



Discussion

Life quality index revisited,  
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This paper requires discussion as some of its assumptions and findings are definitely not in line with the main stream of concepts developed by several other authors. It is probably not so important that the “willingness-to-pay” for a reduction of risk from event-type adverse events as calculated by the author is about half the value determined by others. In many practical problems it is sufficient to estimate the correct order of magnitude. It is probably also not so important that the “willingness-to-pay” as calculated by the author is at the low end of the empirical estimates, for example, as summarized in Viscusi/Aldy [8] or Costa/Kahn [2]. But there are several assumptions which need discussion although they concern matters like human behavior and preferences. Nevertheless, a subject of this importance requires clarification of various aspects.

Nathwani et al. [3] hypothesize that life quality, admittedly simplified, can be measured as a product of two functions, one measuring the utility from consumption and the other measuring the utility from the time of leisure. The utility from consumption on a societal level is quantified as (a part) of the gross domestic product (GDP per capita). The utility from the time of leisure is quantified as part of life expectancy at birth. The time to raise the GDP, i.e., the time spent in paid work is subtracted. Next, Nathwani et al. hypothesize that people on average optimise work and leisure time. Some very convincing mathematical derivations then lead to the life quality index (which is, in fact, a composite utility function with some awkward dimension) in Ditlevsen’s notation

$$Q = G^w E^{1-w} (1 - w)^{1-w} \quad (1)$$

reasonably assuming that the actual value of  $w$  is at or close to its optimum value. The willingness-to-pay is given by Eq. (11) in the paper with  $c = w$  and  $w$  the observed fraction of life expectancy

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for work. A variant of derivation leaves  $E$  in Eq. (1) without an exponent. Interesting further developments can be found in [6] and elsewhere.

Ditlevsen, however, did not start from the work-leisure optimisation principle and the life quality index Eq. (1). He started from Eq. (3) in the form of his Eq. (2) by introducing some other coefficient  $c$  weighing the monetary and the life expectancy part. This appears admissible. However, in his Eq. (4) he introduced a variable productivity parameter. But productivity as defined in macroeconomics definitely cannot be viewed as a parameter manipulable by the individual or by society via governmental regulation in the work-leisure-optimisation process and, therefore, his Eq. (5) is considered as incorrect. In macroeconomics it is defined by GDP/yearly worked hours by those who work. It is exogenously given and depends on technological progress in a country, labour skills and how the available capital is used. These parameters are essentially determined by those contributing to the GDP. Productivity grows slowly with time (and so does life expectancy and GDP while the work time fraction decreases slowly) as can be seen from the following table (data from Maddison [4]). Productivity grew by roughly 0.5% points faster than economic output. Clearly, fluctuations due to political or economical turbulences are averaged out. The surplus in productivity, obviously, went into less working time indicating that there is slightly larger preference for larger earnings than for less working time.

Country	Yearly working hours 1870/1992	Economic growth per capita 1870/1992 in %	Productivity growth 1870/1992 in %
Austria	2935/1576	1.8	2.3
France	2945/1542	1.9	2.5
Germany	2941/1563	1.9	2.3
Italy	2886/1490	1.9	2.6
Netherlands	2964/1338	1.5	2.1
Norway	2945/1565	2.1	2.6
Sweden	2945/1515	1.9	2.4
Switzerland	2984/1645	1.9	2.2
United Kingdom	2984/1491	1.3	1.8
Australia	2945/1631	1.2	1.6
Canada	2964/1656	2.0	2.3
USA	2964/1491	1.8	2.1

The discussor concludes that Ditlevsen's willingness-to-pay is not the same as Nathwani et al.'s and, even if it looks the same, is not based on the same reasoning.

Ditlevsen further introduced a variable output  $G$  of work  $w$  by assuming  $G = pg(w)$ . This is similar to the so-called production function in economics (see, Barro [1]). But Nathwani et al. considered  $w$  as a lifetime sum (fraction of life expectancy) in which, consequently, such details cannot be considered. In addition, the relation  $G = pg(w)$  opens the question how to select the function  $g(w)$  and its parameters.

The other important but debatable assumption is the definition of  $w$  in his Eq. (1). He argues that sleep time is necessary time in order to work and enjoy life. Therefore, he sets only 16 h per day available for work and leisure. As a result he came up with  $c \approx 0.3$  as compared to

$c \approx w \approx 0.125$  or less in [3]. This implies approximately half the value for the willingness-to-pay as compared with the estimates in [7] and half the value in the empirical estimates in Viscusi/Aldy [8]. But, if related to one year it is,  $G = \text{GDP} = pw$ , with  $p$  defined in the macroeconomic sense and  $w$  taken from official statistics, for example from OECD [5].

Another point concerns Ditlevsen's assumption to simply take  $G = \text{GDP}$ . But the GDP also contains substantial parts of national income which must be reinvested in order to maintain the deteriorating production means and also parts used by the government in order to ensure the basic conditions of a society with good life quality, both of which are not available for risk reduction. Only  $60 \pm 20\%$  of the GDP are really available. This would reduce Ditlevsen's willingness-to-pay further.

A final remark appears in order emerging from an increasing number of applications of Eq. (1) and its variants. So far, no example has been found where a design of a technical facility would require changes if the values in the willingness-to-pay formula vary by a factor less than two, say. In practice, many other, additional aspects most likely need to be considered including stochastic modelling, the matter of risk perception and frequently just politics. A simple, clean, convincing and understandable concept is required to guide applications. In this respect Nathwani, Lind and Pandey [3] have admirably been successful and this discussor really has studied and investigated their work just to find that nothing should be changed or added.

## References

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