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Socio-economically sustainable civil engineering infrastructures by optimization

R. Rackwitz [a,*], A. Lentz [a], M. Faber [b]

[a] *Technische Universität München, München Arcisstrasse 21, D-80290 Muenchen, Germany*
[b] *Eidgenössische Technische Hochschule, Zürich, Switzerland*

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Abstract

Sustainability is an important requirement for civil engineering infrastructures, technically and financially. The financial aspects are discussed. It is proposed to select a design and maintenance strategy where structures are systematically renewed by reconstruction or repair. An appropriate objective function for cost-benefit analyses based on a renewal model is established. An intergenerationally acceptable discounting scheme is proposed. As infrastructures also involve risk to human life and limb a socio-economic acceptability criterion to be added as a constraint to cost-benefit analyses is derived. Various renewal models for deteriorating structures including multiple failure modes are discussed. The paper includes various examples illustrating the developed theory.

Keywords: Structural safety; Sustainability; Discounting; Renewal model

1. Introduction

The notion of life cycle engineering probably has emerged in the military field where, apart from first installation, inspection and maintenance and, finally, removal and replacement of a sys-

* Corresponding author. Tel.: +49 289 23050; fax: +49 289 23096.
E-mail addresses: rackwitz@mb.bv.tum.de (R. Rackwitz), lentz@mb.bv.tum.de (A. Lentz), faber@ibk.baug.ethz.ch (M. Faber).

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tem formed the major cost. Later, it was also used in civil engineering when societies recognized that the infrastructure in a country or region had not only to be built but maintained and finally renewed because failures occurred not only due to extreme external events but primarily due to wear out, deterioration and, in some cases, obsolescence. The strategies adopted by engineers to manage the life cycle of a structure were widely technical, i.e. by improving reliability and durability and by proposing design solutions which should enable a longer time span of full use, possibly with other uses than initially foreseen. This is, no doubt, an important aspect of life cycle engineering. But not the only one and, most likely, not even the most important one.

In addition, there is growing awareness of the fact that our world is a limited world in the sense that it has only limited non-renewable natural resources, even limited renewable resources like water and limited arable land, for example. This lead the so-called Brundland Commission [9] to conclude in 1987 in their famous report "Our Common Future" that a sustainable development is a development "that meets the needs of the present without compromising the ability of future generations to meet their own needs". In the mean time one can say that this statement has widely become a new ethical standard. The immediate implications for the planning, design and operation of civil engineering infrastructures are clear: Save energy, save non-renewable resources and find out about re-cycling of building materials, do not pollute the air, water or soil with toxic substances, save or even regain arable land, do not interfere into the natural water household to an extent disproportionate to the adverse effects of such an interference, and much more.

For civil engineering infrastructures, but not only for those, there is a third aspect and that is the financial aspect. It is assumed that civil engineering infrastructures are financed by the public via taxes, public charges or other. It is in any case the citizen who pays and, of course, also enjoys the benefits derived from their existence. More precisely, intergenerational equity is the core of the new ethical standard the Brundland Commission has set; here we consider this standard with particular reference to the financial aspects of planning, designing, maintaining and replacing civil engineering infrastructure. Our generation must not leave the burden of maintenance or replacement of too short-lived structures to future generations, it must not use more of the financial resources than are really available. It can use only those which are available and affordable in a sustainable manner and discounting with its many myopic aspects must be done with utmost care. In this sense, civil engineering structures should be optimal not only from a technological point of view but also from a sustainability point of view.

The paper will first review a renewal model for setting up suitable objective functions for cost-benefit analysis. After presenting the basic model it is extended to different failure modes and obsolescence, to deteriorating facilities and to a simple case of inspection and maintenance. Then, some recent socio-economic considerations for public risk acceptability are summarized and some thoughts about sustainable and intergenerationally acceptable public interest rates are given. A sustainable, intergenerationally acceptable and affordable public risk acceptance criterion is derived. This is the basis for developing some optimization tools. Several examples illustrate the theory.

2. Cost-benefit optimal technical facilities

Already in 1971 Rosenblueth and Mendoza [50] proposed optimization with respect to benefits and cost as the final goal of setting up structural codes but also for direct design and operation of structures. A technical facility is financially optimal if the following objective is maximized:

$$Z(\mathbf{p}) = B(\mathbf{p}) - C(\mathbf{p}) - D(\mathbf{p}). \tag{1}$$

It is assumed that all quantities in Eq. (1) can be measured in monetary units. $\mathbf{p}$ is the vector of all safety relevant parameters or actions. $B(\mathbf{p})$ is the benefit derived from the existence of the facility, $C(\mathbf{p})$ is the cost of design and construction and $D(\mathbf{p})$ is the cost in case of failure. The quantities $B(\mathbf{p}), C(\mathbf{p})$ but especially $D(\mathbf{p})$ involve uncertainties. Statistical decision theory then dictates that expected values are to be taken [64]. In the following it is assumed that $B(\mathbf{p})$, $C(\mathbf{p})$ and $D(\mathbf{p})$ are differentiable in each component of $\mathbf{p}$. The cost as well as the benefits may differ for the different parties involved. The different parties, e.g. the owner, the builder, the user and society, may also have different economic objectives. A facility makes sense only if $Z(\mathbf{p})$ is positive within certain parameter ranges for all parties involved. In this paper, however, we will primarily focus on an optimization for and in the name of the public.

In view of sustainability one has to distinguish between at least four replacement strategies:

- the facility is given up after service or failure,
- the facility is systematically replaced after failure,
- the facility is renewed (repaired) after deterioration,
- the facility is renewed due to obsolescence.

Further, we distinguish between facilities which can *fail upon completion or never* and facilities which can *fail at a random point in time* later due to service loads, extreme external disturbances or deterioration. The option "failure upon completion or never" implies that loads and resistances on the facility are time-invariant which is not considered further. The option "facility given up after service or failure" is only considered later in an example because infrastructure facilities must remain functioning for all foreseeable future in accordance with the sustainability requirement. Finally, reconstruction times are assumed to be negligibly short as compared with the normal life time of the facility for simplicity. Appropriate extensions for finite renewal times and failure at construction are available [60].

3. The renewal model

3.1. Discounting

The facility has to be optimized during design and construction at the decision point, i.e. at time $t = 0$. Therefore, all cost need to be discounted down to the decision point. For analytical convenience continuous discounting is assumed which is accurate enough for all practical purposes. Let $\gamma(t)$ be a function of time. If damage $D(t)$ occurs at time t its present value $D(0)$ can be calculated from the elementary differential equation

$$\frac{\mathrm{d}D(t)}{\mathrm{d}t} = -\gamma(t)D(t), \tag{2}$$

whose solution is

$$D(0) = D(t)\exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right].$$

For a constant time-averaged discount rate $\gamma = (1/t_s)\int_0^{t_s}\gamma(\tau)\mathrm{d}\tau$ with $t_s > t$ some reference time, one simply has $D(0) = D(t)\exp[-\gamma t]$. Discount rates are understood as real rates net of any taxes. If a interest rate γ' for discrete discounting is given it is simply $\gamma = \ln(1 + \gamma')$.

3.2. Basic renewal model

Assume random events in time forming a renewal process. The times between failure (renewal) events have identical distribution functions $F(t,\mathbf{p})$, $t \geqslant 0$, with probability densities $f(t,\mathbf{p})$ and are independent. Renewal theory allows for a sometimes useful refinement, i.e., the distribution of the time to the first event can have distribution function $F_1(t,\mathbf{p}) \neq F(t,\mathbf{p})$, $t \geqslant 0$ (see [43] for details). The independence assumption needs to be verified carefully. In particular, one has to assume that loads and resistances in the system are independent for consecutive renewal periods and there is no change in the design rules after the first and all consecutive failures (renewals). Even if designs change failure time distributions must remain the same. Consider first the case of a one-mission facility which is given up after service or failure. Obviously, it is for time-dependent benefit function $b(t)$ and time-dependent interest rate $\gamma(t)$ and service time $t_s \to \infty$:

$$Z(\mathbf{p}) = \int_0^{\infty}\int_0^{t} b(\tau)\mathrm{e}^{-\int_0^{\tau}\gamma(\theta)\mathrm{d}\theta}\,\mathrm{d}\tau\, f_1(t,\mathbf{p})\mathrm{d}t - C(\mathbf{p}) - H\int_0^{\infty}\mathrm{e}^{-\int_0^{t}\gamma(\tau)\mathrm{d}\tau} f_1(t,\mathbf{p})\mathrm{d}t. \tag{3}$$

Neglecting finite (re-)construction times the objective function for systematic reconstruction is in full generality

$$Z(\mathbf{p}) = B^* - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^{\infty}\int_0^{\infty}\mathrm{e}^{-\int_0^{t}\gamma(\tau)\mathrm{d}\tau} f_n(t,\mathbf{p})\mathrm{d}t, \tag{4}$$

where $f_n(t,\mathbf{p})$ is the density of the time to the nth renewal. It is assumed that construction cost $C(\mathbf{p})$ are without cost of financing. If they are included one has to multiply $C(\mathbf{p})$ with the factor $(\gamma_M \exp[\gamma_M t_s])/(\exp[\gamma_M t_s] - 1)$, where γ_M is some (constant) market interest rate and t_s the financing time. H is the monetary loss in case of failure including direct failure cost, demolition cost, cost of removal of debris, loss of business and other indirect cost and, of course, the cost to reduce the risk to human life and limb. Therefore, it is useful to decompose H into physical losses H_M and losses H_F associated with losses of human life and limb (see Section 6.3 for details). Also, it is assumed that $C(\mathbf{p})$ and H are independent of time.

The benefit term requires special treatment depending on how the benefit per time unit evolves in time. If it is assumed that the benefit function $b(t)$ is unaffected by any renewal in the future one obtains

$$B^* = \int_0^{\infty} b(t)\mathrm{e}^{-\int_0^{t}\gamma(\tau)\mathrm{d}\tau}\,\mathrm{d}t. \tag{5}$$

For systematic reconstruction one, alternatively, can assume that at each renewal the benefit function starts from $b(0)$. Then, according to [22]

$$B^* = \int_0^{\infty} B_D^{\#}(t)f(t)\mathrm{d}t\left[1 + \left(\sum_{n=2}^{\infty}\int_0^{\infty}\mathrm{e}^{-\int_0^{t}\gamma(\tau)\mathrm{d}\tau\, t} f_{n-1}(t)\mathrm{d}t\right)\right] \tag{6}$$

with

$$B_D^{\#}(t) = \int_0^{t}\mathrm{e}^{-\int_0^{u}\gamma(\tau)\mathrm{d}\tau} b(u)\mathrm{d}u. \tag{7}$$

3.3. Constant benefit and discount rates

For constant benefit per time unit $b(t) = b$ and a constant discount rate $\gamma(t) = \gamma$ the objective function simplifies greatly, especially because one can make use of the convolution theorem for Laplace transforms in the damage term [1]

$$\begin{aligned} Z(\mathbf{p}) &= \int_0^{\infty} b\mathrm{e}^{-\gamma t}\mathrm{d}t - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^{\infty}\int_0^{\infty}\mathrm{e}^{-\gamma t} f_n(t,\mathbf{p})\mathrm{d}t \\ &= \int_0^{\infty} b\mathrm{e}^{-\gamma t}\mathrm{d}t - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^{\infty} f^*(\gamma,\mathbf{p})^{n-1} f^*(\gamma,\mathbf{p}) \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{f^*(\gamma,\mathbf{p})}{1 - f^*(\gamma,\mathbf{p})} = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)r^*(\gamma,\mathbf{p}), \end{aligned} \tag{8}$$

where $f^*(\gamma,\mathbf{p})$ is the Laplace transform of $f(t,\mathbf{p})$ and $r^*(\gamma,\mathbf{p}) = [f^*(\gamma,\mathbf{p})]/[1 - f^*(\gamma,\mathbf{p})]$ is the Laplace transform of the renewal density (renewal intensity) $r(t,\mathbf{p}) = \sum_{k=1}^{\infty} f_k(t,\mathbf{p})$. If the density of the time to the first renewal is different from all other renewal times, i.e. $f_1(t) \neq f(t)$, one derives $r^*(\gamma,\mathbf{p}) = [f_1^*(\gamma,\mathbf{p})]/[1 - f^*(\gamma,\mathbf{p})]$ [51]. For the one-mission facility we have instead:

$$\begin{aligned} Z(\mathbf{p}) &= \int_0^{\infty}\int_0^{t} b\mathrm{e}^{-\gamma\tau}\mathrm{d}\tau f_1(t,\mathbf{p})\mathrm{d}t - C(\mathbf{p}) - H\int_0^{\infty}\mathrm{e}^{-\gamma t} f_1(t,\mathbf{p})\mathrm{d}t \\ &= \frac{b}{\gamma}(1 - f_1^*(\gamma,\mathbf{p})) - C(\mathbf{p}) - Hf_1^*(\gamma,\mathbf{p}). \end{aligned} \tag{9}$$

If failures occur according to a Poisson process with occurrence rate $\lambda(\mathbf{p})$ Eq. (8) simplifies to [50]

$$Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{\lambda(\mathbf{p})}{\gamma} \tag{10}$$

because $f^*(\gamma,\mathbf{p}) = [\lambda(\mathbf{p})]/[\gamma + \lambda(\mathbf{p})]$ for $f(t,\mathbf{p}) = \lambda(\mathbf{p})\exp[-\lambda(\mathbf{p})t]$. This result is especially relevant because the parameter $\lambda(\mathbf{p})$ may be replaced asymptotically by the stationary outcrossing rate $\nu^+(\mathbf{p})$ frequently used in time-variant structural reliability analysis [42]. If $\nu^+(\mathbf{p})$ depends on an uncertain parameter vector $\mathbf{R}$ and/or a random sequence $\mathbf{Q}$ one should use $E_{\mathbf{R},\mathbf{Q}}[\nu^+(\mathbf{p},\mathbf{R},\mathbf{Q})]$ instead.

[1] Laplace transforms are defined by $f^*(\gamma) = \int_0^{\infty}\mathrm{e}^{-\gamma t}f(t)\mathrm{d}t$ and there is $0 \leqslant f^*(\gamma) \leqslant 1$ if $f(t), t \geqslant 0$, is a probability density for which $f^*(0) = 1$ and $f^*(\infty) = 0$. In the transformed space there is $h^*(\gamma) = f(\gamma)^* g^*(\gamma)$ for $h(t) = \int_0^t f(t-\tau)g(\tau)\mathrm{d}\tau$, an operation necessary to determine $f_n(t) = \int_0^t f_{n-1}(t-\tau)f(\tau)\mathrm{d}\tau$.

$$Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{E_{\mathbf{R},\mathbf{Q}}[\nu^+(\mathbf{p},\mathbf{R},\mathbf{Q})]}{\gamma}. \quad (11)$$

It is seen that continuous discounting and continuous failure models lead to relatively simple, analytical results. Completely parallel results can be obtained for discrete failure models and discrete discounting [63].

If, further, at an extreme Poissonian loading event (e.g. flood, wind storm, earthquake, explosion) failure occurs with probability $P_f(\mathbf{p})$ one obtains for independent failure events and systematic reconstruction [21,51]:

$$r^*(\gamma,\mathbf{p}) = \sum_{n=1}^{\infty} f^*(\gamma)P_f(\mathbf{p})[f^*(\gamma)R_f(\mathbf{p})]^{n-1} = \frac{P_f(\mathbf{p})f^*(\gamma)}{1-f^*(\gamma)} = \frac{\lambda P_f(\mathbf{p})}{\gamma} \quad (12)$$

in Eq. (8) with $R_f(\mathbf{p}) = 1 - P_f(\mathbf{p})$, λ the occurrence rate of the disturbing event and $g(t,\mathbf{p}) = \sum_{n=1}^{\infty} P_f(\mathbf{p})(1-P_f(\mathbf{p}))^{n-1} f_n(t,\mathbf{p})$ the failure time density. Analogously, one derives for the one-mission case an objective function as in Eq. (9):

$$Z(\mathbf{p}) = \frac{b}{\gamma}\left(1 - \frac{P_f(\mathbf{p})f^*(\gamma)}{1-R_f(\mathbf{p})f^*(\gamma)}\right) - C(\mathbf{p}) - H\frac{P_f(\mathbf{p})f^*(\gamma)}{1-R_f(\mathbf{p})f^*(\gamma)} = \frac{b - HP_f(\mathbf{p})\lambda}{\gamma + P_f(\mathbf{p})\lambda} - C(\mathbf{p}). \quad (13)$$

An important asymptotic result for arbitrary failure models is [11]

$$\lim_{t\to\infty} r(t,\mathbf{p}) = \lim_{\gamma\to 0} \gamma r^*(\gamma,\mathbf{p}) = \frac{1}{E[T(\mathbf{p})]}, \quad (14)$$

where $E[T(\mathbf{p})]$ is the mean time between renewals.

The precise details of this and more general renewal models can be found in [43]. Many other objective functions can be formulated. For example, serviceability failure, obsolescence, aging, deterioration and inspection and maintenance, finite renewal times, repeated renewal during construction and finite service times can be dealt with. Benefit and damage terms can be functions of time [22,43,59,60]. Also, multiple failure modes can been considered [60]. Some more important cases especially relevant for life cycle costing and sustainability are summarized in the following.

3.4. Non-constant discounting

For non-constant discount rates Eq. (4) is still valid but the elegant methodology with the Laplace transforms no more applies because the convolution theorem no more holds. As will be shown consideration of time-variant discount rates is necessary in the context of sustainable life cycle cost-benefit analysis. Therefore, we study first the behavior of the term $\sum_{n=1}^{\infty}\int_0^{\infty} e^{-\int_0^t \gamma(\tau)d\tau} f_n(t,\mathbf{p})dt$ in Eq. (4). A general analysis appears not feasible but we can study it at an important example. Assume that the times between renewals are exponentially distributed with parameter λ. Then, the density function to the nth renewal is the density of the Γ-distribution, i.e. $f_n(t) = [\lambda(\lambda t)^{n-1}/\Gamma(n)]\exp[-\lambda t]$. It is, therefore, easy to determine the sum-terms of $\sum_{n=1}^{\infty}\int_0^{\infty} e^{-\int_0^t \gamma(\tau)d\tau} f_n(t,\mathbf{p})dt$. Clearly, we have to require that all integrals in Eq. (4) converge. This

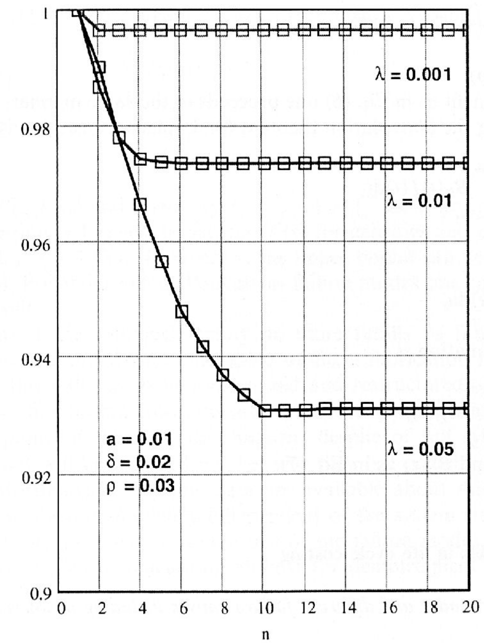

Fig. 1. Ratio of exact terms in Eq. (3) and by Laplace convolution theorem using Eq. (12) for $\gamma(t) = \rho\exp[-at] + \delta$ and exponential times between failures with rate λ.

is the case if the integral $\int_0^t \gamma(\tau)d\tau$ asymptotically grows linearly or $\gamma(t) \underset{t\to\infty}{\longrightarrow} \gamma_0 > 0$. Alternatively, one could apply the convolution theorem for Laplace transforms for the new "transformation" $f_n^{\#}(\gamma) = \int_0^{\infty} e^{-\int_0^t \gamma(\tau)d\tau} f_n(t,\mathbf{p})dt$, i.e. $f^{\#}(\gamma)^n$. Fig. 1 shows the ratio of the exact result $\sum_{j=1}^{n} f_j^{\#}(\gamma)$ and the application of the convolution theorem $\sum_{j=1}^{n} f^{\#}(\gamma)^j$ for increasing n up to $n = 20$ for the discount rate $\gamma(\tau) = \rho\exp[-a\tau] + \delta$. It is seen that (i) only a few terms in the sum need to be considered in Eq. (4) and (ii) the application of the convolution theorem for Laplace transforms to the modified transformation yields an excellent approximation, even for mean failure times and decay times for the interest rate in the same order of magnitude ($1/a \approx 1/\lambda$) although slightly on the unconservative side. Parameter studies show that this still is valid for relatively large variations in the parameters. At most an error of a few percent has been found under realistic conditions. Only if the decay time, i.e. $1/a$, of the discount rate is much smaller than the mean time between renewals one must expect larger errors and must include a larger number of sum-terms in Eq. (4). This suggests to take all results of the last section and the additional models studied in the next section as very good approximations provided that the new transformation

$$f^{\#}(\gamma, \mathbf{p}) = \int_0^{\infty} e^{-\int_0^t \gamma(\tau)d\tau} f(t, \mathbf{p})dt \tag{15}$$

is used instead of $f^*(\gamma, \mathbf{p})$.

For non-constant benefit as in Eq. (6) one proceeds in the same manner. The solution for constant interest rate using the convolution theorem for Laplace transforms is

$$B^* = \frac{1}{1 - f^*(\gamma)} \int_0^{\infty} B_D(t) f(t)dt \tag{16}$$

with

$$B_D(t) = \int_0^t e^{-\gamma u} b(u)du, \tag{17}$$

so that a good approximation is

$$B^{\#} \approx \frac{1}{1 - f^{\#}(\gamma)} \int_0^{\infty} B_D^{\#}(t) f(t)dt \tag{18}$$

with $f^{\#}(\gamma)$ in Eq. (15) and $B_D^{\#}(t)$ as in Eq. (7).

4. Further renewal models in life cycle costing

4.1. Independent failure modes and different failure causes including obsolescence in series systems

Assume for the moment two independent failure modes, denoted by "V_1" and "V_2", respectively, each requiring renewal of the whole system after failure. The times between renewals then are distributed as $F(t) = 1 - (1 - F_{V_1}(t))(1 - F_{V_2}(t)) = 1 - \bar{F}_{V_1}(t)\bar{F}_{V_2}(t)$. The corresponding density is $f(t) = f_{V_1}(t)\bar{F}_{V_2}(t) + f_{V_2}(t)\bar{F}_{V}(t)$ and its Laplace transform is $f^{**}(\gamma, \mathbf{p}) = f^{**}_{V_1|\bar{V}_2}(\gamma) + f^{**}_{V_2|\bar{V}_1}(\gamma)$. It follows that

$$D(\mathbf{p}) = \frac{(C_1(\mathbf{p}) + H_1) f^{**}_{V_1|\bar{V}_2}(\gamma) + (C_2(\mathbf{p}) + H_2) f^{**}_{V_2|\bar{V}_1}(\gamma)}{1 - (f^{**}_{V_1|\bar{V}_2}(\gamma) + f^{**}_{V_2|\bar{V}_1}(\gamma))}, \tag{19}$$

where $f^{**}_{V_i|\bar{V}_j}(\gamma) = \int_0^{\infty} \exp[-\gamma t] f_i(t)\bar{F}_j(t)dt$. This equation is derived as follows: let $\theta_i = t_i - t_{i-1}$ be the times between renewals with density $f_{V_1,V_2}(t)$ and, for example, C_{V_1} and C_{V_2} the cost associated with the two types of renewals. Then, the expected cost is

$$\begin{aligned} D &= E\left[\sum_{n=1}^{\infty} (C_{V_1} + C_{V_2}) \exp\left[-\gamma \sum_{k=1}^{n} \theta_k\right]\right] = \sum_{n=1}^{\infty} E[\exp(-\gamma\theta)]^{n-1} E[(C_{V_1} + C_{V_2}) \exp[-\gamma\theta]] \\ &= \frac{E[(C_{V_1} + C_{V_2}) \exp[-\gamma\theta]]}{1 - E[\exp(-\gamma\theta)]} = \frac{C_{V_1} f^{**}_{V_1|\bar{V}_2} + C_{V_2} f^{**}_{V_2|\bar{V}_1}(\gamma)}{1 - (f^{**}_{V_1|\bar{V}_2}(\gamma) + f^{**}_{V_2|\bar{V}_1}(\gamma))}. \end{aligned} \tag{20}$$

One can generalize to more (independently caused) renewals:

$$D(\mathbf{p}) = \frac{\sum_{i=1}^{s} C_i(\mathbf{p}) f^{**}_{V_i|\cap_{j\neq i}\bar{V}_j}(\gamma)}{1 - \sum_{i=1}^{s} f^{**}_{V_i|\cap_{j\neq i}\bar{V}_j}(\gamma)} \leqslant \frac{\sum_{i=1}^{s} C_i(\mathbf{p}) f^{*}_{V_i}(\gamma)}{1 - \sum_{i=1}^{s} f^{*}_{V_i}(\gamma)} \tag{21}$$

with $f(t) = \sum_{i=1}^{s} f_i(t) \prod_{j\neq i} \bar{F}_j(t)$ and, therefore $f^{**}_{V_i|\cap_{j\neq i}\bar{V}_j}(\gamma) = \int_0^{\infty} \exp[-\gamma t] f_i(t) \prod_{j\neq i} \bar{F}_j(t)dt$. Here, we distinguish between ordinary Laplace transforms $f^*(\gamma)$ for densities and modified Laplace transforms $f^{**}(\gamma)$ for which $f^{**}(\gamma) \leqslant f^*(\gamma)$. Frequently, the upper bound can be used and this has also been proposed in [28]. For independent Poissonian failure modes one can show that the upper bound is the exact result.

Obsolescence occurs if the technical facility no more fulfills its function. For example, a bridge may become too narrow for the increasing traffic, a fabrication hall is replaced because the machinery inside this hall has to be modernized and restructured, certain vehicles are put out of service because they become too uncomfortable, too uneconomical or unserviceable because of outdated equipment. Usually, this happens despite of full system integrity. In fact, most structural facilities will be replaced not because they fail or deteriorate but because they become obsolete. Unfortunately, very few data are available about this well-known fact [24]. Obsolescence is almost always completely independent of the system state. But this is just the case dealt with in the foregoing section where one of the failure modes, i.e. cause for renewal, is treated as obsolescence. With A denoting all cost for demolishment and removal of debris it is:

$$D(\mathbf{p}) = \frac{(C(\mathbf{p}) + H) f^{**}_{V_1|\bar{A}_2}(\gamma) + (C(\mathbf{p}) + A) f^{**}_{A_2|\bar{V}_1}(\gamma)}{1 - (f^{**}_{V_1|\bar{A}_2}(\gamma) + f^{**}_{A_2|\bar{V}_1}(\gamma))}. \tag{22}$$

4.2. Dependent failure modes

Multiple mode failures (series systems) with stationary failure models or even non-stationary failure models with dependent modes can also be considered [59,60]. Here, only the case of deteriorating components is presented. Assume that there are s time-dependent failure modes whose state functions are given by $g_k(\mathbf{u}, t) \approx \boldsymbol{\alpha}_k^T(t)\mathbf{u} + \beta_k(t)$ so that $V_k(t) = P(T_k \leqslant t) = P(g_k(\mathbf{U}, t) \leqslant 0) = P(Z_k \leqslant -\beta_k(t))$. Here, $\mathbf{U}$ is a vector of mutually independent standard normal variables, $\boldsymbol{\alpha}_k$ the (normalized) gradient of the (linearized) failure surface and $\beta_k(t)$ its distance from the origin. Hence, Z_k is a standard normal variable. The failure probability at time t_j then is $F(t) = P(\bigcup_{k=1}^{s}\{Z_k \leqslant -\beta_k(t)\}) = 1 - P(\bigcap_{k=1}^{s}\{Z_k \leqslant \beta_k(t)\}) \approx 1 - \Phi_s(\boldsymbol{\beta}(t); \mathbf{R})$ where $\boldsymbol{\beta}(t) = \{\boldsymbol{\alpha}_k^T \mathbf{u}_k^*(t); k = 1, 2, \ldots, s\}$, $\|\boldsymbol{\alpha}_k\| = 1, k = 1, 2, \ldots, s$; $\mathbf{u}_k^*(t) = \min \|\mathbf{u}\|\}$ for $\mathbf{u} : g_k(\mathbf{u}, t) \leqslant 0\}$ and $0.35em\mathbf{R} = E[\mathbf{Z}\mathbf{Z}^T] = \{\rho_{ij}\} = \{ ha_i^T \boldsymbol{\alpha}_j; i, j = 1, 2, \ldots, s\}$. In good approximation it is assumed that the matrix of correlation coefficients $\mathbf{R}$ varies little with time so that $(\partial/dt)\boldsymbol{\alpha}_k(t) \approx 0$ and, hence, $(\partial/dt)\rho_{ij}(t) \approx 0$ and there is $g_k(\mathbf{0}, t) > 0$ for all k. The failure density is

$$
\begin{aligned}
f_s(t) &= \frac{d}{dt}(1-\Phi_s(\beta(t);\mathbf{R})) = -\sum_{k=1}^{s}\frac{\partial}{\partial\beta_k(t)}\Phi_s(\beta(t);\mathbf{R})\frac{\partial\beta_k(t)}{\partial t}\\
&= -\sum_{k=1}^{s}\frac{\partial}{\partial\beta_k(t)}\int_{-\infty}^{\beta_k(t)}\Phi_{s-1}(\beta(t);\mathbf{R}\mid Z_k=\beta_k(t))\varphi_1(z_k)\mathrm{d}z_k\frac{\partial\beta_k(t)}{\partial t}\\
&= -\sum_{k=1}^{s}\varphi_1(\beta_k(t))\Phi_{s-1}(\hat{\mathbf{c}}_k;\hat{\mathbf{R}}_k)\frac{\partial\beta_k(t)}{\partial t} = \sum_{k=1}^{s}\varphi_1(\beta_k(t))\Phi_{s-1}(\hat{\mathbf{c}}_k;\hat{\mathbf{R}}_k)\left(\frac{-\frac{\partial}{\partial t}g_k(\mathbf{u}^*,t)}{\|\nabla_{\mathbf{u}}g_k(\mathbf{u}^*,t)\|}\right)\\
&\leqslant \sum_{k=1}^{s}\varphi_1(\beta_k(t))\left(\frac{-\frac{\partial}{\partial t}g_k(\mathbf{u}^*,t)}{\|\nabla_{\mathbf{u}}g_k(\mathbf{u}^*,t)\|}\right)
\end{aligned}
\tag{23}
$$

with $\hat{\mathbf{c}}_k = \beta^k(t) - \beta_k(t)\rho_k^k$; and $\hat{\mathbf{R}}_k = \mathbf{R} - \rho_k^k(\rho_k^k)^T$, where ρ_k is the kth column vector of $\mathbf{R}$ and the superscript means that the k-th row and column, respectively, are deleted from the original vector and matrix, respectively. This result is obtained from regression analysis. Note that $\hat{\mathbf{R}}_k$ needs to be re-normalized and therefore also $\hat{\mathbf{c}}_k$. The result $[\partial/\partial\beta_k(t)\Phi_s(\beta(t);R)] = \varphi_1(\beta_k(t))\Phi_{s-1}(\hat{\mathbf{c}}_k;\hat{\mathbf{R}}_k)$ is due to [52]. Here, $s-1$-dimensional normal integrals have to be evaluated for each t. Suitable computation schemes for $\Phi_r(\mathbf{b};\mathbf{B})$ have been given in [18] and elsewhere. Due to the substantial numerical effort when computing multi-normal probabilities this scheme can only be applied to smaller systems. Dropping the terms $\Phi_{s-1}(\hat{\mathbf{c}}_k;\hat{\mathbf{R}}_k)$, i.e. the survival probabilities in the other failure modes, corresponds to the upper bound solution:

$$
D = \sum_{i=1}^{s}(C_i+H_i)\frac{f_{1,i}^{**}(\gamma)}{1-\sum_{i=1}^{s}f_{1,i}^{**}(\gamma)} \leqslant \sum_{i=1}^{s}(C_i+H_i)\frac{f_{1,i}^{*}(\gamma)}{1-\sum_{i=1}^{s}f_{1,i}^{*}(\gamma)}, \tag{24}
$$

where

$$
\begin{aligned}
f_{1,i}^{**}(\gamma) &= \int_0^{\infty}\exp[-\gamma t]\varphi_1(\beta_i(t))\Phi_{s-1}(\hat{\mathbf{c}}_i;\hat{\mathbf{R}}_i)\left(\frac{-\frac{\partial}{\partial t}g_i(\mathbf{u}^*,t)}{\|\nabla_{\mathbf{u}}g_i(\mathbf{u}^*,t)\|}\right)\mathrm{d}t \leqslant f_{1,i}^{*}(\gamma)\\
&= \int_0^{\infty}\exp[-\gamma t]\varphi_1(\beta_i(t))\mathrm{d}t.
\end{aligned}
$$

The trivial upper bound may be useful but its application is limited to smaller systems because Laplace transforms of densities must remain smaller than unity. Clearly, a trivial lower bound is formed by the largest member in the sum. A better lower bound is found by replacing $\Phi_{s-1}(\hat{\mathbf{c}}_k;\hat{\mathbf{R}}_k)$ by $\prod_{k=1}^{s}\Phi(\hat{c}_k)$ because $\Phi_m(\mathbf{x};\mathbf{R}) \leqslant \Phi_m(\mathbf{x};\mathbf{K})$ if for some ij there is $\{\rho_{ij}\} \leqslant \{\kappa_{ij}\}$ but $\mathbf{R} \geqslant \mathbf{0}$ and $\mathbf{K} \geqslant \mathbf{0}$ [55].

4.3. Deteriorating structures and numerical laplace transforms

Deteriorating structures are characterized by an increasing hazard function $\rho(t) = f(t)/[1-F(t)]$. Let the failure probability at a given time t be computed by FORM or SORM [23] so that

$$
P_f(t) = P(T \leqslant t) = F_T(\mathbf{p},t) = \Phi(-\beta(\mathbf{p},t))C_{\mathrm{SORM}},
$$

where

$$
\beta(\mathbf{p},t) = \|\mathbf{u}^*\| = \min\{\|\mathbf{u}\|\} \quad \text{for } g(\mathbf{u},\mathbf{p},t) \leqslant 0,
$$

with $g(\mathbf{u},t)$ a monotonically decreasing state function, $\mathbf{u} = \mathbf{T}(\mathbf{x})$ a vector of independent standard normal variables and $\mathbf{x}$ the vector of uncertain variable in the original space. $\mathbf{u} = \mathbf{T}(\mathbf{x})$ is a unique probability distribution transformation. C_{SORM} is the second-order correction which usually is neglected. The mean time to failure is

$$
E[T(\mathbf{p})] = \int_0^{\infty}(1-F_T(\mathbf{p},t))\mathrm{d}t = \int_0^{\infty}\Phi(\beta(\mathbf{p},t))\mathrm{d}t. \tag{25}
$$

It can be shown that the density of the time to failure is to first order

$$
f_T(\mathbf{p},t) = -\varphi(\beta(\mathbf{p},t))\frac{\mathrm{d}\beta(\mathbf{p},t)}{\mathrm{d}t} = -\varphi(\beta(\mathbf{p},t))\frac{\frac{\partial}{\partial t}g(\mathbf{u}^*,\mathbf{p},t)}{\|\nabla_{\mathbf{u}}g(\mathbf{u}^*,\mathbf{p},t)\|} \tag{26}
$$

so that the Laplace transform can be determined numerically from

$$
f^*(\gamma,\mathbf{p}) \approx \Delta\sum_{j=0}^{m}w_j\exp[-\gamma t_j]f_T(\mathbf{p},t_j), \tag{27}
$$

with w_j are the weights of a suitable integration formula (trapezoid, Simpson, Newton, etc.) and Δ is an appropriate spacing in time. For the variant with time-dependent interest rate one has only slightly more complicated:

$$
f^{\#}(\gamma,\mathbf{p}) \approx \Delta\sum_{j=0}^{m}w_j\exp\left[-\sum_{i=0}^{j}\gamma(t_i)\Delta\right]f_T(\mathbf{p},t_j). \tag{28}
$$

Of course, more refined numerical integration formulae can be used.

4.4. Inspection and repair of aging components

In the literature maintenance cost frequently have been assumed to increase continuously with time. More realistic in the structures area is the case where maintenance cost are the sum of inspection and possible repair cost. Assume inspections at regular intervals $a, 2a, 3a, \ldots$ Repairs occur only at these points in time (or with some delay, say at $a+\Delta, 2a+\Delta, 3a+\Delta, \ldots$) Inspections and repairs occur only if renewals have not occurred before due to obsolescence or failure. Assume further that repairs, if undertaken, restore the properties of a component to its original (stochastic) state, i.e. repairs are equivalent to renewals. Inspection and repair times are assumed negligibly short. Of course, it makes only sense to consider aging components with increasing risk function $\rho(t)$.

A renewal (repair) occurs either after failure or at times $a, 2a, 3a, \ldots$ Renewal (repair) times are assumed negligibly short and, therefore, $\Delta = 0$. In [3] this is denoted by age replacement. Then, one can apply Eq. (20) and one obtains

$$
Z(\mathbf{p},a) = B - C(\mathbf{p}) - \frac{(C(\mathbf{p})+H)f_V^{***}(\gamma,\mathbf{p},a) + I_1(\mathbf{p})\exp[-\gamma a]\bar{F}_V(\mathbf{p},a)}{1-(f_V^{***}(\gamma,\mathbf{p},a)+\exp[-\gamma a]\bar{F}_V(\mathbf{p},a))} \tag{29}
$$

with $I_1(\mathbf{p}) < (C(\mathbf{p}) + H)$ the cost of repair, $\bar{F}_V(\mathbf{p}, a)$ the probability of survival up to a and $f_X^{***}(\gamma, \mathbf{p}, a) = \int_0^a \exp[-\gamma t] f_X(t)\mathrm{d}t$ the incomplete Laplace transform of $f_X(t)$. Note that $f^*(\gamma) = \exp[-\gamma a]/(1 - \exp[-\gamma a])$ is the Laplace transform of a deterministic density function $f(t) = \delta(a)$. It is seen that repair is treated as a second failure (and renewal) mode. This important result was already obtained by Fox [12]. Under suitable conditions the quantity $f_X^{***}(\gamma, \mathbf{p}, a)$ can be replaced by $f_X^{\#\#\#}(\gamma, \mathbf{p}, a)$.

If there are regular inspections there is not necessarily a repair because inspections are uncertain (or the signs of deterioration are vague). Denote the failure model for the aging component by "V" whereas "A" stands for any other (independent) failure mode (or obsolescence as another cause for renewal). Then, inspection and repair cost must also be included in the damage term

$$Z(\mathbf{p}, a) = B(\mathbf{p}, a) - C(\mathbf{p}) - D(\mathbf{p}, a), \tag{30}$$

where

$$D(\mathbf{p}, a) = \frac{ND}{D}, \tag{31}$$

$$ND = (C(\mathbf{p}) + A)(f_{A|\bar{V}}^{***}(\gamma, a) + A11) + (C(\mathbf{p}) + H)(f_{V|\bar{A}}^{***}(\gamma, \mathbf{p}, a) + A12) + I_0((1 - P_R(a)) \times \exp[-\gamma a]\bar{F}_A(a)\bar{F}_V(\mathbf{p}, a) + A21) + (I_0 + I_1(\mathbf{p}))(P_R(a)\exp[-\gamma a]\bar{F}_A(a)\bar{F}_V(\mathbf{p}, a) + A22),$$

$$D = 1 - (f_{A|\bar{V}}^{***}(\gamma, a) + A11 + f_{V|\bar{A}}^{***}(\gamma, \mathbf{p}, a) + A12 + P_R(a)\exp[-\gamma a]\bar{F}_A(a)\bar{F}_V(\mathbf{p}, a) + A22),$$

$$A11 = \sum_{n=2}^{\infty}\prod_{j=1}^{n-1}(1 - P_R(ja))f_{A|\bar{V}}^{****}(\gamma, \mathbf{p}, (n-1)a \leqslant t \leqslant na),$$

$$A12 = \sum_{n=2}^{\infty}\prod_{j=1}^{n-1}(1 - P_R(ja))f_{V|\bar{A}}^{****}(\gamma, \mathbf{p}, (n-1)a \leqslant t \leqslant na),$$

$$A21 = \sum_{n=2}^{\infty}(1 - P_R(na))\prod_{j=1}^{n-1}(1 - P_R(ja))\exp[-\gamma(na)]\bar{F}_A(na)\bar{F}_V(\mathbf{p}, na),$$

$$A22 = \sum_{n=2}^{\infty}P_R(na)\prod_{j=1}^{n-1}(1 - P_R(ja))\exp[-\gamma na]\bar{F}_A(na)\bar{F}_V(\mathbf{p}, na),$$

and $P_R(a)$ is the probability of repair after inspection increasing in a, $\bar{P}_R(a) = 1 - P_R(a) =$ is the probability of no repair after inspection, a is deterministic inspection interval, I_0 is cost per inspection, $I_1(\mathbf{p})$ is repair cost including inspection cost, $f_{X|\bar{Y}}^{***}(\gamma, \mathbf{p}, a) = \int_0^a \exp[-\gamma t]f_X(t)\bar{F}_Y(t)\mathrm{d}t \leqslant \int_0^a \exp[-\gamma t]f_X(t)\mathrm{d}t$ is the incomplete, modified Laplace transform of $f_X(t)$ and $f_{X|\bar{Y}}^{****}(\gamma, \mathbf{p}, (n-1)a \leqslant t \leqslant na) = \int_{(n-1)a}^{na} \exp[-\gamma t]f_X(t)\bar{F}_Y(t)\mathrm{d}t$.

Here, one has to extend the renewal interval to $2a, 3a, \ldots$ if an inspection is not followed by repair. The terms $A11$, $A12$, $A21$ and $A22$ vanish for $P_R(a) \to 1$ and are significant only for relatively small a. Note that the renewal cost $C(\mathbf{p})$ can also be different in the two cases. If the benefit is constant in time we simply have $B(\mathbf{p}, a) = b/\gamma$. The case of non-constant benefit $b(t)$ is dealt with in [60].

The repair probability depends on the magnitude of a suitable damage indicator. For cumulative damage phenomena $P_R(a, \mathbf{p})$ increases with a. For example, $P_R(a, \mathbf{p}) = P(S(a, \mathbf{X}, \mathbf{p}) > s_c)$ with $S(a, \mathbf{X}, \mathbf{p})$ a monotonically increasing damage indicator, $\mathbf{X}$ a random vector taking into account of all uncertainties during inspection and s_c a given threshold level. Frequently, the length of inspection intervals is taken as an optimization parameter. The case without inspection and $P_R(a, \mathbf{p}) = 1$ is already dealt with in the literature [12,63]. Repair after inspection is interpreted as preventive renewal (replacement of an aging component after a finite time of use a). Renewal after failure is called corrective renewal. It must be mentioned that optimal inspection/repair intervals do not always exist. Preventive renewals must, in fact, be substantially cheaper than corrective renewals for optimal repair intervals to exist. Also, the repair probability must be sufficiently high at a. As can be seen from Eq. (31) failure and repair events are assumed independent. This is an approximation which, however, appears to be very difficult to remove.

4.5. Discussion

The renewal model applied herein turned out to be very powerful in deriving suitable objective functions for various applications. As mentioned many more extensions and refinements are possible. They are all based on the concept of systematic reconstruction (or renewal by repair) which clearly is a sustainable concept. The assessment of cost appears to be easy and straightforward for construction cost and direct physical damage cost. More difficult is the quantification of indirect failure cost such as loss of use by the public, loss of business, etc.. Even more difficult but related is the assessment of the benefit derived from a civil engineering infrastructure. But most difficult is the quantification of the losses in human life and limb in case of failure. Some results on this subject will be summarized in Section 6. Similarly difficult is the assessment of the appropriate interest rate. An attempt to do this is summarized in the following section.

5. Sustainable public interest rates

5.1. Two bounds on public interest rates

In accordance with economic theory benefits and (expected) cost should be discounted by the same rate as done above. While the owner or operator may take interest rates from the financial market the assessment of the interest rate for an optimization in the name of the public is difficult. The requirement that the objective function must be non-negative leads immediately to the conclusion that the (constant) interest rate must have an upper bound γ_{max} depending on the benefit rate $b = \beta C(\mathbf{p})$ (see [22]). For the model in Eq. (12) we have

$$\frac{\beta C(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{\lambda P_f(\mathbf{p})}{\gamma} \geqslant 0, \quad (32)$$

and, therefore, by solving the equality for γ and given (optimal) $\mathbf{p} = \mathbf{p}^*$

$$\gamma < \gamma_{max} < \beta - \lambda P_f(\mathbf{p})\left(1 + \frac{H}{C(\mathbf{p})}\right), \quad (33)$$

implying $\gamma < \beta$ for $\lambda P_f(\mathbf{p}) \ll \beta$. It follows that the benefit rate β must be slightly larger than γ_{max}. From Eq. (32) one also concludes that there must be $\gamma > 0$ because the left-hand side tends to $\pm\infty$ in the limit $\gamma \to 0^+$ or at least is undefined.

5.2. *Public interest rates based on economic growth theory*

The cost for saving life years in Eq. (58) enters into the objective function (8) and with it the question of discounting those cost arises. Later, we will see that the derivation of a suitable acceptance criterion for risks for human life and limb also involves some discounting. At first sight discounting of human lives is not in agreement with our moral value system. However, a number of studies summarized in [39,31] express a rather clear opinion based on ethical and economical arguments and on public opinion. The cost for saving life years must be discounted at the same rate as other investments. Otherwise serious inconsistencies cannot be avoided.

In view of the time horizon of some 20 to more than 100 years (i.e. several generations) for civil engineering infrastructures the interst rate should be a long-term average. It should be net of inflation and taxes. In the private sector a long term real interest rate is roughly identical to the (maximum) return rate one could get from an investment. But can the public also adopt such a strategy? The public does not make financial profit except by its economical growth. And what is financial profit of public life saving interventions? Public interest rates should be close to the long-term economic growth rate (per capita) as this is the rate with which a member of the public becomes more wealthy. In the economics literature this is sometimes called the "natural interest rate". Traditionally, it has been argued that public investments should be financed within some mean residual life expectancy of the population, i.e. within 40–50 years. For larger financing horizons the burden of financing would be left in part to the next generation. If this time is viewed as the time of amortization of a public investment, rates of 2–2.5% are implied.

Recently, further discussions took place in the context of sustainable development, long term public investments in general and intergenerational justice – aspects which appear very relevant in our context. Discounting for sustainability should at least be consistent with discounting for risk reduction investments. Weinstein/Stason [66] and others require that interest rates for life saving investments should be the same as for other cost and thus equal to the real market interest rate, simply for consistency reasons. This appears to be an extreme point of view. The other extreme of not discounting intergenerationally at all is expressed, for example, in [8,53], based primarily on ethical grounds in the context of CO_2 – induced global warming, nuclear waste disposals, depletion of natural resources, etc.. In this case the rationale of our basic optimization model Eq. (8) and its generalizations break down which must be considered as highly unrealistic.

Due to the requirement $\beta > \gamma_{max}$ stated just below Eq. (33), the interest rate is strongly related to the benefit a society earns from the various activities of its members, i.e. its real economic growth per capita (see also [44] where the public benefit and interest rate has been set equal to the growth rate). The United Nations Human Development Report 2001 [62] gives values between 1.2% and 1.9% for industrialized countries during 1975–1998. If one considers the last 100 years and the data in [35] for some selected countries one determines a growth rate $\delta = \ln(g_{1992}/g_{1870})/1992\text{–}1870$ of about 1.8% for Western Europe, the so-called Western Offshoots, USA, Canada and Australia, and Japan. For Southern Europe, Latin America and Asia one finds from the same data similar growth rates $\delta \approx 1.7\%$, for Eastern Europe still $\delta \approx 1.4\%$ but for Africa only $\delta \approx 0.9\%$. The growth data in [35] are per capita. Therefore, the growth rate of the respective economy is by the population growth rate larger.

Modern economic growth theory widely applied to sustainability financing can provide more insight. Nordhaus [34] and others (see [61] for an overview but also the other papers in Energy Policy, 23, 3/4, 1995) follow the classical Ramseyan approach (see [49,54,4]) for optimal stable economic growth in perfect markets

$$\gamma = \rho + \epsilon\delta > 0, \quad (34)$$

where γ is the real market interest rate, ρ the rate of pure time preference of consumption, $\epsilon > 0$ the elasticity of marginal consumption (income) and δ the consumption (income) growth rate per capita. Clearly, the subjective element is the quantity ρ. The rate ρ has been interpreted as the effect of human impatience, myopia, egoism, lack of telescopic faculty, etc. Its existence in human behavior has been widely demonstrated in human ethology and economics [13,40]. It is partially justified because there is uncertainty about one's future. On the basis of Eq. (34) Nordhaus [34] obtains $\gamma \approx 0.05$, Arrow [1] estimates $\gamma \approx 0.03$. In many other studies for sustainable development discount rates γ cluster around 5%. All those values are close to the real market rates or a little smaller. However, there are many authors in economics as well as philosophical and political sciences including Ramsey who refuse convincingly to accept a rate $\rho > 0$ in intergenerational contexts on ethical grounds (see, for example, [49,53,8,41]) while it is considered fully acceptable for intragenerational discounting. On the other hand, intergenerational equity arguments in Arrow [1] indicate that there should be $\rho > 0$ in order to remove an "*...incredible and unacceptable strain on the present generation*". Rabl [41], too, who sets $\rho = 0$, argues that there must be $0 < \gamma < \epsilon\,\delta$ in the framework of long-term public investments. In [45] the following bounds have been proposed:

$$n + \delta(1 - \epsilon) < \rho < \gamma \leqslant \gamma_{max} < \beta = n + \epsilon\delta, \quad (35)$$

n is the population growth rate. Values for ρ and β are presented in Table 1. It is then possible to compute $\gamma_{max} < \beta$ from Eq. (33). γ_{max} usually is only insignificantly (1–20%) smaller than β depending on the specific case at hand, i.e. the particular sensitivities of $C(\mathbf{p})$ and $r(\mathbf{p}, t)$ with respect to $\mathbf{p}$. The interest rates γ_{max} implied by the value of β (see Eq. (33)) are considerably lower than the usual real market interest rates in developed countries. More discussion on discounting is provided in [47].

5.3. Generation adjusted discounting

In the economical and ecological literature the adequacy of the Ramseyan model has been seriously questioned. So-called overlapping generation models or generation adjusted discounting models are advocated instead [4]. The main idea is to discount for living generations at the rate in Eq. (35) with $\rho > 0$, i.e. with $\gamma = \rho + \varepsilon\delta$ but diminish the rate for all yet unborn generations down to $\varepsilon\delta$ or even lower, thus also facilitating the transition into a sustainable state of economy [41,6,5,17]. The model by Bayer/Cansier [6] is especially appealing because of its simplicity. It is pretended that it captures the main issues of long time horizons and intergenerational equity. It requires that each future generation should be treated as the present generation (see also [15]) and, thus, each generation discounts with $\gamma = \rho + \epsilon\delta$, $\rho > 0$, with ρ selected according to the preferences of that generation. At each point in time m far in the future there are $G = L$ overlapping generations where L is the mean renewal time in years for generations. L can be taken as a time somewhere between 25 and 40 years. It may be taken as the time when reproduction of a new generation is completed to a large extent. For $L = 40$ almost a complete children generation has been generated. Therefore, exactly G generations are affected by a consumption or loss effect c_i in the future. Intragenerational equity further requires that this effect is distributed equally among all generations concerned. It follows that discounting must be performed first with γ down to m in accordance with Arrow's argument [1] mentioned above, and if $m > L$ discounting down to the decision point $t = 0$ is only performed with $\epsilon\delta$ in accordance with Rabl's [41] and many others' intergenerational equity requirement. The discounting idea in [6] is illustrated at an example. Assume that four generations are living at each point in time. If a generation dies a new one is born. The generations which are affected by the consumption or loss effect are concerned with it depending on their residual life time (see Fig. 2). Starting from generation D this and all subsequent generations will be affected over their full life time. The consumption or loss effects c_0, c_1, c_2 and c_3 are discounted down to time $t = t_0 = 0$ with rate $\gamma = \rho + \epsilon\delta$. For the generations A, B, C the same discounting procedure applies but they die earlier and, therefore, have only a shorter time for discounting. Things change when generation A has died and generation E is born. Generation E now discounts down to t_1 with rate $\gamma = \rho + \varepsilon\ \delta$. Discounting down to time t_0 is done only with the intergenerationally acceptable rate, i.e. with the effective growth rate $\gamma = \epsilon\delta$. For simplicity, all generations are assumed to have the same size.

Generation	t_0	t_1	t_2	t_3	t_4	t_5	t_m	Sum
A	c_0							c_0
B	c_0	$c_1\theta^{-1}$						$c_0 + c_1\theta^{-1}$
C	c_0	$c_1\theta^{-1}$	$c_2\theta^{-2}$					$c_0 + c_1\theta^{-1} + c_2\theta^{-2}$
D	c_0	$c_1\theta^{-1}$	$c_2\theta^{-2}$	$c_3\theta^{-3}$				$c_0 + c_1\theta^{-1} + c_2\theta^{-2} + c_3\theta^{-3}$
E		c_1	$c_2\theta^{-1}$	$c_3\theta^{-2}$	$c_4\theta^{-3}$			$c_1 + c_2\theta^{-1} + c_3\theta^{-2} + c_4\theta^{-3}$
F			c_2	$c_3\theta^{-1}$	$c_4\theta^{-2}$	$c_5\theta^{-3}$	...	$c_2 + c_3\theta^{-1} + c_4\theta^{-2} + c_5\theta^{-3}$
G				c_3	$c_4\theta^{-1}$	$c_5\theta^{-2}$	...	$c_3 + c_4\theta^{-1} + c_5\theta^{-2} + c_6\theta^{-3}$
H					c_4	$c_5\theta^{-1}$	...	$c_4 + c_5\theta^{-1} + c_6\theta^{-2} + c_7\theta^{-3}$
Sum	$4c_0$	$c_1(1 + 3\theta^{-1})$	$c_2(1 + \theta^{-1} + 2\theta^{-2})$	$c_3(1 + \theta^{-1} + \theta^{-2} + \theta^{-3})$	$c_4(1 + \theta^{-1} + \theta^{-2} + \theta^{-3})$	$c_5(1 + \theta^{-1} + \theta^{-2} + \theta^{-3})$		

Fig. 2. Intra- and intergenerational consumption effects, $\theta = (1 + \gamma)$ (after [6]).

Assuming further that all generations have the same preferences with respect to ρ (i.e. ρ = constant) then leads to the present value for discrete discounting:

$$PV_{\mathrm{OLG}} = \sum_{j=0}^{L-1} \frac{\frac{c_j}{G}(L-j)}{(1+\rho+\epsilon\delta)^j} + \sum_{k=1}^{PV} \frac{\sum_{i=k}^{k+(L-1)} \frac{\frac{c_i}{G}}{(1+\rho+\epsilon\delta)^{i-k}}}{(1+\epsilon\delta)^k}, \tag{36}$$

where PV is the planning horizon which can and should be set equal to infinity for all sustainability related investments. The first sum covers all generations living at $t = 0$. For $k = 1$ the generation born at time $t = 0$ has disappeared and the generation born at $j = 1$ should only discount with rate $\epsilon\delta$ in the interval [0, 1]. The factor $(L - j)$ in the first term of Eq. (36) reflects the transient phase of the model.

Assume now that a single consumption or loss effect c_m occurs at time m. Formula (36) then simplifies for the stationary state to

$$PV_{\mathrm{OLG}}(m) = \begin{cases} \sum_{j=0}^{m-1} \frac{c_m/G}{(1+\epsilon\delta)^{m-j}(1+\rho+\epsilon\delta)^j} + \sum_{j=m}^{L} \frac{c_m/G}{(1+\rho+\epsilon\delta)^m} & \text{for } m \leqslant L, \\ \sum_{j=0}^{L} \frac{c_m/G}{(1+\epsilon\delta)^{m-j}(1+\rho+\epsilon\delta)^j} & \text{for } m > L. \end{cases} \tag{37}$$

The stationary state is defined as a continuous stream of overlapping generations so that the decision point $t = 0$ can be chosen arbitrarily. If $m \leqslant L$ the first sum covers all generations born later than $t = 0$ whereas the second term deals with those alive at $t = 0$. For $m \geqslant L$ the first term in the expression for PV_{OLG} is zero, if $m < L$ the second term is zero. The present value by discounting with $\gamma' = \rho + \varepsilon\delta$ is

$$PV_R(m) = \frac{c_m}{(1+\gamma')^m}. \tag{38}$$

The discounting rule Eq. (37) is complicated and not readily applied to our context but an equivalent time-dependent discount rate $\gamma'_E(t)$ can be determined by equating the corresponding present values of Eq. (37) and (38) for given m, i.e. from

$$\frac{c_m}{(1+\gamma'_E(t))^m} - PV_{\mathrm{OLG}}(m) = 0. \tag{39}$$

Clearly, the present value according to Eq. (37) which is denoted as OLG-model is larger than Eq. (38).

Fig. 3 shows $\gamma'_E(m)$ for $\rho = 0.03$, $\delta = 0.02$, $\epsilon = 1$ and $L = 40$. The dotted line corresponds to the stationary state. The equivalent rate Eq. (39) for the stationary state has been truncated at $\rho + \epsilon\delta$

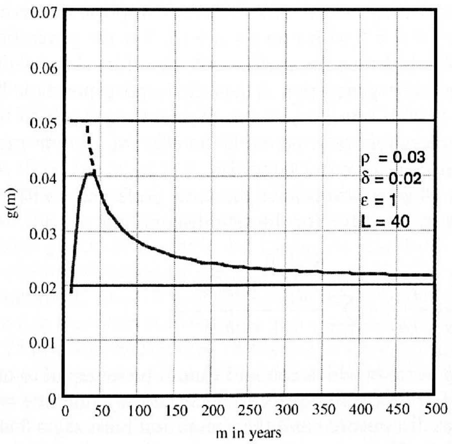

Fig. 3. Equivalent discount rate according to Eq. (36) with (solid line) and without transient phase (dashed line).

for $j < L$. $\gamma'(m)$ can be shown to converge to $\varepsilon\delta$ for $m \to \infty$. The rate can well be approximated by $\gamma'_E(m) \approx \varepsilon\delta + \rho\exp[-am]$ with a a suitably chosen constant. The discount rate for continuous discounting is $\gamma_E(t) = \ln(1 + \gamma'_E(t))$.

Alternatively, one can distribute the consumption or loss effect c_m among all age groups affected by it but weighted by the probability mass function of ages $h(a,n)$ obtainable from complete life tables at age a and where n is the population growth rate (see Eq. (50)). Each member of society then carries the same share of the consumption or loss effect c_m. Otherwise, the same concepts as before are applied. Then, the present value for overlapping age groups (OLAG) can be determined from

$$PV_{\mathrm{OLAG}}(m) = \begin{cases} \sum\limits_{a=0}^{m-1} \frac{c_m h(a,n)}{(1+\varepsilon\delta)^{m-a}(1+\rho+\varepsilon\delta)^a} + \sum\limits_{a=m}^{a_u} \frac{c_m h(a,n)}{(1+\rho+\varepsilon\delta)^m} & \text{for } m \leqslant a_u, \\ \sum\limits_{a=0}^{a_u} \frac{c_m h(a,n)}{(1+\varepsilon\delta)^{m-a}(1+\rho+\varepsilon\delta)^a} & \text{for } m > a_u, \end{cases} \tag{40}$$

where a_u is the largest age considered in the life table. Here again one can determine an equivalent time-dependent discount rate which is in analogy with Eq. (39):

$$\gamma'_E(m) = \left(\frac{c_m}{PV_{\mathrm{OLAG}}(m)}\right)^{1/m} - 1. \tag{41}$$

Some numerical results are shown in Fig. 4. The solid line corresponds to the model in Eq. (40). Fig. 4 also contains a curve showing the equivalent interest rate if all age groups receive the same consumption or loss effect (dashed line) and a line showing the equivalent discount rate for the original stationary model in Eq. (37) (dotted line). A recent Swiss life table has been used but it can be shown that other life tables produce almost the same results. Also, predictive mortalities produce the same results. Surprisingly, all strategies yield almost the same time-dependent equivalent discount rate except for $m \leqslant L$ according to Eq. (37) as shown in Fig. 4. From a practical point of view it is, therefore, irrelevant which model to use. The OLAG-model with age-weighting may, however, be preferred because no specification of a generation time and no truncation at L is necessary. Age-weighting also avoids strange effects for very small m (see dashed line in Fig. 4) and is intragenerationally acceptable. The most critical parameter still is the time preference rate ρ which may be determined from a long term market interest rate $\gamma - \delta$ or $\varepsilon\delta$.

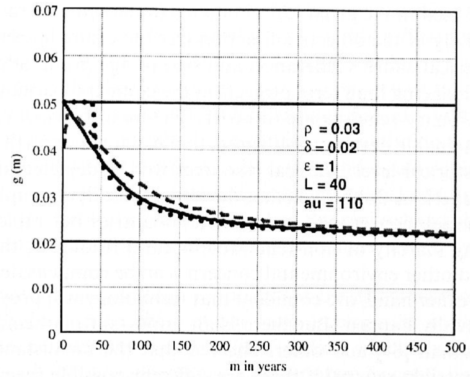

Fig. 4. Equivalent time-dependent discount rate according to OLAG-model (solid line), OLAG-model without age-weighting (dashed line) and original OLG-model (dotted line).

These findings need to be discussed and evaluated. The Ramsey model assumes a central "social planner" not only for a sustainable development but, as we will see, also for the assessment of a public risk acceptability criterion. It assumes optimal economical behavior of society and all its members in perfect markets. This implies a steady state of economy and, thus, constant discount rates. The generation-adjusted or age-group-adjusted models as discussed before assume that each generation acts on its own with all its preferences. However, whenever intergenerational aspects enter into the cost-benefit analysis in the name of the public the discounting rate is adjusted to the "natural discount rate", i.e. the effective, real economic growth rate. By doing so no extra burden is transferred from one generation to the next. This is certainly a much more realistic and ethically defensible setting than that by Ramsey and his followers. Recall that Ramsey himself found it ethically indefensible to discount future generations by the pure time preference rate. An equivalent discount rate decaying from something like the real market discount rate down to the real economic growth can be defined. This decay must exist under otherwise stable conditions if intergenerational equity is required. It must exist even for a time-dependent economic growth rate unless this increases strongly over a long period, a very unlikely scenario locally and worldwide. It complicates cost-benefit analyses but it has been shown in Section 3.4 that sustainable cost-benefit analyses using the renewal model are still possible with minor modifications of the mathematical tools. The requirement

Eq. (32) can no more be used in the given form but a maximum interest rate function still exists in order to maintain positivity of the objective function (see the example section). Its determination may involve some numerical effort. Generation-adjusted or age-group-adjusted models also help solving the problem of financing long term projects in the context of sustainability.

Some further precautionary remarks are in order. The main body of environmental and economics literature on sustainable development agrees that economic growth will not persist, at least not at the long-term historical level. Natural resources will be depleted and arable land will become scarce. Many raise serious doubts whether the foreseeable demographic changes (aging populations and negative population growth in industrial countries but rapid growth in developing countries), the increasing scarcity of non-renewable natural resources, the increase of pollution of water, soil and air and other environmental concerns can be compensated by technological progress. Optimists, on the other hand, are confident that technology will provide solutions. It is hard to predict what will actually happen. But there is an important mathematical result which may guide our choice. Weitzman [67] and others showed that the far-distant future should be discounted at the lowest possible rate >0 if there are different possible scenarios each with a given probability of being true. Interestingly, it is not even necessary to know those probabilities. Exactly this strategy has been pursued in the foregoing and it should be emphasized that lowest possible interest rates so far have been chosen only for the subjective part ρ of the real interest rate γ. Of course, Weitzman's [67] precautionary result also holds for generation adjusted discounting. Time-dependent (decaying) ρ's and δ's can be considered as long as there is still some positive limiting constant discount rate as discussed in Section 3.4.

Realistic discounting models for long term public investments into sustainability and/or risk reduction must take account of the economics of a changing world with limitations. The model in [6] captures those aspects with respect to intergenerational equity. Realistic economic and regulatory mechanisms should be added. An example of a more complicated but also conceptually more comprehensive OLG-model directly associated with the financial aspects of greenhouse gas abatement is proposed in [17]. It is beyond the scope of this paper to discuss this model in detail. It incorporates aging and growing populations, a decreasing economic growth rate and some standard mechanisms in a competitive economy with special regulatory policies for the distribution of property rights of environmental qualities over the generations. Important is that it also gives decaying interest rates – partially for additional reasons than in the model in [6] and with different decay rate. Besides Gerlagh/van der Zwaan [17] can demonstrate that a significant reduction of CO_2 is economically possible which has been denied by many other economists. Other similar and even more realistic models must be expected to emerge in the future. For civil engineering infrastructures these economic mechanisms certainly are different but some of the factors in [17] are likely to be also relevant for the building industry and infrastructure financing.

6. Additional constraints based on societal criteria for risk acceptance

6.1. Societal willingness-to-pay

At this point we may ask whether monetarily optimal facilities automatically lead to acceptable facilities with respect to risk. Clearly, the monetary value of a human life must be discussed in this context and whether the failure probability corresponding to an optimal solution is also acceptable from a societal point of view. Let us first define risk. Graham/Wiener [19] define: Risk = The chance of an adverse outcome to human health, the quality of life, or the quality of the environment. Other definitions have also been proposed. But it is important that risk now is understood as something far more general than just risk to human life and limb. Other aspects of life quality are also involved. Risk control is not only the scientific understanding and management of a risk but according to common understanding its cost efficiency and its affordability to the individual as well as to society. And it should be clear that only involuntary risks to human life and limb from technical installations or the natural environment by an anonymous member of society can be discussed. The cost of any risk reduction is carried by the public, i.e. by us all via taxes or public charges.

Recently, interesting concepts have been proposed for the assessment of public risk acceptance [31,33,44,32,37,45,46,38]. In essence, they set out from a composite social indicator, the *societal life quality index*, also to be interpreted as a utility function which encompasses three important indicators of life quality, that is life expectancy, consumption (income net of taxes) and the time necessary to raise the total income by paid work, i.e. the time not available for leisure. In the following it is rederived making use of somewhat different concepts.

The basic ideas of evaluating the time to enjoy life and its quality in an economical sense by the level of consumption can be extended by making use of concepts in economics. Denote by $c(\tau) > 0$ the consumption rate at age τ and by $u[c(\tau)]$ the utility derived from consumption.

$$U(a,t) = \int_a^t u[c(\tau)]\mathrm{d}\tau. \tag{42}$$

Individuals tend to undervalue a prospect of future consumption as compared to that of present consumption. This is taken into account by some discounting. The life time utility for a person at age a until she/he attains age $t > a$ then is

$$U(a,t,\rho) = \int_a^t u[c(\tau)] \exp\left[-\int_a^\tau \rho(\theta)\mathrm{d}\theta\right]\mathrm{d}\tau. \tag{43}$$

In contrast to [46] and [47] the discount rate is explicitly made dependent on time. ρ should be conceptually distinguished from a financial interest rate and is referred to as rate of time preference of consumption. Exponential population growth with rate n should be considered by replacing ρ by $\rho - n$ taking into account that families are by a factor $\exp[nt]$ larger at a later time $t > 0$. Constant exponential population growth can be verified for the last 100 years in good approximation from the data collected in [35]. The economics literature also states that if no such "discounting" is applied more emphasis is placed on the well being of future generations rather than improving welfare of those alive at present, assuming economic growth but no population growth. Future generations are wealthier. Therefore, one should add the real, exponential growth rate ζ of an economy. The growth rate per capita then is $\delta = \zeta - n$. Exponential economic growth at a nearly constant rate can again be verified from the data in [35] for at least the last 100 years if short term fluctuations for δ are averaged out. Therefore, if δ is constant but the time preference rate ρ (θ) is taken as a function decreasing in time according to Fig. 4 the discount factor is effectively $\exp\left[-\int_a^\tau(\rho(\theta)+\delta)\mathrm{d}\theta\right]=\exp\left[-\left(\int_a^\tau\rho(\theta)\mathrm{d}\theta+\delta(\tau-a)\right)\right]$. The expected remaining present value life time utility at age a (conditional on having survived until a) then is (see, for example, [2,56])

$$L(a)=E[U(a)]=\int_a^{a_u}\frac{f(t)}{\ell(a)}U(a,t)\mathrm{d}t$$

$$=\int_a^{a_u}\frac{f(t)}{\ell(a)}\int_a^t u[c(\tau)]\exp\left[-\left(\int_a^\tau\rho(\theta)\mathrm{d}\theta+\delta(\tau-a)\right)\right]\mathrm{d}\tau\,\mathrm{d}t$$

$$=\frac{1}{\ell(a)}\int_a^{a_u}u[c(t)]\exp\left[-\left(\int_a^t\rho(\theta)\mathrm{d}\theta+\delta(t-a)\right)\right]\ell(t)\mathrm{d}t, \tag{44}$$

where $f(t)\mathrm{d}t=(\mu(t)\exp[-\int_0^t\mu(\tau)\mathrm{d}\tau])\mathrm{d}t$ is the probability of dying between age t and $t+\mathrm{d}t$ which can be computed from life tables for given mortalities $\mu(t), a_u$ is some maximum age and $\ell(t)=\exp[-\int_0^t\mu(\tau)\mathrm{d}\tau]$ is survival probability. The last member of Eq. (44) is obtained by partial integration. Note that $e(a)=\int_a^{a_u}\ell(t)\mathrm{d}t$ is (unconditional) life expectancy at age a. In economics this function is used to find the optimal consumption path $c^*(t)$ by dynamic optimization [30] given rational behaviour of the consumer and subject to relevant budget constraints. Under perfect market conditions it has been shown that the optimal consumption path is constant over time because insurance annuities can compensate for outliving one's assets or temporary overconsumption and because a surplus of earnings over consumption is best invested into an insurance, i.e. $c^*(t)=c$ (see [56] for details). This leads to

$$L(a)=u(c)e_d(a,\rho,\delta), \tag{45}$$

where the "discounted" life expectancy $e_d(a,\rho,\delta)$ at age a is given by

$$e_d(a,\rho,\delta)=\frac{\exp[\delta a]}{\ell(a)}\int_a^{a_u}\exp\left[-\left(\int_0^t\mu(\tau)\mathrm{d}\tau+\int_a^t\rho(\tau)\mathrm{d}\tau+\delta t\right)\right]\mathrm{d}t. \tag{46}$$

Note that $L(a)$ is finite throughout due to $a_u<\infty$. "Discounting" affects $e_d(a,\rho,\delta)$ primarily when $\mu(\tau)$ is small (i.e. at young age) while it has little effect for larger $\mu(\tau)$ at higher ages. It is important to recognize that "discounting" by $\rho(\tau)+\delta$ is initially with respect to $u[c(\tau)]$ but is formally included in the life expectancy term due to $u[c(\tau)]=u(c)$. Clearly, there is $e_d(0,\rho,0)\leqslant e$ for $\rho>0$. $L(a)$ varies as $e_d(a,\rho,\delta)$ with age.

Next, assume that an average individual earns his part of the GDP (gross domestic product per year and capita) and he has $g\approx 0.6\,\mathrm{GDP}$ at his personal disposal, for example for private use and consumption but also for investments into risk reduction. The rest of the GDP per capita is used by the state (internal and external safety, jurisdiction, education, etc.) and for investments into deteriorating production means (the most elementary form of sustainability which maintains a functioning economy also in the near and far future). Let annuities and changes in assets cancel each other over the life time. Then, $i=g=c$.

We are now faced with the problem of choosing an appropriate function $L(a)$ or, more specifically, $u(c)$. This is essentially the question of how much to sacrify of the utility of consumption and other aspects of life quality in favor of some more life years which can be achieved by some payment for risk reduction. We will return to this question a little later. Income is produced by work, all considered at a national level. More income will be produced for more work but this will leave less time for leisure. It will be proportional to productivity of work defined as earnings per time unit of work. Some recent ideas are repeated here in all brevity. Nathwani et al. [33] now hypothetized that "*...people on the average work just enough so that the marginal value of wealth produced, or income earned, is equal to the marginal value of the time they lose when at work*" and defined a measure for life quality at age a as $L(a)=f(g)h(t)$, where $t=(1-w)e(a)$ is leisure time and w $(0<w<1)$ the fraction of life expectancy lost due to (paid) work which is spread equally over the remaining life time. Switching to relative changes in life quality $\mathrm{d}L(a)/L(a)$ Nathwani et al. found that life quality can be described by $L(a)=g^r((1-w)e(a))^s$. Assuming $g=cw$ with c productivity (with appropriate dimensions) and finding the optimum from $\mathrm{d}L(a)/\mathrm{d}w=0$ together with $r+s=1$ $(0<r,s<1)$ and noting that $\mathrm{d}^2L(a)/\mathrm{d}w^2<0$ yields

$$L(a)=g^w e(a)^{1-w}(1-w)^{1-w}.$$

Dropping the term $(1-w)^{1-w}$ almost constant in a given country and taking the $(1-w)$th root finally gives

$$L(a)=g^q e(a), \tag{47}$$

where $q=w/1-w$. Much further discussion about this principle can be found in [46] where it is also supported by some empirical evidence. This simple isoelastic power function implying constant relative risk aversion (CRRA) according to Arrow-Pratt is widely used in economics. It is valid for all $a\geqslant 0$. $L(0)$ corresponds to the original derivations in [33].

Following the reasoning for Eq. (45) the conditional life expectancy $e(a)/\ell(a)$ should now be replaced by discounted conditional life expectancy $e_d(a,\rho,\delta)$. Integrating over the age distribution $h(a,n)$ in a (stable) population finally produces the so-called societal life quality index (SLQI) as introduced earlier by Pandey/Nathwani [37,38] or the average remaining life time utility of consumption:

$$L_{\bar{E}}=g^q\int_0^{a_u}e_d(a,\rho,\delta)h(a,n)\mathrm{d}a=g^q\bar{E} \tag{48}$$

with

$$\bar{E}=\int_0^{a_u}e_d(a,\rho,\delta)h(a,n)\mathrm{d}a, \tag{49}$$

$$h(a,n)=\frac{\exp[-na]\ell(a)}{\int_0^{a_u}\exp[-na]\ell(a)\mathrm{d}a}. \tag{50}$$

Note that Eq. (47) is recovered from Eq. (48) if no discounting and no age-averaging is performed. All quantities like g,q, the life tables and at least δ are observable. It should be clear by its derivation that the risk aversion parameter q as derived before must only be used in the context of risk reduction. It has nothing to do with the same parameter used in economics in connection with intertemporal consumption choices.

Willingness-to-pay (WTP) measures a person's willingness to sacrifice one desired attribute, wealth or consumption, in order to obtain another desired attribute, improved survival. Let dm

denote a marginal change in (crude) mortality (m = number of people dying in a given year/number of people in a given year), $d\ell$ the marginal change in life expectancy and dg the marginal change in consumption. Shepard/Zeckhauser and others [56,31] derived the *willingness-to-pay* from the condition that a change in life expectancy and the corresponding change in consumption balance each other by keeping the life quality index (or the remaining life time utility) constant.

$$\mathrm{d}L(a) = \frac{\partial L(a)}{\partial e(a)}\mathrm{d}e(a) + \frac{\partial L(a)}{\partial g}\mathrm{d}g \geqslant 0$$

so that after rearrangement

$$\mathrm{WTP} = \mathrm{d}g \geqslant -\frac{\frac{\partial L(a)}{\partial e(a)}}{\frac{\partial L(a)}{\partial g}}\mathrm{d}e(a) \tag{51}$$

or after age-averaging and expanding $\mathrm{d}e(a)/e(a)$ in terms of a small change $\mathrm{d}m$ in mortality into a Taylorseries around $\mathrm{d}m = 0$ [26]

$$\mathrm{SWTP} = \mathrm{d}g = -E\left[\frac{\frac{\partial L(A)}{\partial e(a)}}{\frac{\partial L(A)}{\partial g}}\right]\mathrm{d}e(a) = -E\left[\frac{g}{q}\frac{\mathrm{d}e_d(a)}{e_d(a)}\right] \approx -\frac{g}{q}E\left[\frac{\frac{\mathrm{d}e_d(A,\rho,\delta,\mathrm{d}m)}{\mathrm{d}m}\big|_{\mathrm{d}m=0}}{e_d(A,\rho,\delta)}\right]\mathrm{d}m$$
$$= -\frac{g}{q}C_{x\bar{E}}(\rho,\delta)\mathrm{d}m = -G_{x\bar{E}}(\rho,\delta)\mathrm{d}m. \tag{52}$$

The quantity SWTP will be denoted as *societal willingness-to-pay*. In Eq. (52) the (small) change in discounted life expectancy is replaced by (small) change in mortality. The constant $G_{x\bar{E}}(\rho,\delta)$ is numerically identical with an equivalent constant derived directly from the societal life quality index [47]. It also appears justified to denote it by *societal value of a statistical life* (SVSL) as done in many health-related economical studies [25]. The constant $G_{x\bar{E}}(\rho,\delta)$ depends on the mortality reduction scheme x of a particular intervention, for example whether the intervention reduces mortality proportional to age-dependent mortality, only in certain age ranges or simply as a constant at all ages. In the following only constant (age-independent) mortality changes denoted by scheme Δ will be considered, i.e. $\mu_\Delta(a) = \mu(a) + \mathrm{d}m$. This scheme is intragenerationally equitable in technical applications. Others are discussed in [45,47]. The acceptability criterion Eq. (52) is *necessary*, *affordable* and *efficient* from a societal point of view [33]. And it is also intergenerationally equitable.

6.2. Use of predictive cohort life tables and time-dependent discounting

Setting risk acceptability criteria also has a sustainability aspect because the next or even the far future is concerned. Period life tables have been used in [44] and [46]. So-called cohort life tables certainly would be better as they reflect the common trend towards larger life expectancies and more compact age distributions. Since we are interested in the future we have to extrapolate into the future. Time- and age-dependent mortalities can be obtained by extrapolating from a sequence of historical period life tables so that $\mu_{\vartheta,y}(a) = \mu_y(a)b(a)^{\vartheta+a-y}$ where y is the reference year, i.e. the last year for which a period table is available and $\vartheta \lesseqgtr y$ is the year of birth. Unfortunately,

cohort life tables do exist only for a few countries. Cohort life tables, for example, yield 7% larger life expectancies at present. Predictive cohort table must be constructed. Results are collected in Table 1 for eight developed countries. An uninterrupted sequence of period life tables must be available for at least the last 50 years so that extrapolations for the age dependent mortalities can be performed. The data used are all from [7] where some more countries are included. Clearly, such extrapolations are based on the assumption that the observed demographic trends continue throughout the next 100 years. Trends in all other parameters are not taken into account but must be expected. Some numerical values for various economic and demographic quantities entering Eq. (62) or Eq. (63) are given in Table 1.

The data for GDP, g, δ, n and w are discussed in detail in [47]. Monetary quantities are given in PPPUS$ (corrected for purchasing power parity). For all considered countries $\rho = 0.03$ is assumed in Eq. (41). Life expectancy e is computed from the predictive cohort tables. The values for ρ and β given in the table correspond to Eq. (35). The results $G_{\Delta\bar{E}}$ for the USA are comparatively small because a rather high value for w has been reported. For Sweden they also are comparatively low because the data give only a relatively small g for the given GDP. If the discounting scheme according to Eq. (35) is used the constant $G_{\Delta\bar{E}}$ is a little larger than if the scheme in Eq. (41) is used which is a consequence of a smaller discount rate for the near future. But these differences are of no practical relevance. The values of SLSC introduced below in Eq. (57) are larger than those in [47] because of larger future life expectancy. For information, the values of G_Δ without any discounting and age-averaging are also given. All in all, the results for SLSC and $G_{\Delta\bar{E}}$ show remarkable consistency among the countries considered. The demographic constant C_x computed from the table by $C_x = G_x(q/g)$ varies only by ±10%, suggesting that one can also ignore the

Table 1
Social indicators for some countries

Country	GDP[a], g[b]	δ[c]	n[d]	e[e]	w[f]	ρ[g]	β[g]	SLSC[h]	G_Δ[i], $G_{\Delta\bar{E}}$[j], $G_{\Delta\bar{E}}$[k]
Canada	27330, 16040	2.0	0.99	84	0.13	1.3	3.0	7.6×10^5	4.6×10^6, 1.9×10^6, 1.8×10^6
USA	34260, 22030	1.8	0.90	86	0.15	1.3	2.3	1.0×10^6	5.6×10^6, 2.4×10^6, 2.3×10^6
France	24470, 14660	1.9	0.37	85	0.12	0.7	1.9	6.5×10^5	4.7×10^6, 2.1×10^6, 1.8×10^6
Germany	25010, 14460	1.9	0.27	87	0.12	0.5	2.2	6.5×10^5	4.9×10^6, 2.1×10^6, 1.8×10^6
Japan	26460, 15960	2.7	0.17	92	0.13	0.7	2.3	7.2×10^5	4.8×10^6, 1.8×10^6, 1.6×10^6
Sweden	23770, 12620	1.9	0.02	82	0.12	0.3	1.6	5.1×10^5	3.8×10^6, 1.6×10^6, 1.4×10^6
Switzerland	29000, 17700	1.9	0.27	85	0.12	0.6	1.8	7.7×10^5	5.8×10^6, 2.5×10^6, 2.2×10^6
UK	23500, 15140	1.3	0.23	87	0.13	0.4	1.5	6.0×10^5	4.1×10^6, 1.9×10^6, 1.7×10^6

[a] In PPPUS$ [65].
[b] Private consumption in PPPUS$ [62].
[c] Average yearly economic growth per capita in % for 1870–1992 after [35].
[d] Population growth (2000) in % [10].
[e] Life expectancy for those born in 2000.
[f] Estimates based on [36] including 1 h travel time per working day and a life working time of 45 years.
[g] ρ and β according to Eq. (35).
[h] SLSC computed with g and age-averaged life expectancies according to Eq. (57).
[i] Without any discounting and age-averaging.
[j] Computed from predictive cohort life tables and ρ = const. in column 7, Δ indicates constant additive mortality changes.
[k] Same as for but discount rate according to Eq. (41) with $\rho = 0.03$ and $\epsilon = 1$.

demographic differences in practical applications. Those last conclusions might no more hold for countries with significantly differing demographic and economic conditions.

6.3. Application to civil engineering infrastructures

Application to technical objects requires that the mortality change is expressed in terms of changes in the failure rate. Let $\mathrm{d}m$ be proportional to the increment in the mean failure rate $\mathrm{d}r(p)$, i.e. it is assumed that the process of failures and renewals is already in a stationary state that is for $t \to \infty$ (see Eq. (14)). Rearrangement and introducing the incremental cost and the failure rate as a function of a (scalar) parameter p yields

$$\frac{\mathrm{d}C_Y(p)}{\mathrm{d}r(p)} \geqslant -kG_{\Delta\bar{E}}(\rho,\delta), \tag{53}$$

where

$$\mathrm{d}m = k\,\mathrm{d}r(p), \quad 0 < k \leqslant 1, \tag{54}$$

the proportionality constant k relating the changes in mortality to changes in the failure rate. More specifically, k is the probability of being killed in a failure event. Note that for any reasonable risk reducing intervention there is necessarily $\mathrm{d}r(p)/\mathrm{d}p < 0$. k $(0 \leqslant k \leqslant 1)$ must be determined by careful failure consequence analysis. For simplicity, a scalar design parameter p is assumed.

The life saving cost (LSC) or implied cost of averting a fatality (ICAF) can be obtained from the equality of Eq. (51) after replacing $e(a)$ by $e = e(0)$, separation and integration from g to $g + \Delta g$ and e to $e + \Delta e$, i.e. the cost $\Delta C = -\Delta g$ per year to extend a person's life by Δe is

$$\Delta C = -\Delta g = g\left[1 - \left(1 + \frac{\Delta e}{e}\right)^{-1/q}\right].$$

Because ΔC is a yearly cost and the (undiscounted) LSC has to be spent for safety related investments into technical projects at the decision point $t = 0$, one should multiply by $e_r = \Delta e$ and

$$\mathrm{LSC}(e_r) = g\left[1 - \left(1 + \frac{e_r}{e}\right)^{-1/q}\right]e_r \tag{55}$$

follows. The societal equality principle prohibits to differentiate with respect to special ages within a group. The conditional (remaining) life expectancy given that the person has survived up to age a is

$$e(a) = \int_a^{a_u} \frac{\ell(t)}{\ell(a)}\mathrm{d}t = \frac{1}{\ell(a)}\int_a^{a_u} \exp\left[-\int_0^t \mu(\tau)\mathrm{d}\tau\right]\mathrm{d}t. \tag{56}$$

Therefore, averaging the remaining life expectancy over the age distribution leads to the societal life saving cost (SLSC)

$$\mathrm{SLSC} = \int_0^{a_u} \mathrm{LSC}(e(a))h(a,n)\mathrm{d}a, \tag{57}$$

where $h(a,n)$ is the density of the age distribution of the population with n its population growth rate. In countries with a fully developed social system SLSC is approximately the amount to support the (not working) surviving dependents of a victim in an adverse event by the social system, mostly by redistribution. If no social system is present, it is useful to think of the amount an insurance should cover after an event. For example, if GDP $\approx$ 25,000 PPP US\$ and thus, $g \approx 15{,}000$ PPP US\$, $e \approx 77$ years and $w \approx 0.15$, one calculates SLSC $\approx$ 600,000 PPP US\$.

The criterion equation (53) is derived for safety-related regulations for a larger group in a society or the entire society. For a specific project it makes sense to apply criterion (53) to the specific group exposed. Therefore, the "life saving cost" of a technical project with N_{PE} potential endangered persons is

$$H_F = \mathrm{SLSC}\,kN_{\mathrm{PE}}. \tag{58}$$

The monetary losses in case of failure are decomposed into $H = H_{\mathrm{M}} + H_F$ in formulations of the type Eq. (1) with H_{M} all losses not related to human life and limb. k is the probability of being killed by an adverse event followed by failure. N_{PE} as well as k must be estimated taking account of the number of persons endangered by the event, the cause of failure, the severity and suddenness of failure, possibly availability and functionality of rescue systems, etc. It is important to note the difference of *SLSC*, which can also be interpreted as compensation cost, and the quantity $G_{\Delta\bar{E}}(\rho,\delta)$ in criteria of the type Eq. (52) as the societal willingness-to-pay per unit mortality reduction. Whether SLSC is included in cost-benefit analyses or not depends on the particular legal system in a country.

Criterion (53) changes accordingly into:

$$\frac{\mathrm{d}C_Y(p)}{\mathrm{d}r(p)} \geqslant -G_{\Delta\bar{E}}(\rho,\delta)kN_{\mathrm{PE}}. \tag{59}$$

All quantities in Eq. (59) are related to one year. For a particular technical project all design and construction cost, denoted by $\mathrm{d}C(p)$, must be raised at the decision point $t = 0$. The yearly cost must be replaced by the erection cost $\mathrm{d}C(p)$ at $t = 0$ on the left-hand side of Eq. (59) and discounting is necessary. The method of discounting is the same as for discharging an annuity. As the public is involved in life saving $\mathrm{d}C_Y(p)$ may be interpreted as cost of societal financing of $\mathrm{d}C(p)$ such that $\mathrm{d}C_Y(p) = \mathrm{d}C(p)\{\gamma\exp[\gamma t_s]/(\exp[\gamma t_s] - 1)\}$. The (real) interest rate to be used must then be a societal interest rate. g in $G_{\Delta\bar{E}}(\rho,\delta)$ also grows in the long run approximately exponentially with rate δ, the rate of effective economic growth in a country (see [35] for an empirical verification). It can also be taken into account by discounting. Consideration of the time preference rate ρ is ethically indefensible [49] if future generations are concerned. The acceptability criterion for individual technical projects then is (discount factor for discounted erection cost moved to the right-hand side)

$$\frac{\mathrm{d}C(p)}{\mathrm{d}r(p)} \geqslant -\frac{\exp[\gamma t_s] - 1}{\gamma\exp[\gamma t_s]}G_{\Delta\bar{E}}(\rho,\delta)\frac{\delta\exp[\delta t_s]}{\exp[\delta t_s] - 1}kN_{\mathrm{PE}} \underset{t_s\to\infty}{\to} -G_{\Delta\bar{E}}(\rho,\delta)kN_{\mathrm{PE}}\frac{\delta}{\gamma}, \tag{60}$$

where t_s is service time or, better, the credit period. For $\delta \to 0$ as well as $\gamma \to 0$ we have the interesting limiting result for arbitrary t_s:

$$\frac{dC(p)}{dr(p)} \geqslant \underset{\delta\to 0,\gamma\to 0}{\to} -G_{\bar{E}}(\rho,\delta)kN_{\mathrm{PE}}. \tag{61}$$

But γ also approaches δ (or $\varepsilon\delta$) for $t_s \to \infty$ as can be seen in Fig. 4. Therefore, Eq. (60) approaches Eq. (61) in the limit according to Eq. (41). Consequently, there is no discounting in the acceptability criterion except for discounting in the demographic constant $C_{x\bar{E}}$ in $G_{\Delta\bar{E}}(\rho,\delta)$ by $\rho+\delta$ as in Eq. (52).

Generalizing now to a vectorial parameter **p** we have

$$\nabla_p C(\mathbf{p}) + G_{\Delta\bar{E}}(\rho,\delta)kN_{\mathrm{PE}}\nabla_p r(\mathbf{p}) \geqslant 0, \tag{62}$$

which is easily seen to be equivalent to the solution of the following optimization task:

$$\text{Minimize}: \quad Z'(\mathbf{p}) = C(\mathbf{p}) + G_{\Delta\bar{E}}(\rho,\delta)kN_{\mathrm{PE}}r(\mathbf{p}). \tag{63}$$

Conversely, Eq. (62) is nothing else than the optimality condition $\nabla'_{pZ}(\mathbf{p}) = 0$ of the (unconstrained) optimization problem equation (63). Eq. (63) allows solving for vectorial parameter **p**. A solution to Eq. (62) or (63) can almost always be found because ∇_p c(**p**) usually grows approximately linearly in **p** whereas ∇_pr(**p**) decays exponentially. The criterion (62) is, thus, independent of any interest rate or benefit rate.

7. Numerical techniques of optimization

7.1. Principles of a one-level approach

Let **p** be a parameter vector which enters the cost function and the limit state function $g(\mathbf{u},\mathbf{p}) = 0$. Benefit, construction and damage function as well as the limit state function(s) are differentiable in **p** and **u**. The conditions for the application of FORM/SORM hold. In the so-called β-point $\mathbf{u}^*$ the optimality conditions (Kuhn–Tucker conditions) are [27]:

$$g(\mathbf{u},\mathbf{p}) = 0,$$

$$\frac{\mathbf{u}}{\|\mathbf{u}\|} = -\frac{\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})}{\|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\|}. \tag{64}$$

The geometrical meaning of (64) is that the gradient of $g(\mathbf{u},\mathbf{p}) = 0$ is perpendicular to the vector of direction cosines of $\mathbf{u}^*$. The basic idea mentioned first in [16] and elaborated in [27] now is to use these conditions as constraints in the cost optimization problem thus avoiding a bi-level optimization. It will turn out that this concept is crucial for further numerical analysis as described below.

It is important to reduce the set of the gradient conditions in the Kuhn–Tucker conditions by one. Otherwise the system of Kuhn–Tucker conditions is overdetermined. It is also important that the remaining Kuhn–Tucker conditions are retained under all circumstances, for example, if one or more gradient Kuhn–Tucker conditions become co-linear with one or more of the other constraints possibly included in the cost-benefit optimization task. Otherwise the so-called β-point conditions are not fulfilled.

7.2. Formulations for time-variant problems

In the simplest stationary, one-component case and constant discount rates we have for the case of random disturbances:

$$Z(\mathbf{p}) = B - C(\mathbf{p}) - (C(\mathbf{p}) + H_{\mathrm{M}} + H_F)\cdot\frac{\lambda P_f(\mathbf{p})}{\gamma} \tag{65}$$

subject to

$$g(\mathbf{u},\mathbf{p}) = 0,$$
$$u_i\|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\| + \nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})_i\|\mathbf{u}\| = 0, \quad i = 1,\ldots,n-1,$$
$$h_\ell(\mathbf{p}) \leqslant 0, \quad \ell = 1,\ldots,q,$$
$$\lambda P_f(\mathbf{p}) \leqslant r_{\text{admissible}}$$

or

$$\nabla_p C(\mathbf{p}) + G_{x\bar{E}}(\delta,\rho)kN_{\mathrm{PE}}\nabla_p v^+(\mathbf{p}) \geqslant 0,$$

where the $h_\ell(\mathbf{p})$ are contraints for the parameter vector **p**. The same scheme applies to the full Laplace transform of non-stationary problems as in Eq. (27) or (28).

$$Z(\mathbf{p}) \approx B - C(\mathbf{p}) - (C(\mathbf{p}) + H_{\mathrm{M}} + H_F)\cdot\frac{f^*(\gamma,\mathbf{p})}{1-f^*(\gamma,\mathbf{p})}, \tag{66}$$

$$g(\mathbf{u}_j,\mathbf{p},t_j) = 0 \quad \text{for } j = 0,1,\ldots,m,$$
$$u_{i,j}\|\nabla_{\mathbf{u}} g(\mathbf{u}_j,\mathbf{p},t_j)\| + \nabla_{\mathbf{u}} g(\mathbf{u}_j,\mathbf{p},t_j)_i\|\mathbf{u}_j\| = 0, \quad i = 1,\ldots,n-1, \; j = 0,\ldots,m,$$
$$h_\ell(\mathbf{p}) \leqslant 0, \quad \ell = 1,\ldots,q,$$
$$\frac{1}{E[T(\mathbf{p})]} \leqslant r_{\text{admissible}},$$

or

$$\nabla_p C(\mathbf{p}) + G_{x\bar{E}}(\delta,\rho)kN_{\mathrm{PE}}\nabla_p\left(\frac{1}{E[T(\mathbf{p})]}\right) \geqslant 0,$$

depending on whether a reliability constraint $h_{\text{admissible}}$ is imposed exogenously or criterion (62) is used. Here, the failure rate acceptability criterion must use the asymptotic failure rate equation (14). Eq. (27) is used resulting in a large number of equality constraints which may cause numerical difficulties in extreme cases. The mean value of interarrival times of renewal in Eq. (66) is computed by Eq. (25) or for dependent series systems from

$$E[T_s(\mathbf{p})] = \int_0^\infty (1 - F_{T,s}(\mathbf{p},t))\mathrm{d}t = \int_0^\infty \Phi_s(\beta(\mathbf{p},t))\mathrm{d}t. \tag{67}$$

It should be clear that $f^*(\gamma,\mathbf{p})$ should be replaced by $f^{\#}(\gamma,\mathbf{p})$ whenever a time-dependent discount rate must be used.

The optimization procedure proposed before is by no means the only one. Alternatives can also be used. More technical details can be found in [59,60].

8. Examples

The following examples have been selected with special emphasis to sustainability aspects. They illustrate various aspects of the extended theory, especially of time-dependent discounting.

8.1. Random demand versus random capacity [47]

The example has already been given in [44] in somewhat different form and with different parameters. A single-mode system is considered where failure is defined if a random resistance or capacity is exceeded by a random demand. The demand is modelled as a one-dimensional, stationary marked Poissonian rectangular wave renewal process of disturbances (earthquakes, wind storms, explosions, etc.) with stationary renewal rate λ and random, independent sizes of the disturbances S_i, $i = 1,2,\ldots$ The disturbances are assumed to be short as compared to their mean interarrival times. The resistance is log-normally distributed with mean p and a coefficient of variation V_R. The disturbances are also independent and log-normally distributed with mean equal to unity and coefficient of variation V_S so that p can be interpreted as central safety factor. A disturbance causes failure with probability:

$$P_f(p) = \Phi\left(-\frac{\ln\left\{p\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right\}}{\sqrt{\ln\left((1+V_R^2)(1+V_S^2)\right)}}\right). \tag{68}$$

An appropriate objective function then is with $b = b(\mathbf{p})$:

$$\begin{aligned} Z(p) &= \frac{b}{C_0}\int_0^\infty e^{-\int_0^t \gamma(\tau)d\tau}dt - \left(1+\frac{C_1}{C_0}p^a\right) \\ &\quad - \left(1+\frac{C_1}{C_0}p^a+\frac{H_M}{C_0}+\frac{H_F}{C_0}\right)\int_0^\infty e^{-\int_0^t \gamma(\tau)d\tau}g(t,h)dt \\ &\approx \frac{b}{C_0}\int_0^\infty e^{-\int_0^t \gamma(\tau)d\tau}dt - \left(1+\frac{C_1}{C_0}p^a\right) - \left(1+\frac{C_1}{C_0}p^a+\frac{H_M}{C_0}+\frac{H_F}{C_0}\right)\frac{P_f(p)f^{\#}(\gamma)}{1-f^{\#}(\gamma)} \end{aligned} \tag{69}$$

with

$$\begin{aligned} g(t,p) &= \sum_{n=1}^{\infty} P_f(p)f_n(t)(1-P_f(p))^{n-1}, \\ f_n(t) &= [\lambda(\lambda t)^{n-1}/\Gamma(n)]\exp[-\lambda t], \\ f^{\#}(\gamma) &= \int_0^\infty e^{-\int_0^t \gamma(\tau)d\tau}\lambda\exp[-\lambda t]dt, \\ \gamma(t) &= \epsilon\delta + \rho\exp(-at), \quad \epsilon\delta = 0.02, \quad \rho = 0.03, \quad a = 0.012. \end{aligned} \tag{70}$$

The criterion (60) has the form

$$\frac{d}{dp}(C_0 + C_1p^a) \geqslant -G_{\Delta\bar{E}}kN_{PE}\frac{d}{dp}(\lambda P_f(p)). \tag{71}$$

Some more or less realistic, typical parameter assumptions are: $C_0 = 10^6$, $C_1 = 10^4$, $a = 1.25$, $H_M = 3\cdot C_0$, $V_R = 0.2$, $V_S = 0.3$, $b = 0.05C_0$ and $\lambda = 1$ [1/year]. The LQI-data are $e = 77$, GDP = 25,000, $g = 15{,}000$, $w = 0.13$. Also there is $N_{PE} = 100$, $k = 0.1$ so that $H_F = \text{SLSC}kN_{PE} = 5.4\times10^6$ and $G_{\Delta\bar{E}}kN_{PE} = 5.0\times10^7$, respectively. The value of N_{PE} is chosen relatively large for demonstration purposes. Monetary values are in US\$.

A maximum discount rate ρ_{max} is determined in analogy to Eq. (33) and using Eq. (70) from

$$\rho_{max} = \text{solution of}(Z(p_{opt}, \rho) = 0),$$

which is $\rho_{max} = 0.035$ with corresponding $p_{opt} = 4.04$ ($r(p_{opt}) = 3.0\times10^{-5}$). The approximation produces almost the same result $p_{opt} = 4.00$ ($r(p_{opt}) = 3.4\times10^{-5}$). It is therefore concluded that the approximation can be used for all further analysis. Criterion (71) requires $p_{lim} = 3.61$ ($r(p_{lim}) = 1.1\times10^{-4}$). It is interesting to see that in this case one can do better in adopting the optimal solution rather than just realizing the facility at its acceptability limit (see Fig. 5). Note that the figure has been computed using $b = 0.05C_0$ and the corresponding $\rho_{max} = 0.035$. For smaller ρ (or larger benefit rate) the maximum of the objective function is well above zero as shown in Fig. 5.

The location of the optimum varies very little with the discount rate ρ as shown in Fig. 6.

The stochastic model and the variability of capacity and demand also play an important role for the magnitude and location of the optimum as well as on the position of the acceptability limit. The specific marginal cost (rate of change) of a safety measure and its effect on a reduction of the failure rate are equally important as pointed out already in [44]. This is shown in Fig. 7 where the acceptable failure rates are plotted over the parameters C_1 and V_R for $G_{\Delta\bar{E}}kN_{PE} = 5.0\times10^6$, i.e. $kN_{PE} = 1$.

This example also allows to derive risk-consequence curves by varying the number of fatalities in an event. For the same data as before we first vary the cost effectiveness of the safety measure. Here, only the ratio C_1/C_0 is changed. The upper bounds (solid lines) are derived from Eq. (71). The lower bounds (dashed lines) correspond to the optimum according to

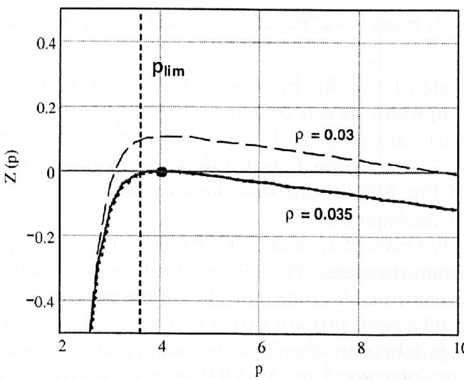

Fig. 5. Objective function and acceptability limit for capacity-demand example.

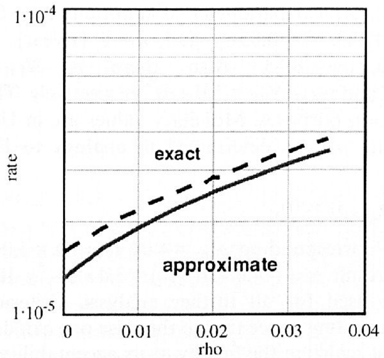

Fig. 6. Location of optimum failure rate as a function of ρ.

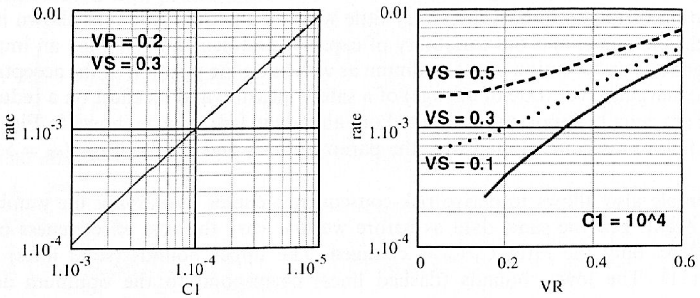

Fig. 7. Location of acceptable failure rate as function of marginal cost for safety and coefficients of variation.

Eq. (69) (see left hand side of Fig. 8). In general, the use of a time-dependent discount rate moves the lower bounds upwards as can be compared with the results given in [47]. According to Eq. (33) $\rho_{\max}$ also must vary with the failure consequences (number of fatalities N_f) for a given ratio C_1/C_0. In fact, as shown in Fig. 9 it decreases with N_f. This is taken into account on the right-hand side of Fig. 8 indicating that the area between solid and dashed lines broadens for very high failure consequences.

Most realistic is probably a ratio of around $C_1/C_0 = 0.001$. A ratio of $C_1/C_0 = 0.01$ or higher may apply for earthquake resistant structures. The failure rate of approximately 10^{-4} per year for $N_F = 1$ corresponds well with the controllable crude mortality of the same magnitude.

In Fig. 10 the demographic constants are varied indicating that this is of only moderate influence. In this figure the region between the upper bound(s) and the lower curve derived from the societal optimum may be interpreted as ALARP-region (ALARP = As Low As Reasonably Practicable).

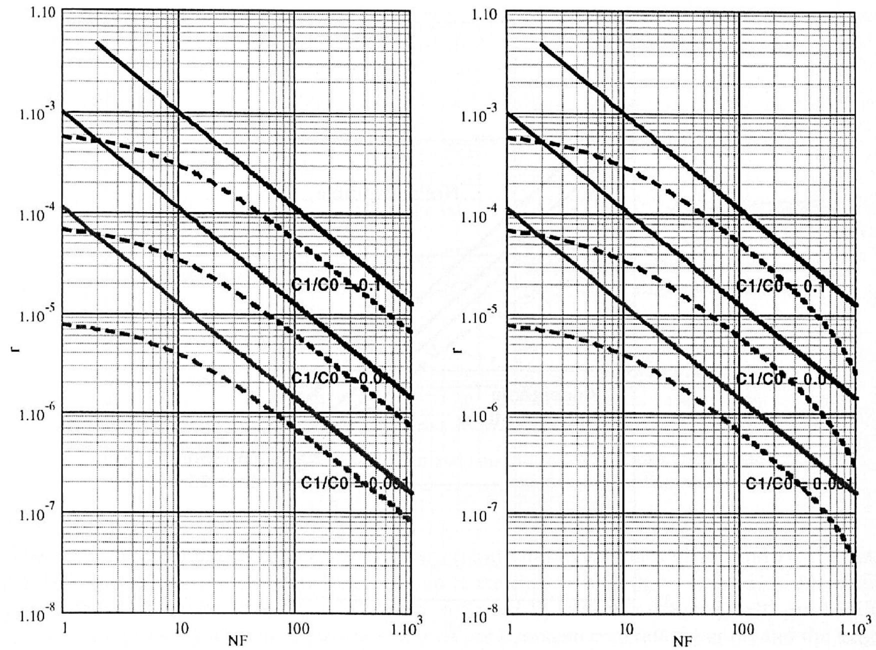

Fig. 8. Risk-consequence curves for various ratio C_1/C_2.

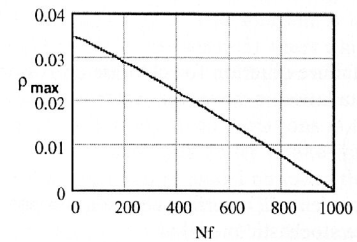

Fig. 9. Maximum ρ as a function of failure consequences (fatalities).

Note that in these figures the failure rate is given by λP_f and the number of fatalities is given by $N_F = kN_{PE}$. Therefore, these figures cover the full range of λ and P_f and k and N_{PE}, respectively.

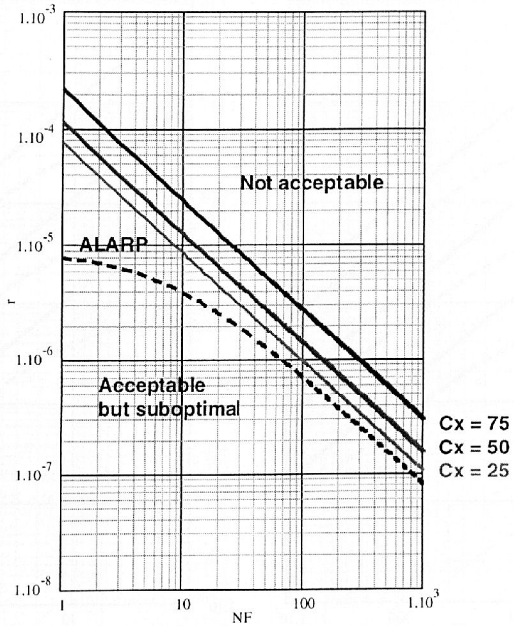

Fig. 10. Acceptable risk for different mortality regimes – definition of ALARP – region.

8.2. Optimal replacement of a reinforced concrete structure subject to chloride corrosion in warm sea water [58]

Following [48] a simplified failure criterion for chloride corrosion in the splash zone of some reinforced concrete harbor installation in warm sea water is

$$C_{cr} - C_s\left(1 - \mathrm{erf}\left(\frac{c}{2\sqrt{Dt}}\right)\right) \leqslant 0,$$

where C_{cr} is a critical chloride content, C_s is surface chloride content, c is concrete cover in cm and D is a diffusion parameter. The stochastic model is

Variable	Distribution function	Parameters
C_{cr}	Uniform $[a,b]$	0.125, 0.175
C_s	Uniform $[a,b]$	0.2, 0.4
c	Log-normal	$m_c, \sigma_c = 1$
D	Uniform $[a,b]$	0.1, 0.315

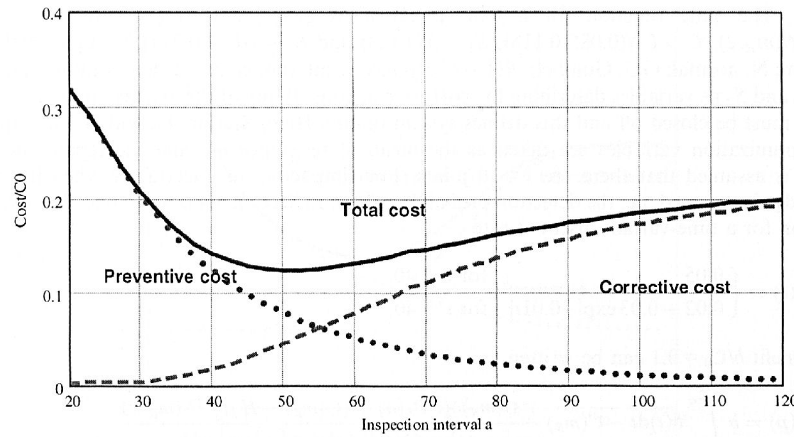

Fig. 11. Total cost for regular inspections and renewals for chlorid corrosion example.

The uniform distributions reflect the large uncertainty in the variables C_{cr},C_s and D. The standard deviation of concrete cover is 1 cm. The units are chosen such that t is in years. Inspection are performed at regular intervals a. They are followed by renewals (repairs) with probability $P_R(a) = 1-\exp[-a_R\, a^2]$. The optimization variables are the mean concrete cover m_c and the length a of the inspection interval and the theory outlined in Section 4.4 is applied. Erection cost are $C(m_c) = C_0 + C_1 m_c^2$, inspection cost are $I_0 = 0.1C_0$, repair cost are $I_1 = 0.5C_0$ and we have $C_0 = 10^6$, $C_1 = 10^4$, $H = 10C_0$, a benefit function decreasing in time and a time-variant interest rate, i.e. $B = C_0 \int_0^\infty b(t)\exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right]\mathrm{d}t$, $b(t) = b_0\Phi\quad[-(t-50)/25]$, $b_0 = 0.1$, $\gamma(t) = 0.02 + 0.03\exp(-0.012t)$, and $a_R = 0.005$. The solution is $a^* = 51$ and $m_c^* = 6.0$. It turns out that preventive repairs should be performed every 51 years which saves up to 35% of the cost. The same example with constant discount rate $\gamma = 0.03$ gave $a^* = 66$ and $m_c^* = 6.5$ in [58]. These results comply well with practical experience with such structures. The contributions to the total damage cost are shown in Fig. 11. Relatively small variations in the repair model or in the cost factors will, however, result in cases where it is better not to inspect and repair but just wait for failure. It is noted that for the given failure model no mean time to failure exists. Therefore, no acceptability criterion can be defined theoretically. But it is not necessary because there are clear signs of deterioration and, thus, pre-warnings. For the same reason the parameter a_R has been chosen such that $P_R(51) \approx 1$.

8.3. Optimal replacement of a series system of corroding dependent expansion joints [60]

For illustration purposes a simple failure criterion is assumed which may model the deterioration of s expansion joints in a long multi-span bridge exposed to corrosion due to heavy winter salting. The state function for a joint is taken as $g(\mathbf{X}) = R(1 - C\sqrt{t}) - (S_1 + S_2)$, where $R \sim LN(m_R, 2)$, $C \sim UN(0.085, 0.115)$, $S_1 \sim N(1, 0.3)$ and $S_2 \sim GU(0, 0.2)$ (LN, lognormal; UN, uniform; N, normal; GU, Gumbel). $R(1 - C\sqrt{t})$ may be interpreted as the deteriorating resistance and S_1 and S_2 as variables describing the corrosive actions. If any of the expansion joints fails the bridge must be closed off and this defines system failure. Here, Section 4.2 with 4.4 are applied. The optimization variables are taken as the mean of resistance m_R and the repair interval a and it is assumed that there are $s = 10$ joints. For simplicity, no uncertainty when inspecting the bridge is assumed, i.e. the detection probability $P_R(a)$ in Section 4.4 is set to one. The objective function for a time-variant discount rate

$$\gamma(t) = \begin{cases} 0.05 & \text{for } t \leqslant 40, \\ 0.02 + 0.03\exp[-0.01t] & \text{for } t > 40, \end{cases}$$

and benefit $b/C_0 = 0.1$ can be written as

$$Z(p) = b\int_0^\infty \delta(t)\mathrm{d}t - C(m_R) - \frac{C(m_R)\delta(t)P_S(a) + (C(m_R) + H)f_S^{\#\#\#}(m_R, \gamma)}{1 - (\delta(t)P_S(a) + f_S^{\#\#\#}(m_R, \gamma))},$$

$$\delta(t) = \exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right],$$

$$f_S^{\#\#\#}(m_R, \gamma) = \int_0^a \exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right] f_s(t)\mathrm{d}t,$$

$$P_S(a) = P\left(\bigcap_{k=1}^{s}\{R(1 - C\sqrt{t}) - (S_{1,k} + S_{2,k}) > 0\}\right),$$

$$f_s(t) = \frac{\partial}{\partial t}F_S(t) = \frac{\partial}{\partial t}P\left(\bigcup_{k=1}^{s}\{R(1 - C\sqrt{t}) - (S_{1,k} + S_{2,k}) \leqslant 0\}\right).$$

R and C are common to all spots while the other variables are assumed to be independent from spot to spot and, therefore, $\rho_{ij}(t) = \alpha_R^2(t) + \alpha_C^2(t) \geqslant 0$. For b/C_0 between 0.10 and 0.09 the objective function will be close to zero. In particular, $C(m_R) = C_0 + C_1 m_R^2$, $C_0 = 10^6$, $C_1 = 10^4$, $H = 10^7$ is assumed and one determines $m_R^* \approx 7.8$, $a^* \approx 40$ and the benefit is 2.28. Fig. 12 shows the total damage cost, the corrective cost and the preventive cost.

It can be demonstrated that a constant discount rate of $\gamma = 0.03$ leads to roughly the same results for m_R^* and a^* but the benefit differs. The objective function is positive at the optimum. We see that more than 50% of the total cost can be saved by a suitable preventive repair strategy.

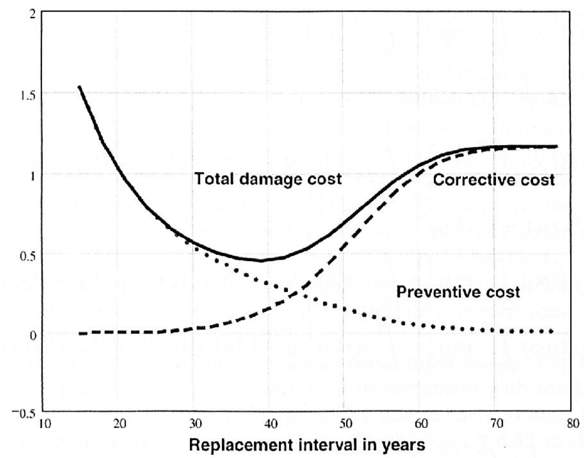

Fig. 12. Optimization of replacement interval for expansion joint example, all cost divided by C_0.

8.4. Dam failure by overtopping

This example is based on a real application but the data are modified for illustration purposes. A dam is primarily used for electricity production of a larger isolated rural downstream area which is subject to flash floods roughly once a year. The occurrence of floods can be modelled by a stationary Poissonian process with parameter $\lambda = 1.0$. A spillway usually is blocked by a gate but the system for opening the gate has failure probability of $P_{\text{Gate Failure}} = 5.2\times10^{-3}$ in case of demand although it has built-in redundancies. The flood level above the still water level SWL has exceedance probability $P(l) = P(L \geqslant l) = \exp[-(l/w)^k]$, $w = 2.0$, $k = 1.25$, at each occurrence of a flood where l is the height above SWL. Therefore, the probability of overtopping and destruction of the dam is $P_f(h) = P_{\text{Gate Failure}}P(L \geqslant h)$ where h is dam height above SWL. Two strategies for sustainable design are investigated. In the first strategy (with objective function Z_1) it is assumed that dams are reconstructed systematically after possible future failures whereas in the second strategy (with objective function Z_2) dams are abandoned after the first failure. Reconstruction times are taken into account approximately by the stationary "availability" $A(\infty)$. A (re)-construction time of 5 years is assumed so that

$$A(\infty) = \frac{T_{\text{mean failure time}}}{T_{\text{mean failure time}} + 5} = \frac{1}{1 + 5\lambda P_f(h)},$$

which is a number close to unity. In the first case one has with sustainable discount rate $\gamma(t) = \delta + \rho\exp(-at)$, $(\delta = 0.02,\ \rho = 0.03,\ a = 0.012)$

$$Z_1(h) = b_N(h)A(\infty)\int_0^\infty \exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right]\mathrm{d}t - C(h) - (C(h) + H_M + H_F)A(\infty)$$
$$\times \int_0^\infty \mathrm{e}^{-\int_0^t \gamma(\tau)\mathrm{d}\tau} g(t,h)\mathrm{d}t$$
$$\approx b_N(h)A(\infty)\int_0^\infty \exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right]\mathrm{d}t - C(h) - (C(h) + H_M + H_F)\sum_{A(\infty)n=1}^{\infty} P_f(h)f_1^{\#}(\gamma)$$
$$\times (f^{\#}(\gamma)(1-P_f(h)))^{n-1}$$
$$= b_N(h)A(\infty)\int_0^\infty \exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right]\mathrm{d}t - C(h) - (C(h) + H_M + H_F)A(\infty)\frac{g_1^{\#}(\gamma)}{1-g^{\#}(\gamma)}$$
$$= b_N(h)A(\infty)\int_0^\infty \exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right]\mathrm{d}t - C(h) - (C(h) + H_M + H_F)A(\infty)\frac{P_f(h)f_1^{\#}(\gamma)}{1-f^{\#}(\gamma)}$$

with

$$f^{\#}(\gamma) = \int_0^\infty \exp\left[-\int_0^t \gamma(\tau)\mathrm{d}\tau\right] f(t)\mathrm{d}t$$

and $f(t) = \lambda\exp[-\lambda t]$. In the second strategy a "follow up" consequence (see also [14]) is included to take into account the effect that if the dam is lost the population normally benefiting from the power production have to buy electricity from elsewhere.

$$Z_2(h) = b_N(h)\int_0^\infty \int_0^t \mathrm{e}^{-\int_0^\tau \gamma(\theta)\mathrm{d}\theta}\mathrm{d}\tau\, g_1(t,h)\mathrm{d}t - \ell_N(h)\int_0^\infty \int_t^\infty \mathrm{e}^{-\int_0^\tau \gamma(\theta)\mathrm{d}\theta}\mathrm{d}\tau\, g_1(t,h)\mathrm{d}t$$
$$- C(h) - (H_M + H_F)\int_0^\infty \mathrm{e}^{-\int_0^t \gamma(\tau)\mathrm{d}\tau} g_1(t,h)\mathrm{d}t$$
$$\approx \frac{b_N(h)}{\gamma_\infty}\left(1 - \frac{P_f(h)f_1^{\#}(\gamma)}{1-(1-P_f(h))f^{\#}(\gamma)}\right) - \frac{\ell_N(h)}{\gamma_\infty}\frac{P_f(h)f_1^{\#}(\gamma)}{1-(1-P_f(h))f^{\#}(\gamma)}$$
$$- C(h) - (H_M + H_F)\frac{P_f(h)f_1^{\#}(\gamma)}{1-(1-P_f(h))f_1^{\#}(\gamma)}.$$

In both cases the acceptability criterion is

$$\frac{\mathrm{d}}{\mathrm{d}h}C(h) - \left(C_{\Delta E}g\frac{1-w}{w}kN_{PE}\right)\frac{\mathrm{d}}{\mathrm{d}h}(\lambda P_f(h)) = 0.$$

Note that the time to first failure has density $g_1(t,h) = \sum_{n=1}^{\infty} P_f(h)f_n(t)(1-P_f(h))^{n-1}$. Remember that $f_n(t) = [\lambda(\lambda t)^{n-1}/\Gamma(n)]\exp[-\lambda t]$. For larger n it must be computed by $f_n(t) = \exp\left[\ln(\lambda) + (n-1)\ln(\lambda t) - \lambda t - \sum_{j=1}^{n-1}\ln(j)\right]$. Because Poissonian floods are assumed there is $f_1^*(\gamma) = f^*(\gamma)$ and $f_1^{\#}(\gamma) = f^{\#}(\gamma)$. Also, the approximation $1/\gamma_\infty = \int_0^\infty \mathrm{e}^{-\int_0^\tau \gamma(\theta)\mathrm{d}\theta}\mathrm{d}\tau = \int_0^\infty \exp[-(\delta\tau a - \rho\exp(-a\tau) + \rho)/a]\mathrm{d}\tau$ is introduced which is to be interpreted as an equivalent

constant discount rate producing the same benefit in $Z_1(h)$. Construction cost are $C(h) = C_0 + c(SWL + h)^2$ as the cost grow roughly quadratically with the height of the dam. The time-independent benefit is $b(h) = \beta C_0 SWL$ because the benefit grows linearly with the long-term water head. The dam needs to be financed. Financing cost reduce the benefit (or increase construction cost). For comparison of the two replacement strategies the discount rate for financing is taken as $\gamma_F = 0.03$ over a period of $t_s = 40$ years according to the German standard [29] implying that the net benefit is $b_N(h) = \beta C_0\mathrm{SWL} - C(h)\gamma_F\exp[\gamma_F t_s]/(\exp[\gamma_F t_s]-1)$. The real discount rate for financing the dam is probably a little larger. The loss after failure for the second strategy is assumed to be $10b_N(h)$. Further, the following data are assumed: SWL = 11 m, $C_0 = 2\times10^7$, $c = 40{,}000$, $\beta = 0.015$, $H_M = 1.5\times10^9$, $g = 15{,}000$, $w = 0.13$, $G_{\Delta E} = 50(g/q)$, $e = 77$, $k = 0.1$, $N_{PE} = 10{,}000$ implying $N_F = 1000$ or roughly 300 flooded buildings, SLSC = 580,000, and $H_F = 5.7\times10^8$. All monetary quantities are in appropriate currency units. The probability k has been estimated making use of [57] and [20].

Optimization for the two objective functions shows very similar results (Fig. 13). The exact and approximate objective functions differ only very little in agreement with the findings in Section 3.4. The second objective is about 10% lower primarily due to the loss after failure introduced in $Z_2(h)$. Systematic reconstruction results in a dam height $h_{\mathrm{opt},1} = 6.42$ m (by approximation 6.30 m) with a mean return period for failure of 14,210 years and roughly the same (re-)construction cost and benefit as the second option. If the option without reconstruction is chosen a dam height of $h_{\mathrm{opt},2} = 6.52$ m (6.35 m) corresponding to a return period for failure of 15400 years is obtained. The acceptability criterion requires a minimum dam height of $h_{\lim} = 4.53$ m corresponding to a return period for failure of 3098 years. The design with or without systematic reconstruction at the limiting height or, better, at the respective optima should be chosen. The objective

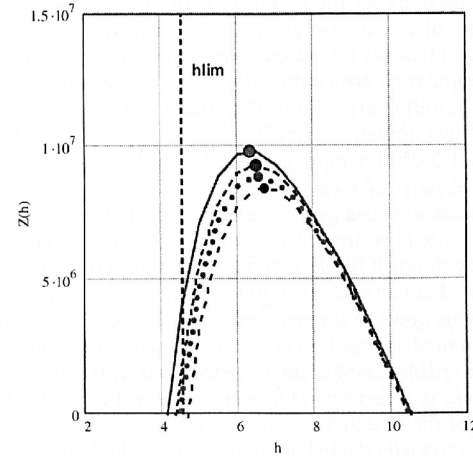

Fig. 13. Objective functions for dam example (solid line = exact $Z_1(h)$, dashed line = appr. $Z_1(h)$, dotted line = exact $Z_2(h)$, dashed–dotted line = appr. $Z_2(h)$).

function is positive between approximately 4 m and 10 m but the domain is limited downwards by h_{lim}. The maximum ρ in $\gamma(t) = \delta + \rho \exp(-at)$ in order to keep the objective function $Z_1(h)$ positive is calculated as $\rho_{max} = 0.043$ which happens to be also the equivalent discount rate γ_∞. If this equivalent constant rate is used in Eq. (10) one determines $h_{opt,1} = 6.47$ m but for Eq. (13) one obtains $h_{opt,2} = 7.26$ m indicating again that the option with systematic reconstruction is slightly preferable. Nevertheless, in this example the difference between the two replacement strategies is remarkably small.

9. Conclusion

Technical facilities should be optimal with respect to benefits and cost. Some suitable objective functions relevant for sustainable building activities are developed based on the renewal model. It is found that the only replacement strategy fulfilling the requirement of sustainability is systematic reconstruction after failure or obsolescence or preventive repair. If the structure is deteriorating a suitable inspection and maintenance strategy has to be designed. Repairs must be planned already in the design phase and must be such that they correspond to a renewal, i.e. re-establish the (stochastically) initial state. The repair intervals together with other design parameters can also be optimized under appropriate conditions. A special technique for solving the optimization problem is presented.

If time is involved all monetary quantities need to be discounted down to the decision point. Discount rates γ must be long term averages in view of the time horizon of some 20 to more than 100 years for the facilities of interest and net of inflation and taxes. While the operator may use long term averages from the financial market for his cost-benefit analysis the assessment of interest rates for investments of the public into sustainability and risk reduction is more difficult. The classical Ramsey model decomposes the output growth rate into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is found that the rate of time preference of consumption should be a little larger than the long term population growth rate if used for the determination of parameters in the acceptability criterion. The output growth rate (= public interest rate) on the other hand should be smaller than the sum of the population growth rate and the long term growth rate of a national economy which is around 2–3% for most industrial countries which is also intergenerationally acceptable from an ethical point of view.

Alternatively, one can use so-called generation-adjusted discounting models. These models assume that all generations should be treated according to their own preferences but discounting is only by the economic growth rate if future generations are concerned. The discounting scheme for such a model is complicated but an equivalent time-dependent discount rate can be defined decaying from some market value down to the economic growth rate. Such schemes, in fact, appear to be far more realistic than models based on economic growth theory only. All in all, discounting plays an important role in public cost-benefit considerations. It is also shown that given a certain interest rate function there is a corresponding maximum real interest rate function in order to maintain non-negativity of the objective function in the public's interest.

The risk of failure and especially the risk to human life and limb must be limited. A general risk acceptability criterion is derived. The SLSC (societal life saving cost = implied cost of averting a fatality) to be used in optimization as live saving or compensation cost and the *societal willingness-to-pay* based on the *societal value of a statistical life* are derived. The acceptability criterion, which is *necessary*, *affordable* and *efficient* from a societal point of view, depends on the marginal cost to reduce the risk, the corresponding marginal decrease in risk, the GDP, the life working time and on demographic factors obtainable from life tables. Predictive cohort life tables should be used. Discounting also enters into the considerations about the societal willingness-to-pay. It turns out that the exact value of the discount rate (or its variation in time) is less important for the public risk acceptability criterion. Key parameters such as the societal life saving cost (SLSC) and the societal value of a statistical life (SVSL) or an equivalent quantity also derivable from the life quality index cluster around 700,000 PPPUS\$ and 2.0 Mill. PPPUS\$, respectively, with very little variation for industrialized countries. They can be appreciably smaller in developing countries with smaller GDP, higher economic and population growth rates and different demographic characteristics. It also appears remarkable that cost-benefit optimal facilities usually provide more safety than the acceptability criterion in most cases if life saving cost are included in the analysis.

Several examples illustrate that the developed methodology is well suited even for complicated problems.

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