SONDERFORSCHUNGSBEREICH 96

LABORATORIUM FÜR DEN KONSTRUKTIVEN INGENIEURBAU (LKI)
TECHNISCHE UNIVERSITÄT MÜNCHEN

BERICHTE
zur
ZUVERLÄSSIGKEITSTHEORIE DER BAUWERKE

RESPONSE SURFACES IN STRUCTURAL RELIABILITY

R. Rackwitz

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Vorwort

Die nachstehende Abhandlung ist das Resultat einer ganzen Reihe von Überlegungen, die in den letzten Jahren vom Verfasser angestellt wurden, wenn die Zuverlässigkeitsmethode 1. bzw. 2. Ordung auf komplexe mechanische Probleme angewandt werden sollte. Sie stellen dennoch nur erste Überlegungen, die in weiteren Beispielen ausprobiert und weiterentwickelt werden müssen, dar. Viele Anwendungen lassen sich jedoch so beherrschen - vielleicht aber noch nicht in der optimalen Weise. Die Thematik wird sicher in weiteren Forschungsarbeiten aufgenommen werden.

Preface

The following paper is the result of the author's attempts in recent years to apply the first-or second order reliability technique to mechanically complex problems. The ideas presented herein still have to be applied at more examples and need further development. Yet, many practical problems can be handled - presumably in a still suboptimal manner. The topic certainly will be subject to future research.

München, Herbst 1982 Der Autor

Introduction

Essential steps forward in formulating and numerical treatment of structural reliability have been made in the very past. This is in part due to the achievements in so-called first- or second-order reliability methods which now, in principle, can deal with arbitrary systems with many components and with quite arbitrary nature of the uncertainties represented by random vectors. (See, for example, 1-7). The perhaps most important result is that multidimensional integration of probability masses over complex domains has essentially been reduced to a problem of mathematical programming together with an application of certain statistical facts for normal variables. Yet, the use of this methodology in complex engineering problems may still require a numerical effort which is hardly justified in practical applications. Additional measures have to be taken in order to make such problems numerically feasible. The following

difficulties are typical for many such problems. Firstly, the number of basic uncertainty variables is very large. Their stochastic model can be rather complex exhibiting dependencies between variables and severe departures from normality. Secondly, the failure surfaces (or the state functions) of the components or the system are available only in implicit form, i.e. involving themselves considerable, generally iterative numerical calculations. Finally, there can be convergence problems in the reliability part when searching for the so-called most likely failure point (failure point, Hasofer-Lind point, ß-point) The first type of problem is perhaps most difficult to overcome except by sound prior knowledge of the dominating variables and by pooling the other variables into one or several random "model simplification" factors whose statistical properties must be estimated in an appropriate manner. The second type of problem is encountered quite frequently in practice and deserves special attention. Since, for computational reasons, the physical model can only be evaluated in a few points, the task is essentially one of finding "optimal" approximations of the physical context on the basis of these points and to optimally design the "experiment", i.e. select only points which are of interest. Their investigation is the main purpose

of this paper. It also usually implies the necessity to decouple the physical and probabilistic part of an analysis and define the way and the ingredients of mutual interaction between these parts. It is believed that a interactive, iterative approach is most effective. The third type of problem is typical for any method involving mathematical programming. Therefore, even if the problems of the first and second type would not exist this "technical" problem requires some thought.

In the following a first attempt is made to outline a procedure which enables a practical solution to such problems. It is based on a combination of the first- resp. second-order reliability method with certain "response surface" techniques well-known in a number of technical and scientific fields. (See [8] for a survey of the methodology). The solutions proposed are still widely empirical and/or intuitive and certainly deserve further consideration. They undoubtedly have been implicit in many previous investigations in the structures area (see. for example, [9]), but, apparently, have never been studied systematically. It will be seen, indeed, that both methodologies well melt together and that first- or second-order methods can be interpreted as a special response surface method particularly suitable for reliability problems.

Basic Formulation

Let the state of a structural component be described by a function

$$M = g(\underline{X},\underline{\Pi}) \qquad (1)$$

where $(\underline{X},\underline{\Pi})=(X_1,\ldots,X_n,\Pi_1,\ldots\Pi_m)$ is a vector of variables. Among these variables the first n variables are uncertain with joint probability distribution $F_{\underline{X}}(\underline{x})=P(\bigcap_{i=1}^{n}\{X_i\leq x_i\})$. The rest of m variables is a set of deterministic variables. As usual, the failure probability is

$$P_f(\Pi) = P(g(\underline{X},\Pi)\leq 0) = \int_{F(\Pi)} dF_{\underline{X}}(\underline{x}) \qquad (2)$$

with the failure domain described by

$$F(\Pi) = \{\underline{X}\in(g(\underline{x},\Pi)<0)\} \qquad (3)$$

Eq. 2 can advantageously be evaluated by the first-(second-) order reliability method, i.e. by transforming $\underline{X}$ into an independent standard normal vector $\underline{U}$ and then using the well-known results for linearly or quadratically bounded failure domains in normal variates. These planes or quadratics

are fitted to the true failure surface in the most likely failure point.

However, if $g(\underline{X},\underline{\Pi})$ is difficult to evaluate and, therefore, must be approximated by a simpler "response surface" $s(\underline{X},\underline{\Pi})$ in a number of knot-points $(\underline{x}_i,\underline{\Pi}_i)$ several questions arise. A first such questions is to select the general type of response surface. A natural choice is to use polynomial surfaces [10]. For reasons which will be explained later on we will restrict ourselves to linear or quadratic surfaces. The next question is how to approximate the original state function or failure surface, i.e. by a Taylor expansion, by interpolation surfaces or by fitting regression surfaces according to a least squared deviation criterion. We might require that any approximation strategy yields, if applied iteratively, the correct probability result, at least in the sense of the first-or second-order reliability method. Once the type(s) of approximating surface(s) is (are) chosen the selection procedure of the sequence of knot-points is of utmost importance as their total number determines the feasibility of a reliability study. Finally, rules are required to reduce the space of uncertainty variables as early as possible in the analysis.

Types of Response Surfaces

For the moment, assume that $\underline{U}$ is an independent standard normal vector. Fitting an arbitrary surface by linear or quadratic approximations in principle can be done in three ways. If the original surface is at least twice differentiable in some domain a simple Taylor expansion in a given point may be obtained.

$$m \approx t(\underline{u}) = g(\underline{u}^o) + \sum_{i=1}^{n} \frac{\partial g(u)}{\partial u_i} \Big|_{\underline{u}^o} (u_i - u_i^o)$$

$$+ \frac{1}{2!} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial g(\underline{u})}{\partial u_i \partial u_j} \underline{u}^o (u_i - u_i^o)(u_j - u_j^o) \qquad (4)$$

Unfortunately, the closeness of fit of this form highly depends on the expansion point $\underline{u}^o$ which generally is unknown before hand. In the context of first-order reliability methods the optimal point is the ß-point denoted by $\underline{u}^*$. Also, differentiability is seldomly assured. Furthermore, $t(\underline{u})$ does take account of the behaviour of $g(\underline{u})$ only in the immediate vicinity of the expansion point $\underline{u}^o$.
If the derivatives have to be evaluated numerically, by simple tangent or secant differential quotients, a minimum of $1+2n+n(n-1)/2$ points have to be evaluated while $1+n$ points are needed for the linear expansion.

An alternative to simple Taylor expansions is, with the same number of knot-points, the first-order Lagrangian interpolation which for small distances between the knot-points reproduces the Taylor expansion under suitable continuity conditions for the state function. It has the following functional form [10]

$$M \approx h(\underline{u}) = A + \sum_{j=1}^{n} \{[B_j + C_j(u_j - u_j^*) + \sum_{k=j+1}^{n} D_{jk}(u_k - u_k^*)](u_j - u_j^*)\}$$

with

$$A = G(\underline{u}^*);\quad B_j = R_{j1}(u_j^* - u_{j2}) + R_{j2}(u_j^* - u_{j1}); \qquad (5)$$

$$C = R_{j1} + R_{j2};\quad D = \frac{A + g(\underline{u}_{jk}) - g(\underline{u}_{j1}) - g(\underline{u}_{k1})}{(u_{j1} - u_j^*)(u_{k1} - u_k^*)};$$

$$R_{j1} = \frac{g(u_{j1}) - A}{(u_{j1} - u_j^*)(u_{j1} - u_{j2})};\quad R_{j2} = \frac{g(\underline{u}_{j2}) - A}{(u_{j1} - u_j)(u_{j2} - u_{j1}^*)}$$

For simplicity, the knot-points have been selected analogously to the "secant" Taylor expansion. It is worth mentioning that the coefficients of eq. (5) can be given explicitly.

Alternatively, the method of least squares may be used. Approximation of $g(\underline{u})$ then is virtually a regression problem.

$$m \approx r(\underline{u}) = b_o + \sum_{i=1}^{n} b_i u_i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} u_i u_j + \varepsilon \qquad (6)$$

where the b's are determined such that $g(\underline{u})$ is fitted by $r(\underline{u})$ in a redundant number of knot-points such that

the sum of squared differences $\sum_k [g(\underline{u}_k)-r(\underline{u}_k)]^2$ becomes minimal and, hence, yielding the smallest variance σ_ε^2 of the randomised residual lack-of-fit term ε. Any such model is linear in the unknown coefficients and, therefore, can be obtained by standard procedures which, for the sake of completeness are repeated here (see [13] for further details). In slightly generalizing eq. (6) we wish to determine the matrix of coefficients $\underline{\underline{b}}$ for several output quantities $\underline{m} \approx \underline{r} = (r_1,\ldots,r_p)^T$ given the observation matrix

$$\underline{\underline{u}} = \begin{bmatrix} 1 & u_{11} & u_{21}\cdots u_{11}^2 & u_{21}^2\cdots u_{11}\,u_{21}\cdots \\ 1 & u_{12} & u_{22}\cdots u_{12}^2 & u_{22}^2\cdots u_{12}\,u_{22}\cdots \\ 1 & & & \\ & & & \\ & & & \\ 1 & u_{1k} & u_{2k}\cdots u_{1k}^2 & u_{2k}^2\cdots u_{1k}\,u_{2k}\cdots \end{bmatrix}$$

for k observations of the vector $\underline{u} = (u_1,\ldots u_n)$, each yielding also the necessary observations for $\underline{m}$, i.e. eq. (6) is generalised to

$$\underline{r} = \underline{\underline{u}}\,\underline{\underline{b}} + \underline{\varepsilon} \qquad (6a)$$

The least square estimates for $\underline{\underline{b}}$ are

$$\hat{\underline{\underline{b}}} = (\underline{\underline{u}}^T\underline{\underline{u}})^{-1}\underline{\underline{u}}^T\underline{m}$$

if $E[(\underline{m}-\underline{r})^T(\underline{m}-\underline{r})] = \underline{\underline{I}}\sigma^2$ is assumed. We further have $E[\hat{\underline{\underline{b}}}] = \underline{\underline{b}}$, $E[(\underline{m}-\underline{r})] = E[\underline{\varepsilon}] = \underline{0}$ and

$$s_\varepsilon^2 = \frac{1}{(k-n-1)}(\underline{m}^T\underline{m} - \hat{\underline{\underline{b}}}^T\underline{\underline{u}}^T\underline{\underline{u}}\,\hat{\underline{\underline{b}}}) \qquad (7)$$

Later, we shall need also least square estimates for weighted observations. In the statistical literature weighting normally is used to discriminate observations which are less "reliable" [12]. Here, we understand weighting as having observed a particular observation several times, i.e., if the weights p_i fulfilling $\sum_{i=1}^{k} p_i = 1$ are introduced, the number of identical observations is $k_i = p_i \cdot k$. With this in mind application of the foregoing to weighted observations is straight forward. Similar approximations may, of course, be devised with respect to the parameter vector $\underline{\Pi}$.
The best approximation scheme for both types of variables, however, depends on the context. While regression surfaces might frequently be preferred for the description of the dependence of $g(\underline{u},\Pi)$ on Π insofar as it introduces a random model uncertainty factor which, in fact, augments the dimension of $\underline{U}$ by one, interpolation surfaces appear at first sight more suitable for the reliability part since, if applied iteratively, the central knot-point

should finally coincide with the ß-point and the distances between knot-points are made "small" enough so that the interpolation surface coincides with the Taylor expansion in that point. Only then, optimal use of the concepts of first-(second-) order reliability methods can be made, in particular, the fact that at least asymptotically only this point and its immediate vicinity are of interest [14]. However, we shall see that there can be better alternatives.

The selection of the knot-points for $\underline{\Pi}$ is primarily determined by the range of values to be covered. The central point $\underline{\Pi}^*$, in turn, might be selected to optimally fulfill certain performance criteria under the condition that $\underline{U}=\underline{u}^*$. In many cases, especially if the failure probability has a maximum at some value of $\underline{\Pi}$, this value ought to be taken as the central point. It must be left to further problem related studies to investigate appropriate methods for determining optimal knot-points for $\underline{\Pi}$.

Before investigating approximation and knot-point selection strategies any further it is necessary to point out that the physical and the reliability part generally need to be separated except for simple problems. The physical part must be carried out in the original x-space of basic uncertainty variables. Only then, the calculation methods presently available can directly be used. Results allow for immediate physical interpretations.

On the other hand, reliability calculations ought to be made in the u-space of independent standard normal variables for well-known reasons the most important of which will shortly be explained. Thus, a probability distribution transformation is required which uniquely maps $\underline{X}$ into $\underline{U}$. However, due to the generally non-linearity of this transformation and dependencies among the original variables physical interpretation of the results is seldomly retained in the u-space but will be recovered if results in the u-space are mapped back into the original space.

Knot-Point Selection and Approximation Strategies

In very few cases the analyst has almost no information of the problem at hand. Then, he should plan the knot-points such that a large range of values is covered by his first "experiments". The same situation can also arise in system analysis where the same uncertainty variable acts favourable in one system component but unfavourably in another componen Knot-points should, of course, be selected in the u-space where the reliability considerations are carried out. This space quite naturally has certain advantageous properties i.e. rotability which says that the reliability properties of any approximation will not be changed under rotations. Furthermore, the reliability characteristics of a given failure surface are mainly determined by its distance to the coordinate origin and, here, the region of greatest interest

is where the failure surface is closest to the origin because of the rotational symmetry of the multinormal density. Therefore, it is assumed that a transformation $\underline{U}=\underline{T}(\underline{X})$ and its inverse $\underline{X}=T^{-1}(\underline{U})$, e.g. according to [6] is known. For the case of complete ignorance about the region of interest a natural choice of the central point is $\underline{u}^{o}=\underline{O}$. Due to the rotational symmetry of the u-space and the assumed complete disorientation with respect to the region of interest it is also natural to choose the first set of additional knot-points to lie on a central hypersphere (Figure 1). The radius R of the hypersphere might be chosen such that it contains a certain probability p close to unity. We have $R^2 = F^{-1}_{\chi^2_n}(p)$ with $F_{\chi^2_n}$ the chi-suqared distribution with n degrees of freedom A possible choice of points is also given in figure 1. The quadrant for the interaction point may be selected at random at this state. Oftenly, it is advisable first to determine only knot-points sufficient for a plane response surface and, then, if this plane is still uninformative to proceed to quadratic interpolation surfaces.

In most other cases there is some information on the region of interest. It may be derived from rules of thumb or qualitative engineering judgement (or intuition) but most frequently from simple idealizing models for the physical context; for example, by simple equilibrium considerations for structures, analytical linear-elasticity models, simple rigid-plasticity models, etc. This model

should at least be as realistic as to include the variables known to be important a priori and which have the same type of sensitivity with respect to changes in the variables. A further simplification might consist in assuming a simplified stochastic model, e.g. $\underline{X}$ as a normal-lognormal vector with the same first and second probabilistic moments. A first-order reliability evaluation then will produce a first estimate of the region of interest which simply is the ß-point of the elementary analysis. Figure 2 demonstrates the importance of selecting a suitable starting point at an example typical in complex structural analysis. It shows the state function of an initially curved column under uncertain load U_1 and with random yield strength U_2. The problem is known to be highly non-linear beyond a certain state. Any approximation with knot-points around $\underline{u}=\underline{0}$ would be completely misleading. Since the state function has a pole at $T^{-1}(u_2)=c_3$, the ß-point of a plane or a quadratic failure surface derived from the corresponding approximate state function fitted to points near $\underline{u}=\underline{0}$ would even fall into a domain where the state function is undefined. This example is also an indication that any selection of points must observe possible physical constraints. However, this is usually not easy to fulfill in practice. Any such preliminary analysis frequently turns out to be very useful since it localizes the region where further investigations should be made. From now on, any further analysis should be based on the desired stochastic model and the

complete physical model.

The first-order reliability method now would proceed in further searching for the region of interest. Finally, the failure surface $g(\underline{x})=0$ resp. $g(T^{-1}(\underline{u}))=0$ would be linearized by a hyperplane perpendicular to the vector spanning the shortest distance between $g(T^{-1}(\underline{u}))=0$ (most likely failure point) and the coordinate origin [1-6]. First probability estimates would be obtained by $\phi(-\beta)$ if β is this shortest distance, i.e. $\beta=\min\{\|\underline{u}\|\}$ for $\{\underline{u}:g(\underline{u})=0\}$. If $g(T^{-1}(\underline{u}))=0$ is twice differentiable in this point, quadratic expansion forms can also be used at the expense of some more effort yielding better failure probability estimates [3,14]. Hence, the first-or second-order reliability method is a response surface method with either first-or second-order expansion surfaces fitted to the failure surface in a point of greatest interest for reliability. Some of the algorithms to find the most likely failure point are, this is worth mentioning, direct analogue to the method of steepest descent (ascent) in classical response surface techniques where expansion points are sough such that the response attains some desired extremum. Clearly, in order to make this procedure practical which actally means rapidly converging, one has to require relatively well-behaved state functions $g(\underline{T}^{-1}(\underline{u}))$. Nevertheless, this algorithm would represent a first possibility to approach the reliability problem.

Occasionally, the gain in accuracy when proceeding towards the most likely failure point (the interesting region) becomes less and less and will soon be dominated by the gain which could be achieved by locally fitting quadratic forms. Note, however, that the number of determinations of $g(\underline{x})$ in each step is approximately proportional to n in the linear response surface case. Furthermore, all methods presently in use first-order reliability methods or classical steepest ascent (descent) response surface methods, appear to lack incorporation of the information on $g(\underline{x})$ gained in previous steps of the iteration. This is certainly one of the important aspects to be considered in complex problems where the calls for $g(\underline{x})$ must be kept at a minimum. In order to allow for early truncation of the iteration we, therefore, propose, as a second alternative, to use quadratic response surfaces at the very first steps. One probably would select this alternative if one expects convergence problems. In order to smooth the actual state function but retain those points on $g(\underline{T}^{-1}(\underline{u}))$ which actually have been determined exactly, one now should apply Lagrangian interpolation surfaces. The first step is the selection of additional knot-points in the u-space having distance Δ (1) to the central point determined previously. Δ (1) generally should be chosen rather large, about Δ (1) = 1, say (see figure 3). Then, the corresponding knot-points in the x-space must be evaluated by $\underline{x}_k=\underline{T}(u_k)$ which are used to determine $m_k=g(\underline{x}_k)$ for k=1 to $k=1+2n+n(n-1)/2$. The resulting quadratic interpolation surface

normally requires a non-linear transforamtion. It is no more quadratic if transformed. Therefore, a new ß-point $\underline{u}_o$ (1) on $g(\underline{T}^{-1}(\underline{u}))=0$ must be found and a probability estimate is obtained either by expanding a $(\underline{T}^{-1}(\underline{u}))=0$ into a plane $l(\underline{u})=0$ or into a quadratic $q(\underline{u})=0$ provided that the transformation is continuously differentiable. According to [3] the resulting quadratic must be brought into one of the standard forms. Its probability content can then be evaluated by one of the methods reviewed in the same reference. If this probability is large a particularly useful asymptotic formula given in [14] is worth mentioning:

$$P_f = P(g(\underline{X})\leq 0) \approx P(q(\underline{U})\leq 0) \sim \phi(-\beta) \prod_{i=1}^{n-1} (1-|\beta|\kappa_i)^{-1/2}$$

β^t

:he κ_i' s are the principal curvatures < 1/ß of $g(\underline{u}))=0$ in the ß-point. The formula describes the probability of an approximating paraboloid with an axis going through the ß-point. The result is interesting quite generally since it emphasizes the dominance of the value of ß, i.e. the region of interest against other characteristics of the failure surface. Thus, finding the most likely failure region is essential in any analysis. In reducing the distances Δ in an appropriate manner when applying this procedure iteratively one should finally find the result of the first alternative. However, one should have in mind that each iteration step now involves

approximately $n^2/2$ instead of n calls of $g(\underline{x})$ and, therefore, the method might be inferior to the first alternative. An intermediate procedure consists in starting with a plane interpolation surface and adding consecutively quadratic terms only for those variables which appear to be dominating i.e. have largest sensitivity with respect to the failure probability expressed by the direction cosine of the ß-point in that variable.

The third alternative combines the first steps of alternative ii. with a least-square fitting in the consecutive steps for the search of the region of interest. For $k>1+2n+n(n-1)/2$ the interpolation surface needs not be any more a simple quadratic surface since more than the necessary points are available. Higher-order polynomials could in principle be used. Unfortunately, such polynomials can oscillate considerably between points and thus being a potential cause of non-convergence (local minima!) when used later on. A perhaps better alternative to incorporate more information on the shape of $g(\underline{x})$ into approximation surfaces is to perform a weighted linear or quadratic regression analysis. Clearly, the weights in the least square criterion should reflect both the value of $g(\underline{x}_k)$ and the distance of $\underline{x}_k$ to $\underline{x}^*$. For example, one could use

$$p_k = \frac{1}{N} \frac{1}{(|g(\underline{T}^{-1}(\underline{U}_k))|+\delta)} \exp\left[-\frac{1}{2}||\underline{U}_k||^2\right]$$

in

$$\sum_k p_k [g(\underline{T}^{-1}(\underline{u}_k)) - r(\underline{u}_k)]^2 \rightarrow \min$$

where N is a necessary normalizing factor such that the p_k' s sum up to unity, δ a small positive number close to zero and r(.) the response surface as given by eq.(6). The only reason to introduce δ is to avoid divisions by zero. The first factor reflects the fact that only those values of $\underline{u}_k$ are of primary interest for which $g(\underline{T}^{-1}(\underline{u}_k))$ is or is close to zero. The peak of weights for $g(\underline{T}^{-1}(\underline{u}))$ close to zero can be made sharper in raising the first factor to powers > 1. The second factor takes account of the distance $\| \underline{u}_k \|$ and simply describes the decay of the multinormal density function with distance from the origin. Note that monotony of $g(\underline{T}^{-1}(\underline{u}))$ in each of the components of $\underline{u}$ is somehow a condition of this procedure to work. A disadvantage is that it does not necessarily converge to the asymptotically exact result as given in [14] although the differences should, in practice, be negligible small.

The error term of the regression surface can or cannot be included in the final probability analysis. Its mean and variance can easily be estimated using the standard formulae for $\varepsilon_k = p_k (g(\underline{T}^{-1}(\underline{u}_k)) - r(\underline{u}_k))$, i.e. by $\bar{\varepsilon} = (\Sigma \varepsilon_k)/\Sigma k_i$ and $s_\varepsilon^2 = (\Sigma (\varepsilon_k - \bar{\varepsilon})^2)/(\Sigma k_i - n - 1)$. Due to the weighing $\bar{\varepsilon}$ is a biased estimator for $E[\varepsilon]$. Also, the distribution function of ε

is unknown. Nevertheless, since this lack-of-fit term should be small it may without too large error be assumed normally distributed.

The iteration for the search of the region of interest can now be done in the usual way, i.e. finding the new ß-point on the regression surface in each step. In fact, one could add only one new knot-point, the ß-point of the actual regression surface in each step having in mind that the laborious part of the analysis is in the mechanics. It appears though that no general rule can be given how many new points to select in each step and when to truncate the iteration. Obviously, the proposed method is a method of linear to quadratic interpolation with memory to find the region of interest yielding probability estimates at each step in the iteration. Quadratic interpolation should be faster convergent than any linear interpolation scheme (e.g. method of steepest descent, method of Lagrangian multipliers, elementary iteration schemes based on $\underline{x}^{(i+1)} = h(\underline{x}^{(i)})$, etc.). The keeping of memory has its justification in the fact that the majority of structural problems is somehow well-behaved in that $g(\underline{T}^{-1}(\underline{u}))$ is monotonic. The use of regression surfaces (with or without error term) might introduce a certain robustness of the algorithm. The key problem here and for the earlier alternatives simply is a suitable selection of the central starting point.

Knot-point selection in system analysis

The problem becomes even more involved if the response surface technique has to be applied simultaneously to various components of a system with given logical structure. Such components could, for example, be the elements in a finite-element model for a complex structure. Failure of the system could be defined by any element or a given set of elements failing. For each element a different state function and failure domain is formulated, e.g. for the r-th element in a set of s elements it is: $F_r = \{\underline{X} \in (g_r(\underline{X}) \leq 0)\}$. System failure is defined as $F = \bigcup_{\nu} \bigcap_{\mu} F_{\nu\mu}$ which is the minimal cut set representation of the failure event. In this case the procedure outlined before can, of course, be applied s-times, for each individual element, respectively. The failure domains would finally be replaced by equivalent half-spaces and the system probability can be calculated in the usual way [7, 15]. If this is not possible for computational reasons the selection strategy for the knot-points is of utmost importance since one set of knot-points optimal for an initially "critical" component may not be optimal for other components which might have become critical if another set of knot-points had been chosen. It should also be clear that, in general, it is not immediately obvious which component or which component

of the basic variable vector plays the most important role if there is any. In recognition of the fact that the state function has to be known over a much larger range of values of the uncertainty vector it is proposed to start in a systematic manner as described at the beginning of the last section.

In order to find the next central knot-point one can make use of the equivalent plane concept as given in [15]. Therein, an equivalent hyperplane is defined as a plane which cuts off the same probability content as the original failure domain and has the same sensitivity against (small) changes in the components of the uncertainty vector $\underline{U}$ with respect to the failure probability. Clearly, the point on that equivalent hyperplane with shortest distance to the origin is such a new central knot-point. If this concept is combined with the regression fitting as outlined above sufficiently accurate results may be expected already for the very first iterations. What appears important to note is that the logical structure of the system must be formulated beforehand.

Comments on possible reduction strategies of the basic variable space

One of the most effective means to simplify the numerical analysis is, of course, an early reduction of the space of basic uncertainty variables. Unfortunately, very little is known about objective criteria upon which such reductions could be based so that several remarks must suffice for the moment. They exclusively refer to insights which can be gained in the reliability part, i.e. in the u-space. Obviously, if certain variables in either of the approximating surfaces have (almost) identical coefficien in the linear terms in two consecutive iteration steps and this remains so during the iteration, $g(\underline{T}^{-1}(\underline{u}))$ is linear in these variables and nothing is gained by repeating the determination of the gradient of $g(\underline{T}^{-1}(\underline{u}))$ with respect to these variables. Also, if the direction cosines for some variables are small compared to others indicating that the influence of variations in these variables on the failure probabilities is small these variables can be collected in one single variable $Y = \Sigma\alpha_i U_i$ with mean $E[Y] = 0$ and variance $Var[Y] = \Sigma\alpha_i^2$. This variable can then be treated similar to the residual ε in regression surfaces. Note that this does not necessarily mean that the calculations in the x-space are carried out for constant values of these variables since the probability

distribution transformation may very well require changes for dependent variables. What is kept constant is, in fact, only the partial gradient for these variables in the u-space. Certainly, these or other types of simplifications normally should be checked at the end of the iteration.

Summary and Conclusions

First- or second-order reliability methods need to be combined with certain techniques in classical response surface methodology if applied to complex structural problems. A rigorous implementation of the first methodology which, in fact, can be interpreted as a special version of the latter can numerically be no more feasible since the region of interest, i.e. the most likely failure point(s) can only be found iteratively with large numerical effort if it can be found at all. It is therefore proposed to determine the region of interest in first approximation by use of simplified physical models and, then, instead of using local expansion surfaces approximating the original state function by interpolation or better by weighted regression surfaces supported by appropriate number of knot-points. These surfaces are used in the reliability analysis allowing in general also considerations of multi-state componental behaviour. Second-order surfaces, at least in the dominating variables, are recommended. Yet, much more experience has still to be gained at practical examples since theoretical considerations appear to be only of limited value. It is hoped that the review of concepts and methods presented herein will help in designing such examples in a somehow structured manner.

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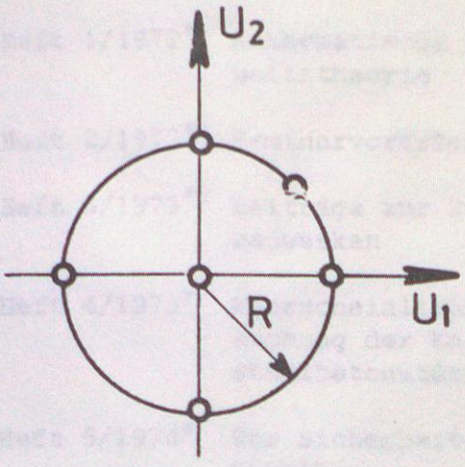

Figure 1

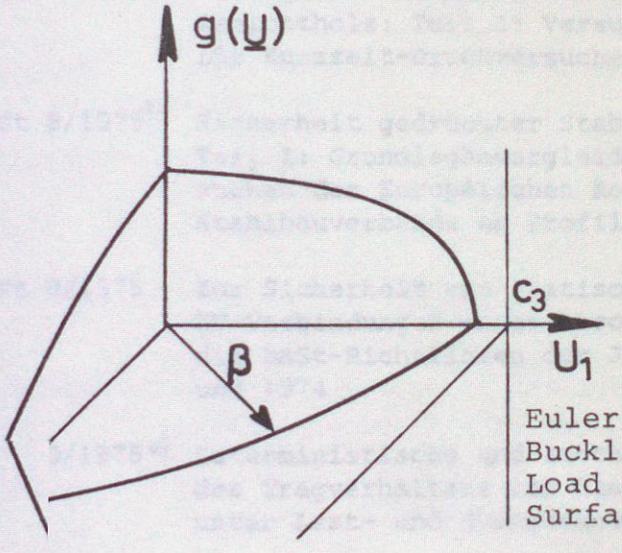

Figure 2

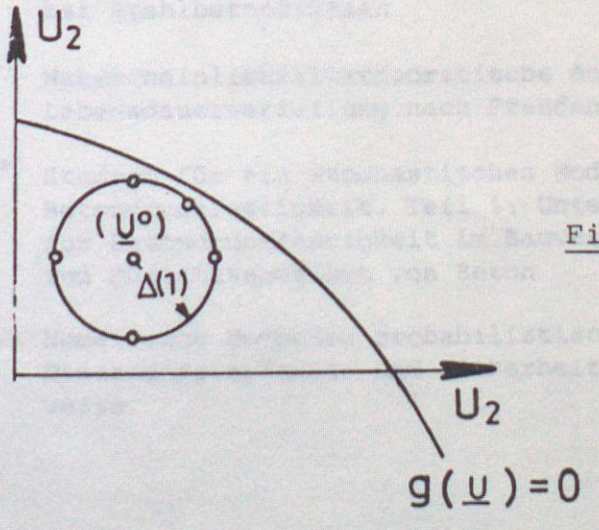

Figure 3

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