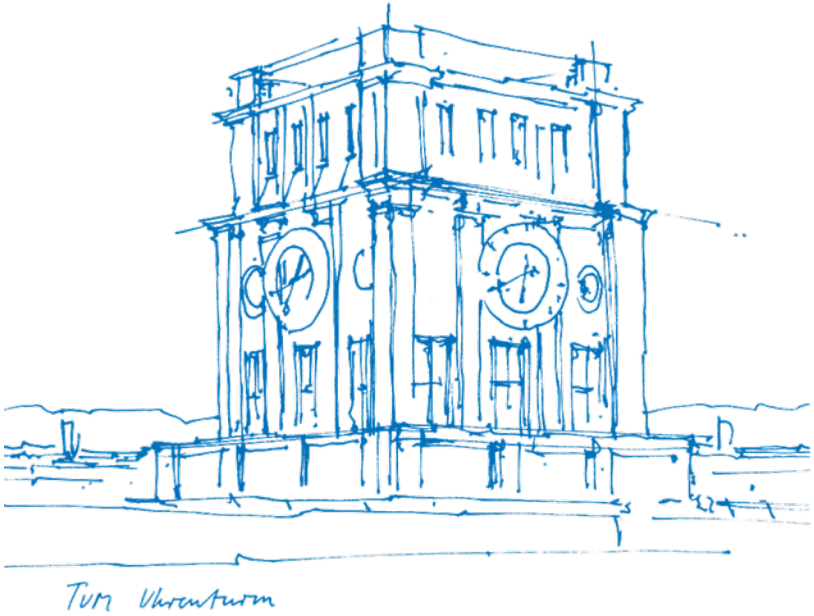
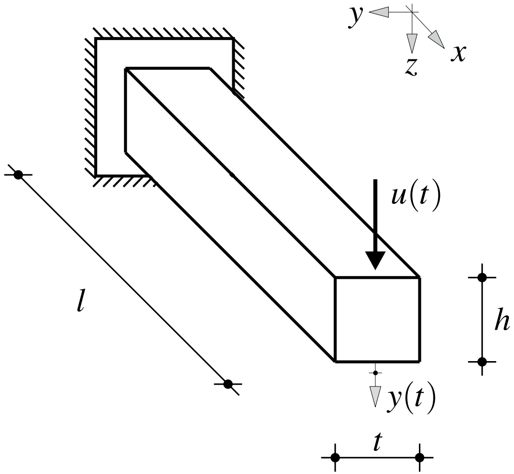
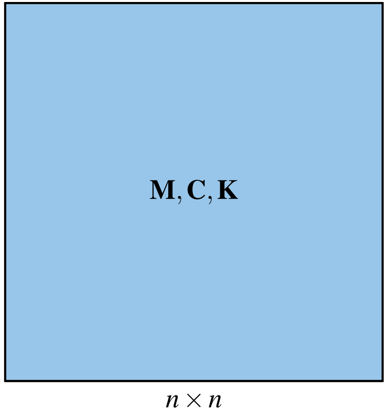


Motivation





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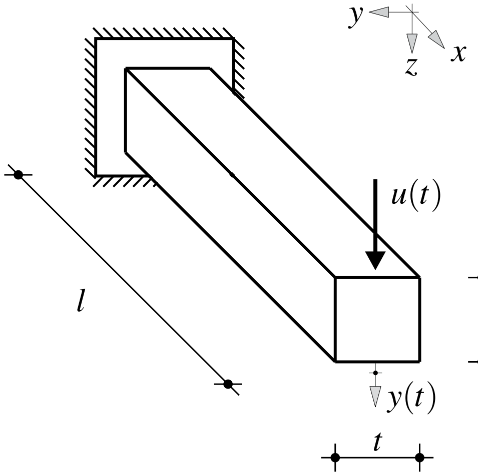

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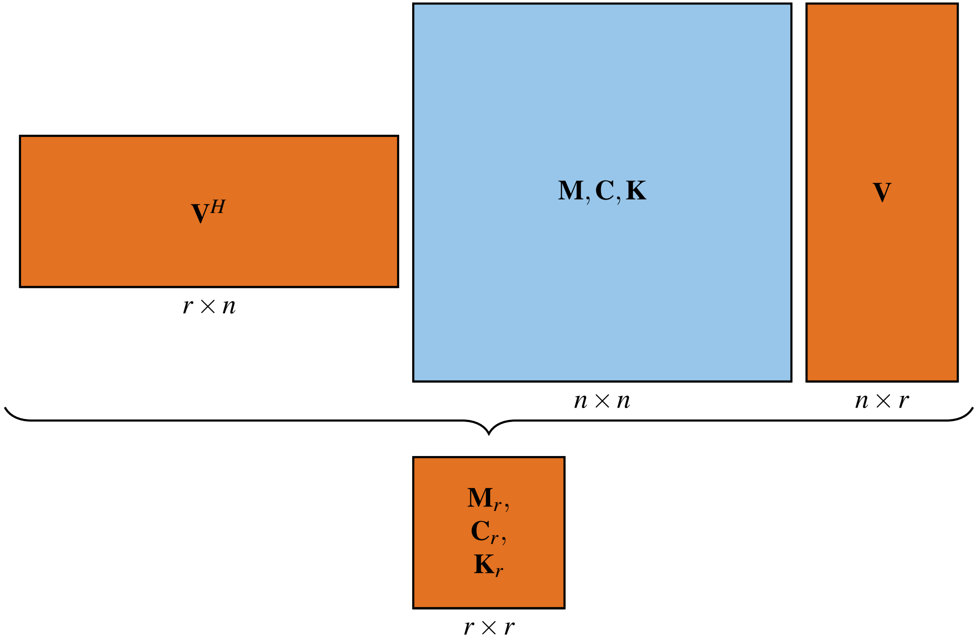

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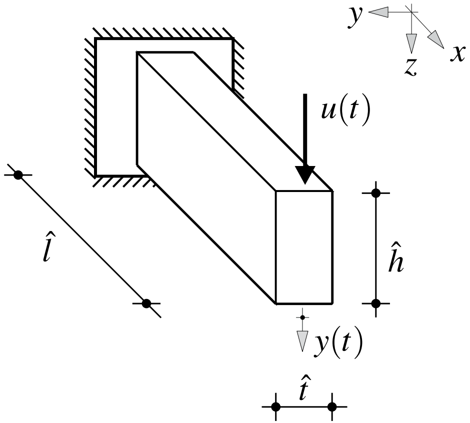

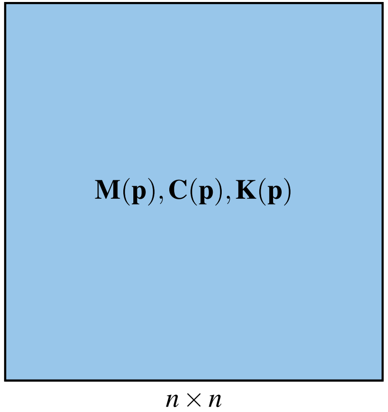

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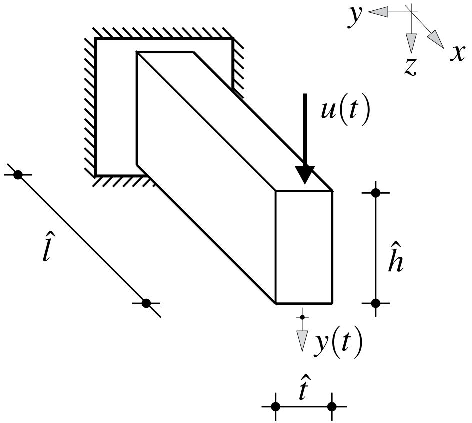

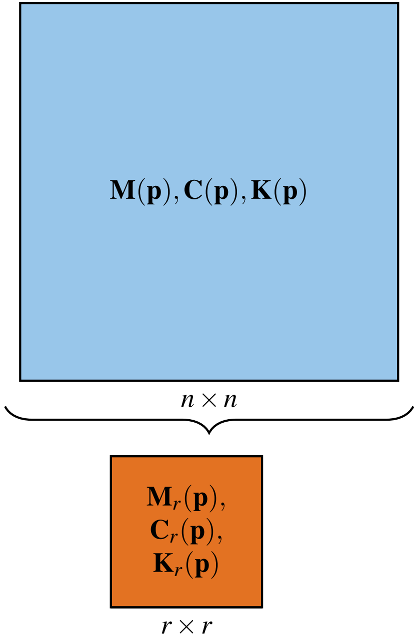

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Outline

- Mathematical System Description
- Parametric Model Order Reduction by Matrix Interpolation
- Removal of Inconsistencies via Adaptive Sampling and Clustering
- Results
- Conclusion and Future Work

Mathematical System Description

Parametric Dynamic Systems

Linear-time invariant, parametric dynamical systems with single input and single output (SISO) in second-order form are regarded:

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with mass, damping and stiffness matrix $\mathbf{M}(\mathbf{p}), \mathbf{C}(\mathbf{p}), \mathbf{K}(\mathbf{p}) \in \mathbb{R}^{n\times n}$, and input and output mapping $\mathbf{f}(\mathbf{p}) \in \mathbb{R}^{n\times 1}$ and $\mathbf{g}(\mathbf{p}) \in \mathbb{R}^{1\times n}$, which depend on d parameters $\mathbf{p} = [p_1, p_2, \dots, p_d]$. The vectors $\mathbf{x}(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ denote the state, input and output.

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After performing a Laplace transformation, the transfer function of the system can be computed as

$$H(s,\mathbf{p}) = \mathbf{g}(\mathbf{p}) \left(s^2\mathbf{M}(\mathbf{p}) + s\mathbf{C}(\mathbf{p}) + \mathbf{K}(\mathbf{p})\right)^{-1} \mathbf{f}(\mathbf{p}), \tag{2}$$

with the complex frequency $s \in \mathbb{C}$.

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with the complex frequency $s \in \mathbb{C}$.

We assume that **no** reasonable affine representation of the parametric dependency of the following form is available (exemplarily shown for the stiffness matrix):

$$\mathbf{K}(\mathbf{p}) = \mathbf{K}_0 + \sum_{i=1}^{M} f_i(\mathbf{p})\mathbf{K}_i, \qquad i = 1,\dots,M, \tag{4}$$

where $f_i(\mathbf{p})$ are scalar functions. [BGW15]

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- does not require an affine representation of the parametric dependency,
- is valid for a large range of the parameters, and
- is generated via an adaptive algorithm.

Parametric Model Order Reduction by Matrix Interpolation

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We follow the approach of pMOR by matrix interpolation by [PMEL10]:

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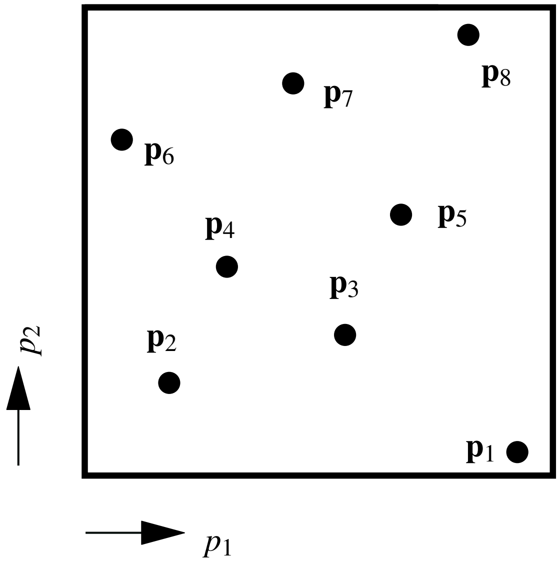

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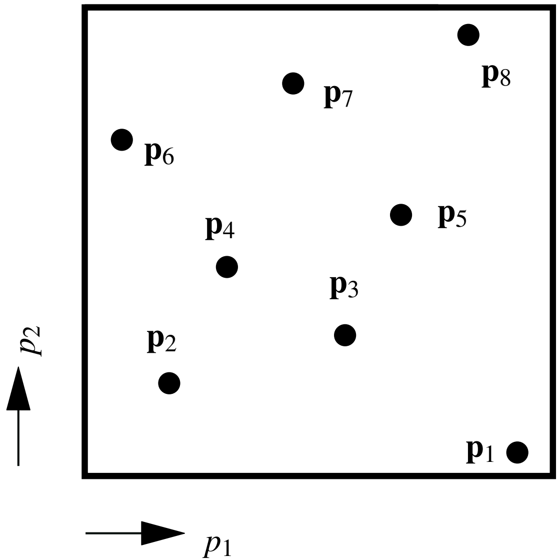

$$\{\mathbf{M}(\mathbf{p}_k), \mathbf{C}(\mathbf{p}_k), \mathbf{K}(\mathbf{p}_k), \mathbf{f}(\mathbf{p}_k), \mathbf{g}(\mathbf{p}_k)\}$$

$\Downarrow$ Project into $\mathbf{V}_k \in \mathbb{C}^{n\times r}$, $(\mathbf{x}(\mathbf{p}_k) \approx \mathbf{V}_k \mathbf{x}_r(\mathbf{p}_k))$

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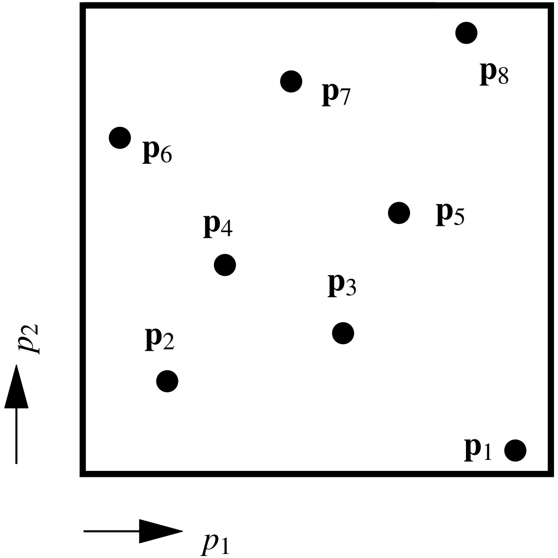

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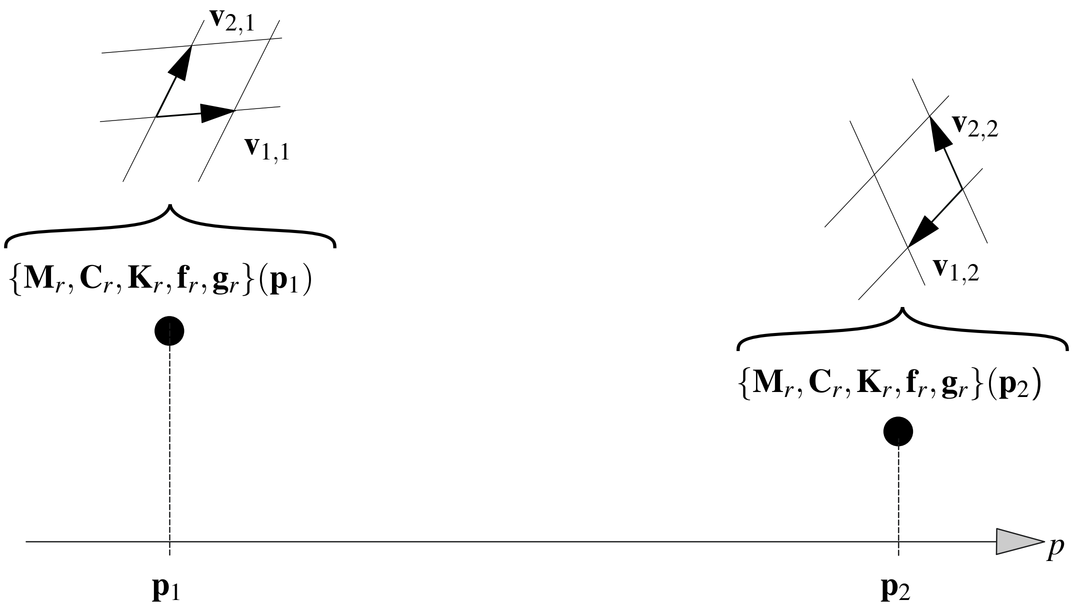
where

$$(\mathbf{K}(\mathbf{p}_k) - \omega^2\mathbf{M}(\mathbf{p}_k))\boldsymbol{\phi} = \mathbf{0}$$
$$\mathbf{V}_k = [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \dots, \boldsymbol{\phi}_r].$$

Parametric Model Order Reduction by Matrix Interpolation

1. Sampling of local reduced systems



Parametric Model Order Reduction by Matrix Interpolation

1. Sampling of local reduced systems

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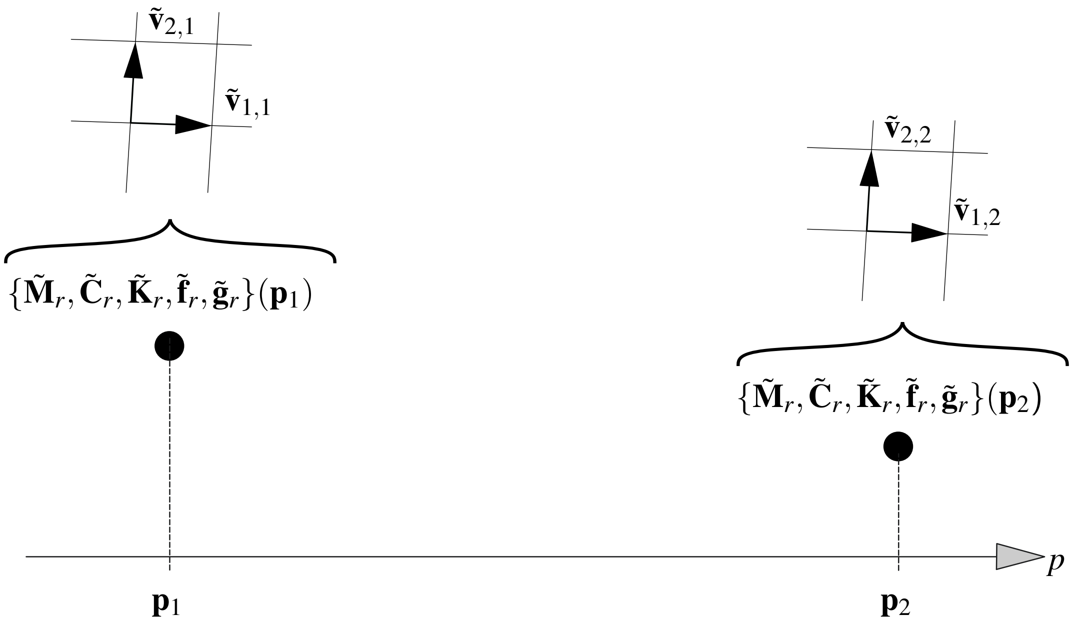
2. Transformation to generalized coordinate system



Parametric Model Order Reduction by Matrix Interpolation

For a meaningful interpolation, the reduced operators should be in the same coordinate system. To achieve this, the following approach was suggested in [PMEL10]:

1. Find a generalized coordinate system. For this purpose, find the most significant basis vectors by concatenating all N sampled bases and then performing an SVD:

   $$[\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_N] = \mathbf{U\Sigma Y}, \qquad \mathbf{V}_k \in \mathbb{C}^{n\times r}, \quad k = 1, \ldots, N \tag{6}$$

   The most significant basis vectors are the first r columns in $\mathbf{U}$ and denoted with $\mathbf{R}$:

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2. Transform the individual reduced operators from their individual bases $\mathbf{V}_k$ to the generalized coordinate system $\mathbf{R}$:

$$\tilde{\mathbf{K}}_r(\mathbf{p}_k) = \mathbf{T}_k^\mathsf{T}\mathbf{K}_r(\mathbf{p}_k)\mathbf{T}_k, \quad \tilde{\mathbf{C}}_r(\mathbf{p}_k) = \mathbf{T}_k^\mathsf{T}\mathbf{C}_r(\mathbf{p}_k)\mathbf{T}_k, \quad \tilde{\mathbf{M}}_r(\mathbf{p}_k) = \mathbf{T}_k^\mathsf{T}\mathbf{M}_r(\mathbf{p}_k)\mathbf{T}_k, \quad \tilde{\mathbf{f}}_r(\mathbf{p}_k) = \mathbf{T}_k^\mathsf{T}\mathbf{f}_r(\mathbf{p}_k), \quad \tilde{\mathbf{g}}_r(\mathbf{p}_k) = \mathbf{g}_r(\mathbf{p}_k)\mathbf{T}_k, \tag{8}$$

   with

$$\mathbf{T}_k = (\mathbf{R}^T\mathbf{V}_k)^{-1}, \quad \tilde{\mathbf{V}}_k = \mathbf{V}_k\mathbf{T}_k. \tag{9}$$

Parametric Model Order Reduction by Matrix Interpolation

1. Sampling of local reduced systems

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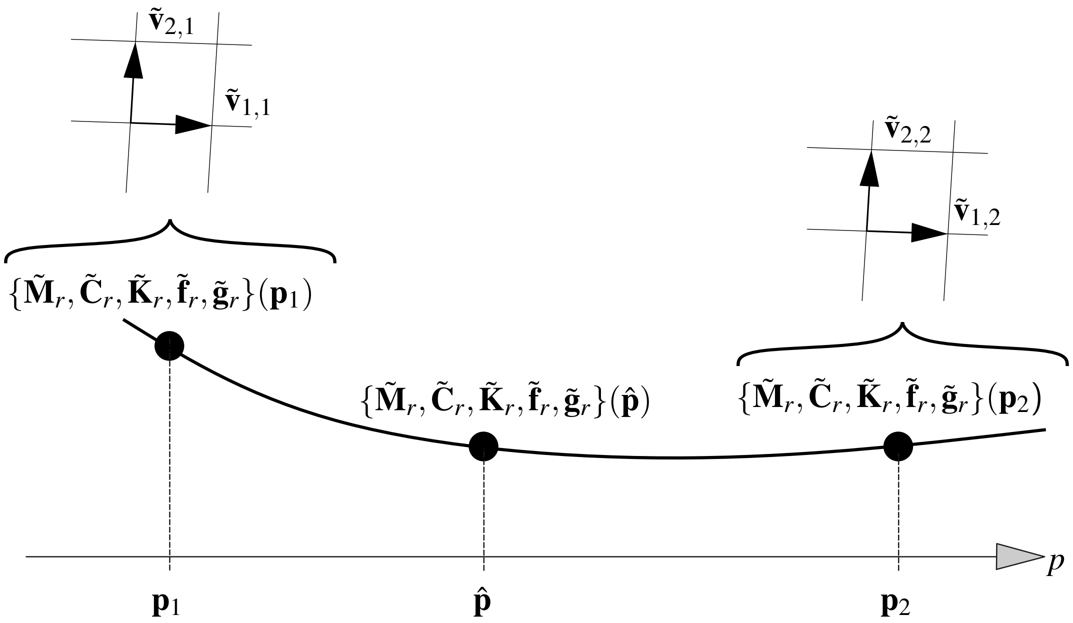
2. Transformation to generalized coordinate system

↓

3. Interpolation/ regression of reduced operators



Inconsistencies in Reduced Bases

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Strong changes in the reduced bases introduce inconsistencies in the training data for the matrix interpolation. They can occur due to several reasons:

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- Model Order Reduction method used [FE15]
- Change of the system dynamics [BNN$^+$15]
- Mode switching and truncation [ATF15]

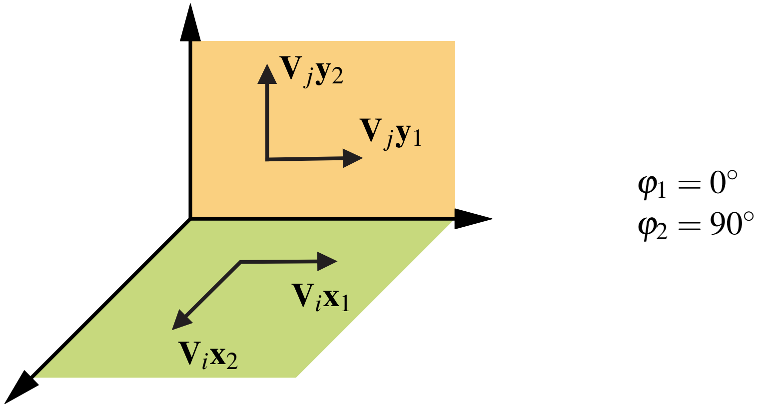
Detection of Inconsistencies

The angles between the subspaces spanned by the two orthonormal bases $\mathbf{V}_i$ and $\mathbf{V}_j$ are computed by first performing an SVD on the following product [ATF15]:

$$\mathbf{V}_i^{\mathrm{H}}\mathbf{V}_j = \mathbf{X}\boldsymbol{\Sigma}\mathbf{Y}^{\mathsf{T}}, \qquad i,j = 1,\ldots,N \tag{12}$$

The subspace angles can then be found as

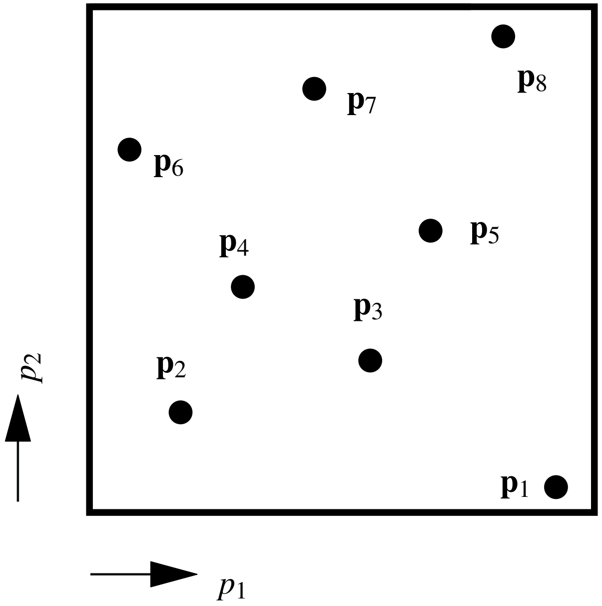
$$\varphi_l = \arccos(\sigma_l), \qquad l = 1,\ldots,r. \tag{13}$$



Removal of Inconsistencies via Adaptive Sampling and Clustering

Adaptive Sampling and Clustering

1. Sampling of local reduced systems
2. Transformation to generalized coordinate system
3. Interpolation/ regression of reduced operators



Adaptive Sampling and Clustering

1. Sampling of local reduced systems → 1.1 Triangulation

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2. Transformation to generalized coordinate system

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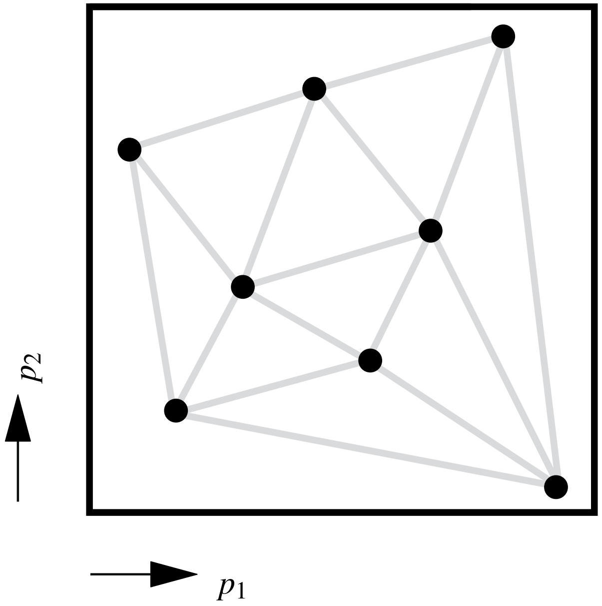
3. Interpolation/ regression of reduced operators



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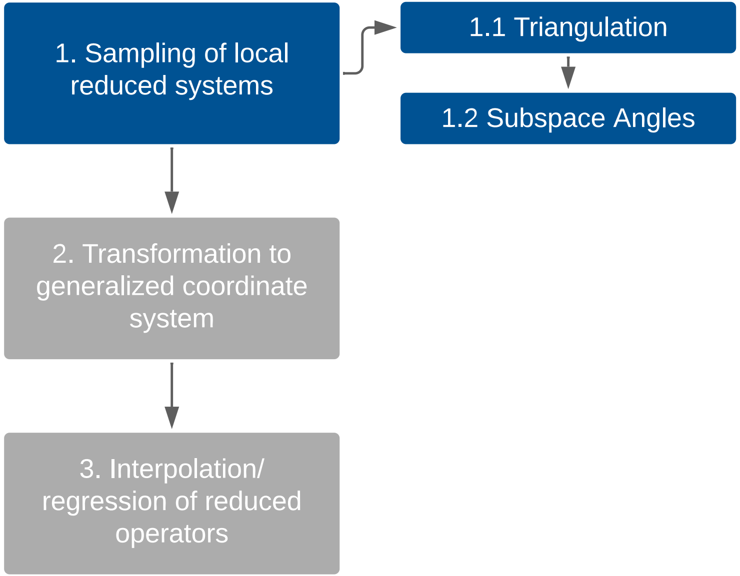

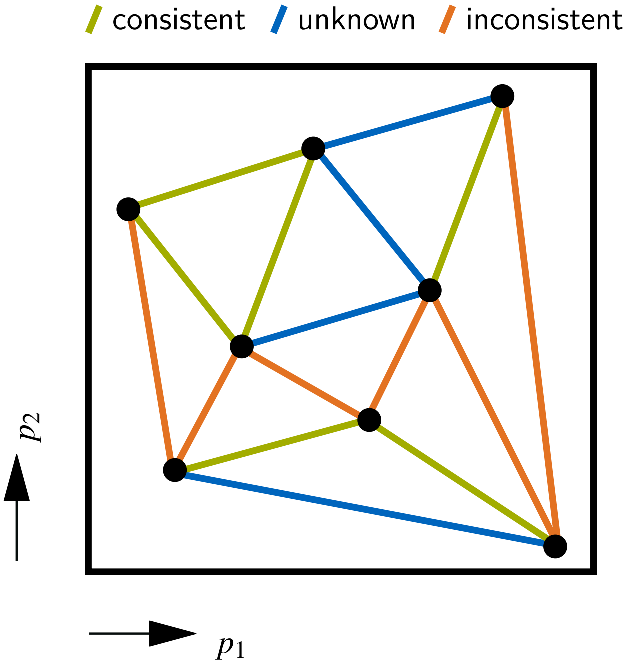

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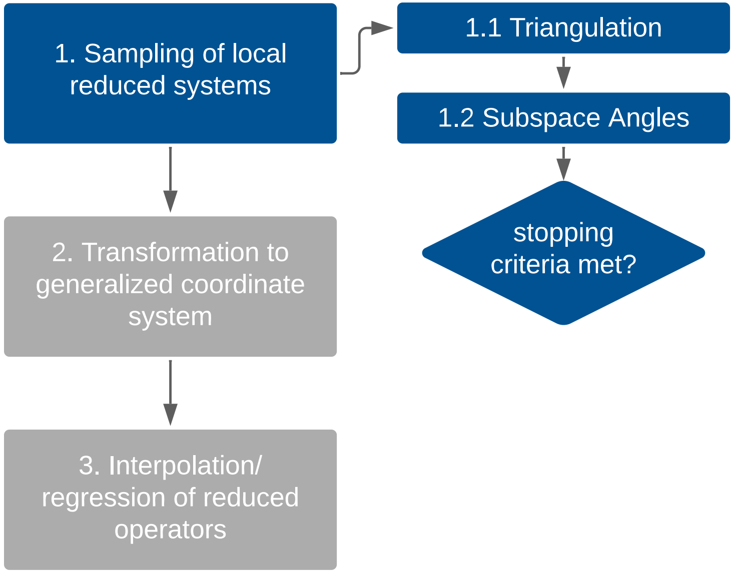

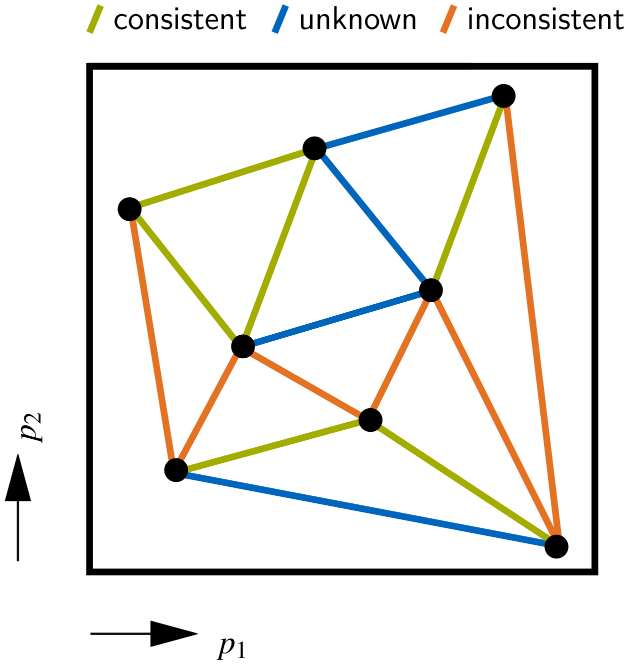

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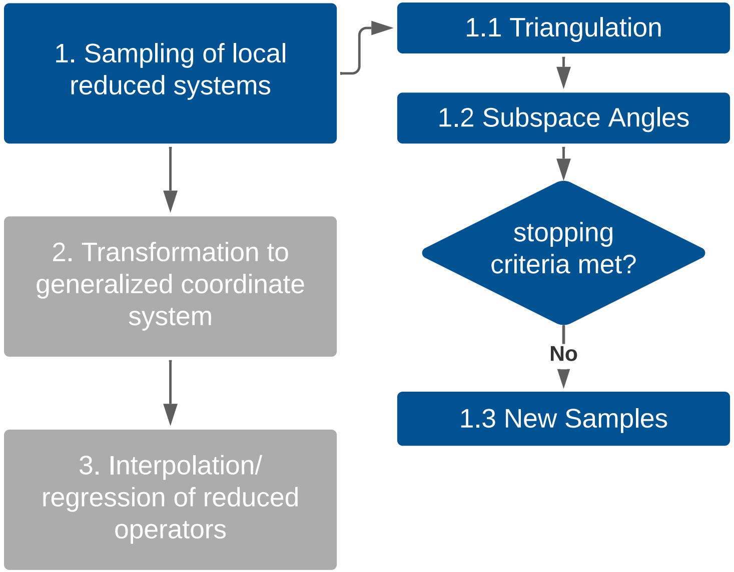

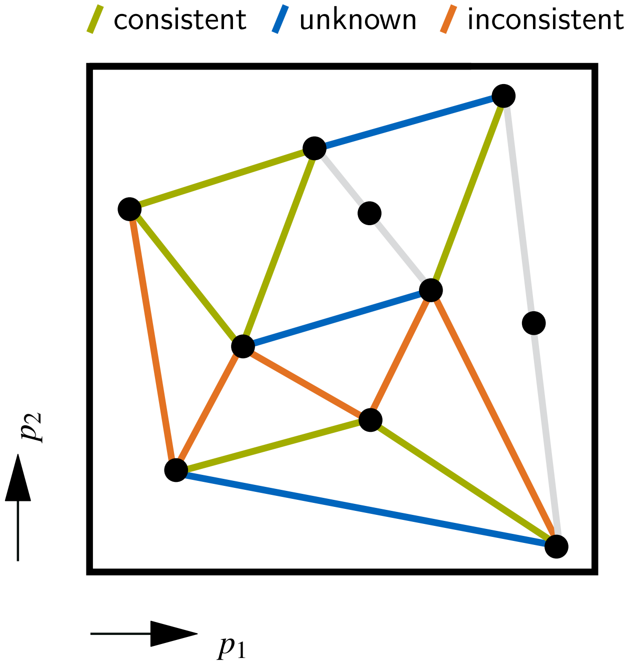

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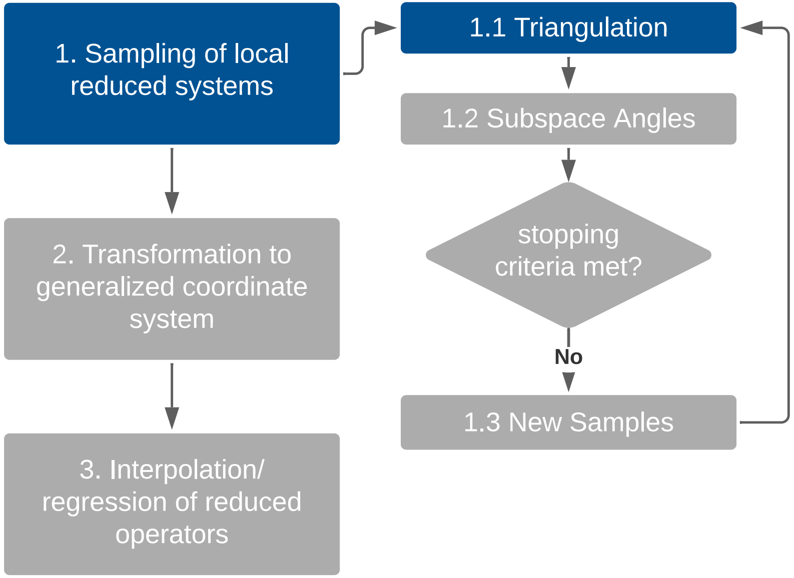

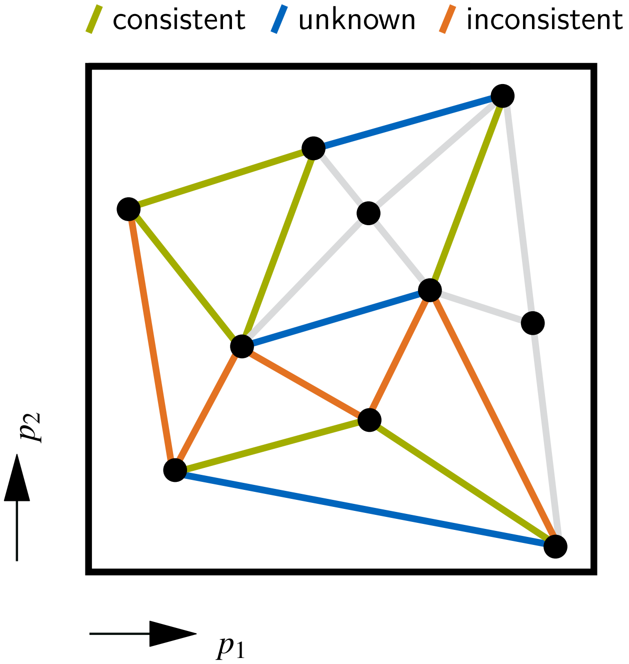

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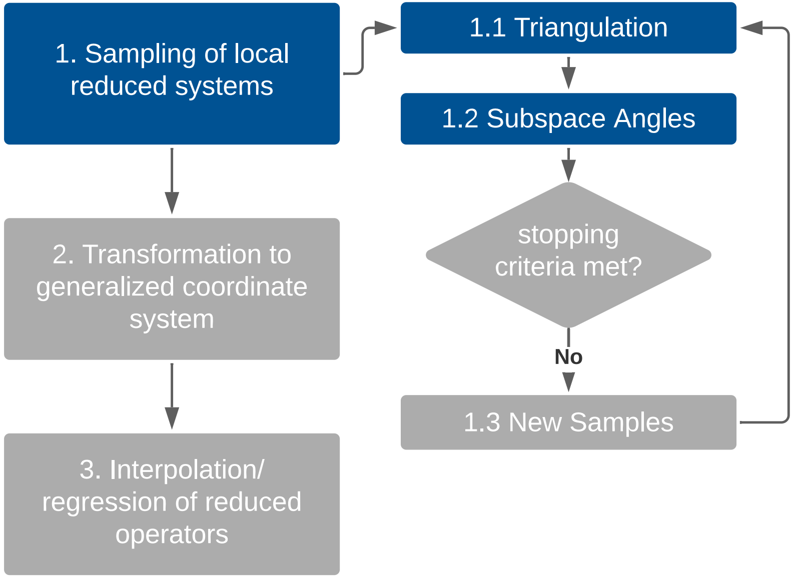

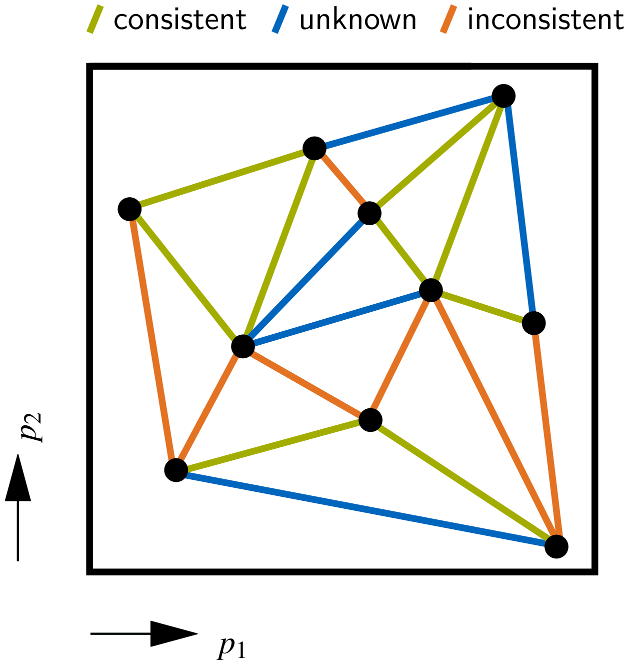

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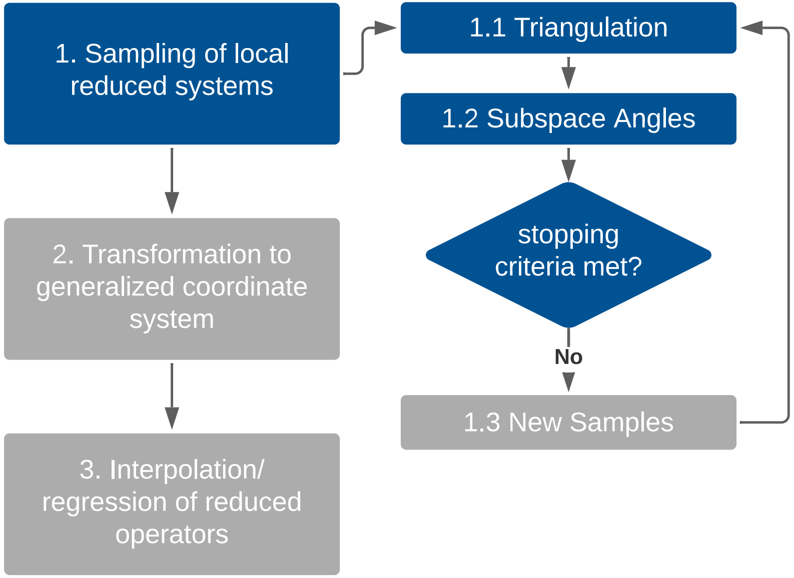
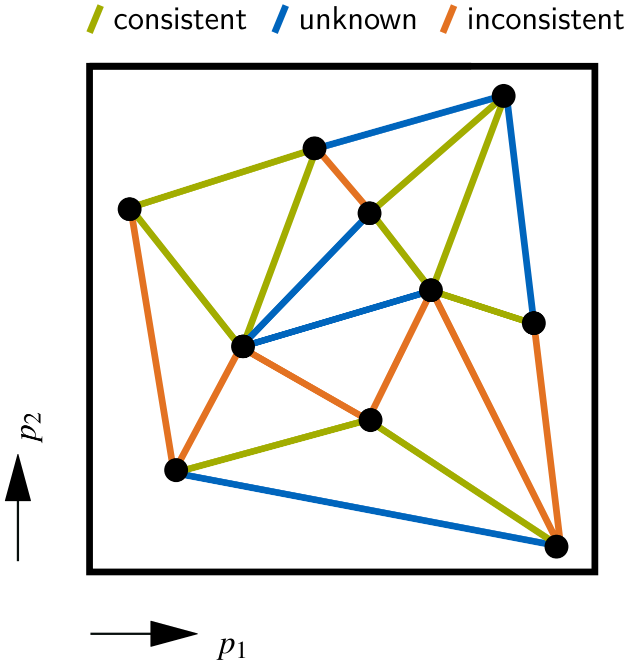
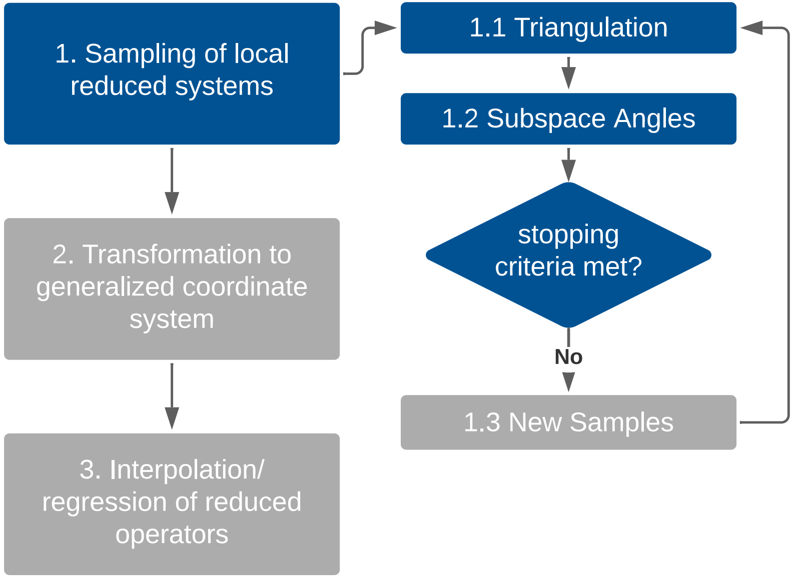
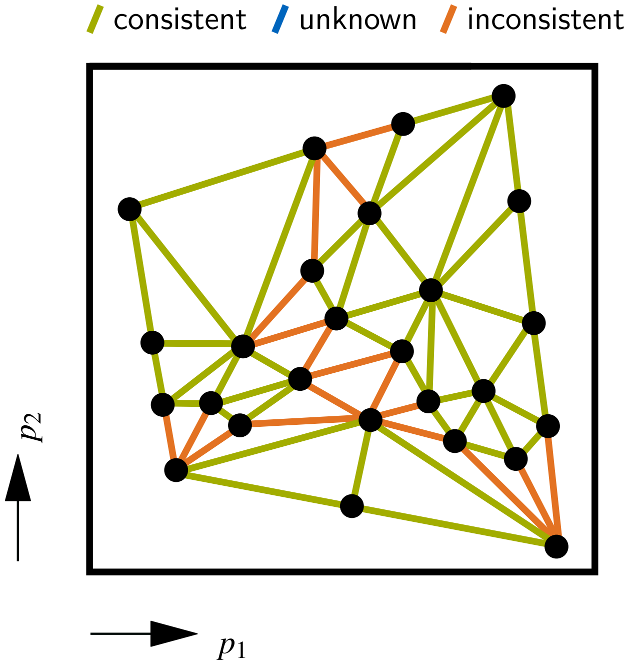


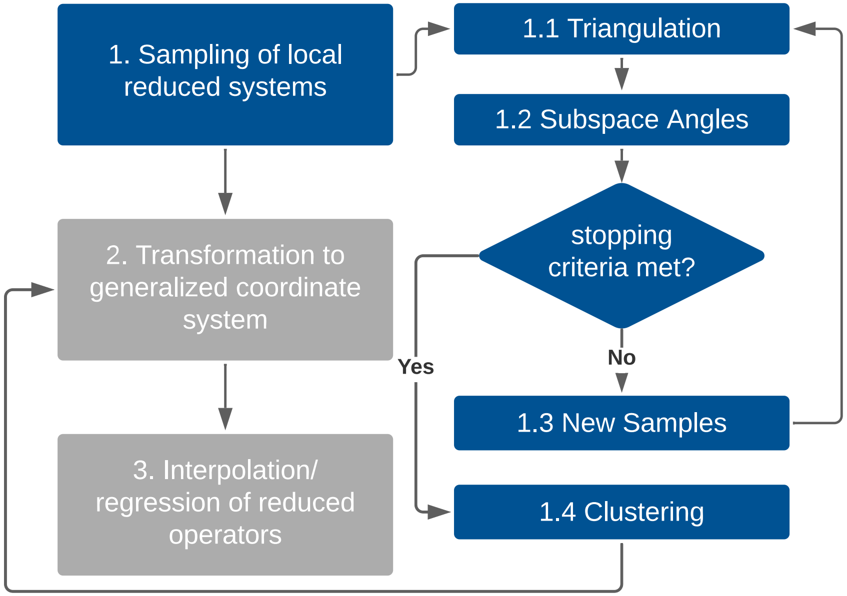


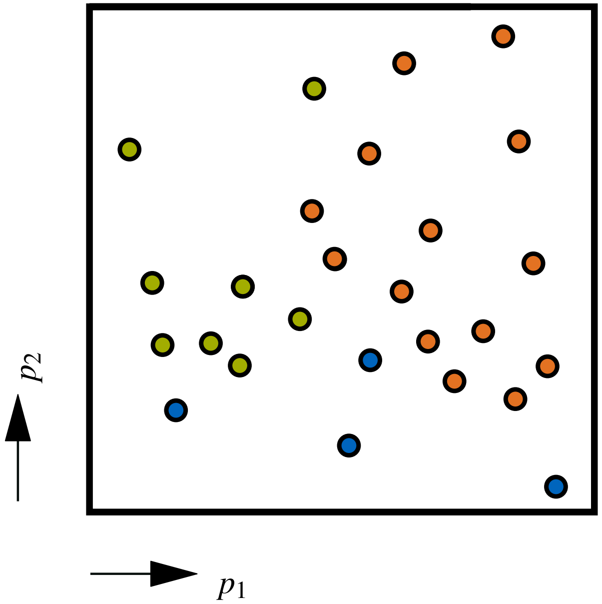
Adaptive Sampling and Clustering





Adaptive Sampling and Clustering





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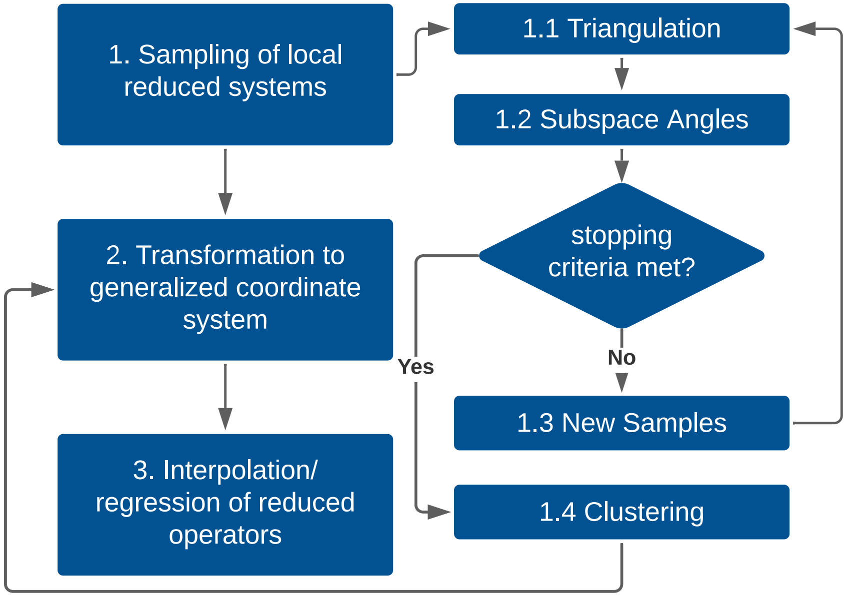
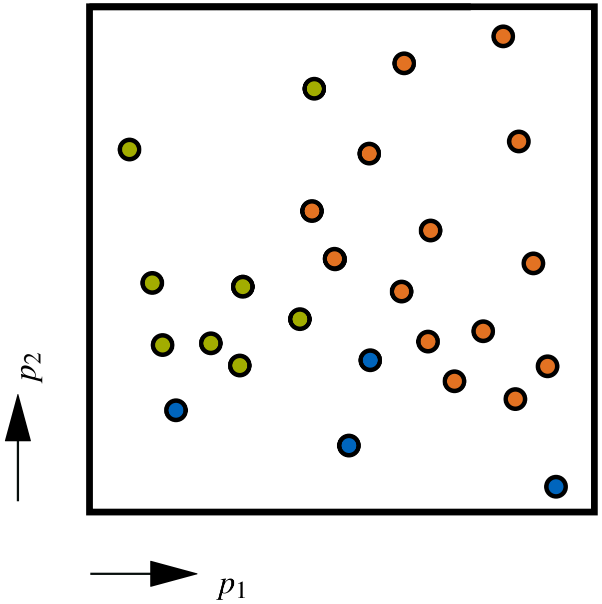
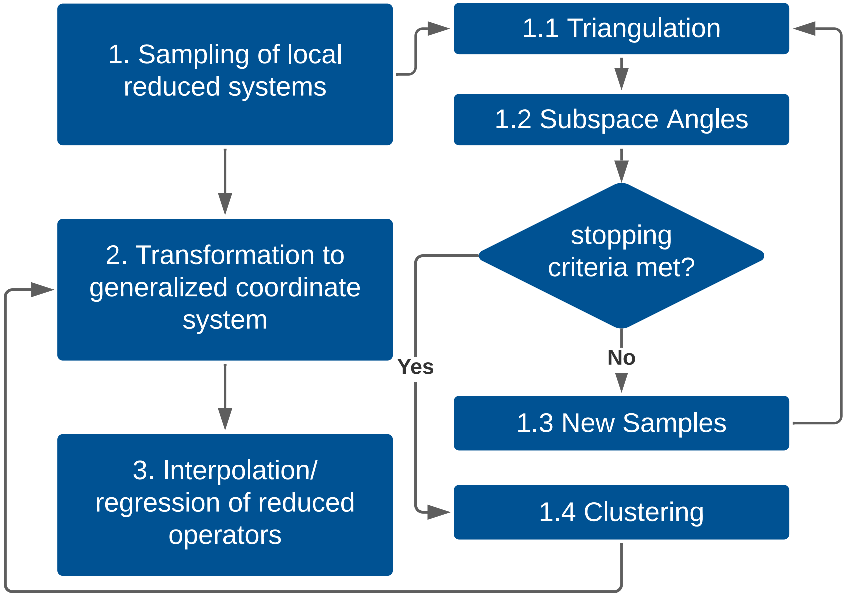
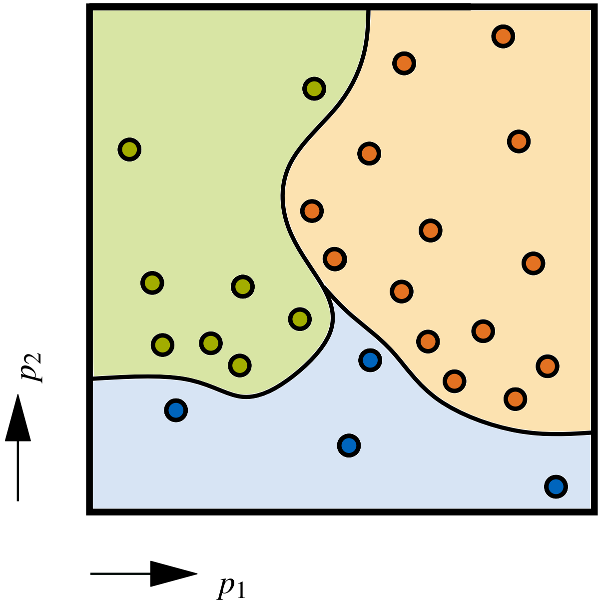




Adaptive Sampling and Clustering

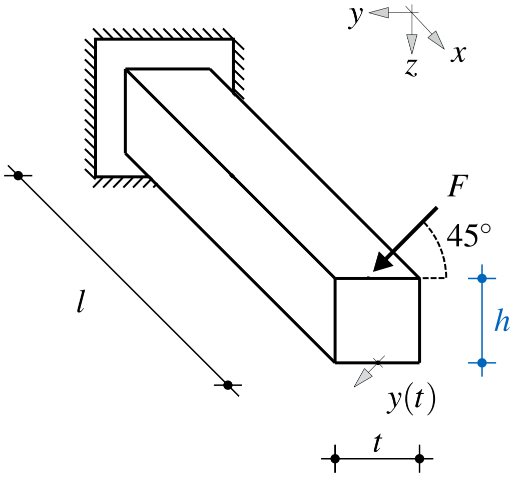




Results

Results – Timoshenko Beam – Beam Height h

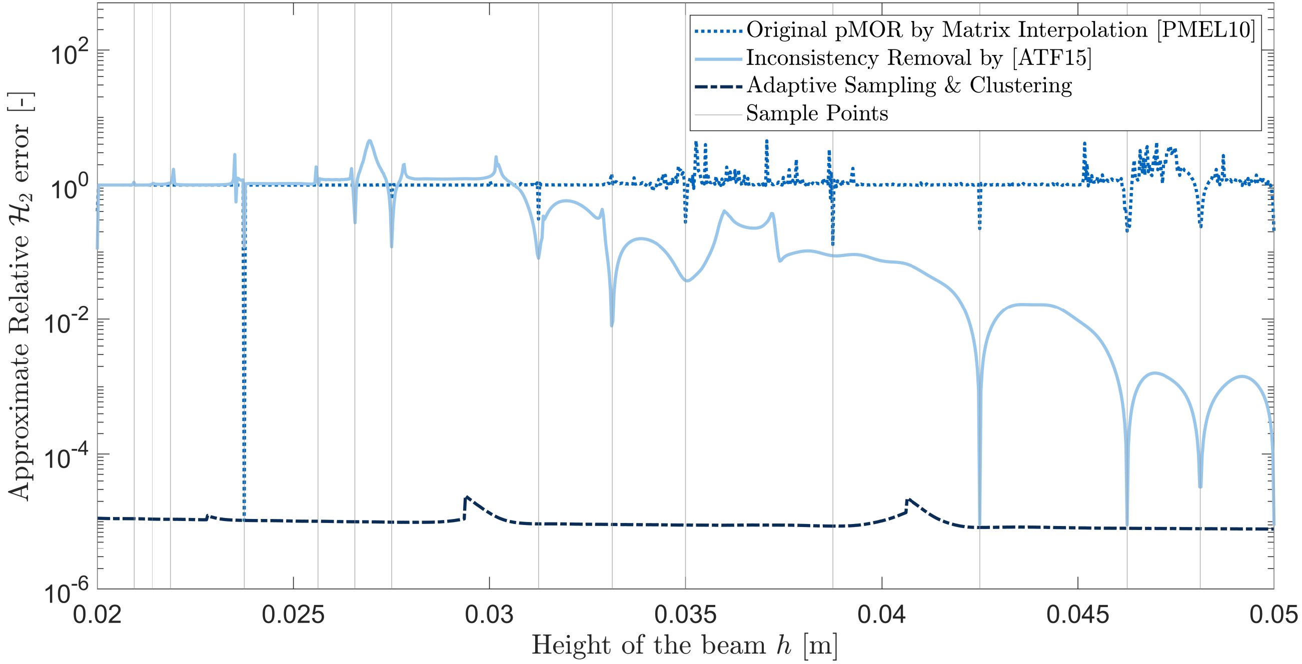
A 3D cantilevered beam discretized with Timoshenko beam elements is investigated. The beam is excited at the tip with a harmonic force of varying frequency ($[0, 1000]$ Hz). The adaptive sampling and clustering algorithm is compared to the original version of pMOR by Matrix interpolation [PMEL10] and a method for inconsistency removal by [ATF15].



Parameter	Range/Value	Unit
Height h	$[0.02, 0.05]$	m
Thickness t	0.01	m
Length l	1.0	m
Young's modulus E	$2.1 \cdot 10^{11}$	N/m^2
Poisson's ratio ν	0.3	-
Density ρ	7860	kg/m^3
Rayleigh damping α	$8 \cdot 10^{-6}$	$1/s$
Rayleigh damping β	8	s

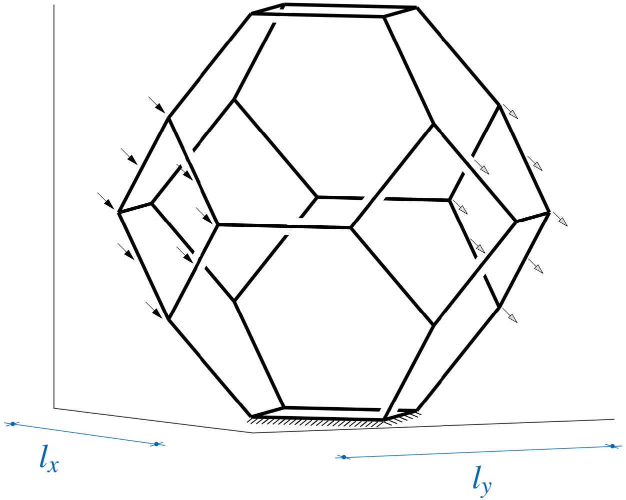
Table: Geometry and material parameters of the 3D cantilevered beam.

Results – Timoshenko Beam – Beam Height h



Results – Kelvin Cell – Dimensions l_x and l_y

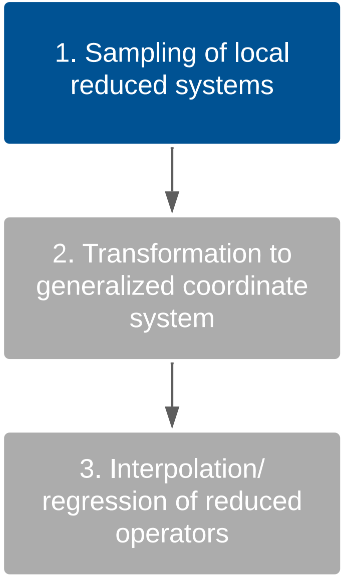
A 3D cantilevered beam discretized with Timoshenko beam elements is investigated. The beam is excited at the tip with a harmonic force of varying frequency ($[0, 1000]$ Hz). Rayleigh damping is used: $\mathbf{C} = \alpha\mathbf{K} + \beta\mathbf{M}$.

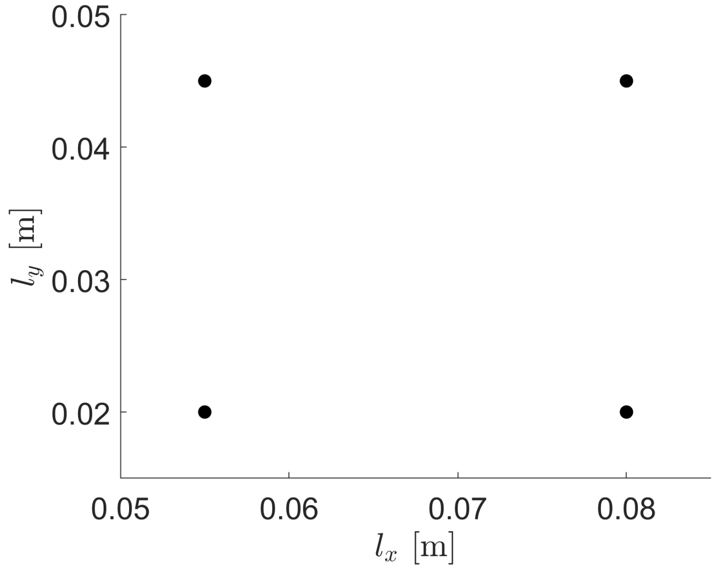


Parameter	Range/Value	Unit
Length l_x	$[0.055, 0.080]$	m
Length l_y	$[0.020, 0.045]$	m
Length l_z	0.05	m
Beam thickness t	0.001	m
Young's modulus E	$4.35 \cdot 10^9$	N/m^2
Poisson's ratio ν	0.3	-
Density ρ	1180	kg/m^3
Rayleigh damping α	$8 \cdot 10^{-6}$	$1/s$
Rayleigh damping β	8	s

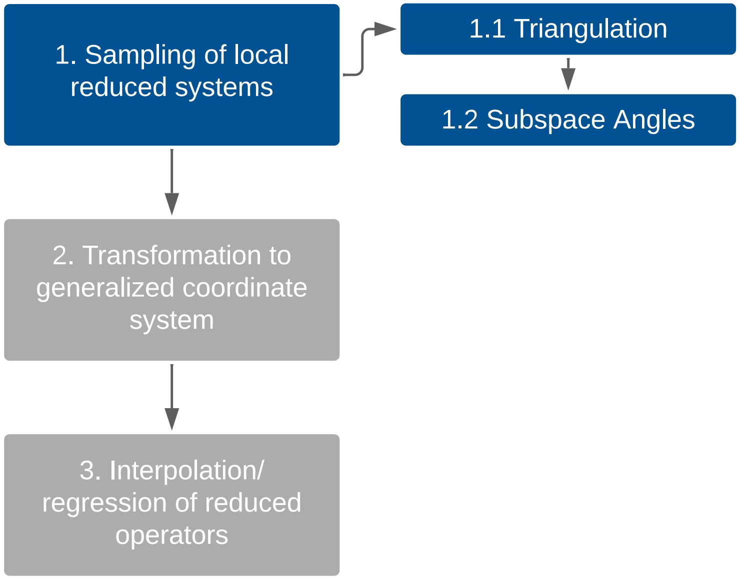
Table: Geometry and material parameters of the Kelvin Cell.

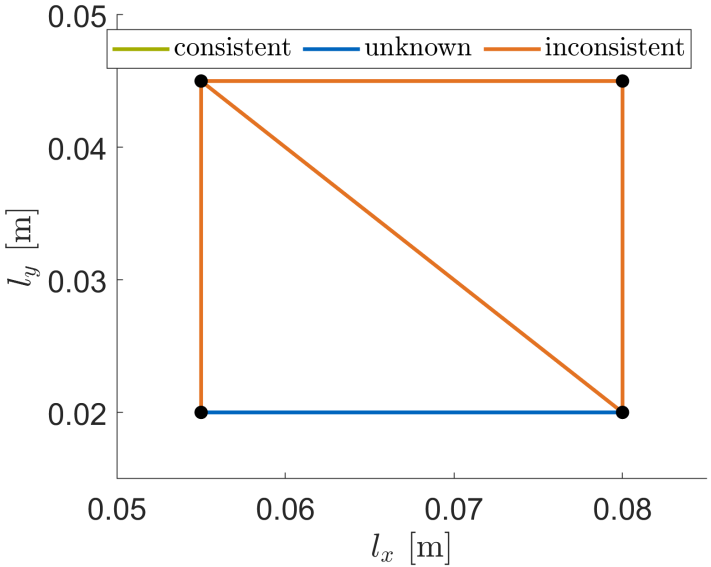
Results – Kelvin Cell – Adaptive Sampling & Classification



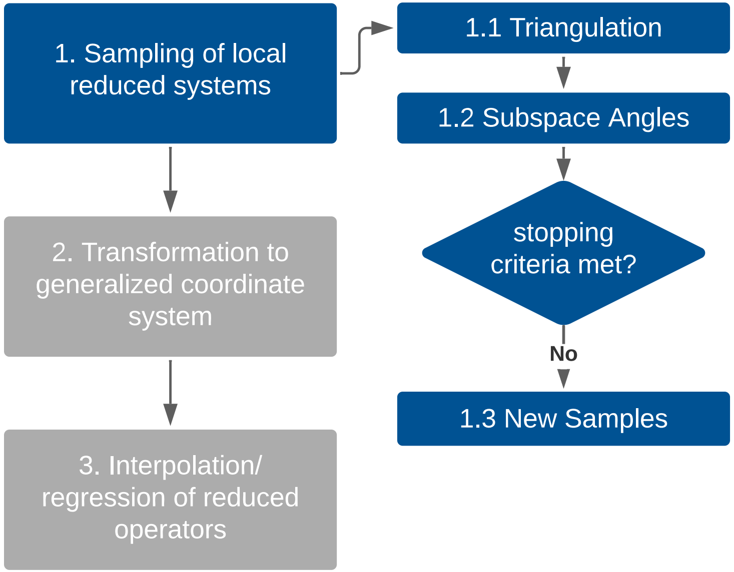


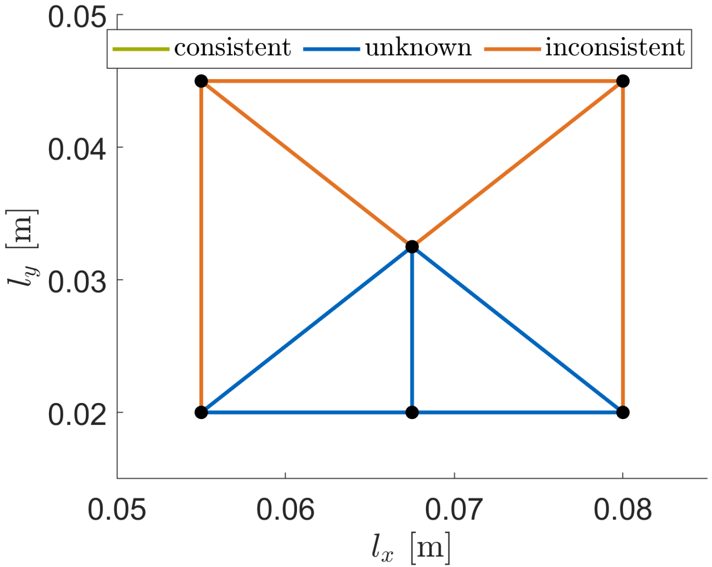
Results – Kelvin Cell – Adaptive Sampling & Classification



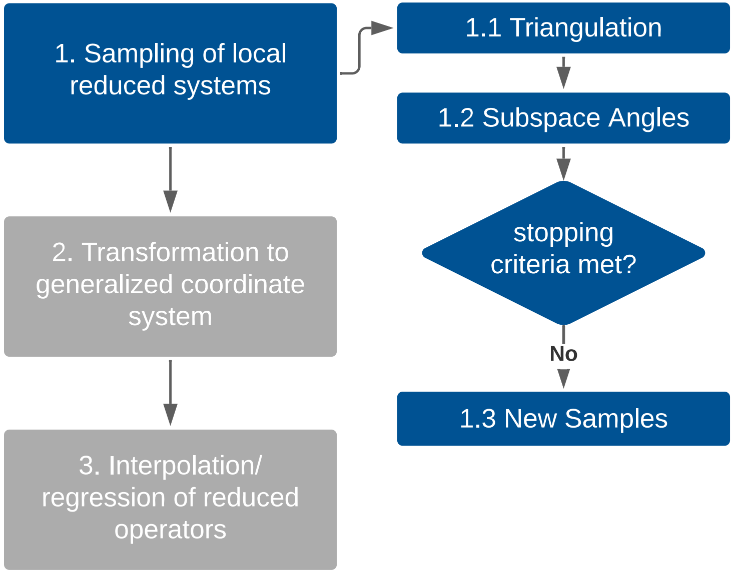


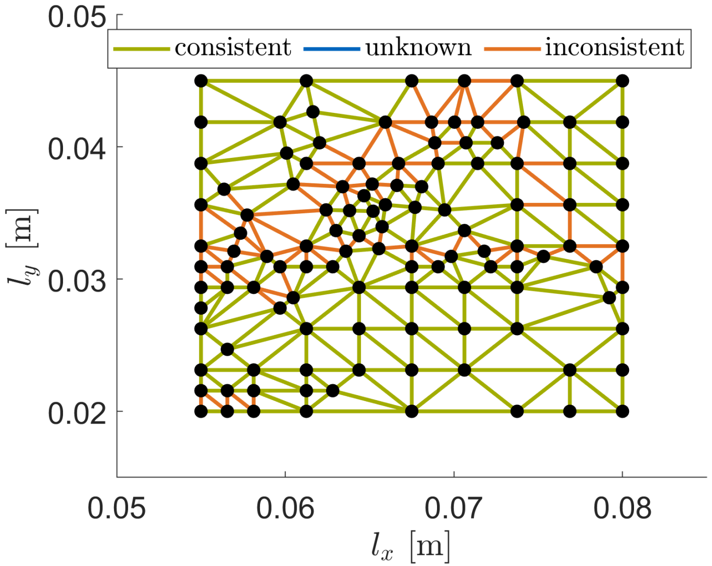
Results – Kelvin Cell – Adaptive Sampling & Classification



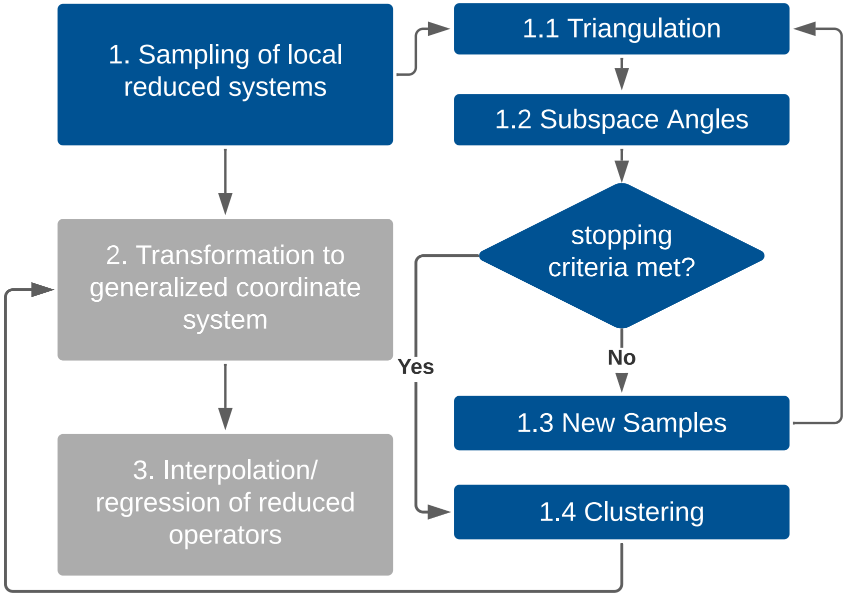


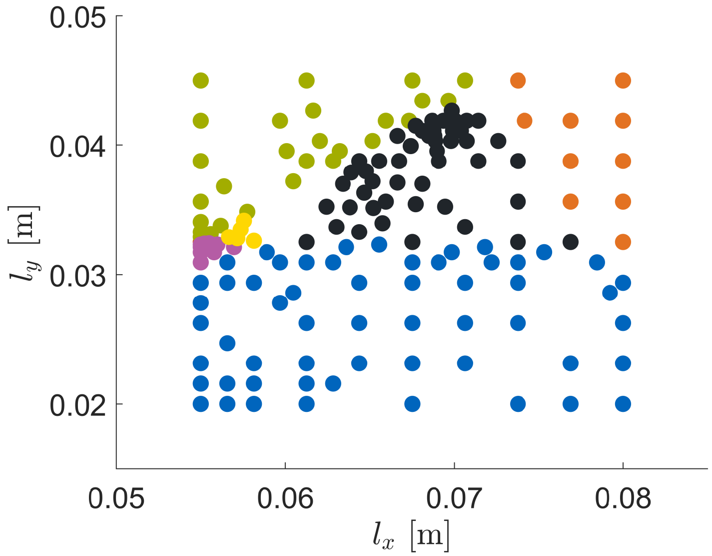
Results – Kelvin Cell – Adaptive Sampling & Classification



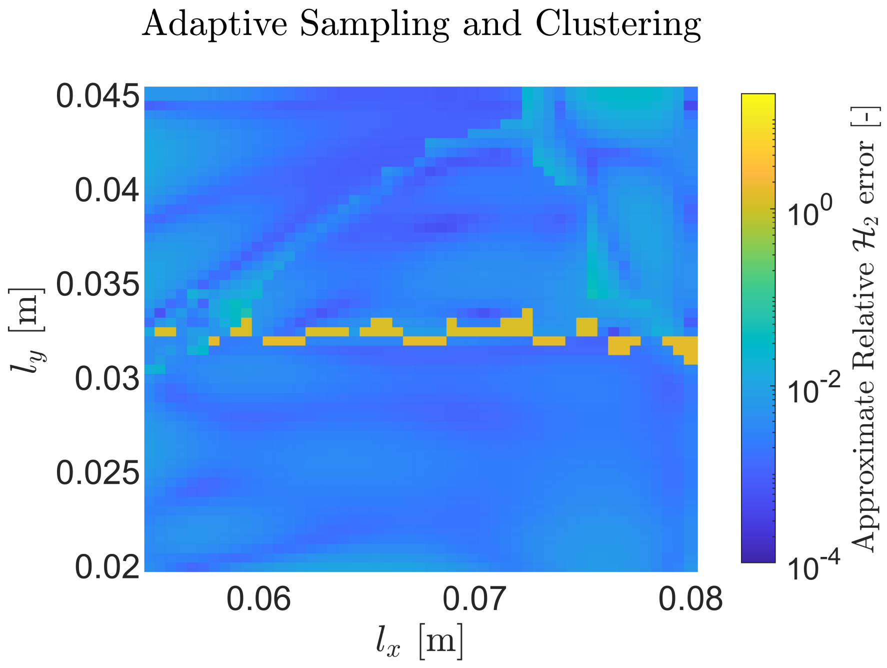


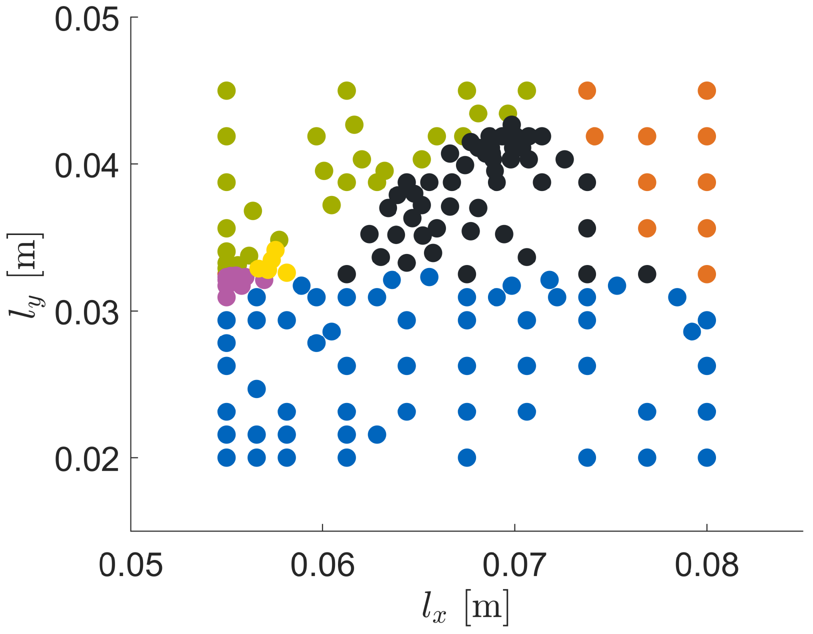
Results – Kelvin Cell – Adaptive Sampling & Classification



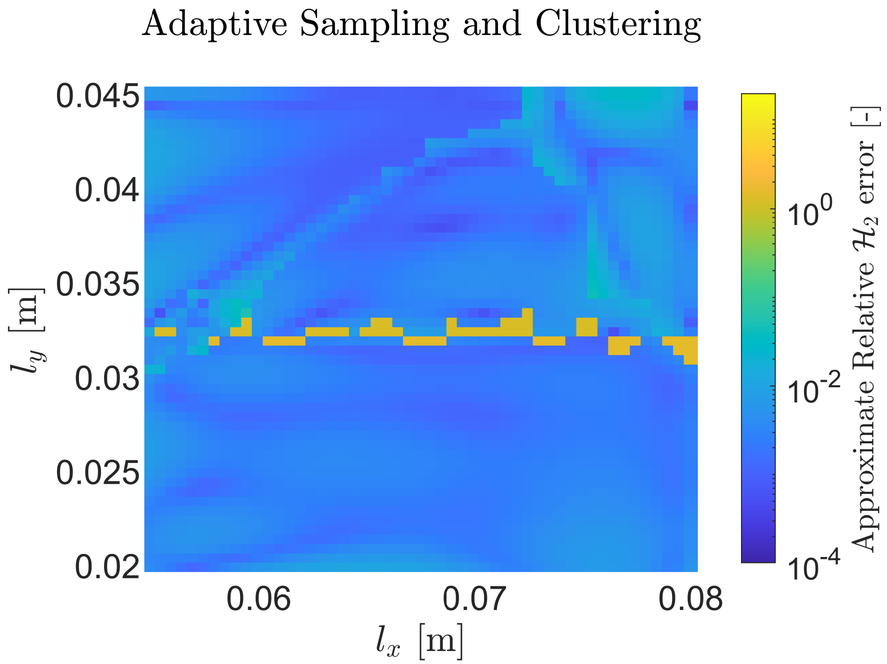


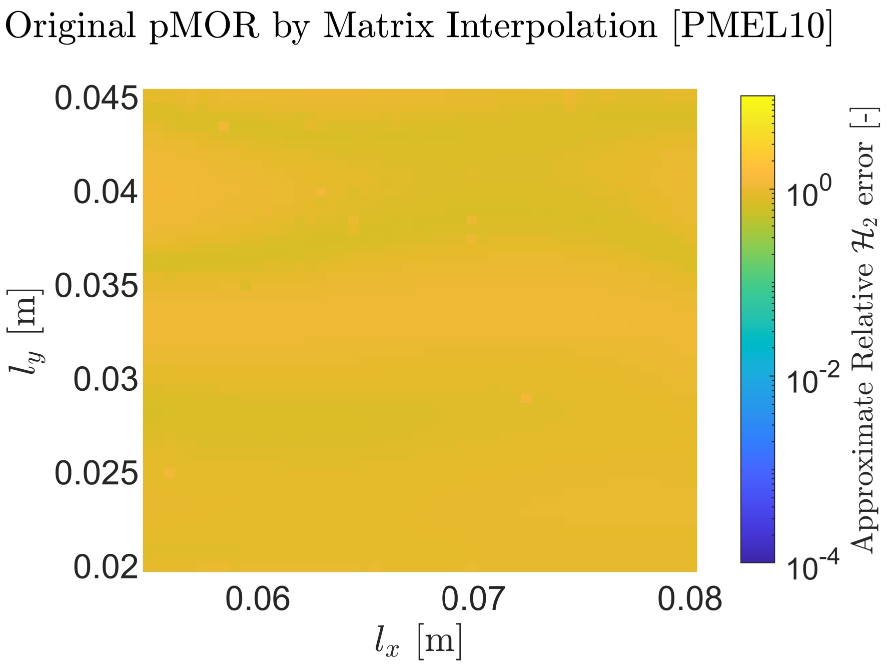
Results – Kelvin Cell – Dimensions l_x and l_y





Results – Kelvin Cell – Dimensions l_x and l_y





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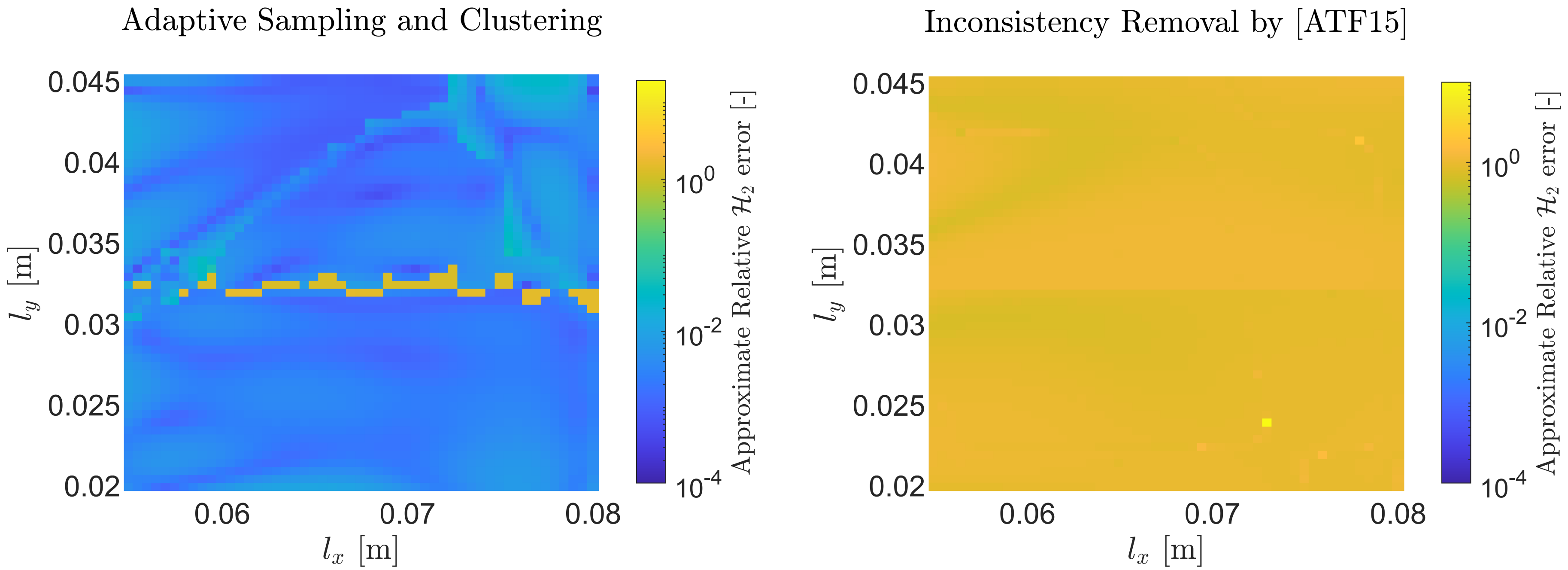

Conclusion and Future Work

Conclusion & Future Work

Our objective was to generate a parametric reduced-order model (pROM) that allows to solve the transfer function

$$H(s,\mathbf{p}) = \mathbf{g}(\mathbf{p})\left(s^2\mathbf{M}(\mathbf{p}) + s\mathbf{C}(\mathbf{p}) + \mathbf{K}(\mathbf{p})\right)^{-1}\mathbf{f}(\mathbf{p}), \tag{14}$$

efficiently and

- does not require an affine representation of the parametric dependency,
- is valid for a large range of the parameters
- is generated via an adaptive algorithm.

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efficiently and

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→ Partitioning of the parameter space, generation of several local pROMs

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Conclusion & Future Work

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$$H(s,\mathbf{p}) = \mathbf{g}(\mathbf{p}) \left(s^2\mathbf{M}(\mathbf{p}) + s\mathbf{C}(\mathbf{p}) + \mathbf{K}(\mathbf{p})\right)^{-1} \mathbf{f}(\mathbf{p}), \tag{14}$$

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$\rightarrow$ Parametric Model Order Reduction by Matrix Interpolation

- is valid for a large range of the parameters

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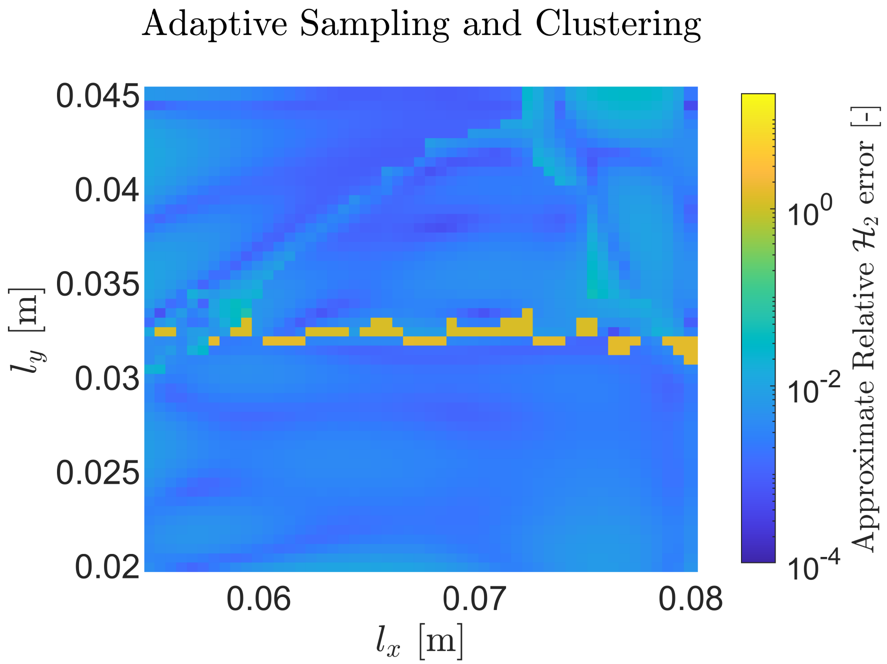
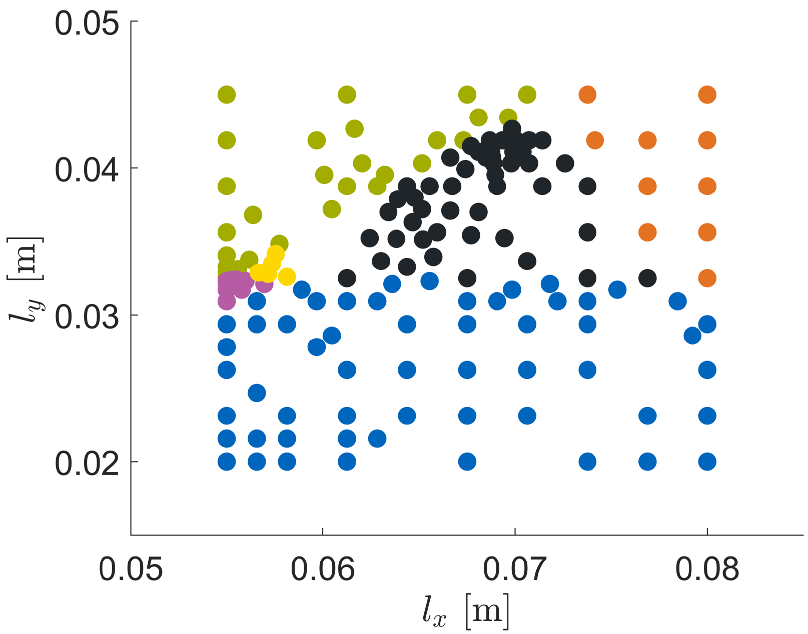
- is generated via an adaptive algorithm.

$\rightarrow$ Adaptive sampling

Future Work



Adaptive Sampling and Clustering
0.045
0.04
0.035
0.03
0.025
0.02
l_y [m]
0.06
0.07
0.08
l_x [m]
10^0
10^{-2}
10^{-4}
Approximate Relative $\mathcal{H}_2$ error [-]



0.05
0.04
0.03
0.02
l_y [m]
0.05
0.06
0.07
0.08
l_x [m]

References

[ATF15] David Amsallem, Radek Tezaur, and Charbel Farhat. Real-time solution of computational problems using databases of parametric linear reduced-order models with arbitrary underlying meshes. *Journal of Computational Physics*, 326, 06 2015.

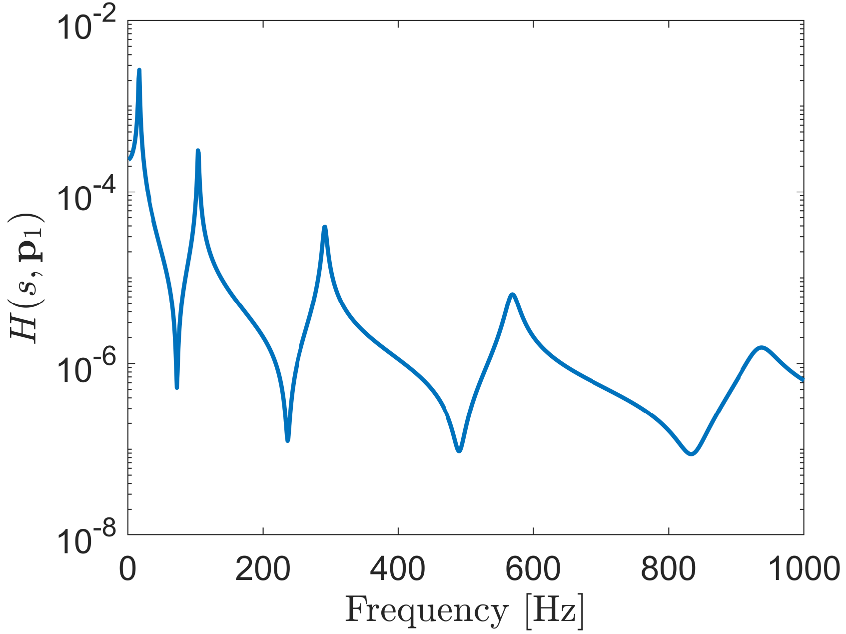
[BGW15] Peter Benner, Serkan Gugercin, and Karen Willcox. A survey of projection-based model reduction methods for parametric dynamical systems. *SIAM Review*, 57(4):483–531, jan 2015.

[BNN+15] M. A. Bazaz, S. A. Nahve, M. Nabi, S. Janardhanan, and M. U. Rehman. Adaptive parameter space sampling in matrix interpolatory pmor. In *2015 International Conference on Recent Developments in Control, Automation and Power Engineering (RDCAPE)*, pages 83–89, March 2015.

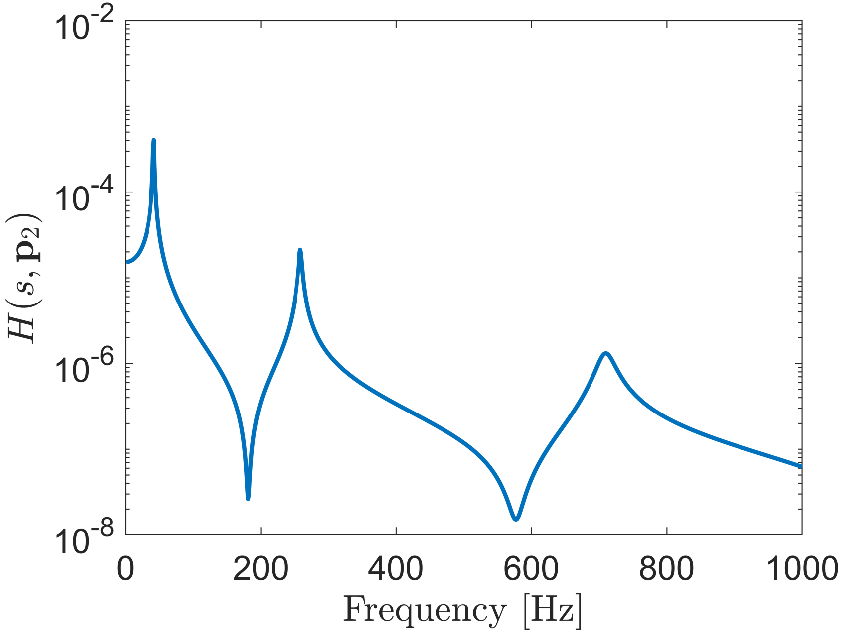
[FE15] Michael Fischer and Peter Eberhard. Application of parametric model reduction with matrix interpolation for simulation of moving loads in elastic multibody systems. *Advances in Computational Mathematics*, 41(5):1049–1072, October 2015.

[PMEL10] Heiko Peuscher, Jan Mohring, Rudy Eid, and Boris Lohmann. Parametric model order reduction by matrix interpolation. *Automatisierungstechnik*, 58:475–484, 08 2010.

MOR Method



10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
200
400
600
800
1000
$H(s, \mathbf{p}_1)$
Frequency [Hz]



10^{-2}
10^{-4}
10^{-6}
10^{-8}
0
200
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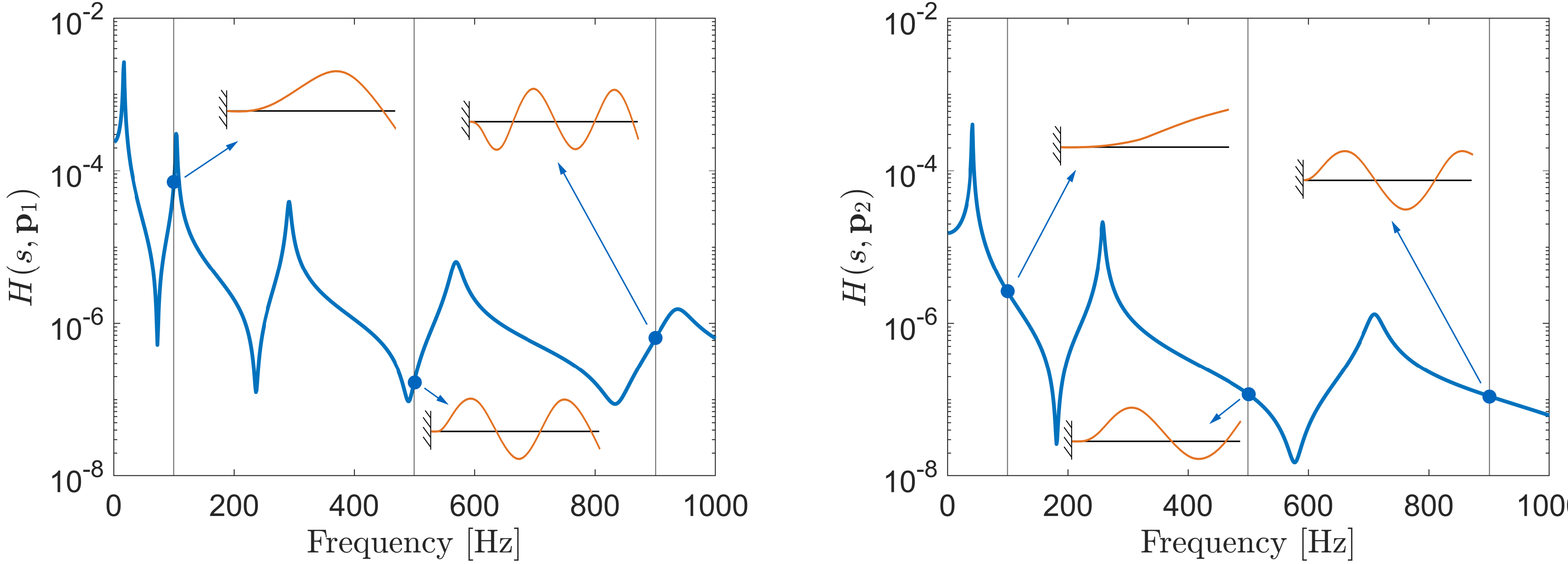
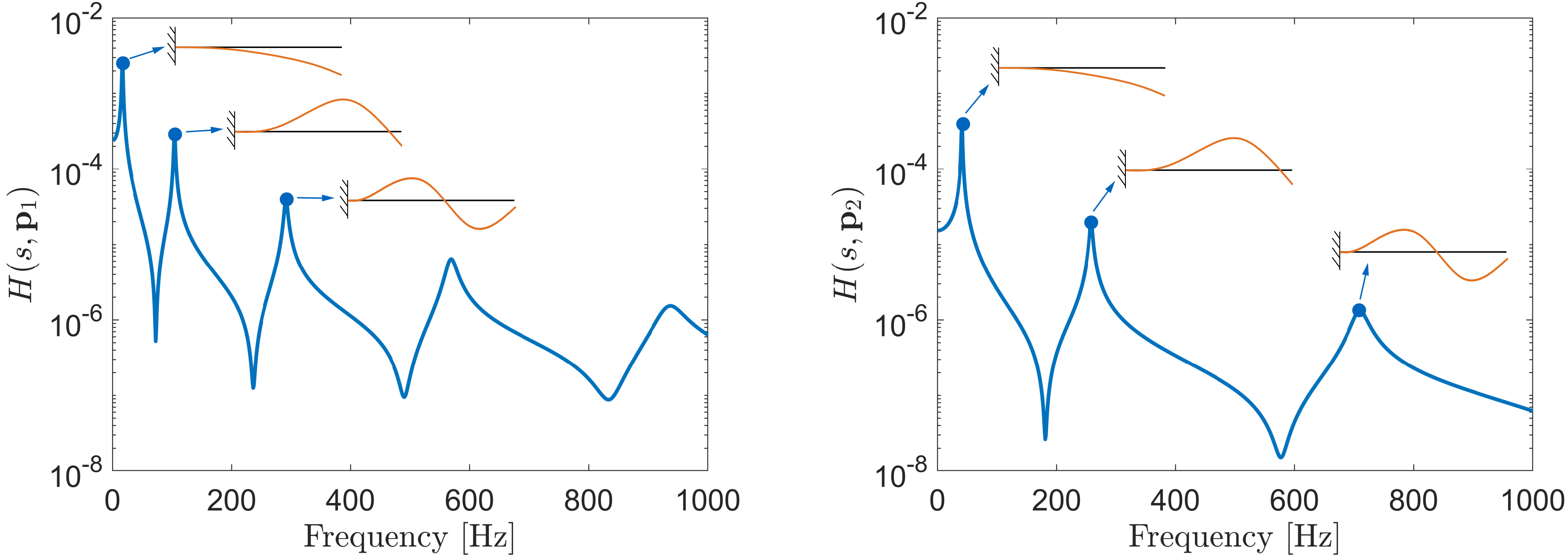
600
800
1000
$H(s, \mathbf{p}_2)$
Frequency [Hz]

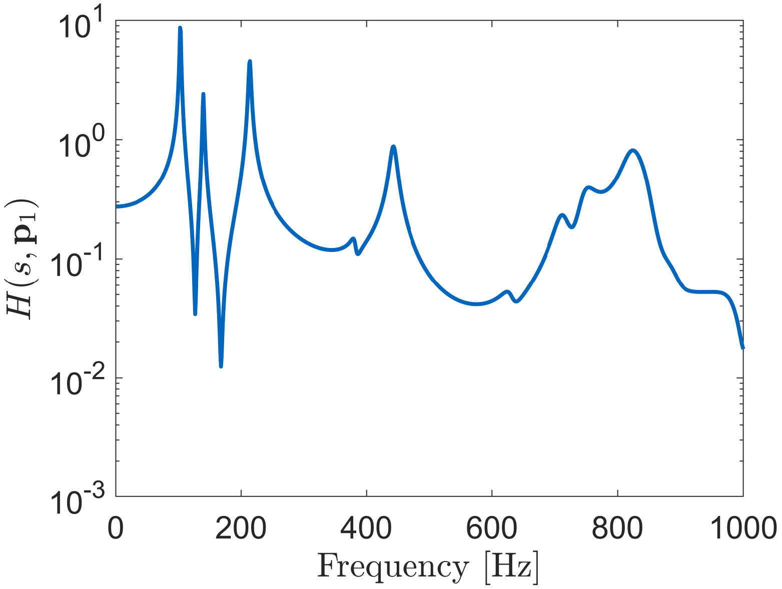
MOR Method – Proper Orthogonal Decomposition

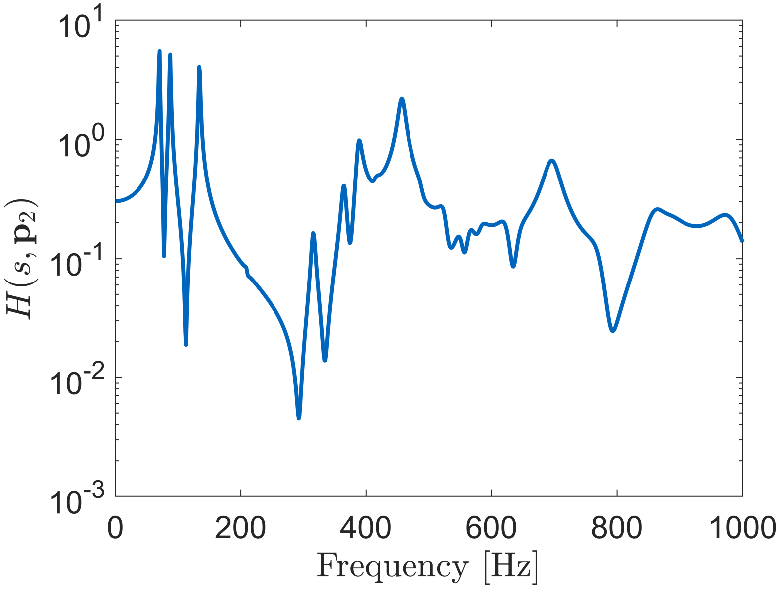


MOR Method – Modal Truncation

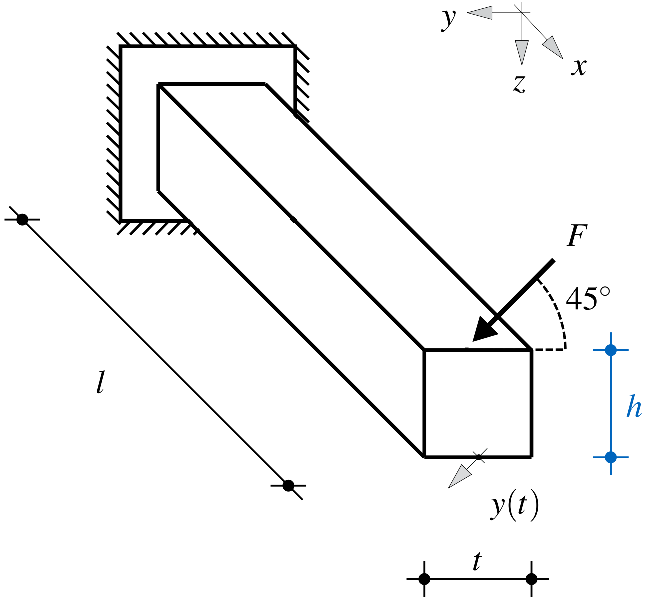


Change of System Dynamics

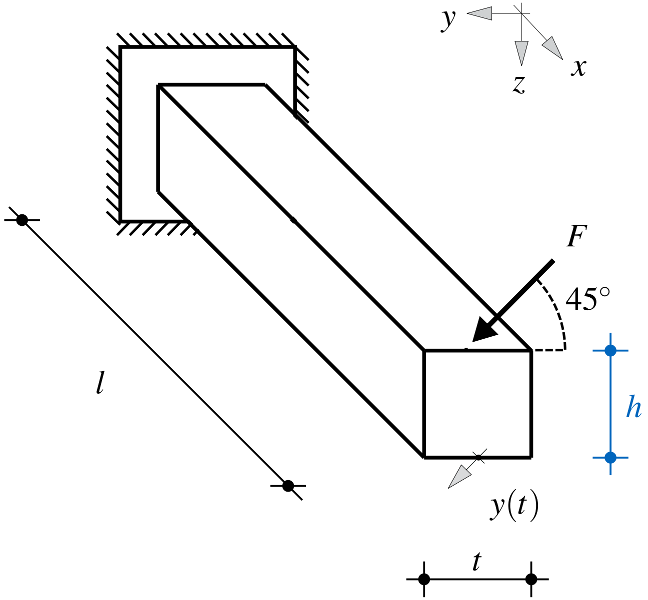


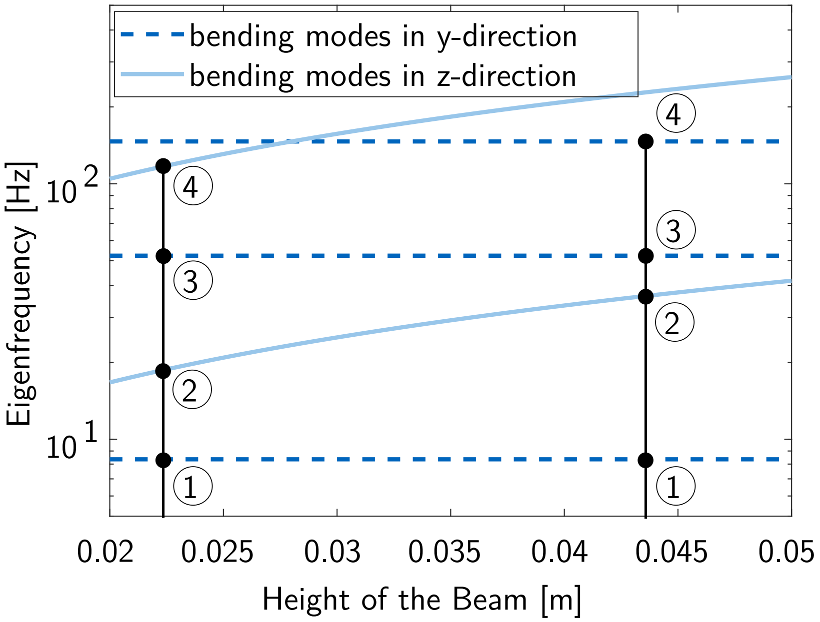


Mode Switching and Truncation

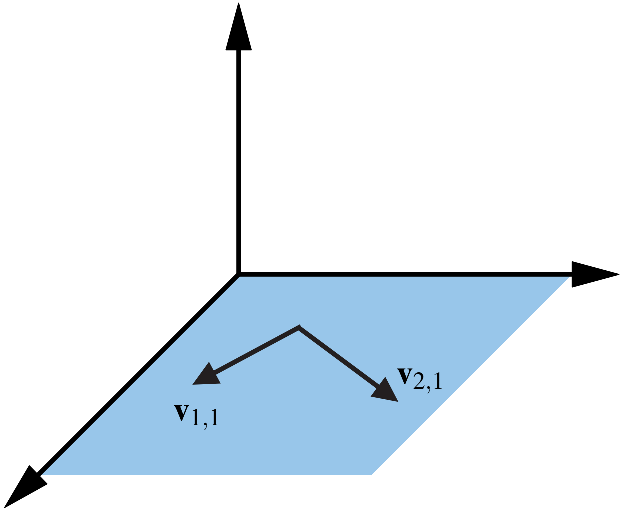


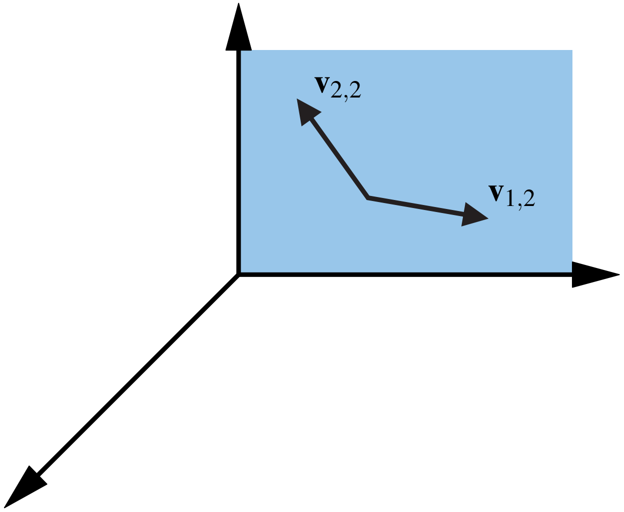
Mode Switching and Truncation





Mode Switching and Truncation





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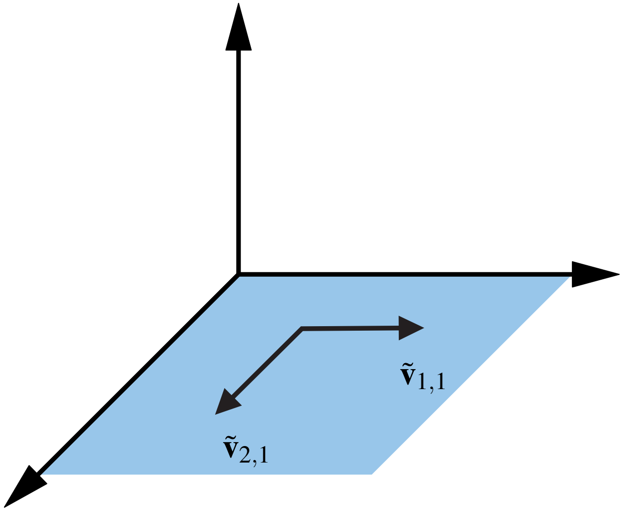
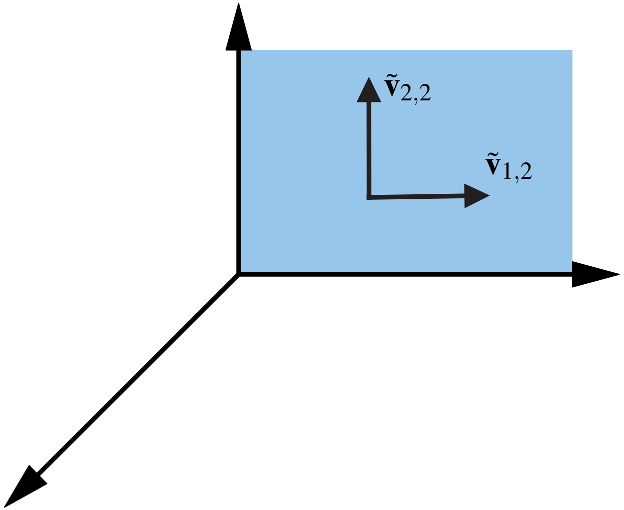
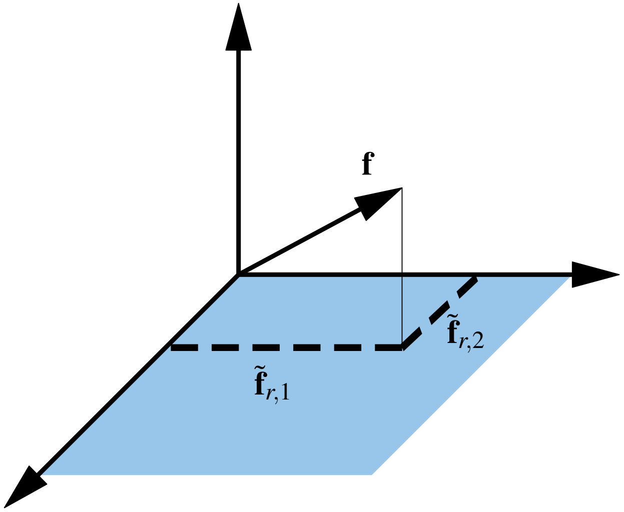
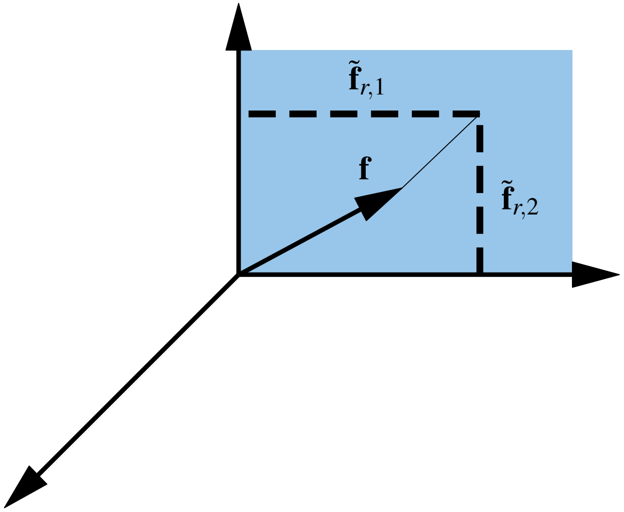


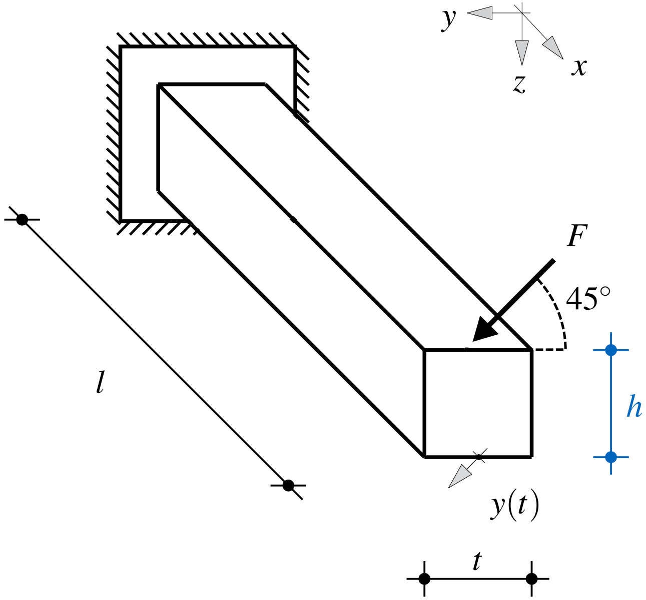


Mode Switching and Truncation

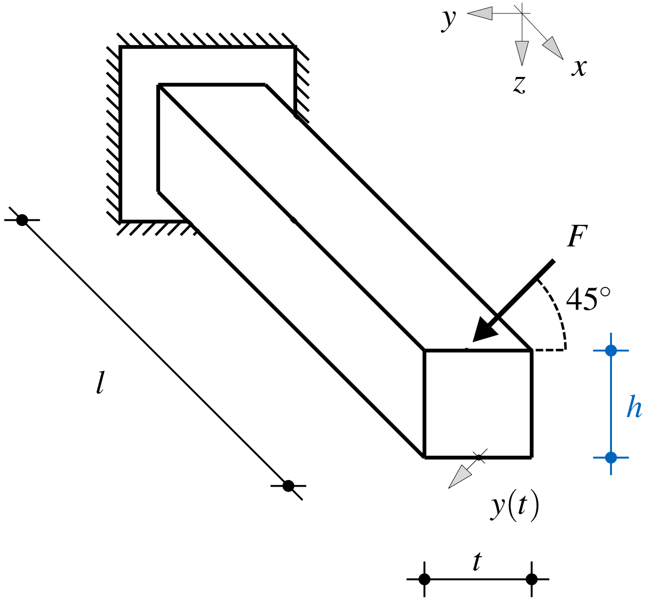


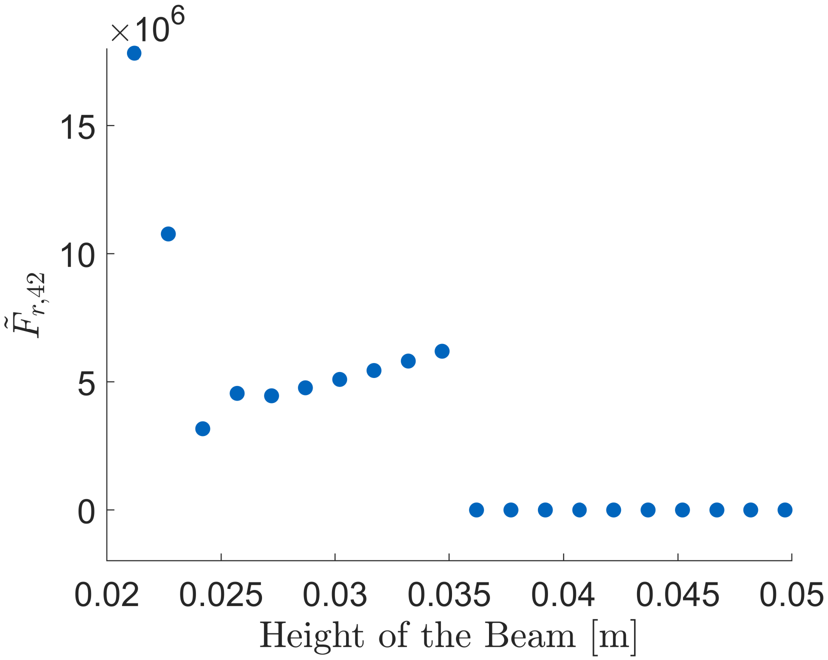


Mode Switching and Truncation

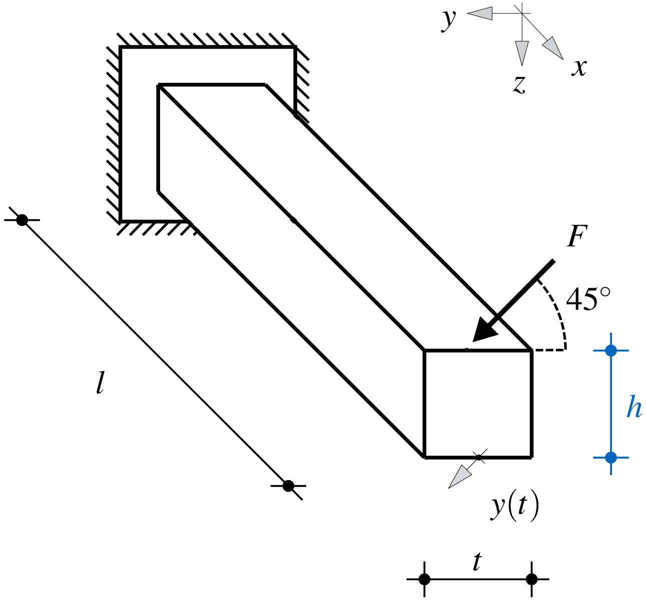


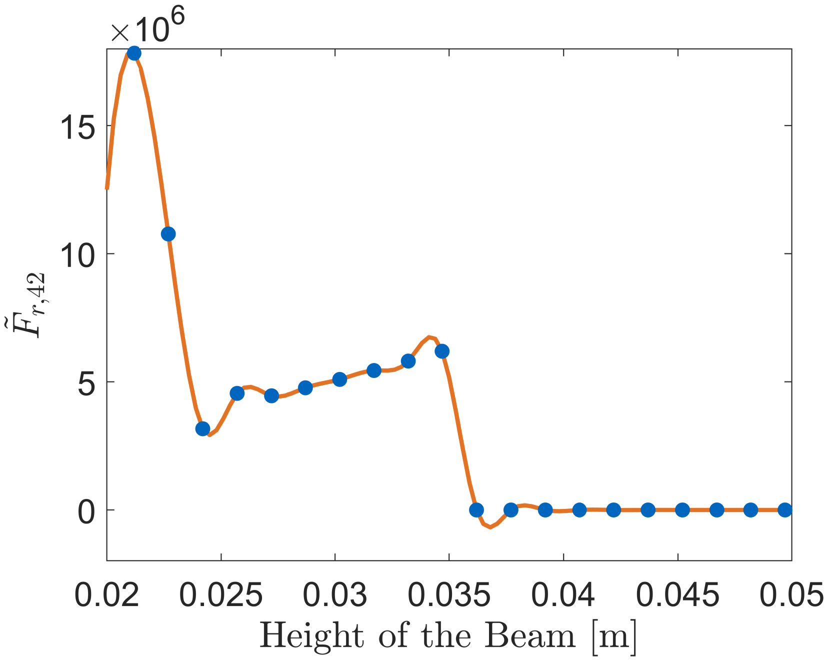
Mode Switching and Truncation



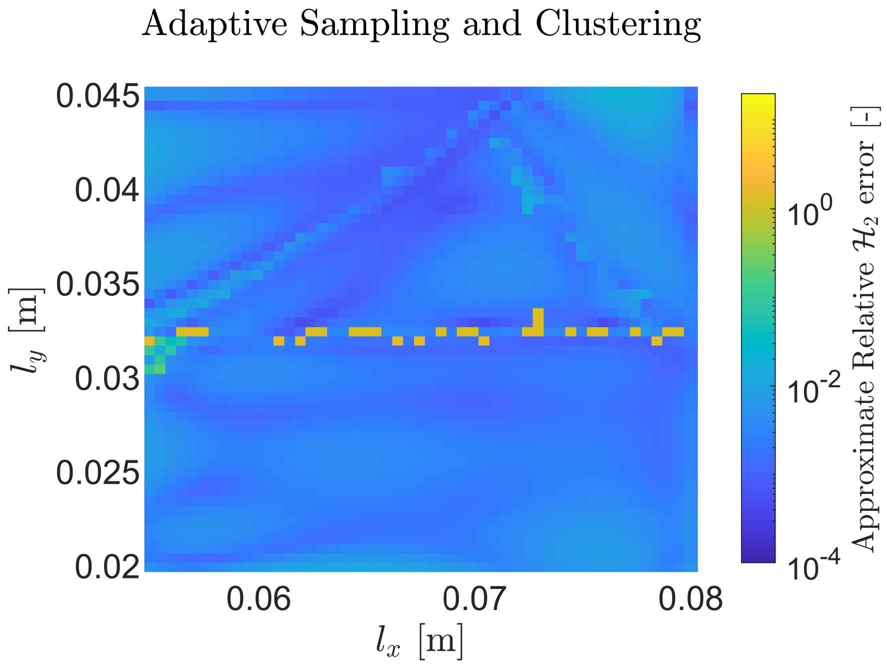


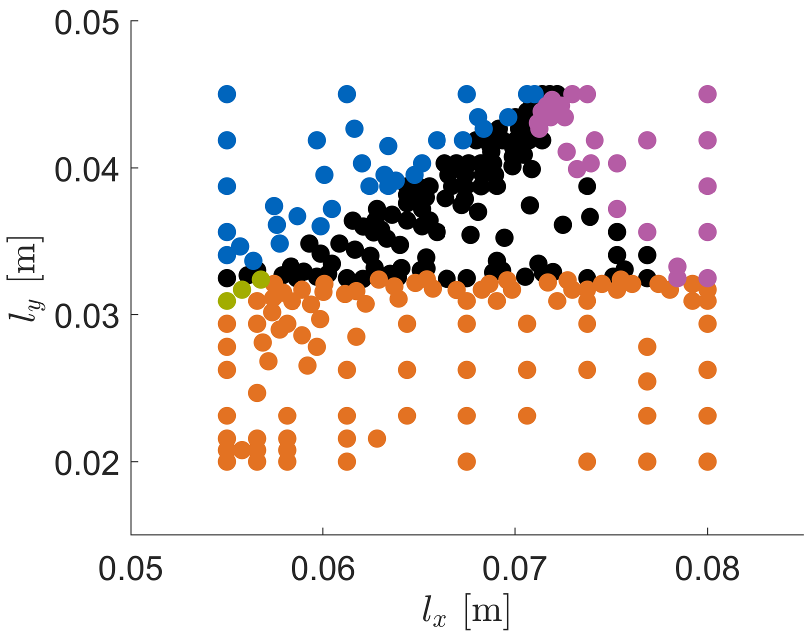
Mode Switching and Truncation



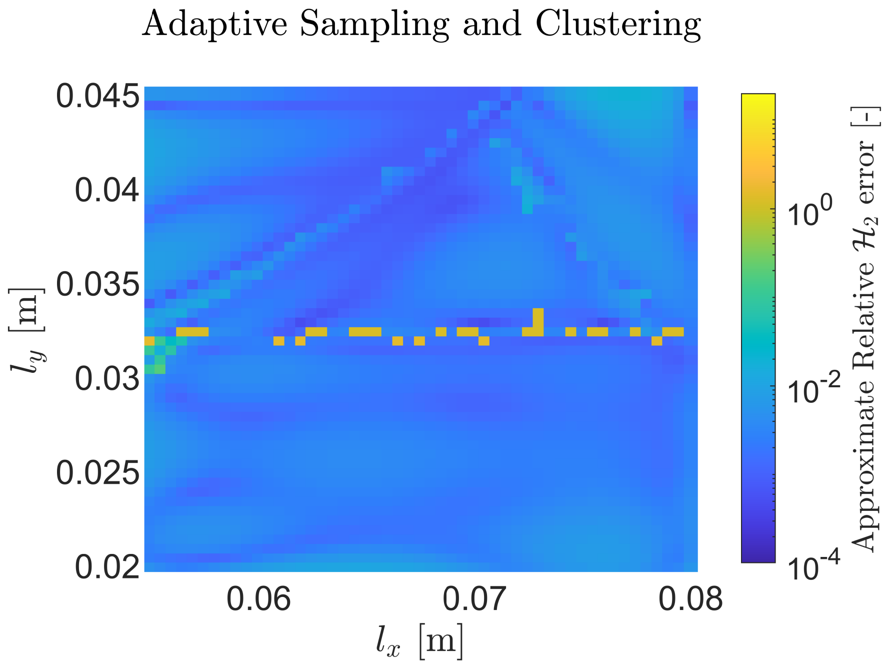


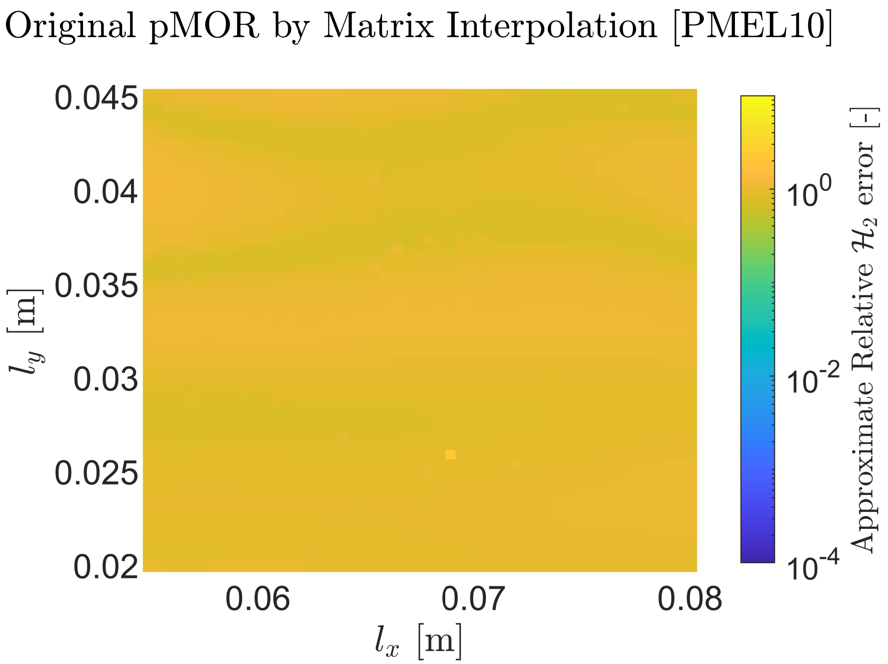
Results – Kelvin Cell – Dimensions l_x and l_y



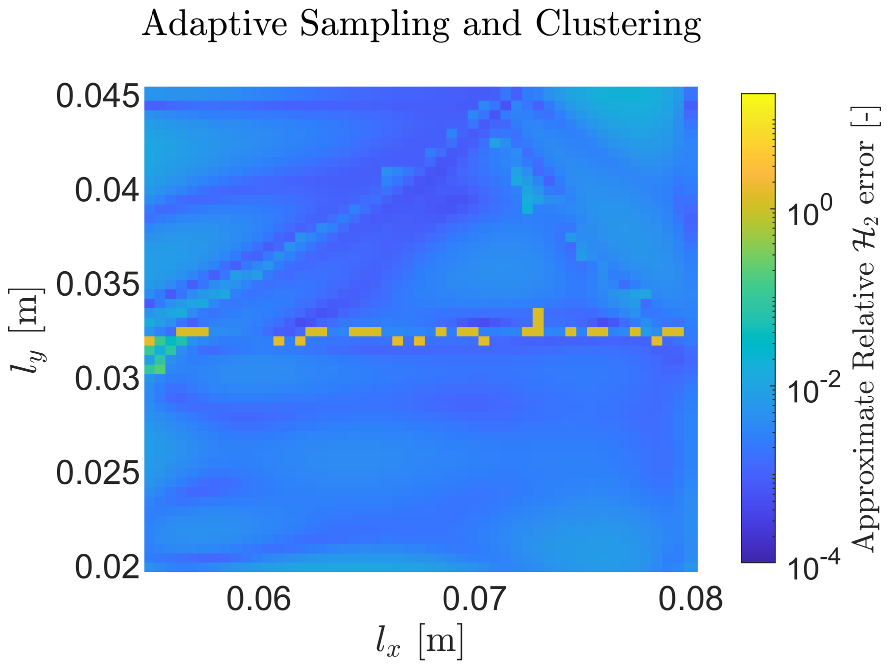


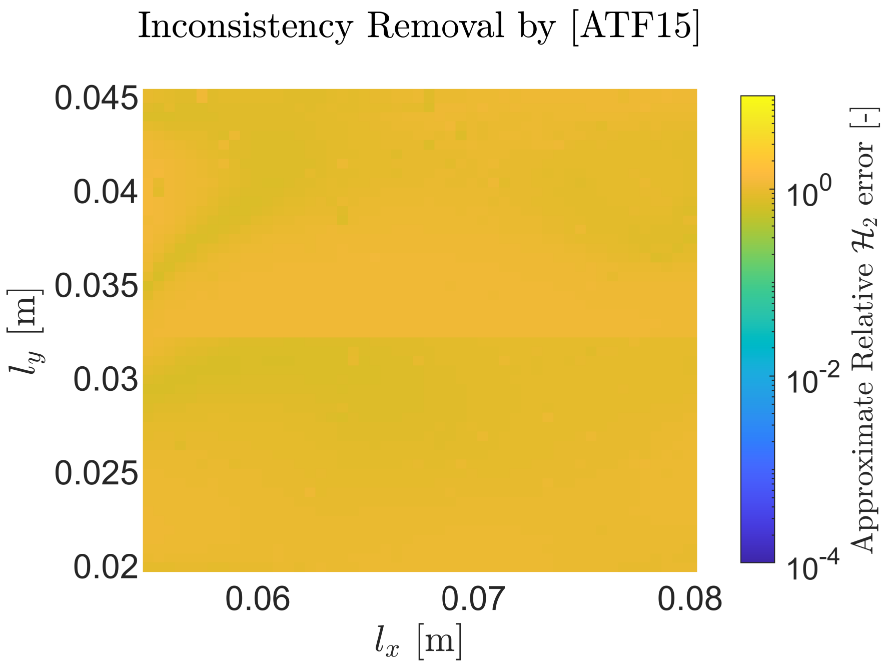
Results – Kelvin Cell – Dimensions l_x and l_y





Results – Kelvin Cell – Dimensions l_x and l_y





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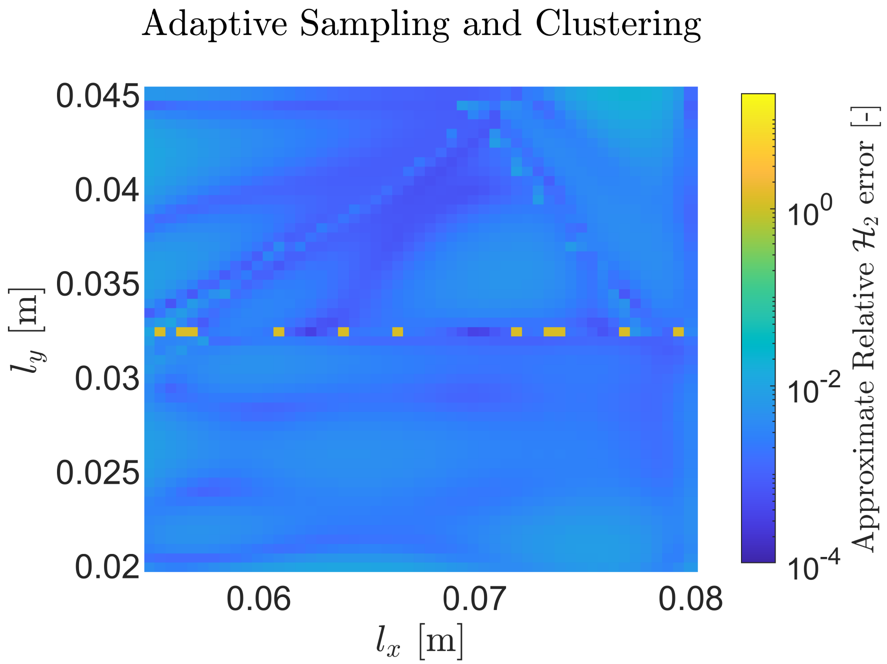
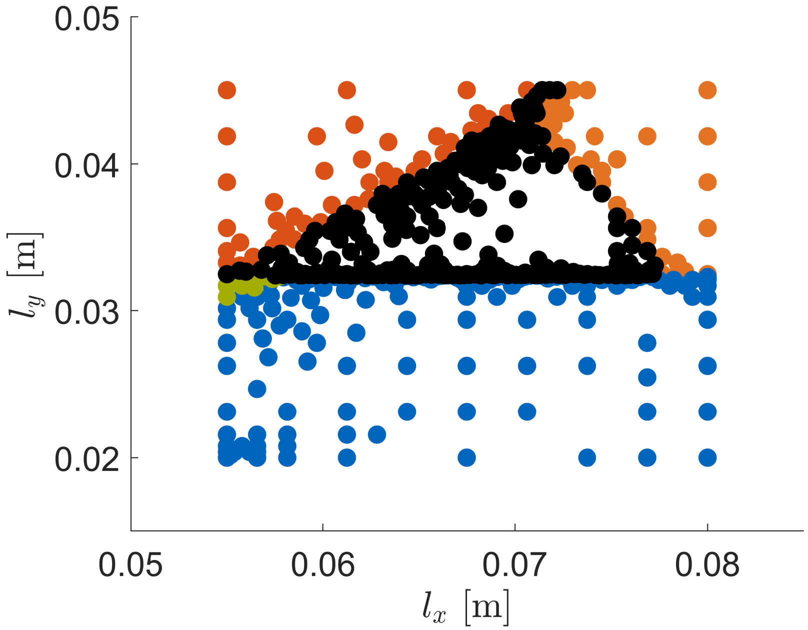
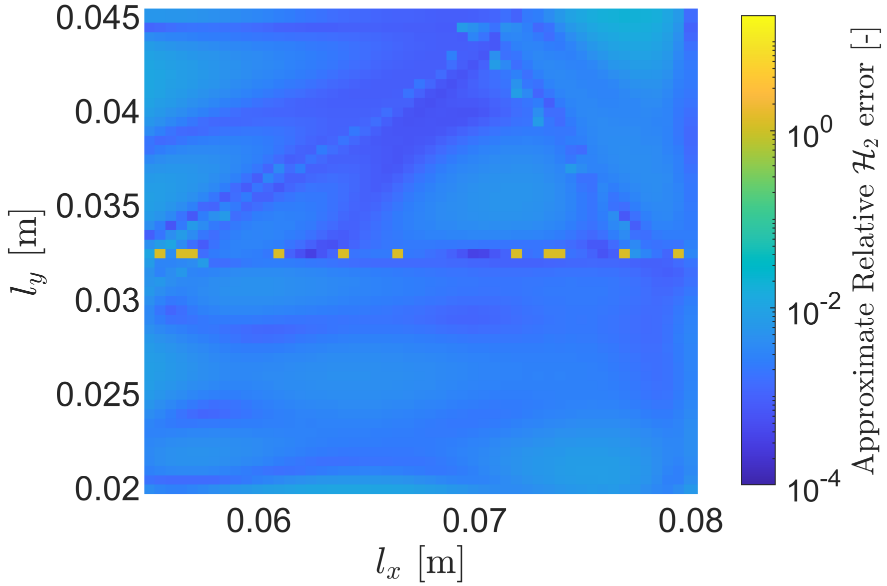


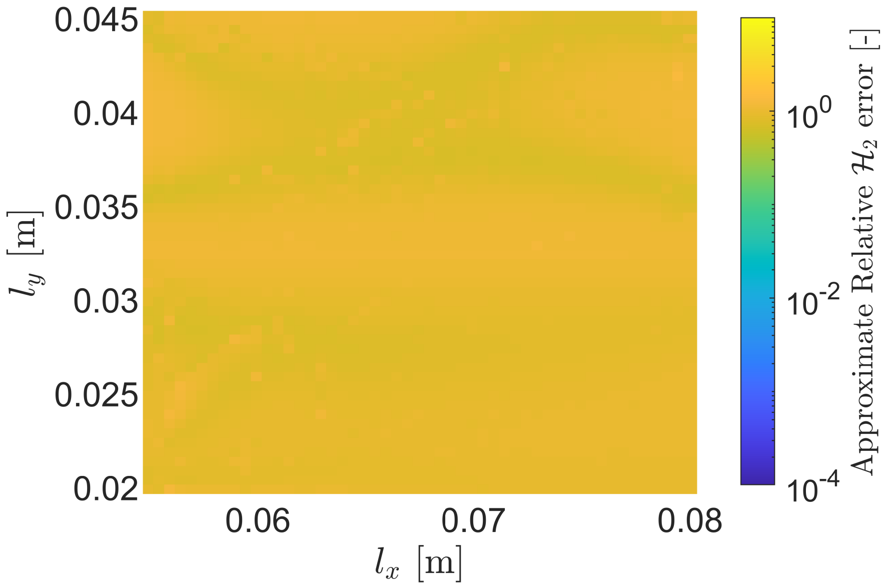


Results – Kelvin Cell – Dimensions l_x and l_y

Adaptive Sampling and Clustering



Original pMOR by Matrix Interpolation [PMEL10]



Results – Kelvin Cell – Dimensions l_x and l_y