B-Splines for flexible and robust multirate time stepping

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preCICE Workshop 2023

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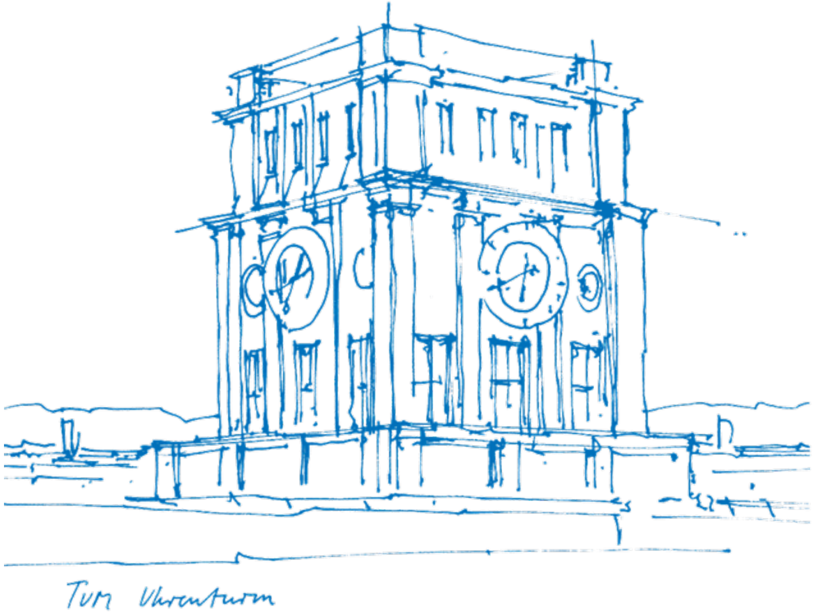
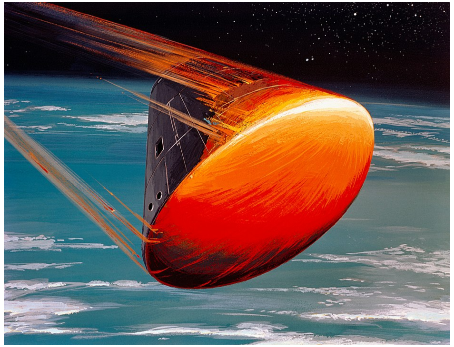


Why time interpolation and subcycling?



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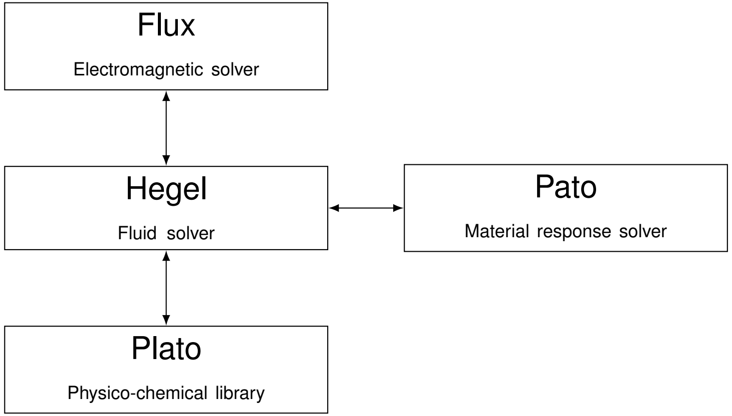


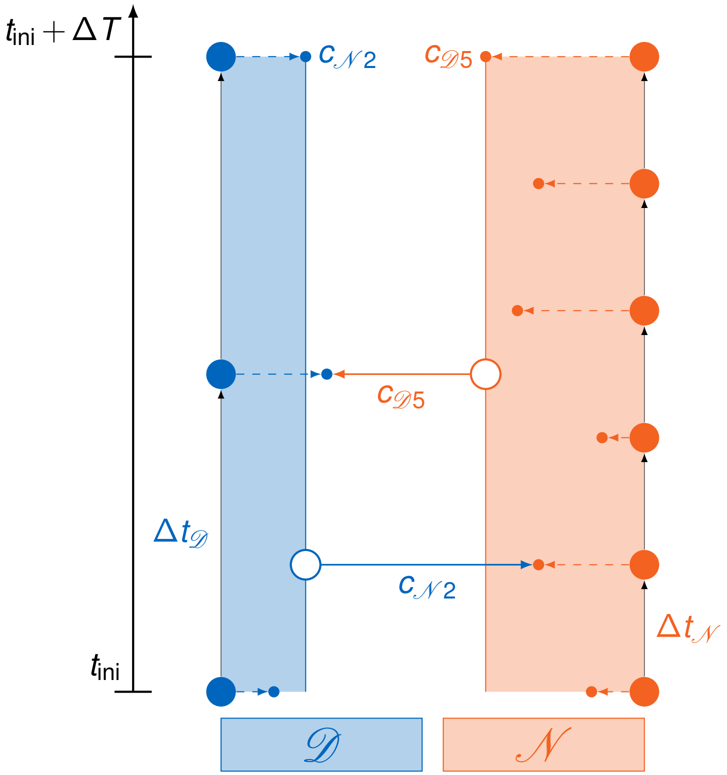
Partitioned solver for ICP[1] wind tunnels

- $\Delta t_{\text{FLUX}} \approx 10 \Delta t_{\text{HEGEL}} \approx 1000 \Delta t_{\text{PATO}}$
- HEGEL uses 3rd order RK
- PATO uses $\Delta t_{\text{PATO}} = 10^{-4} s = \Delta T_{\text{preCICE}}/100$
- HEGEL uses $\Delta t_{\text{HEGEL}} = 10^{-2} s = \Delta T_{\text{preCICE}}$
- FLUX uses $\Delta t_{\text{FLUX}} = 10^{-1} s = 10 \Delta T_{\text{preCICE}}$ (???).

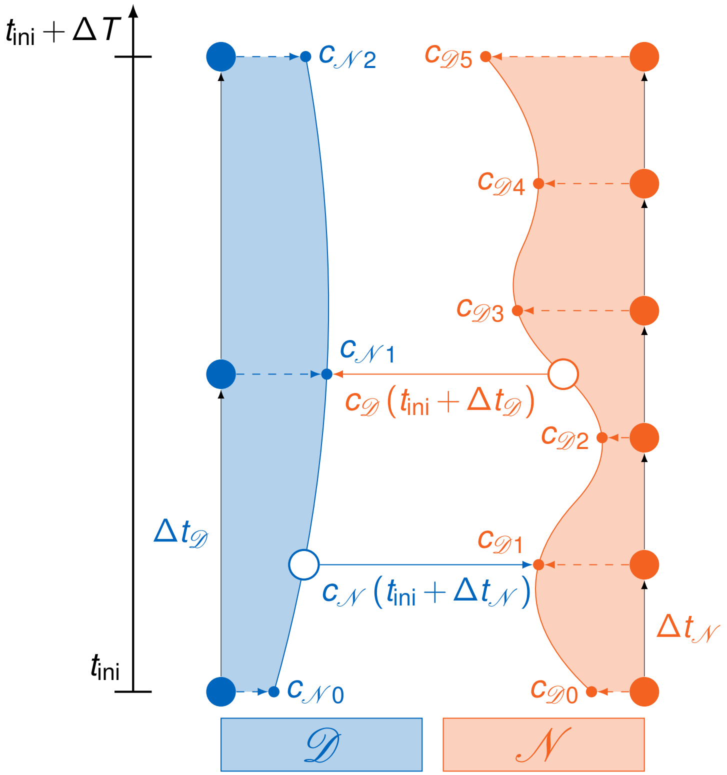
from *Alessandro Munafò, et al. A Multi-Physics Modeling Framework for Inductively Coupled Plasma Wind Tunnels. 2022.* `https://doi.org/10.2514/6.2022-1011`

[1] Inductively Coupled Plasma

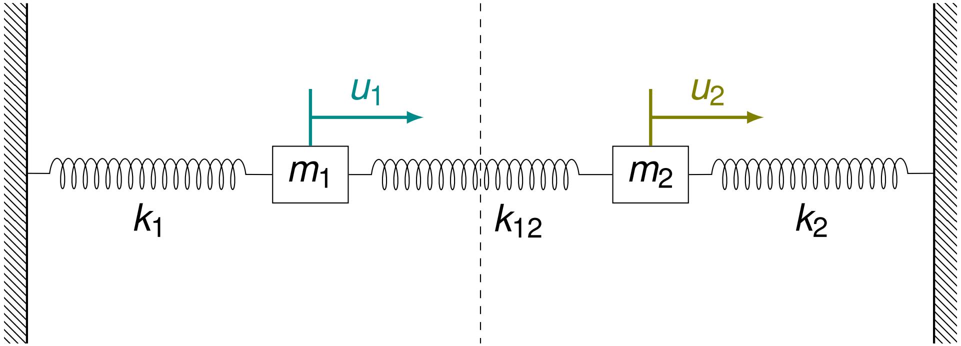
preCICE v2: Constant in time window



Implementation idea: Waveform iteration

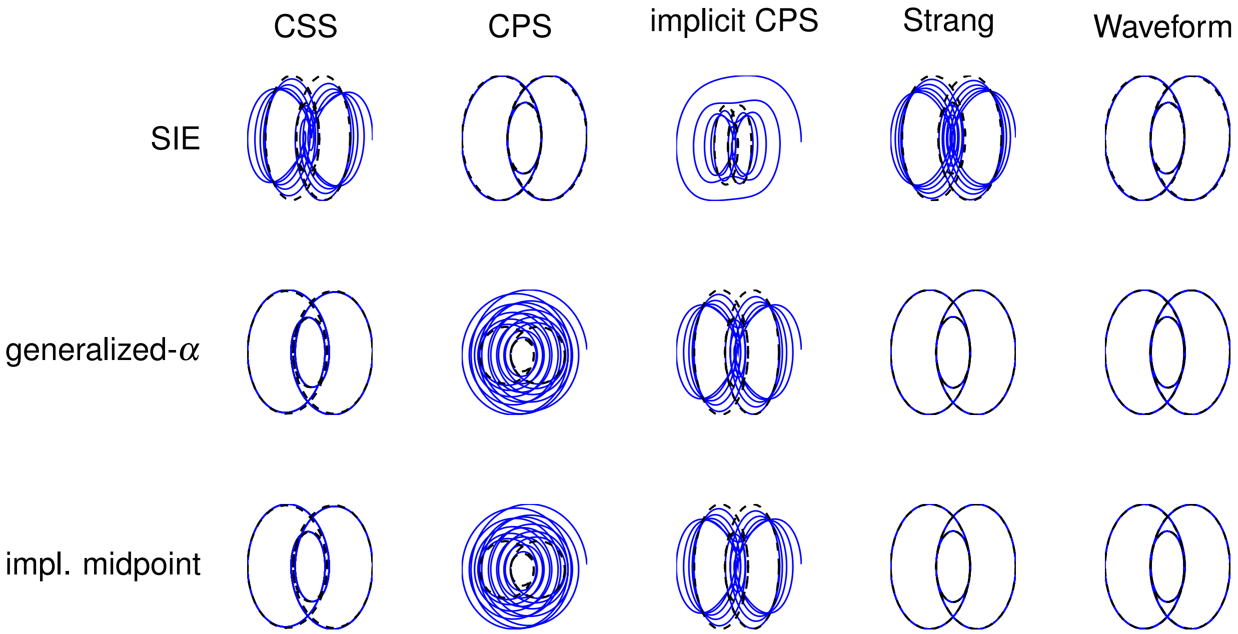


Proof of concept: Test case



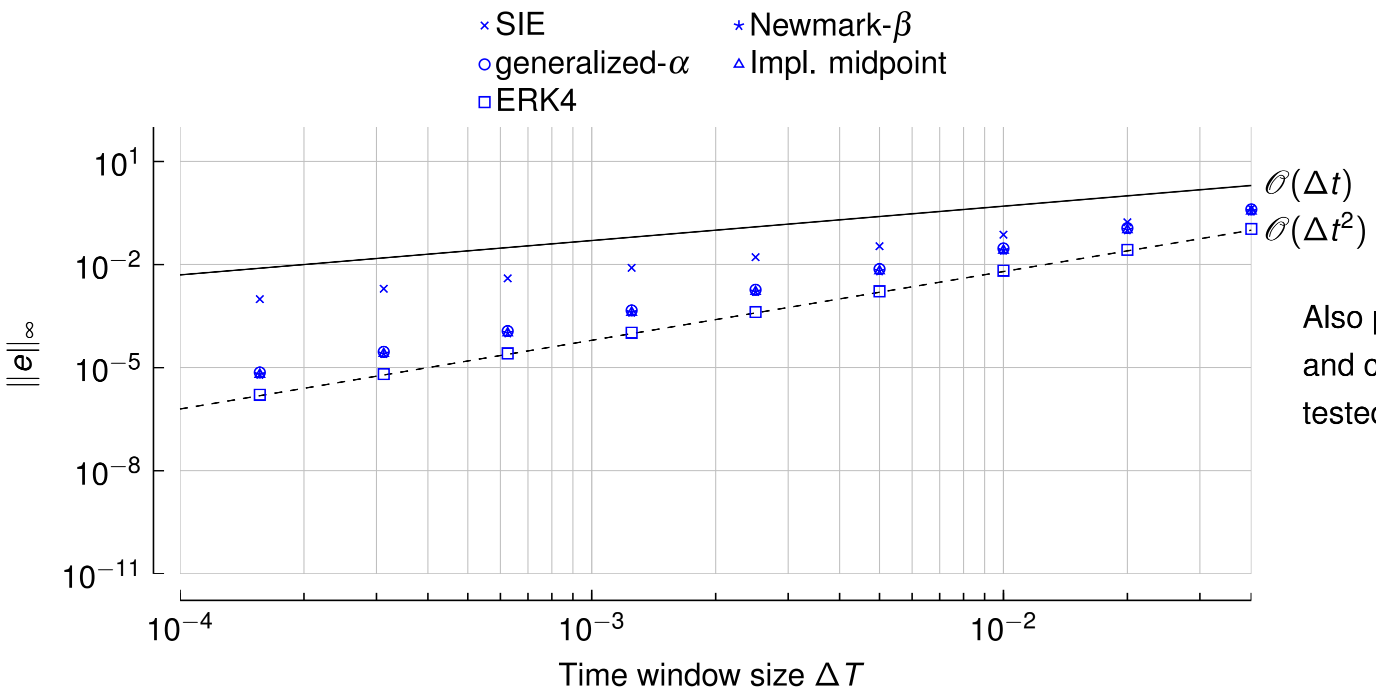
More details will follow in talk "preCICE-FMI Runner to couple controller models to PDEs" by Leonard Willeke.

Proof of concept: Energy Conservation



A Simple Test Case for Convergence Order in Time and Energy Conservation of Black-Box Coupling Schemes. 2022. `https://doi.org/10.23967/wccm-apcom.2022.038`

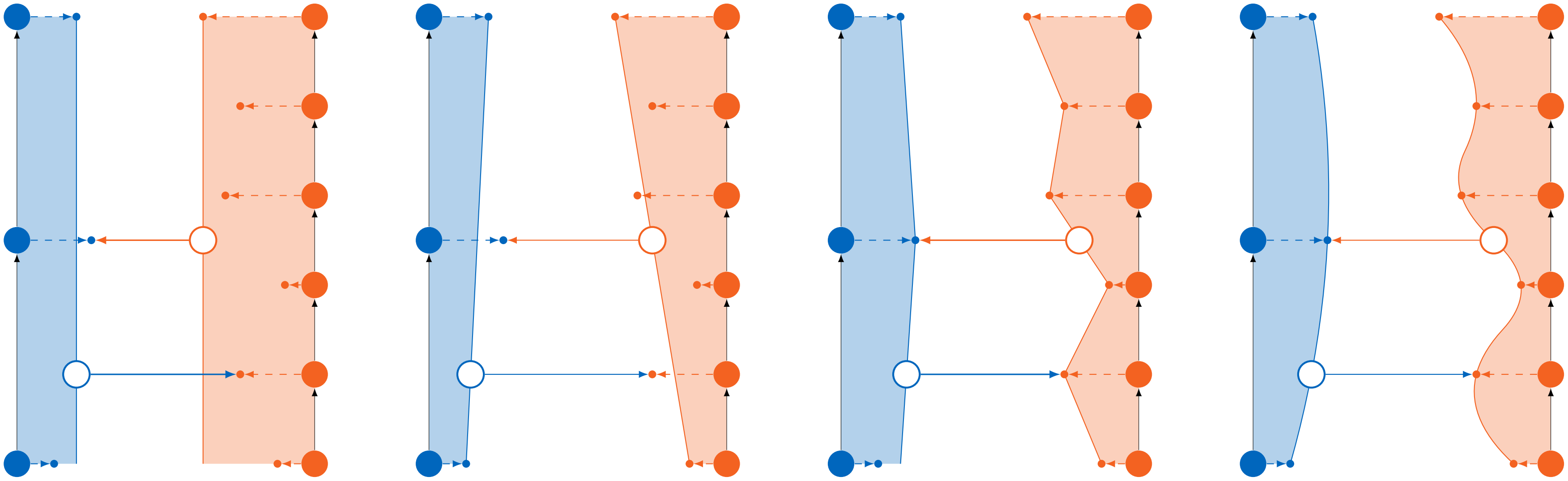
Proof of concept: Order Conservation



Also piecewise linear and cubic interpolation tested in other publication.

A Simple Test Case for Convergence Order in Time and Energy Conservation of Black-Box Coupling Schemes. 2022. `https://doi.org/10.23967/wccm-apcom.2022.038`

Bringing waveforms to preCICE



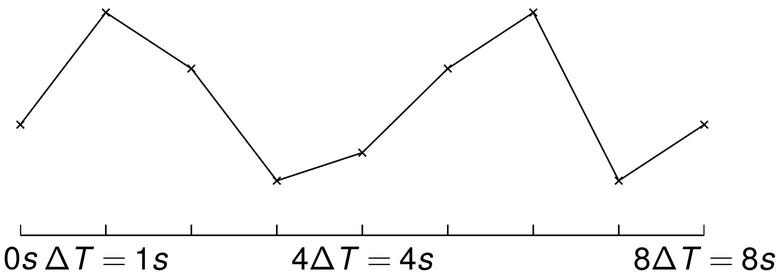
`v1.0` | `v2.4` (parallel coupling)[1] `develop` (serial coupling) | `PR #1523` | `PR #1523`

[1] my talk *From low-order to high-order coupling schemes* at preCICE Workshop 2022

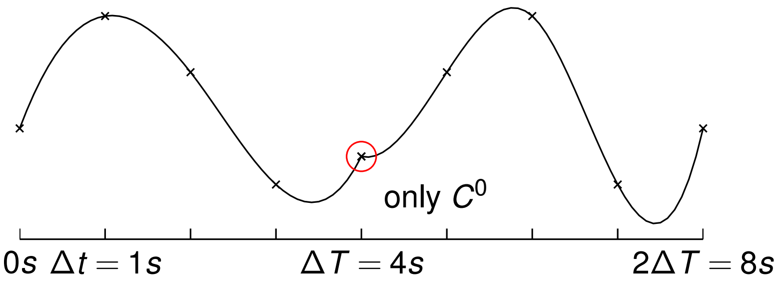
Subcycling

- Time window size $\Delta T \geq$ time step size Δt_1 and Δt_2.
- Do n time steps in window: $\Delta T = n_1 \Delta t_1 = n_2 \Delta t_2$
- Allows to create BSpline of degree $n-1$. (Goal reached: Something better than linear interpolation)
- Restriction: Only use data of current window!
- Larger window + subcycling has impact on number of QN iterations[1]

Without subcycling

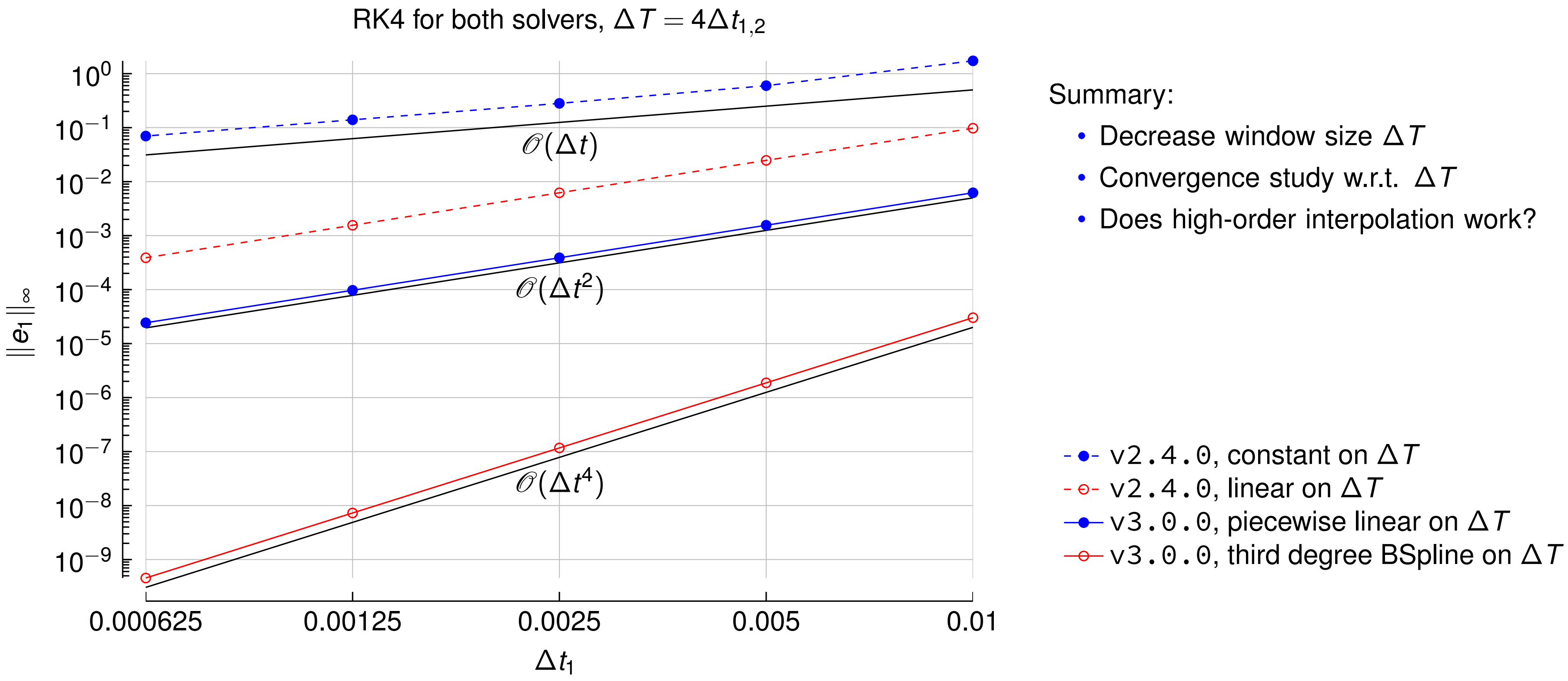


With subcycling (third order BSpline)



[1] Rüth, B, Uekermann, B, Mehl, M, Birken, P, Monge, A, Bungartz, H-J. Quasi-Newton waveform iteration for partitioned surface-coupled multiphysics applications. Int J Numer Methods Eng. 2021; 122: 5236– 5257. `https://doi.org/10.1002/nme.6443`

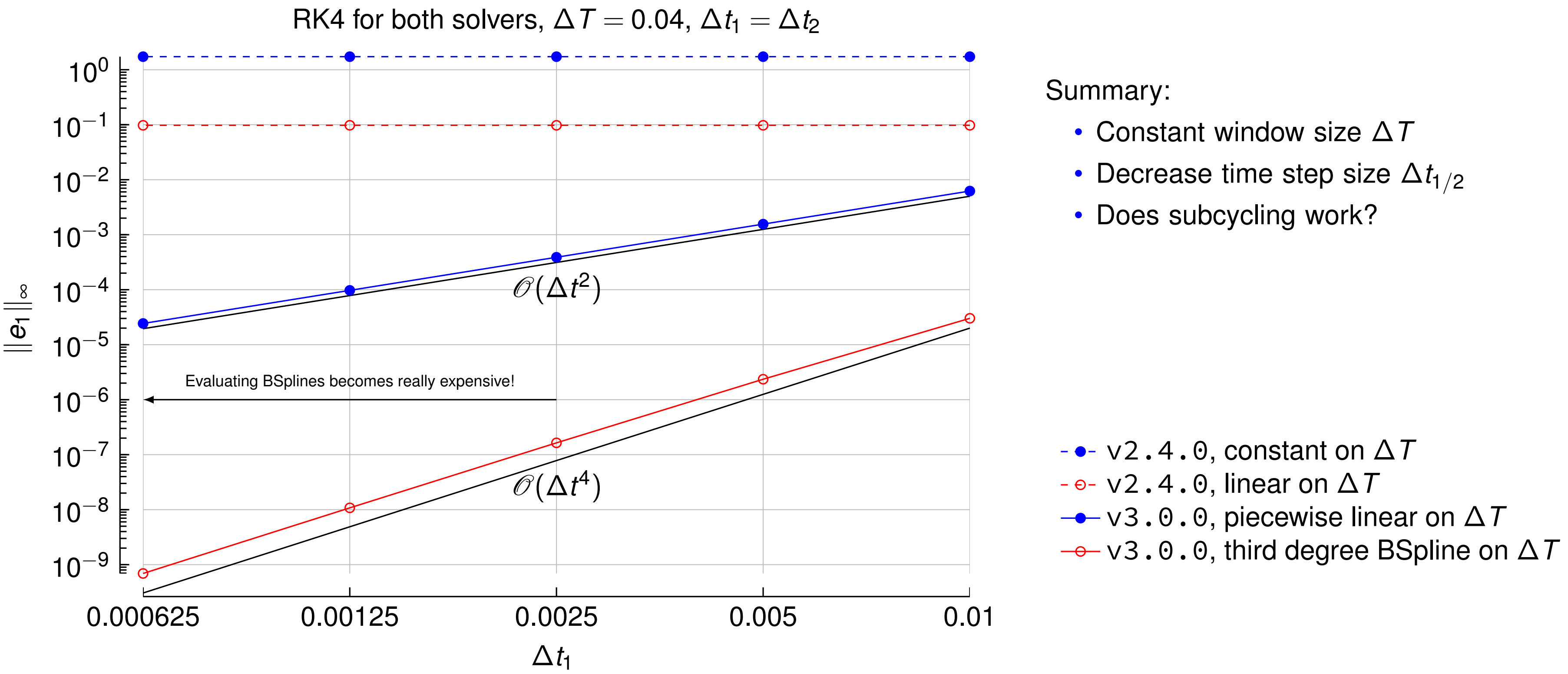
Comparison v2.4 vs. v3.0



Summary:

- Decrease window size ΔT
- Convergence study w.r.t. ΔT
- Does high-order interpolation work?

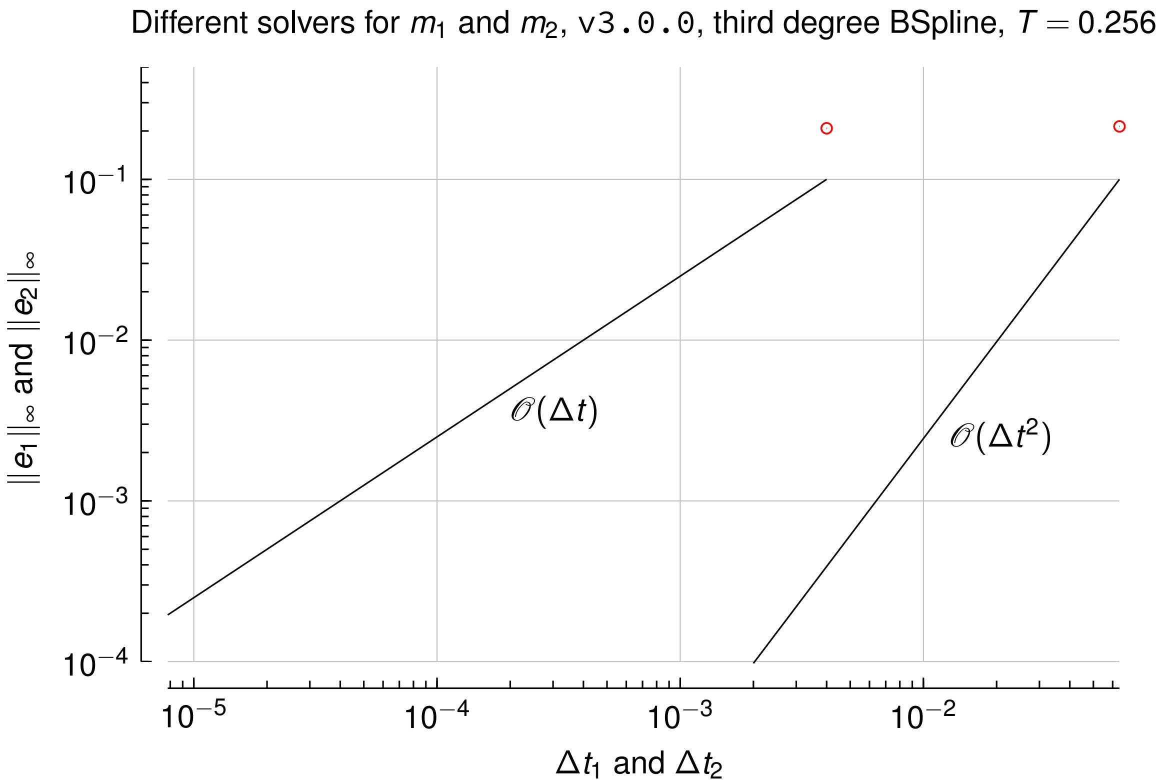
Comparison v2.4 vs. v3.0



Summary:

- Constant window size ΔT
- Decrease time step size $\Delta t_{1/2}$
- Does subcycling work?

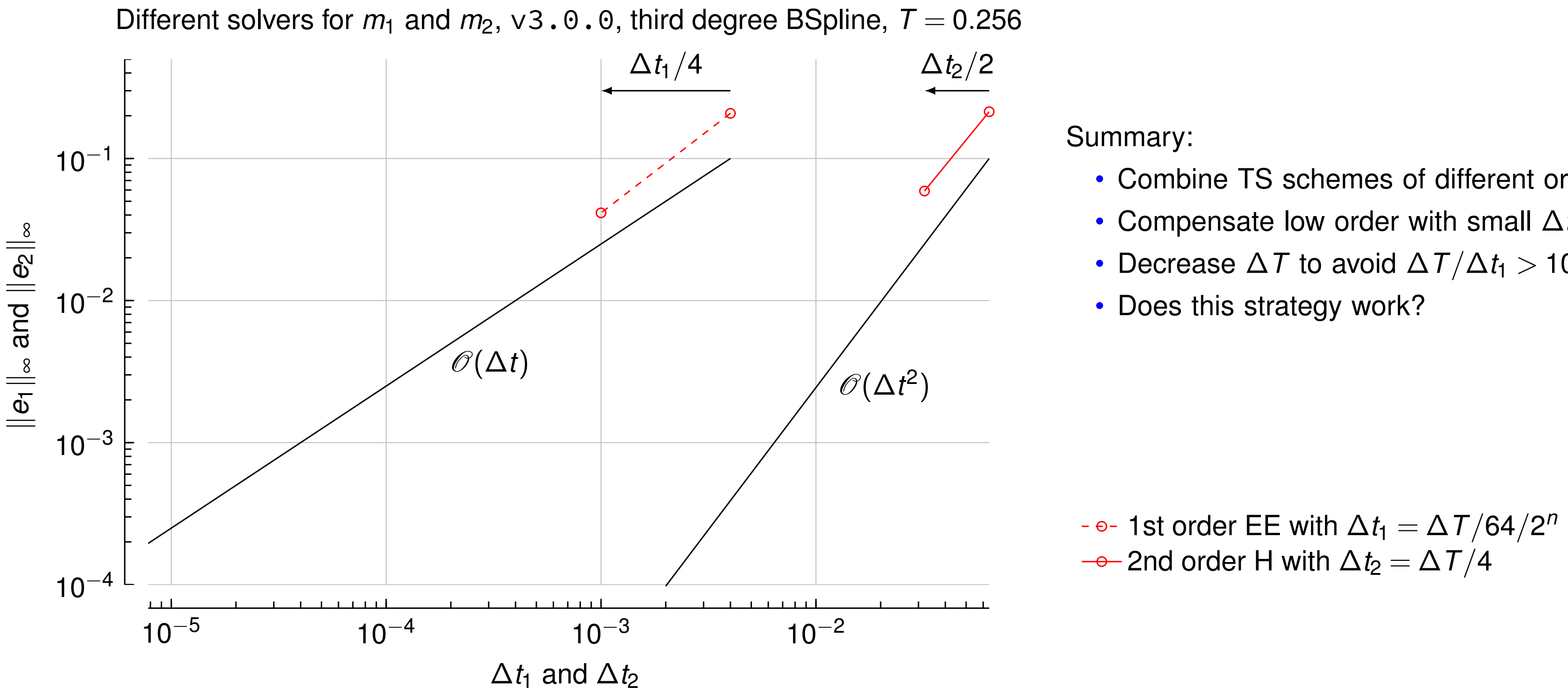
Possible real-world setup?



Summary:

- Combine TS schemes of different order
- Compensate low order with small Δt_1
- Decrease ΔT to avoid $\Delta T/\Delta t_1 > 1000$
- Does this strategy work?

Possible real-world setup?



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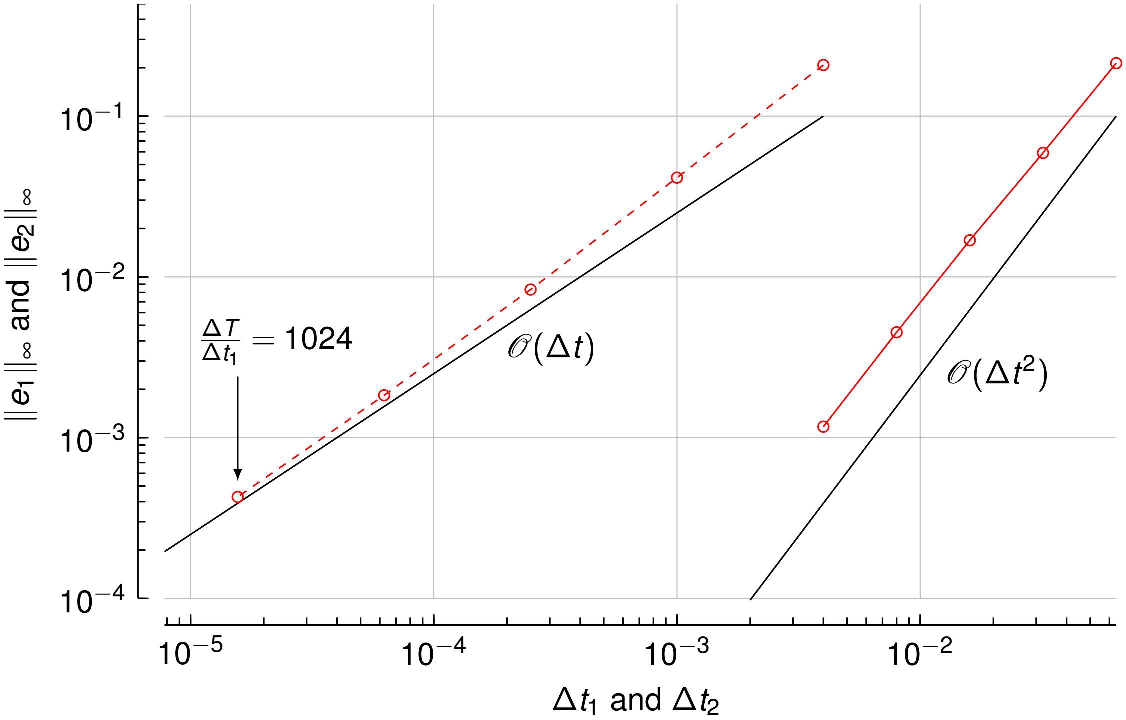
Different solvers for m_1 and m_2, `v3.0.0`, third degree BSpline, $T = 0.256$



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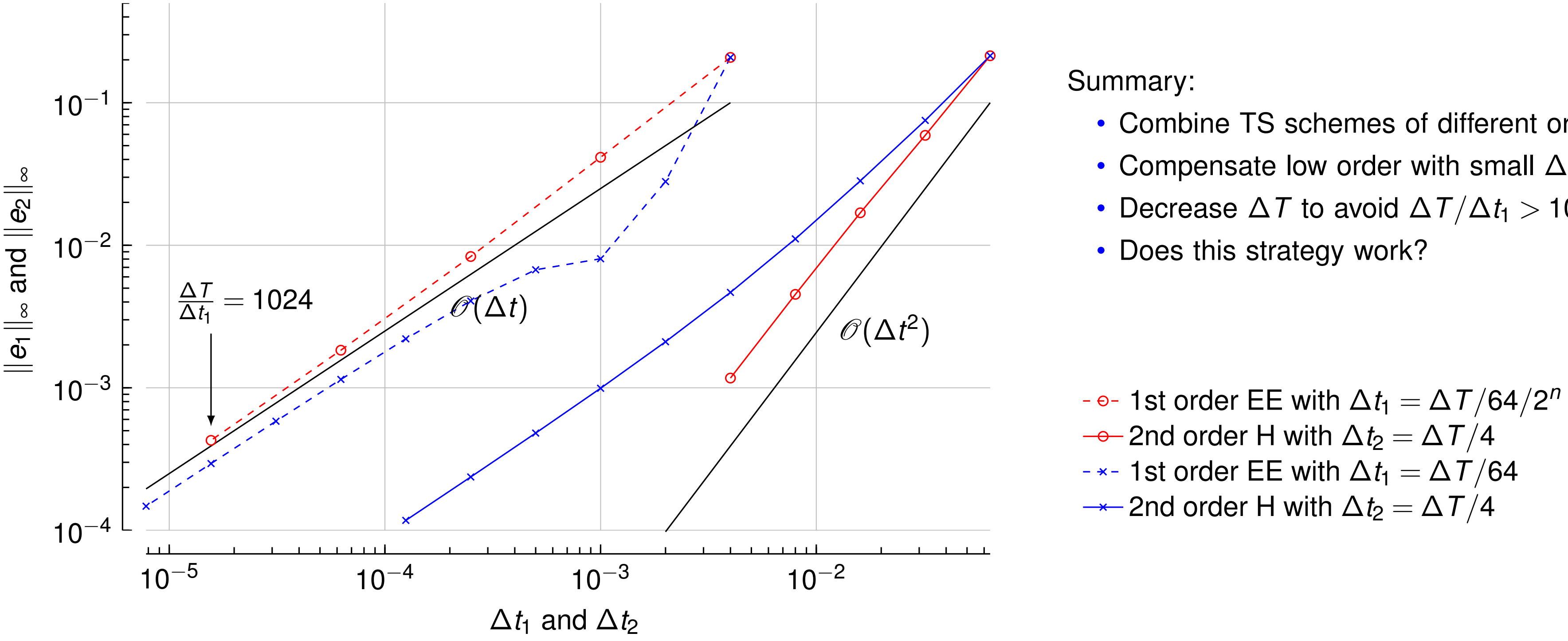

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Conclusion

State of development

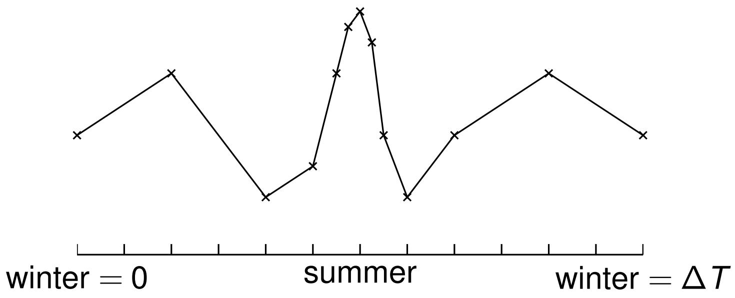
- Goal: Finalize implementation of all this for preCICE v3.0.0.
- If you want to try experimental version now: Talk to me!
- Restriction: High order only possible with subcycling. We think this is a good solution.

Where to be careful:

- BSpline interpolation works well for up to 100s of samples per window. From 1000+ samples it becomes very slow.
- Only very simple test case so far!

Many questions:

- Does less frequent synchronization due to subcycling improve performance?
- Are more Quasi-Newton iterations harmful?
- Better parallel performance and higher order vs. (slightly) higher effort
- Adaptivity (inside window / across windows)



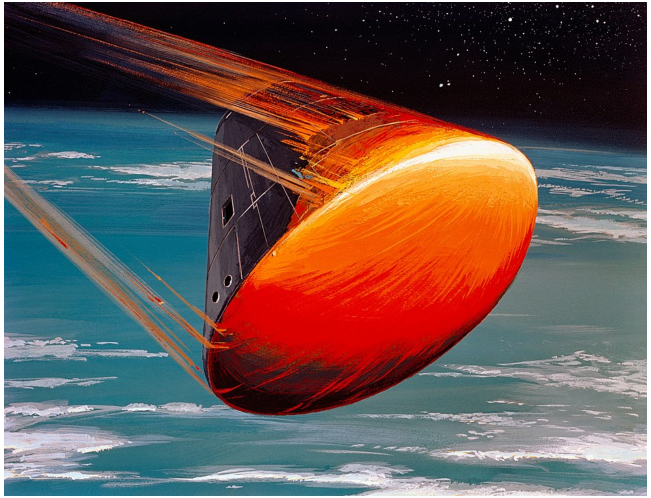
World-Cafe table "Time stepping"

Discuss with us your time stepping setup!

- We cannot promise that we can solve it today or soon
- But: Helps us to consider real cases early on
- Plus: Helps me in writing my thesis

Homework questions:

- Which time stepping schemes?
- Different rates?
- What interpolation requirements?



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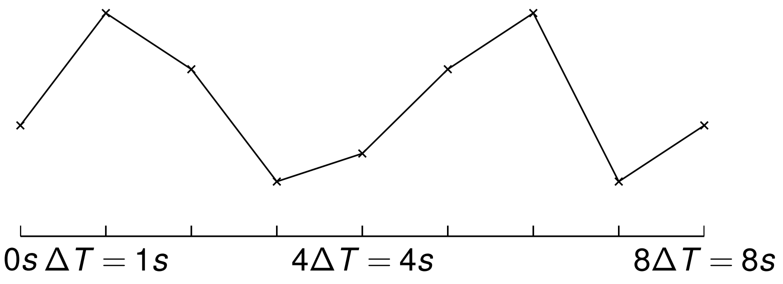
QN iterations

Without subcycling



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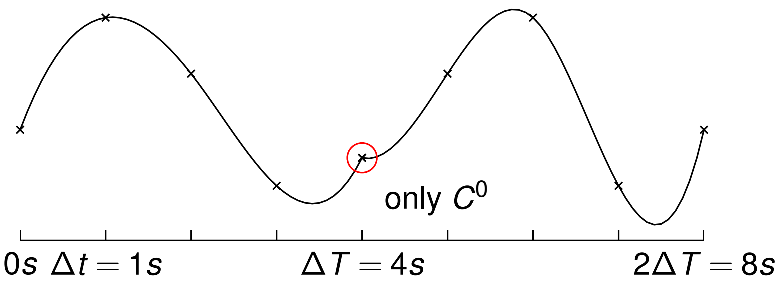

rQN-WI | ΔT	0.5	0.1
WI$(1, 1; 1)$	7.85	$\boxed{5.45}$
WI$(5, 5; 1)$	$\boxed{10.95}$	7.48

rQN-WI means we only use the data at the end of the window for Quasi-Newton. Different example case, but similar implementation. More possibilities shown in[1].

[1] Rüth, B, Uekermann, B, Mehl, M, Birken, P, Monge, A, Bungartz, H-J. Quasi-Newton waveform iteration for partitioned surface-coupled multiphysics applications. Int J Numer Methods Eng. 2021; 122: 5236– 5257. `https://doi.org/10.1002/nme.6443`