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# Optimization and risk acceptability based on the Life Quality Index

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**Abstract**

Optimization techniques are essential ingredients of reliability-oriented optimal designs of technical facilities. Suitable objective functions are presented for different replacement strategies for structural facilities. Within FORM/SORM structural reliability analysis can be reduced to an optimization task and some simple algebra. However, instead of a optimization of cost on top of a optimization for the reliability task, a one-level optimization is proposed by adding the Kuhn–Tucker conditions of the locally stationary reliability problem to general cost-benefit optimization. For locally non-stationary failure phenomena a bi-level optimization must be used. A rational basis to account for the cost of saving lives based on the recently proposed Life Quality Index is presented. Several examples illustrate the methodology. © 2002 Published by Elsevier Science Ltd.

*Keywords:* Optimization; Structural reliability; Life Quality Index; Life saving cost; Human value

## 1. Introduction

Traditionally, target reliabilities in engineering have been set implicitly by calibration at past and present practice. It is tacitly assumed that past practice is already nearly optimal although the development of present rules has been widely by trial and error and most frequently in terms of safety factors, cautiously selected nominal or representative design values, as well as suitable quality assurance rules and not in terms of rational reliability measures. The profession agrees that this cannot give totally wrong numbers because those developments for appropriate targets had already a long history. Explicit probabilities or reliabilities of satisfying structural behavior have then been inferred backwards from deterministically looking codes of practice. Doing such calibration analyses surprisingly reveals that there is great variation between different structural members, materials and design practices in terms of probabilities. Most of these variations are extremely difficult to interpret. It is also recognized that for new, extraordinary buildings the

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argument of a long empirical history might not apply. Further it is obvious that some structural components may be grossly overdesigned while others are underdesigned. In summary, it is difficult to believe that past and present practice produce structures which are economically optimal and, simultaneously, "safe enough".

More modern approaches define a so-called ALARP-region (As Low As Reasonably Practicable) between a region which is considered acceptable and another region which is no more acceptable. Usually this is defined in a log-log plot of the occurrence probability of adverse events versus their consequences. Those regions are mostly determined from data on failures. It is noted that different industries tend to define different ALARP-regions reflecting their experiences and also their special demands. The empirical nature of this approach is also evident.

More recently, two important concepts found increased interest. The first concept requires explicitly that technical facilities should be economically optimal (see, for example, [22]). Designing, erecting, maintaining and replacing structural facilities is viewed as a decision problem where maximum expected benefit and least cost are sought and the reliability requirements are fulfilled simultaneously at the decision point. In what follows the basic formulations of the various aspects of the decision problem are outlined making use of some more recent results. A renewal model proposed as early as 1971 by Rosenblueth/Mendoza [28], further developed in [5,26] and extended in [22] is presented in some detail. The second concept introduces a special social indicator, which helps to quantify the necessary investments into structural safety, i.e. the investments to save human lives [16]. This social indicator is rather general. It is applicable for the investments by the public into health care, into road traffic safety, into fire protection systems and, of course, to structural safety. The public does such investments either by itself or via codes, programs or regulations. More specifically, the recently proposed Life Quality Index (LQI) [16] is discussed and applied to structural facilities in the context of expected value optimization. Since all quantities of interest are expressed in monetary terms and time is involved some remarks are made about appropriate discount rates. The theoretical developments are followed by some illustrative examples.

## 2. Optimal structures

A structural facility is optimal if the following objective is maximized (see Fig. 1):

$$Z(\mathbf{p}) = B(\mathbf{p}) - C(\mathbf{p}) - D(\mathbf{p}) \tag{1}$$

Without loss of generality it is assumed that all quantities in Eq. (1) can be measured in monetary units. $B(\mathbf{p})$ is the benefit derived from the existence of the facility, $C(\mathbf{p})$ is the cost of design and construction and $D(\mathbf{p})$ is the cost in case of failure. $\mathbf{p}$ is the vector of all safety relevant parameters. Statistical decision theory dictates that expected values are to be taken [17]. In the following it is assumed that $B(\mathbf{p})$, $C(\mathbf{p})$ and $D(\mathbf{p})$ are differentiable in each component of $\mathbf{p}$. It is reasonably assumed that $B = B(\mathbf{p})$ and that $C(\mathbf{p})$ increases whereas $D(\mathbf{p})$ decreases in each component of $\mathbf{p}$. This is illustrated in Fig. 1. The cost may differ for the different parties involved, e.g. the owner, the builder, the user and society. A structural facility makes sense only if $Z(\mathbf{p})$ is positive within certain parameter ranges for all parties involved. The intersection of these ranges defines reasonable structures (public or other subsidizing excluded).



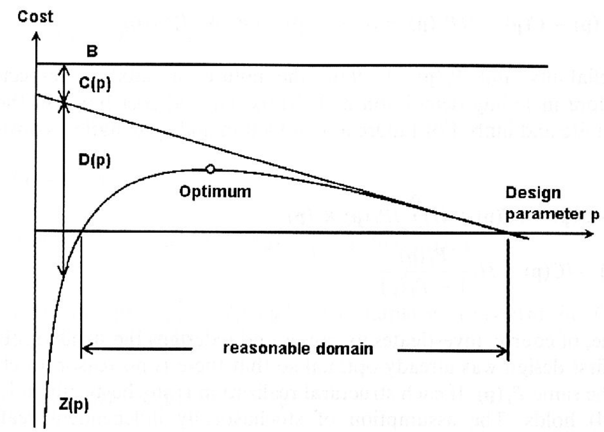


Fig. 1. Objective function after Rosenblueth and Esteva [27].

The structure which eventually will fail after some time will have to be optimized at the decision point, i.e. at time $t = 0$. Therefore, all cost need to be discounted. A continuous discounting function is assumed which is accurate enough for all practical purposes

$$\delta(t) = \exp[-\gamma t] \tag{2}$$

where $\gamma$ is the (tax-free) interest rate. For example, if failure with consequences $D_0$ occurs at time $t$ (in years) the discounted damage is $D(t) = D_0 \exp[-\gamma t]$. If a yearly discount rate $\gamma'$ is defined we have $\gamma = \ln(1+\gamma')$.

It is useful to distinguish between two replacement strategies, one where the facility is given up after service or failure and one where the facility is systematically replaced after failure. Further we distinguish between structures which fail upon completion or never and structures which fail at a random point in time much later due to service loads, extreme external disturbances or deterioration. The first option implies that loads on the structure are time-invariant. Reconstruction times are assumed to be negligibly short. At first sight there is no particular preference for either of the replacement strategies. For infrastructure facilities the second category is a natural strategy. Structures used only once, e.g. special auxiliary construction structures, boosters for space transport vehicles or devices exploiting limited deposits, fall into the first category.

## 3. The renewal model

### 3.1. Failure upon completion due to time-invariant loads

For easy reference the renewal model in structural optimization is first reviewed giving the most important results. The objective function for a structure given up after failure at completion due to time-invariant loads (essentially dead weight) is [28]

$$Z(\mathbf{p}) = B^*R_f(\mathbf{p}) - C(\mathbf{p}) - HP_f(\mathbf{p}) = B^* - C(\mathbf{p}) - (B^* + H)P_f(\mathbf{p}) \quad (3)$$

$R_f(\mathbf{p})$ is the reliability and $P_f(\mathbf{p})=1-R_f(\mathbf{p})$ the failure probability, respectively. $H$ is the direct cost of failure including demolition and debris removal cost but also the cost to reduce the risk to human life and limb. For failure at completion and systematic reconstruction we have [28]:

$$\begin{aligned} Z(\mathbf{p}) &= B^* - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{i=1}^{\infty} iP_f(\mathbf{p})^i R_f(\mathbf{p}) \\ &= B^* - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{P_f(\mathbf{p})}{1 - P_f(\mathbf{p})} \end{aligned} \quad (4)$$

After failure one, of course, investigates its causes and redesigns the structure. However, we will assume that the first design was already optimal so that there is no reason to change the design rules leading to the same $P_f(\mathbf{p})$. If each structural realization is stochastically independent of each other formula (4) holds. The assumption of stochastically independent realizations is also important subsequently.

A certain ambiguity exists when assessing the benefit $B^*$ taken here and in the following as independent of $\mathbf{p}$. If the intended time of use of the facility is $t_s$ it is simply

$$B^* = B(t_s) = \int_0^{t_s} b(t)\delta(t)dt \quad (5)$$

For constant benefit (net of taxes) per time $b(t)=b$ one determines

$$B^* = B(t_s) = \frac{b}{\gamma}[1 - \exp[-\gamma t_s]] \quad (6)$$

and, therefore, for $t_s \to \infty$

$$B^* = \frac{b}{\gamma} \quad (7)$$

Here and in the following we understand $b=\beta C_0$ with $\beta$ a number between 0 and 0.3, say and $C_0$ the construction cost independent of $\mathbf{p}$. It is noted that only very few problems in structural reliability are time-invariant.

### 3.2. Random failure in time

Assume random failure events in time. The time to the first event has distribution function $F_1(t, \mathbf{p})$ with probability density $f_1(t, \mathbf{p})$. If the structure is given up after completion of service at time $t_s$ or failure it is obviously



$$B(t_s) = \int_0^{t_s} b(\tau)\delta(\tau)(1 - F_1(\tau, \mathbf{p}))d\tau \quad (8)$$

$$D(t_s) = \int_0^{t_s} \delta(\tau)f_1(\tau, \mathbf{p})Hd\tau \quad (9)$$

and, therefore, for $b(t)=b$:

$$Z(\mathbf{p}) = \int_0^{t_s} b(\tau)\delta(\tau)(1 - F_1(\tau, \mathbf{p}))d\tau - C(\mathbf{p}) - \int_0^{t_s} \delta(\tau)f_1(\tau, \mathbf{p})Hd\tau \quad (10)$$

For $t_s \to \infty$ and $f_1^*(\gamma, \mathbf{p}) = \int_0^{\infty} e^{-\gamma t}f_1(t, \mathbf{p})dt$ the Laplace transform of $f_1(t, \mathbf{p})$ and using $F_1^*(\gamma, \mathbf{p}) = f_1^*(\gamma, \mathbf{p})/\gamma$ it simplifies to:

$$Z(\mathbf{p}) = \frac{b}{\gamma}\left(1 - f_1^*(\gamma, \mathbf{p})\right) - C(\mathbf{p}) - Hf_1^*(\gamma, \mathbf{p}) \quad (11)$$

If, in particular, the events follow a stationary Poisson process with intensity $\lambda(\mathbf{p})$ we have

$$f_1^*(\gamma, \mathbf{p}) = f^*(\gamma, \mathbf{p}) = \int_0^{\infty} \exp[-\gamma t]\lambda(\mathbf{p})\exp[-\lambda(\mathbf{p})t]dt = \frac{\lambda(\mathbf{p})}{\gamma + \lambda(\mathbf{p})} \quad (12)$$

and therefore:

$$Z(\mathbf{p}) = \frac{b}{\gamma + \lambda(\mathbf{p})} - C(\mathbf{p}) - H\frac{\lambda(\mathbf{p})}{\gamma + \lambda(\mathbf{p})} \quad (13)$$

For the more important case of systematic reconstruction we generalize our model slightly. Assume that the time to first failure has density $f_1(t, \mathbf{p})$ while all other times between failure are independent of each other and have density $f(t, \mathbf{p})$, i.e. failures and subsequent renewals follow a modified renewal process [3]. This makes sense because extreme loading events usually are not controllable, i.e. the time origin lies somewhere between the zeroth and first event. The independence assumption is more critical. It implies that the structures are realized with independent resistances at each renewal according to the same design rules and the loads on the structures are independent, at least asymptotically. For constant benefit per time unit $b(t)=b$ and $f_n(t, \mathbf{p})$ the density of the time to the $n$th renewal an objective function can be derived by making use of the convolution theorem for Laplace transforms

$$\begin{aligned} Z(\mathbf{p}) &= \int_0^{\infty} be^{-\gamma t}dt - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^{\infty}\int_0^{\infty} e^{-\gamma t}f_n(t, \mathbf{p})dt \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{f_1^*(\gamma, \mathbf{p})}{1 - f^*(\gamma, \mathbf{p})} \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)h_1^*(\gamma, \mathbf{p}) \end{aligned} \quad (14)$$

where $h_1^*(\gamma, \mathbf{p})$ is the Laplace transform of the renewal density (renewal intensity) $h_1(t, \mathbf{p})$. For regular renewal processes one replaces $f_1^*(\gamma, \mathbf{p})$ by $f^*(\gamma, \mathbf{p})$ and $h_1^*(\gamma, \mathbf{p})$ by $h^*(\gamma, \mathbf{p})$. For the renewal density and its Laplace transform there is an important asymptotic result [3]:

$$\lim_{t\to\infty} h(t, \mathbf{p}) \lim_{\gamma\to 0} \gamma h^*(\gamma, \mathbf{p}) = \frac{1}{E[T_f(\mathbf{p})]} \tag{15}$$

where $E[T_f(\mathbf{p})]$ is the mean time between renewals (or failures).

Finally, if at an extreme loading event (e.g. flood, wind storm, earthquake, explosion) failure occurs with probability $P_f(\mathbf{p})$ and $f_1(t)$ and $f(t)$, respectively, denote the densities of the times between independent loading events one obtains by similar considerations \cite [26]:

$$h_1^*(\gamma, \mathbf{p}) = \sum_{n=1}^{\infty} f_1^*(\gamma) P_f(\mathbf{p}) \left[f^*(\gamma) R_f(\mathbf{p})\right]^{n-1} = \frac{P_f(\mathbf{p}) f_1^*(\gamma)}{1 - R_f(\mathbf{p}) f^*(\gamma)} \tag{16}$$

with $R_f(\mathbf{p}) = 1 - P_f(\mathbf{p})$. If, in particular, the events follow a stationary Poisson process with intensity $\lambda(\mathbf{p})$ we have

$$h^*(\gamma, \mathbf{p}) = \frac{\lambda(\mathbf{p})}{\gamma} \tag{17}$$

This result is of great importance because structural failures should, in fact, be rare, independent events. Then, the Poisson intensity $\lambda(\mathbf{p})$ can be replaced by the so-called outcrossing rate $\nu^+(\mathbf{p})$ to be described below—even in the locally non-stationary case. For the case treated in Eq. (15) we have for stationary Poissonian load occurrences:

$$h^*(\gamma, \mathbf{p}) = \frac{P_f(\mathbf{p}) \frac{\lambda}{\gamma+\lambda}}{1 - (1 - P_f(\mathbf{p})) \frac{\lambda}{\gamma+\lambda}} = \frac{P_f(\mathbf{p})\lambda}{\gamma + P_f(\mathbf{p})\lambda} \tag{18}$$

Only a few other failure time distributions have analytic Laplace transforms. Taking Laplace transforms numerically requires some effort but taking the inverse Laplace transform must simply be considered as an numerically ill-posed problem. Then, however, one always can resort to the asymptotic result Eq. (14) which can be shown to be accurate enough for all practical purposes [22]. Of course, the mean time between renewals then needs to be determined [23].

The foregoing results can be generalized to cover also multiple mode failure, loss of serviceability and obsolescence of the facility [22]. Also, the important case of non-constant benefit, a special case of obsolescence, has been addressed [6].

All these formulations do not differ substantially from those used in economic cost-benefit analyses under uncertainty. In accordance with economic theory benefits and (expected) cost, whatever types of benefits and cost are considered, should be discounted by the same rate as done above. Different parties, e.g. the owner or the public, may, however, use different rates.



## 4. The value of human life and limb—an unethical question?

Structural and other technical facilities as well as our natural environment involve some risk to human life and limb. Even if it is true that some of the risks in our society are unknown or ignored for whatever reason, the risks which are known must be controlled—with all material and intellectual resources we have. What should then be the rational procedure to take care of the value of human life in cost-benefit analyses for technical projects?

First of all, modern approaches to the question of risk to human lives do not speak of a monetary value of the human life but rather speak of the cost to save lives or, more precisely, the cost to reduce the risk to life. Secondly, any further argumentation must be within the framework of our moral and ethical principles as laid down in our constitutions and elsewhere. We quote as an example two articles from the BASIC LAW of the Federal Republic of Germany:

- Article 2: (1) Everyone has the right to the free development of his personality. (2) Everyone has the right to life and to inviolability of his person
- Article 3: (1) All persons are equal before the law. (2) Men and women have equal rights. (3) No one may be prejudiced or favored because of his sex, his parentage, his race, his language, his homeland and origin, his faith or his religious or political opinions.

The same principles are found in all modern, democratic constitutions including the Universal Declaration of Human Rights of the United Nations adopted without dissent by the General Assembly on December 10, 1948. But H. D. Thoreau (1817–1862) realistically says about the value of human life in his book "*Walden*": "*The cost of a thing is the amount of what I will call life which is required to be exchanged for it, immediately or in the long run...*" (cited in [16]).

We further postulate that it is only the democratic society itself which is authorized to decide on the human value in cost-benefit analyses. Nor are these experts, builders or representatives of the executive power or administration without consulting the rest of the society. On the other hand, the perception of risk in the public, be it voluntary or involuntary risks to health or technical and environmental risks, and the measures to control the risks need some comment in the light of the decision theoretic approach taken up in the last section. Media and politicians appear to play a dominant role. This is dangerous from a decision theoretic point of view if risk controlling measures do not have a rationale basis. Only if society "plays a fair game against risks", i.e. in which it puts its stakes efficiently into risk control according to what society can afford, can society gain improved life quality and safety in the long run.

Can these value fixings be translated into engineering acceptability criteria? This is possible when starting from certain social indicators such as life expectancy, gross domestic (national) product (GDP), state of health, education profile, etc. Cantril [1] concludes from his empirical studies that life expectancy and wealth are among the primary concerns of humans in a modern society. Life expectancy at birth (mean time to death) $e$ is the area under the survivor curve (survival or reliability function) $l(a)$ as a function of age $a$, i.e. $e = e(0) = \int_0^{a_u} l(a)da$ ($a_u$ = largest age considered). It makes sense to adjust it for times in poor health or times in hospital so that the "quality adjusted" (disability adjusted) life expectancy $e_{\mathrm{QALY}}$ is about 90% of $e$. However, this adjustment will not be considered in the following. Another suitable indicator of the quality of life is the GDP per capita $g$. The GDP is roughly the sum of all incomes created by labor and capital (stored labor) in a country during a year. It is a measure for the productivity of a society. It provides the infrastructure of a country, its social structure, its cultural and educational offers, its ecological conditions among others but also the means for the individual enjoyment of life. In most countries some 60% of the GDP is, in fact, used privately, 20% by the state and the rest by investments. Most importantly in our context, it creates the possibilities to "buy" additional life years through better medical care, improved safety in road traffic, more safety in or around building facilities or from hazardous technical activities, more safety from natural hazards etc. In fact, 6–14% of the GDP are spent for health care in one way or another in industrialized countries. In our context it does not matter whether those investments are carried out individually or by the state via taxes.

Lind [11] sets out from a composite social indicator:

$$L = L(a, b, \ldots, q, \ldots) \tag{19}$$

Let it be differentiable so that:

$$\mathrm{d}L = \frac{\partial L}{\partial a}\mathrm{d}a + \frac{\partial L}{\partial b}\mathrm{d}b + \ldots + \frac{\partial L}{\partial q}\mathrm{d}q + \ldots \tag{20}$$

If only the two factors mentioned before, that is $g$ and $e$, are considered $\mathrm{d}L$ vanishes for

$$\frac{dg}{de} = -\frac{\frac{dL}{dq}}{\frac{dL}{dg}} \tag{21}$$

implying that a change in $e$ should be compensated for by an appropriate change in $g$.

Nathwani et al. [16] propose a special composite social indicator, the LQI, in the form:

$$L = f(g)h(t) \tag{22}$$

The LQI is something like an anonymous person's utility function. The time spent in economic activities (including travel times) is $we$ $(0 < w < 1)$, so that the time for the enjoyment of life is:

$$t = (1 - w)e \tag{23}$$

The first factor $f(g)$ measures the level of quality of life and the second factor $h(t)$ quantifies the duration to enjoy life. In order to be "discriminate" both quantities should be independent. This might not be completely true. Economists support the idea of independence, however. For example, they showed that the growth in GDP and life expectancy historically developed quite independently, the latter being mainly influenced by the advances in hygiene, preventive methods and therapeutic techniques which, certainly, are only weakly and indirectly related to the GDP [4]. Let $f(g)$ and $h(t)$ be monotonically increasing, positive functions. Then,

$$\mathrm{d}L = \frac{\mathrm{d}f(g)}{\mathrm{d}g}h(t)\mathrm{d}g + f(g)\frac{\mathrm{d}h(t)}{\mathrm{d}t}\mathrm{d}t \tag{24}$$



and for the relative increment:

$$\frac{\mathrm{d}L}{L} = \frac{g}{f(g)}\frac{\mathrm{d}f(g)}{\mathrm{d}g}\frac{\mathrm{d}g}{g} + \frac{t}{h(t)}\frac{\mathrm{d}h(t)}{\mathrm{d}t}\frac{\mathrm{d}t}{t} = k_g\frac{\mathrm{d}g}{g} + k_t\frac{\mathrm{d}t}{t} \tag{25}$$

In order to be universal Nathwani et al. [16] require that applicability of the LQI, i.e. $\frac{\mathrm{d}L}{L}$ should not depend on absolute values of $g$ and $e$ (= independence of the year, the state of development of a region, etc.). This means that $k_g$ and $k_t$ are constants or, better, $k_g/k_t$ = const. With $c_1$ and $c_2$ certain fixed values the two differential equations

$$k_g \equiv \frac{g}{f(g)}\frac{\mathrm{d}f(g)}{\mathrm{d}g} = c_1 \tag{26}$$

$$k_t \equiv \frac{t}{h(t)}\frac{\mathrm{d}h(t)}{\mathrm{d}t} = c_2 \tag{27}$$

have, apart from some arbitrary constants, the solutions:

$$f(g) = g^{c_1} \tag{28}$$

$$h(t) = t^{c_2} \tag{29}$$

Using Eqs. (26) and (27) then yields for Eq. (22):

$$L = g^{c_1}((1 - w)e)^{c_2} \tag{30}$$

"*Presumably, people on the average work just enough so that the marginal value of wealth produced, or income earned, is equal to the marginal value of the time they lose when at work*" [16]. Accepting this, assume then that $g$ is proportional to the time $we$ spent in economic activities.

$$L = (cwe)^{c_1}((1 - w)e)^{c_2} \tag{31}$$

$c$ is a proportionality constant depending on the productivity of the group of persons considered. Most work is dirty, dull, boring, troublesome and sometimes dangerous. Here, we mean work necessary for raising $g$ and not work done additionally for satisfaction. The time of work (including travel time) $we$ certainly is not available for enjoying life. It is then reasonable to assume that the average individual maximizes its LQI with respect to the quantity $w$. At the maximum it is

$$\frac{\mathrm{d}L}{\mathrm{d}w} = \frac{\mathrm{d}}{\mathrm{d}w}((cwe)^{c_1}((1 - w)e)^{c_2}) = 0 \tag{32}$$

from which, for example:

$$c_1 = c_2 \frac{w}{1 - w} \tag{33}$$

Setting $c_1 + c_2 = \bar{c}$ results in $c_1 = \bar{c}w$ and $c_2 = \bar{c}(1-w)$ and therefore:

$$L = g^{\bar{c}w} e^{\bar{c}(1-w)} \bar{c}(1-w)^{\bar{c}(1-w)} \approx g^{\bar{c}w} e^{\bar{c}(1-w)}$$

In industrialized countries $w$ varies between less than 10 and a little more than 15% of $e$. The factor $(1-w)^{1-w}$ varies only insignificantly for realistic values of $w$ and, therefore, is omitted subsequently. Without loss of generality, we can set $\bar{c} = 1$ so that finally:

$$L = g^w e^{1-w} \tag{34}$$

Using Eq. (21) this finally yields a general acceptance criterion for investments into life saving projects:

$$\frac{dg}{de} = -\frac{\frac{\partial L}{\partial e}}{\frac{\partial L}{\partial g}} \geqslant -\frac{g}{e}\frac{(1-w)}{w} \tag{35}$$

or

$$\frac{dg}{g} + \frac{(1-w)}{w}\frac{de}{e} \geqslant 0$$

It is noted that criterion (35) is independent of $\bar{c}$. It is also independent of the proportion of the $g$ available for private use because such factors cancel out. The work–leisure aspect in Eq. (35), i.e. the factor $\frac{(1-w)}{w}$, increases the ratio $\frac{g}{e}$ by a factor of around 7. Equality in Eq. (35) corresponds to "optimal" investments into life saving which was also the condition for Eq. (21), ">" means that investments into life saving are inefficient and projects having "<" are not admissible because article 2 of the above mentioned BASIC LAW is violated. Criterion (35) gives an indication of what is necessary and also affordable to a society for life saving undertakings. The second version in Eq. (35) is easy to interpret. A relative negative change in $g$ (for a life saving investment, for example) must be compensated for by a relative positive change in $e$ multiplied by $\frac{(1-w)}{w}$. Vice versa—a positive change in $g$ may be associated with a negative change in $e$. Whenever a given incremental increase in life expectancy by some life saving operation (positive d$e$) is associated with larger than optimal incremental cost (negative d$g$) one should invest into alternatives of life saving. From a practical point of view it is important to note that all quantities on the right-hand side of Eq. (35) are easily available and can be updated any time. The democratic equality principle (article 2 of the above mentioned BASIC LAW) dictates that mean values for $g$, $e$ and $w$ have to be taken. Any deviations from mean values for any specific group of people need to be justified carefully if Eq. (35) is applied to projects with involuntary risks. In projects where certain groups of people must take higher risks it should be fair to provide compensation by higher incomes. This compensation can be determined by maintaining the value of the LQI.



There is no doubt that the LQI can be improved by inclusion of additional social indicators. Minor modifications have already been proposed. Most criticism has focused on the reasoning leading to the final functional form of the LQI, especially with respect to the parameter $w$. The need of two exponents for $g$ and $e$ has also been recognized for other composite social indicators such as the human development index (HDI) [35] or the earlier life product index (LPI) [10]. But the concept leading to the special form Eq. (34) provides a straightforward interpretation and, apparently, has numerically the right order of magnitude. The beauty of the LQI in its present form is its simplicity and transparency and strong arguments must be found to make it invalid. It sets the framework for rational debates about investments into life saving projects. Much further discussion is provided in [10,11,16].

There are values which cannot be accounted for directly, such as the loss of cultural heritage, the extinction of a rare species, other environmental aspects or religious beliefs. For example, good family relations have been ranked high as a measure for the quality of life [1] which, as other "intangibles", can hardly be addressed specifically by the LQI. It would be necessary to quantify those values in monetary terms, i.e. the extent to which a society would be willing to reduce $g$ to guarantee those other values. This reduced value of $g$ is no more disposable for "buying" additional life years and also not available for the individual enjoyment of life. Modern societies, no doubt, set some portion of $g$ aside to maintain such values but consensus is always difficult about the amount and the specific value addressed. On the other hand, life quality enjoyed during time $e(1-w)$ might be determined not only by $g$ for many individuals but also by an intact cultural heritage and ecological environment so that the LQI indirectly can measure also these values recognizing the fact that constant factors drop out in Eq. (35).

Practical application of Eq. (35) in a life saving operation is not always easy (see, however, the many examples in [16]). In general, the cost involved in some life saving operation can be estimated with some accuracy. The estimation of the effect of the life saving operation is more difficult. At first, we estimate the cost of averting a fatality in terms of the gain in life expectancy $\Delta e$. Such a value will be important in the optimization considerations to come. The cost of the safety measure is expressed as a reduction $\Delta g$ of the GDP. This *i*mplied *c*ost of *a*verting a *f*atality (*ICAF*) can be obtained from the equality of Eq. (35) after separation and integration from $g$ to $g + \Delta g$ and $e$ to $e + \Delta e$, i.e. the cost $\Delta C = -\Delta g$ per year to extend a persons life by $\Delta e$ is:

$$\Delta C = -\Delta g = g\left[1 - \left(1 + \frac{\Delta e}{e}\right)^{1-\frac{1}{w}}\right] \tag{36}$$

Because $\Delta C$ is a yearly cost and the (undiscounted) *ICAF* has to be spend for safety related investments into technical projects at the decision point $t = 0$, one should multiply by $e$, set $e = \Delta e$, and

$$ICAF = |\Delta g|\, e \tag{37}$$

follows. The value of *ICAF* is shown in Table 1 for some countries with different development status together with the parameter $G_F$ to be explained below. The *ICAF* should never be taken as the value of human life. Such a value does not exist. "*The value of human life is infinite and beyond measure*", irrespective of age, sex, social position, etc. The ICAF, however, is a number which society should be willing to pay for saving lives according to its ethical principles and which society can afford, i.e. for safety-relevant regulations in cost-benefit calculations. It enters into optimization as a fictitious number at the decision point. It is independent of interest or benefit rates.

The ICAF-value can also be an indication for the magnitude of a possible monetary compensation of the relatives of victims in the event. In accordance with [18] a compensation of the relatives of victims is or should be covered by insurance. The premium for it reduces the benefit of an undertaking. In accordance with the decision theoretic concept, it makes sense to recommend that the public specifies a fair compensation for the relatives of victims, due right after the event, by about the value of *ICAF* or the *ICAF* multiplied by the number of lost life years $e_r$ in an event, i.e. by

$$ICAF(e_r) = g\left[1 - \left(1 + \frac{e_r}{e}\right)^{1-\frac{1}{w}}\right]e_r$$

$ICAF(e_r)$ grows approximately linearly with $e_r$. In case of failure of a technical object $e_r$ is between $0.67e$ (for young groups with a triangular age distribution) and approximately $0.5e$ (for aging groups) on average. The societal equality principle mentioned above prohibits to differentiate with respect to special ages with in a group. The details of this proposal remain still to be discussed.

Direct application of criterion (35) can be done if the gain (or loss) in life expectancy is measured by changes in the mortality rate of a group as proposed in [16]. Let the (continuous) age specific death rate (= number of deaths in $[a, a+\mathrm{d}a]$/number of people at age $a$) $m(a)$ in a reference cohort (mostly 100 000) be known from a period life table. The probability of death at age $a$ can be approximated by $q(a) = 1 - \exp[-m(a)]$ $(q(a_u) = 1)$. $l(a + \mathrm{d}a) = l(a)(1 - q(a))$ is the number of survivors at age $a + da$ and $l(0) = 100\,000$. The number of deaths in $[a, a+\mathrm{d}a]$ is $d(\mathrm{a}) = q(a)l(a)$. The numbers of deaths in all age intervals sum up to the cohort size and this means that $q(a)$ can approximately be interpreted as a hazard (risk, failure, mortality or death) rate $\mu(a)$ if we set $l(0) = 1$. Then, the numbers just mentioned transform into proportions. This allows to recover all other distributional characteristics of interest, for example, the survival probability $l(a) = \exp\left[-\int_0^a \mu(\tau)\mathrm{d}\tau\right]$ at age $a$ or the life expectancy $e(a) = \int_a^{a_u} l(t)\mathrm{d}t = \int_a^{a_u} \exp\left[-\int_0^t \mu(\tau)\mathrm{d}\tau\right]\mathrm{d}t$. If $\mu_1(a)$ is the mortality rate before the operation and, by some life saving operation, the mortality rate is changed into $\mu_2(a)$, then, $\mathrm{d}e \approx \Delta e = e_2 - e_1$. The procedure for abridged or full discrete period life tables is slightly different. It is clear that the quantification of changes from $\mu_1(a)$ to $\mu_2(a)$, in general, is most difficult. Changes in mortality at young age change life expectancy significantly whereas there is little change in life expectancy for changes above 60, say. The estimation of $\mathrm{d}e$ must generally be done numerically because the changes from $\mu_1(a)$ to $\mu_2(a)$ can be non-uniform.

However, if some life saving operation reduces mortality uniformly and this is close to what must generally be expected in applications to technical facilities, the impact of a reduction of risk can be measured in terms of a (small) change $\mathrm{d}M$ of (crude) mortality $M$ (= number of deaths due to all causes in a group per year/group size). Let $\delta = \mathrm{d}M/M$. In this case $\mu_2(a) = \mu_1(a)(1+\delta)$. It then follows that $l_2(a) = \exp\left[-\int_0^a \mu_1(\tau)(1+\delta)\mathrm{d}\tau\right] = l_1(a)^{1+\delta}$ and, therefore, $e_2(0) = \int_0^{a_u} l_1(t)^{1+\delta}\mathrm{d}t$. Nathwani et al. [16] relate the quantity $\mathrm{d}e/e$ empirically to changes $\mathrm{d}M$ by



$$\frac{\mathrm{d}e}{e} \approx -C_{FM}\mathrm{d}M \approx -C_{F\delta}\frac{\mathrm{d}M}{M} \tag{38}$$

with $C_{FM} \approx 19$ ($C_{F\delta} \approx 0.14$) for Canada with $M \approx 0.0073$ in 1990. The approximation for $\mathrm{d}e/e$ and a (small) proportional change $\delta = \mathrm{d}M/M$ in $\mu(a)$ can also be derived analytically [12]

$$\frac{\mathrm{d}e}{e} = \frac{\frac{\mathrm{d}}{\mathrm{d}\delta}\int_0^{a_u} l(a)^{1+\delta}\mathrm{d}a}{\int_0^{a_u} l(a)\mathrm{d}a} = \frac{\int_0^{a_u} \ln(l(a))l(a)^{1+\delta}\mathrm{d}a}{\int_0^{a_u} l(a)\mathrm{d}a} \approx -C_{F\delta}\delta \tag{39}$$

where $C_{F\delta} \approx 0.13$ to more than 0.45 depending on the age structure and life expectancy of the group. The constant $0 \leqslant C_{F\delta} \leqslant 1$ is a measure for the shape (convexity) of the curve $l(a)$. $C_{F\delta} = 0$ corresponds to case where all people die at the same age, i.e. for $l(a) = 1(0 \leqslant a \leqslant a_x)$, $C_{F\delta} = 0.5$ to a linearly decreasing function $l(a)$ and $C_{F\delta} = 1$ to the case of constant mortality $\mu$ at all ages, i.e. to $l(a) = \exp[-\mu a]$. Females generally have a smaller $C_{F\delta}$-value than males. For example, by the German period life table of 1993 $C_{F\delta} \approx 0.11$ for females and $C_{F\delta} \approx 0.15$ for males. If by some life saving operation overall mortality is reduced by $x$%, life expectancy is extended by only $C_{F\delta}\,x$%. The approximation on the right hand side of the equation results from a first order expansion of the left hand side around $\delta = 0$. In fact, Eqs. (38) and (39) coincide for the same population, i.e. $C_{FM} = \frac{C_{F\delta}}{M}$. $C_{F\delta}$ changes with time (state of demographic development) as pointed out in [12] and illustrated by Fig. 2 for the USA in the last 100 years [38].

One sees that $\mathrm{d}e/e$ can be estimated if $\delta$ is known. If only $\mathrm{d}M$ is given one needs to know $C_{F\delta}$ and $M$. Table 1 collects some data and the constant $C_{F\delta}$ for countries for which it was possible to obtain complete or abridged, more recent life tables. Table 1 also covers the extremes of socio-economic development. All life tables used for the determination of $C_{F\delta}$ come from years between 1990 and 2000. All are period life tables which give a representative picture of the health and reproductive status in a society for a certain year or small range of years. So-called generation life tables that collect the death data from an cohorte born in year $x$ over the length of the life table $x + a_u$ are difficult to obtain. Only very few exist. But they can be constructed from period life tables after the time changes of $q(x, t)$ have been quantified. They probably would be more informative for our predictive purposes. In fact, due to the advances in geriatry the survivor functions show a tendency to become more compact in recent years (larger proportion of older people) implying smaller values of $C_{F\delta}$, at least for developed countries. For example, $C_{F\delta}$ is by some 5% smaller by the German generation life table for year 1993 as compared to the corresponding period life table. The working time fractions have been estimated taking into account reported average working hours per week, length of holidays, unemployment rate and amount of indirect unpaid work. As $e$ and $g$, the working time fractions $w$ should be average values per capita. The labor force in a country is about 50% of the population which has to be taken into account appropriately. Unfortunately, no appropriate statistics are available. However, the exact value of $w$ is not very important. Crude mortality $M$ and the corresponding birth rate $B$ together with life expectancy $e$ give a first indication about the health state and age structure in a population. Not all recognized but always minor inconsistencies between different data sources could be removed. The values in Table 1 must be viewed as first estimates. All are subject to minor statistical error, variations in time and sometimes the specific way how they have been computed.

It appears that both $C_{F\delta}$ and $e$ approach some asymptote at $C_{F\delta} \approx 0.13$ and a little more than $e \approx 80$ for developed countries, respectively. Countries with high human development have $C_{F\delta} \approx 0.15$, with medium human development $C_{F\delta} \approx 0.25$ and $C_{F\delta} \approx 0.35$ and more with low human development, respectively. Further demographic interpretations of Fig. 2 or Table 1 are beyond the scope of this paper. The *ICAF*-value clearly shows that the various countries can afford quite different cost for life saving interventions. Because *ICAF* as well as $C_{F\delta}$ depend on time (see Fig. 2) it certainly would be a good idea to use time averages. Unfortunately, the data situation does not allow to do this at the present time except for very few countries.

Table 1
Social indicators for several countries

| Country | $g$[a] | $M,B$[b] | $e, e_{QALY}$[c] | $w$[d] | *ICAF*[e] | $G_F$ | $C_{F\delta}$ |
|---|---|---|---|---|---|---|---|
| Brazil | 7320 | 0.00934, 0.01845 | 67, 59 | 0.15 | $4.8 \cdot 10^5$ | $9.3 \cdot 10^5$ | 0.21[f] |
| Canada | 27 330 | 0.00730, 0.01121 | 79, 72 | 0.125 | $2.1 \cdot 10^6$ | $3.4 \cdot 10^6$ | 0.13 |
| Colombia | 5890 | 0.00523, 0.02241 | 70, 61 | 0.15 | $4.0 \cdot 10^5$ | $1.3 \cdot 10^6$ | 0.21[f] |
| Ecuador | 2900 | 0.00544, 0.02544 | 70, 61 | 0.15 | $2.0 \cdot 10^5$ | $6.0 \cdot 10^5$ | 0.20[f] |
| Haiti | 1500 | 0.01500, 0.03168 | 54, 42 | 0.18 | $7.8 \cdot 10^4$ | $1.5 \cdot 10^5$ | 0.34[f] |
| Mexico | 8810 | 0.00502, 0.02277 | 72, 68 | 0.15 | $6.2 \cdot 10^5$ | $1.8 \cdot 10^6$ | 0.18[g] |
| USA | 34 260 | 0.00870, 0.01420 | 77, 70 | 0.15 | $2.6 \cdot 10^6$ | $4.7 \cdot 10^6$ | 0.15 |
| Austria | 26 310 | 0.00980, 0.00974 | 77, 72 | 0.125 | $2.0 \cdot 10^6$ | $3.6 \cdot 10^6$ | 0.15 |
| Czech Republic | 12 900 | 0.01081, 0.00911 | 73, 68 | 0.125 | $9.3 \cdot 10^5$ | $1.3 \cdot 10^6$ | 0.15 |
| France | 24 470 | 0.00909, 0.01210 | 78, 73 | 0.125 | $1.9 \cdot 10^6$ | $2.6 \cdot 10^6$ | 0.14 |
| Germany | 25 010 | 0.01042, 0.00916 | 77, 71 | 0.125 | $1.9 \cdot 10^6$ | $2.2 \cdot 10^6$ | 0.13 |
| Ireland | 25 470 | 0.00807, 0.01457 | 76, 70 | 0.125 | $1.9 \cdot 10^6$ | $3.1 \cdot 10^6$ | 0.14 |
| Norway | 29 760 | 0.00983, 0.01260 | 78, 73 | 0.125 | $2.3 \cdot 10^6$ | $2.7 \cdot 10^6$ | 0.13 |
| Poland | 9030 | 0.00998, 0.01020 | 73, 66 | 0.125 | $6.6 \cdot 10^5$ | $1.4 \cdot 10^6$ | 0.17 |
| Sweden | 23770 | 0.01061, 0.00991 | 79, 73 | 0.125 | $1.9 \cdot 10^6$ | $2.2 \cdot 10^6$ | 0.14 |
| Ukraine | 3710 | 0.01643, 0.00931 | 69, 63 | 0.125 | $2.5 \cdot 10^5$ | $3.2 \cdot 10^5$ | 0.20[g] |
| China | 3940 | 0.00674, 0.01595 | 70, 63 | 0.18 | $2.6 \cdot 10^5$ | $5.1 \cdot 10^5$ | 0.19[g] |
| India | 2390 | 0.00874, 0.02428 | 63, 53 | 0.15 | $1.5 \cdot 10^5$ | $3.7 \cdot 10^5$ | 0.31[g] |
| Indonesia | 2840 | 0.00630, 0.02226 | 65, 60 | 0.15 | $1.8 \cdot 10^5$ | $6.6 \cdot 10^5$ | 0.26[g] |
| Jordan | 4040 | 0.00262, 0.02544 | 71, 60 | 0.15 | $2.8 \cdot 10^5$ | $1.4 \cdot 10^6$ | 0.16[g] |
| Japan | 26 460 | 0.00834, 0.01004 | 80, 74 | 0.15 | $2.1 \cdot 10^6$ | $2.3 \cdot 10^6$ | 0.13 |
| Turkey | 7031 | 0.00595, 0.01831 | 69, 63 | 0.15 | $4.8 \cdot 10^5$ | $1.4 \cdot 10^6$ | 0.21[g] |
| Egypt | 3690 | 0.00770, 0.02489 | 66, 58 | 0.15 | $2.4 \cdot 10^5$ | $6.2 \cdot 10^5$ | 0.23[g] |
| Kenya | 1010 | 0.01435, 0.02850 | 52, 39 | 0.15 | $5.2 \cdot 10^4$ | $1.9 \cdot 10^5$ | 0.48[g] |
| Mozambique | 820 | 0.02421, 0.03720 | 46, 35 | 0.18 | $3.7 \cdot 10^4$ | $9.1 \cdot 10^4$ | 0.59[g] |
| Nigeria | 790 | 0.01391, 0.03969 | 47, 36 | 0.18 | $3.7 \cdot 10^4$ | $1.3 \cdot 10^5$ | 0.52[g] |
| South Africa | 9180 | 0.01577, 0.02112 | 55, 40 | 0.125 | $5.0 \cdot 10^5$ | $1.4 \cdot 10^6$ | 0.43[g] |
| Australia | 25 370 | 0.00718,.01286 | 78/73 | 0.10 | $2.0 \cdot 10^6$ | $4.0 \cdot 10^6$ | 0.16[g] |

[a] In PPP US$ [36].
[b] Birth rate [2].
[c] Rough estimate of quality adjusted life expectancy at birth [37].
[d] Estimated.
[e] ICAF based on not quality adjusted life expectancy.
[f] Calculated from abridged life tables in [29].
[g] Calculated from abridged life tables in [37].



## 5. Application to technical facilities

The application to safety regulations for structures or other technical facilities can be done as follows. We start directly from Eq. (35). It can reasonably be assumed that the life risk in and from such facilities is uniformly distributed over the age and sexes of those affected. Also, it is assumed that everybody uses such facilities and, therefore, is exposed to possible fatal accidents. The total cost of a safety related regulation per member of the group and year is

$$dg = -dC(\mathbf{p}) = -\frac{1}{N}\sum_{i=1}^{n} dC_i(\mathbf{p}) \tag{40}$$

where $n$ is the total number of objects under discussion, each with incremental cost $dC_i$ and $N$ is the group size. Let $N_F$ be the total mean number of fatalities avoided per year by the regulation. Inserting into the inequality following Eq. (35) and using Eq. (38) gives:

$$\frac{-dC(\mathbf{p})}{g} + \frac{(1-w)}{w}(-C_{FM}dM) = \frac{-dC(\mathbf{p})}{g} + \frac{(1-w)}{w}\left(-C_{F\delta}\frac{dM}{M}\right) \geqslant 0$$

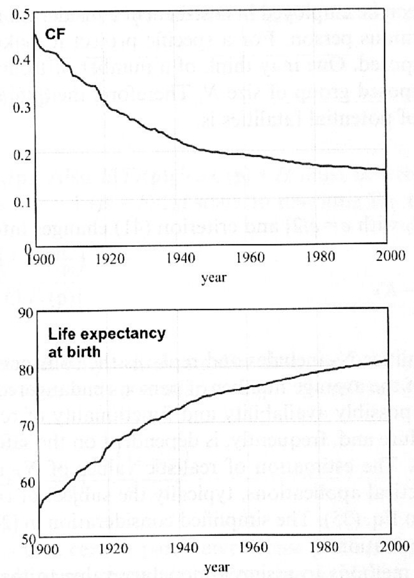


Fig. 2. $C_{F\delta}$ and life expectancy for the USA (1900–2000).

If one uses the relationship in Eq. (38) also for the change in quality adjusted life expectancy and replaces $\mathrm{d}M$ by the failure rate $\mathrm{d}h(\mathbf{p})$ ($t\to\infty$) (or, at least, sets it proportional to $\mathrm{d}M$) one finds

$$\frac{\mathrm{d}C(\mathbf{p})}{\mathrm{d}h(\mathbf{p})} \geqslant -kC_{FM}g\frac{(1-w)}{w} = -k\frac{C_{F\delta}}{M}g\frac{(1-w)}{w} = -k\ G_F \quad (41)$$

where $\mathrm{d}M=k\ \mathrm{d}h(\mathbf{p})$ with $k\leqslant 1$ another constant relating the changes in mortality to the changes in the failure rate. The constant $k$ may be interpreted as a person's probability of actually being killed in case of failure. Note that for any reasonable intervention there is $\mathrm{d}h(\mathbf{p})<0$. The values $G_F$ are also given in Table 1 for comparison. $G_F$ is about twice as large as ICAF. If residual life expectancy is half of life expectancy as given in column 4 the *ICAF*-values are just half of the values given. The *ICAF* is relatively insensitive to the working time fraction $w$. In contrast, $G_F$ is roughly inversely proportional to $w$. If only half of the values for $w$ in Table 1 are taken and this is probably most realistic, the values of $G_F$ in Table 1 must be approximately doubled. The *ICAF* depends explicitely on life expectancy, the constant $G_F$ only indirectly via $C_{F\delta}/M$.

It remains to answer the question whether a criterion like Eq. (41) derived for regulations for a larger group in a society or the entire society can also be applied to individual technical projects. The constant $G_F$ and, similarly, the *ICAF* are derived from general considerations of changes in mortality or directly from Eq. (35). They are valid as long as the ethical principles are maintained, the specific GDP and mortality rate of the considered group are used and the technical project is in the publics interest—remember that saving lives from hazards is in the public interest by its constitution. Therefore, it can be employed in cost-benefit considerations. The $G_F$ as well as *ICAF* were related to one anonymous person. For a specific project it makes sense to apply criterion (41) to the whole group exposed. One may think of a number of technical projects each with $N_F$ potential fatalities in an exposed group of size $N$. Therefore, the "life saving cost" of a technical project with $N_F$ the mean of potential fatalities is

$$H_F = ICAF\ N_F \quad (42)$$

[or, possibly, $ICAF(e_r)\ N_F$ with $e=e/2$] and criterion (41) changes into:

$$\frac{\mathrm{d}C(\mathbf{p})}{\mathrm{d}h(\mathbf{p})} \geqslant -G_F\ N_F = -K_F \quad (43)$$

The mean number of fatalities $N_F$ includes and replaces the constant $k$ in Eq. (41). $N_F$ must be estimated taking account of the average number of persons endangered by the event, the severity and suddenness of failure, possibly availability and functionality of rescue systems, etc. $N_F$ also depends on the cause of failure and, frequently, is dependent on the safety measures, for example, on the design parameter $\mathbf{p}$. The estimation of realistic values of $N_F$ might belong to the most difficult tasks in actual practical applications, typically the subject of risk analysis. Eq. (43) is an efficiency criterion similar to Eq. (35). The simplified consideration in [24] should be considered as superceded by the above derivation.

A few other quantitative methods to assign a monetary value to the human life to be used in cost-benefit analyses exist the most important of which is the human capital method. The cost to



save a life by this method has been set most frequently as $g\ \Delta e$, i.e. the accumulated loss in GDP over $\Delta e$, the mean of the lost life years in the event. In general this amounts to too low values, as judged from the perspectives of the LQI. This section concerns a particular group or a particular country. Whenever hazards by technical objects take place on an international level further considerations are necessary with respect to Eq. (42) as well as Eq. (43).

## 6. Remarks about interest and benefit rates

Cost-benefit optimization as in Eq. (3), (4), (11) or (14) must use interest rates. Considering the time horizon of some 20 to more than 100 years for most structural facilities and other industrial installations involving risks it is clear that average interest rates net of in/deflation and taxes must be chosen, i.e. time averages like $\gamma = E[\gamma(t)] = \frac{1}{t_u}\int_0^{t_u\gg t_s}\gamma(\tau)\mathrm{d}\tau$. If the option with systematic reconstruction is chosen in the asymptotic form

$$Z(\mathbf{p}) = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{1}{\gamma E[T_f(\mathbf{p})]} \quad (44)$$

one immediately sees that the interest rate must be non-zero. Larger interest rates, in general, move the optimum of $Z(\mathbf{p})$ to smaller $\|\mathbf{p}^*\|$ or less safe structures. A constant $b$ does not affect the position of the optimum. From the same equation we see by solving $Z(\mathbf{p})=0$ for $\gamma$ that there is a maximum interest rate $\gamma_{\max}$ for which $Z(\mathbf{p})$ becomes negative for any $\mathbf{p}$ ($b$=const.) [6]

$$\gamma_{\max}(\mathbf{p}) = \frac{E[T_f(\mathbf{p})]b - (C(\mathbf{p}) + H)}{E[T_f(\mathbf{p})]C(\mathbf{p})} \quad (45)$$

and, therefore, $0<\gamma\leqslant\gamma_{\max}(\mathbf{p})$. Also $E[T_f(\mathbf{p})]\,b>C(\mathbf{p})+H$ must be valid for any reasonable project. Further, there must be $\beta/\gamma>1$ ($\beta=b/C_0$) since, in rewriting Eq. (45):

$$\gamma_{\max}(\mathbf{p}) = \beta\left(\frac{C_0}{C(\mathbf{p})}\right) - \frac{\left(1+\frac{H}{C(\mathbf{p})}\right)}{E[T_f(\mathbf{p})]}$$

In general, the second term is only a little smaller than the first, a few percent less than the benefit rate $\beta$, say. $\gamma_{\max}(\mathbf{p})$ varies with $\mathbf{p}$ similar to $Z(\mathbf{p})$. Therefore, we can define another optimization task:

$$\text{Maximize}:\quad \gamma_{\max}(\mathbf{p}) = \beta\left(\frac{C_0}{C(\mathbf{p})}\right) - \frac{\left(1+\frac{H}{C(\mathbf{p})}\right)}{E[T_f(\mathbf{p})]} \quad (46)$$

Solution of Eq. (46) leads to a certain parameter vector $\hat{\mathbf{p}}$. Consequently, solution of Eq. (3), (4), (10) or (14) using the maximum admissible interest rate would first require the solution of Eq. (46). It turns out in many example calculations with a wide range of parameters that $\|\hat{\mathbf{p}}\|$ is close or (numerically) equal to the solution $\|\mathbf{p}^*\|$ of Eq. (3), (4), (10) or (14) and, therefore, $\gamma_{max}(\hat{\mathbf{p}}) \approx \gamma_{max}(\mathbf{p}^*)$ in good approximation. If the maximum rate $\gamma_{max}(\mathbf{p}^*)$ is used, Eq. (44) implies $Z(\mathbf{p}^*) = 0$ at the optimum solution point $\mathbf{p}^*$. Interest rates $\gamma$ less than $\gamma_{max}$ extend the admissible range for $\mathbf{p}$ which may be advisable in applications. The formulae for $\gamma_{max}$ for the other cases dealt with in Section 3.2 are essentially the same with appropriate reinterpretation of the term $1/(\gamma E[T_f(\mathbf{p})])$. Also, non-constant benefit over time may be taken into account similarly. If $\beta$ is expected to vary randomly in a stationary manner we simply have $\beta = E[\beta(t)] = \frac{1}{t_u}\int_0^{t_u \gg t_s} \beta(\tau)\mathrm{d}\tau$, similar to the procedure for randomly varying stationary interest rates. The case of systematically varying benefit rates discussed in [6] will be studied in a separate paper.

Very small interest rates, on the other hand, cause benefit and damage cost to dominate over the erection cost. Then, in the limit [6]

$$Z(\mathbf{p}) = b - \frac{(C(\mathbf{p}) + H)}{E[T_f(\mathbf{p})]} \tag{47}$$

where the interest rate vanishes. Erection cost are normally weakly increasing in the components of $\mathbf{p}$ but $E[T_f(\mathbf{p})]$ grows, according to our assumptions, significantly in $\mathbf{p}$. Consequently, the optimum is reached for $E[T_f(\mathbf{p})] \to \infty$, that is for perfect safety which is not attainable in practice. In other words: the interest rate must be distinctly different from zero but must not be too large.

The cost for saving life years in Eq. (42) also enters into the objective function and with it the question of discounting those cost also arises. At first sight this is not in agreement with our moral value system. However, a number of studies summarized in [11,19] express a rather clear opinion based on ethical and economical arguments. The cost for saving life years must be discounted at the same rate as other investments, especially in view of the fact that our present value system should be maintained for future generations, a goal which is supported by empirical studies on human preferences quoted in [11]. Otherwise serious inconsistencies cannot be avoided.

What should then the discount rate for investments into life saving projects be? It is beyond the scope of this paper to review the relevant economic literature but economists tend to prefer relatively low rates for various reasons. Due to the requirement $\beta/\gamma > 1$ stated just below Eq. (45), the interest rate is strongly related to the benefit a society earns from its various activities. But what is the benefit rate of the public? A first estimate could be based on the long term growth rate of the GDP. The growth rate measures the success of all activities of a society—among them also activities for saving lives. In most developed, industrial countries the growth rate was a little more than 2% over the last 50 years. The United Nations Human Development Report 2000 [34] gives values between 1.2 and 1.9% for industrialized countries during 1975–1998. If one extends the consideration to the last 150 years using data in [15,33,34] one finds an average growth rate $r = \frac{\ln(g_{1998}/g_{1850})}{1998-1850}100$ of about 1.8% (see Table 2). One also could use the rate with which the quantity $ICAF$ (see Eq. (37)) or the like was discounted over the years [32]. The corresponding formula is $\gamma_K = \frac{\ln(ICAF_{1998}/ICAF_{1850})}{1998-1850}100$. The advantage of using this approach is that it also takes account of changes in $e$ and $w$ in first approximation. Table 2 has been compiled from a more detailed table taking into account $w_{1998} = 0.125$ and $w_{1850} = 2\ w_{1998}$. Uncertainties in this table primarily arise from converting the historical value of a monetary unit into present values of US$.



The slightly larger rates $\gamma_K$ as compared to $r$ reflect the favorable changes in life expectancy and working time from year 1850 to 1998. For the last 150 years the equivalent discount rate $\gamma_K$ is between 1.2 and 2.6%. It is noted that the first half of the last century was below average while the second half was well above average. The author is inclined to propose either $\beta \approx r$ or $\beta \approx \gamma_K$. The differences between $\gamma_K$ and $r$ are small enough to be of little practical relevance. Given $\beta$ Eq. (44) then allows to compute the maximum interest rate $\gamma_{max}$ that can be used. $\gamma_{max}$ turns out to be some 5–20% smaller than the benefit rate $\beta$ in the parameter domain of interest.

The considerations in Table 2 can at least define the range of benefit and interest rates to be used in long term investments into life saving operations. Since public benefit and interest rates are close together so that $Z_S(\mathbf{p}^*)$ (index "S" stands for optimization in the society's interest, "O" for optimization in the owner's interest) is positive but close to zero we postulate that the acceptable risk for any undertaking is the risk associated with $\mathbf{p}^*$ as the result of optimization in the public interest. Even if the (long term) growth rate of the GDP is negative (the mean growth rate for the group of countries with low human development in the last 25 years, in fact, is only $r = -0.018$ according to [34]) an optimization of Eq. (3), (4), (10) or (14) is still possible but the requirement of $Z_S(\mathbf{p}^*)$ being positive can no more be maintained. One has to be satisfied with the owner's solution of $Z_O(\mathbf{p})$ being maximum and positive. Note that the public still profits from the undertaking because the owner creates jobs and pays taxes. However, the public may require that Eq. (43) is satisfied. The owner may use interest rates taken from the financial market at the decision point $t = 0$. Nevertheless, the question of public interest and benefit rates appears to require more basic research.

## 7. A one-level optimization for structural components

Let us now turn to the technical aspects of optimization. Cost-benefit optimization according to Eq. (3), (4), (10) or (14) in principle requires two levels of optimization, one to minimize cost and the other to solve the reliability problem if FORM/SORM techniques in the standardized space of

Table 2
Social indices for some developed industrial countries[a]

| Year | 1850 | | | 1998 | | | | |
|---|---|---|---|---|---|---|---|---|
| Country | $g$ | $e$ | $ICAF$ | $g$ | $e$ | $ICAF$ | $\gamma_K(\%)$ | $r(\%)$ |
| UK | 3109 | 39.5 | $1.7\cdot10^5$ | 21 400 | 77.4 | $1.6\cdot10^6$ | 1.5 | 1.3 |
| USA | 1886 | 29.5 | $7.8\cdot10^4$ | 29 680 | 76.7 | $2.3\cdot10^6$ | 2.3 | 1.9 |
| France | 1840 | 40.0 | $1.0\cdot10^5$ | 24 900 | 78.1 | $1.9\cdot10^6$ | 2.0 | 1.8 |
| Netherlands | 2482 | 37.3 | $1.3\cdot10^5$ | 24 800 | 77.9 | $1.9\cdot10^6$ | 1.8 | 1.6 |
| Sweden | 1394 | 43.9 | $8.6\cdot10^4$ | 25 600 | 78.6 | $2.0\cdot10^6$ | 2.1 | 2.0 |
| Germany | 1400 | 37.1 | $7.2\cdot10^4$ | 26 600 | 77.2 | $2.0\cdot10^6$ | 2.3 | 2.1 |
| Australia | 4027 | 46.0 | $2.6\cdot10^5$ | 20 600 | 78.3 | $1.6\cdot10^6$ | 1.2 | 1.1 |
| Japan | 969 | 38.0 | $5.1\cdot10^4$ | 32 600 | 80.1 | $2.6\cdot10^6$ | 2.6 | 2.5 |
| Average | 2138 | 41.3 | $1.2\cdot10^5$ | 28 610 | 78.5 | $2.0\cdot10^6$ | 2.0 | 1.8 |

[a] All monetary values in US$, 1998, not corrected for purchasing power parity.

uncertain variables are used. However, it is possible to reduce it to one level by adding the Kuhn–Tucker condition of the reliability problem to the cost optimization task provided that the reliability task is formulated in the transformed standard space [13]. For the task in Eq. (4) we have

$$\begin{aligned}&\text{Maximize}: \quad Z(\mathbf{p}) = B^* - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)\tfrac{P_f(\mathbf{p})}{1-P_f(\mathbf{p})}\\ &\text{Subject to}:\\ &\qquad g(\mathbf{u},\mathbf{p}) = 0\\ &\qquad u_i\|\nabla_u g(u,\mathbf{p})\| + \nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})_i\|\mathbf{u}\| = 0; \quad i = 1,\ldots,n-1\\ &\qquad h_k(\mathbf{p}) \leqslant 0, k = 1,\ldots,q\end{aligned} \tag{48}$$

where the first and second condition represent the Kuhn–Tucker condition for a valid "critical" point, the third condition some restrictions on the vector **p** of optimization variables. $g(\mathbf{u},\mathbf{p}) \leqslant 0$ denotes the failure domain and **U** the vector of standard normal variables transformed from the original space **X**. Frequently, the term $\frac{P_f(\mathbf{p})}{1-P_f(\mathbf{p})}$ in the objective can be replaced by $P_f(\mathbf{p})$. The interest rate enters into the benefit term. The failure consequences are now decomposed into direct cost $H_M$ (and, possibly, indirect failure cost such as loss of business, service, etc.) and life saving cost $H_F$. The failure probability using FORM/SORM is [7]

$$P_f(\mathbf{p}) \approx \Phi(-\beta(\mathbf{p}))C_{\mathrm{SORM}} \tag{49}$$

and we have to require that $\|\mathbf{u}\| \neq 0$ and $\|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\| \neq 0$. $\Phi(.)$ is the standard normal distribution function and $\beta(\mathbf{p}) = \|u^*\|$ the geometrical reliability index with $\mathbf{u}^*$ the solution point of Eq. (48) in u-space. It is assumed that the second-order correction $C_{\mathrm{SORM}}$ is nearly independent of **p**. $\nabla_p C(\mathbf{p})$ usually must be determined numerically. For $\nabla_p P_f(\mathbf{p})$ one can employ an asymptotic result [8]

$$\frac{\partial P_f(\mathbf{p})}{\partial p_i} = \frac{\partial \Phi(-\beta(\mathbf{p}))}{\partial p_i}C_{\mathrm{SORM}} \approx -\varphi(\beta(\mathbf{p}))\frac{\partial \beta(p)}{\partial p_i}C_{\mathrm{SORM}} \approx -\varphi(\beta(\mathbf{p}))\frac{\frac{\partial}{\partial p_i}g(\mathbf{u},\mathbf{p})}{\|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\|}C_{\mathrm{SORM}} \tag{50}$$

where $\varphi(.)$ is the standard normal density. Further, it is reasonable to assume that $\nabla pC(\mathrm{p}) > 0$ and $\nabla_p P_f(p) > 0$.

For stationary time-variant problems as in Eq. (13) one finds the outcrossing rate for a combination of a rectangular wave renewal process and a differentiable vector process as [21]:

$$\nu^+(\mathbf{p}) \leqslant \left(\sum_{i=1}^{n_J}\kappa_i\Phi(-\beta(p)) + \omega_0\frac{\varphi(\beta(p))}{\sqrt{2\pi}}\right)C_{\mathrm{SORM}} \tag{51}$$

$\kappa_i$ is the jump rate of the $n_J$ components of the rectangular wave renewal process and $\omega_0$ central cycling frequency of the differentiable (vector) process. The optimization task is

$$\begin{aligned}&\text{Maximize}: \quad Z(\mathbf{p}) = \tfrac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)\tfrac{\nu^+(\mathbf{p})}{\gamma}\\ &\text{Subject to}:\\ &\qquad g(\mathbf{u},\mathbf{p}) = 0\\ &\qquad u_i\|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\| + \nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})_i\|\mathbf{u}\| = 0; \quad i = 1,\ldots,n-1\\ &\qquad h_k(\mathbf{p}) \leqslant 0, k = 1,\ldots,q\end{aligned} \tag{52}$$



Now it is:

$$\frac{\partial \nu^+(\mathbf{p})}{\partial p_i} = -\left(\sum_{i=1}^{n_J}\kappa_i\varphi(\beta(\mathbf{p})) + \frac{\omega_0}{\sqrt{2\pi}}\varphi(\beta(\mathbf{p}))\beta(\mathbf{p})\right)\frac{\frac{\partial}{\partial p_i}g(\mathbf{u},\mathbf{p})}{\|\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\|}C_{\mathrm{SORM}} \tag{53}$$

Again, we require $\nabla pC(\mathrm{p}) > 0$ and $\nabla p\nu^+(p) > 0$.

For the case in Eq. (18) one replaces $\frac{\nu^+(\mathbf{p})}{\gamma}$ by $\frac{\lambda P_f(\mathbf{p})}{\gamma + \lambda P_f(\mathbf{p})}$. If the failure process cannot be approximated by a Poisson process or if failure is due to deterioration or the failure density has to be determined numerically one has to use $\frac{1}{\gamma E[T_f(\mathbf{p})]}$, instead. Finally, if the structure can fail in several modes and if it can conservatively be assumed that the modes are uncorrelated, the quantities $P_f(\mathbf{p})$, $\nu^+(\mathbf{p})$, $\lambda P_f(\mathbf{p})$ or $\frac{1}{E[T_f(\mathbf{p})]}$ must be replaced by a sum each term of which corresponds to a failure mode. The different cases are illustrated below at simple examples.

The formulations Eqs. (48) and (52) do not include the LQI-criterion Eq. (43) with good reason. Assume that the conditions $h_k(\mathbf{p}) \leqslant 0$ are not active in the solution point. At the optimum there must be $\nabla_p Z(\mathbf{p}) = \mathbf{0}$. This is written out for the general, asymptotic version of Eq. (52) for $\mathbf{p} = \mathbf{p}^*$

$$\nabla_p Z(\mathbf{p}) = \nabla_p C(\mathbf{p})\left(1 + \frac{1}{\gamma E[T_f(\mathbf{p})]}\right) + \left[\frac{(C(\mathbf{p}) + H_M + H_F)}{\gamma}\nabla_p\left(\frac{1}{E[T_f(\mathbf{p})]}\right)\right] = 0 \tag{54}$$

which is to be compared with the equality of Eq. (43) written as:

$$\nabla_p C(\mathbf{p}) + kK_F\nabla_p\left(\frac{1}{E[T_f(\mathbf{p})]}\right) = 0 \tag{55}$$

Ignoring the small term $\frac{1}{\gamma E[T_f(\mathbf{p})]}$ we see that there is almost always $(C(\mathbf{p}) + H_M + H_F)/\gamma \geqslant kK_F$ under conditions of interest here. Thus, at the optimal solution for Eq. (52) the LQI-criterion Eq. (43) will be fulfilled automatically. In other words: Optimal structures are always safer than the LQI-criterion would require. It is believed that this result is one of the most important insights obtained in this study. It even provides an alternative interpretation of the LQI-criterion because we can recover the following optimization task from Eq. (55)

$$\text{Minimize}: \quad Z'(\mathbf{p}) = C(\mathbf{p}) + kK_F\frac{1}{E[T_f(\mathbf{p})]} \tag{56}$$

which is equivalent to require that the LQI is to be maximized. For the other failure models similar results can be obtained.

The optimization tasks in Eq. (48) or in (52) are conveniently performed by suitable algorithms (for example, [31,20]). For both formulations Eqs. (48) and (52), respectively, gradient-based optimizers require the gradients of the objective as well as the gradients of all constraints. This means that second derivatives are required in order to calculate the gradient of the second condition. This is also the most serious objection against this form of a one level approach. Some good but still laborious approximations are given in [25]. One can, however, proceed iteratively for well-behaved failure surfaces. Initially, one assumes a linear or linearized failure surface and sets $C^{(0)}_{SORM} = 1$. Then, all entries $\frac{\partial^2 g(\mathbf{u},\mathbf{p})}{\partial u_i \partial u_j}$ are zero. After a first solution of problem Eq. (48) or (52) one determines the Hessian once in the solution point ($\mathbf{u}^{*(1)}$, $\mathbf{p}^{(1)}$) and with it also calculates $C^{(1)}_{SORM}$. Problems Eq. (48) or (52) are then solved a second time with fixed Hessian $\mathbf{G}(\mathbf{u}^{*(1)}, \mathbf{p}^{(1)})$. This scheme is repeated until convergence is reached which usually is after a few steps. From a practical point of view it is frequently sufficient to use first-order reliability results and no iteration is necessary. In fact, at the expense of some more numerical effort, one can use any update of the first-order result $\Phi(-\beta(\mathbf{p}))$, for example an update by importance sampling provided that the result of importance sampling is formulated as a correction factor to the first-order result.

In closing this section it is necessary to mention that many technical aspects of optimization are not yet fully understood. The optimization tasks as formulated in Eq. (48) or (52) are among the easiest one can think of. In practice safety related design decisions additionally include changes in the lay-out, in the structural system or in the maintenance strategy. Optimization is over discrete sets of design alternatives. Clearly, this is more difficult and very little is known how to do it formally except in a heuristic, empirical manner in small dimensions.

## 8. Examples

The examples to follow are designed to illustrate the application of the LQI-criterion and various formulations of the objective function. All examples are simplified to some degree but nevertheless can give an impression of the impact of the LQI on optimal design decisions. The calculations are performed for parameters as the public would have to use except stated otherwise. In all cases $dM = dP_f$ is used. Further, $H_F = K_F$ is taken except stated otherwise. All monetary investments are without financing cost.

### 8.1. Example 1: governmental health care program

We first illustrate application of Eq. (35) at an example from governmental health care. A certain disease with an expected yearly death toll of $N_F = 100$ in a population of $N = 6\cdot 10^7$ can be controlled almost perfectly by certain public interventions. This implies a yearly probability of death of $P_f = 1.6\cdot 10^{-6}$ per person and year. The societal data are: $M = 0.01$, $w = 0.125$, $g = 22\ 000$ *EUR*. The relative reduction of the GDP is $\partial g/g = -(\partial cN)/(gN) = -C/(gN)$ with $C$ the total cost for the society. Further, the overall change in (crude) mortality is $dM = N_F/N$ and Eq. (39) can be used to estimate $de/e$. Inserted into Eq. (35) and solved for $C$ with $C_{F\delta} = 0.15$ results in:

$$C = \frac{1 - w}{w}\frac{C_{F\delta}}{M} N_F g = 2.3\cdot 10^8 EUR \tag{57}$$

This is the yearly cost the society should spend to control the disease and which it can afford. If the more realistc value of $w \approx 0.0625$ is taken, the intervention cost is approximately doubled.



### 8.2. Example 2: a (simplified) flood protection problem in a developing country

In a developing country a part of a town will be flooded by a river every 10 years because the existing dams are too low (present dam height is 4.0 m) and in bad shape. For historical and cultural reasons it is not possible to abandon that part of the town. The number of fatalities in a flood is around 500 and the economical losses amount to $H_M = 10^8$ US\$. Both numbers vary significantly from event to event. The alternatives for protection are

- Repair of existing dams
- Repair and heightening of existing dams
- Construction of an additional diversion channel through uninhabited desert land

Simple repair of existing dams would probably avoid flooding of the town due to dam failure. Floods would then still occur with mean return period of about 80 years which is not acceptable. Repair and heightening of the dam could almost eliminate the occurrence of floods depending on the dam height. The proposed diversion channel will reduce the occurrence of floods. It is estimated that they are still likely to occur every 500 years because of failure of operation of the gates in the barrier upstream due to insufficient maintenance. The yearly exceedance probability of the flood level is $P_f(h) = P(H \geq h) = \exp[-(\frac{h}{b})^3]$ with $b = 3.0$ determined from rather inaccurate data and given that suitable dams are present. The cost of dam heightening are $C(h) = c\cdot h^2\cdot L$, $c = 150$, $L = 10\ 000$ m. The cost of the diversion channel have been estimated as $6.0\cdot 10^7$ US\$. All other data are $H_F = 1.4\cdot 10^8$ with $g = 2000$, $w = 0.15$, and $\gamma = 0.05$. It is assumed that after a flood the repair cost are only 20% of the total construction cost. Minimization of

$$Z(h) = ch^2 L + (0.2ch^2 L + H_M + H_F)\frac{P(h)}{\gamma} \tag{58}$$

without any benefit term yields an optimal dam height of $h_{opt} = 5.98$ m. Overtopping then has a mean return period of 2700 years. The construction cost are $5.4\cdot 10^7$ US\$. The alternative of a diversion channel is more expensive. Application of criterion (43)

$$K_F\left|\frac{\partial P_f(h)}{\partial h}\right| - \left|\frac{\partial C(h)}{\partial h}\right| \leq 0 \tag{59}$$

shows that it is fulfilled at a dam height of $h_{min} = 4.27$ m. Checking of criterion (43) with a flood probability of 1/10 before and 1/500 after construction shows that criterion (43) is also fulfilled for the discharge channel option. When the life saving cost are not taken into account the optimum dam height is $h_{opt} = 5.92$ m with return period 2300 years and the construction cost are $5.2\cdot 10^7$ US\$. Whether the project makes socio-economic sense must be determined by further considerations. It may be mentioned that the optimum height for the conditions of a developed country with the same (because of prewarning and evacuation plans then extreme) number of fatalities, i.e. $H_F = 1.4\cdot 10^9$, and $H_M = 5\cdot 10^8$ US\$, is $h_{opt} = 6.25$ m with construction cost $5.9\cdot 10^7$ US\$. In this case the option of a diversion channel becomes more attractive.

*8.3. Example 3: simple log-normal resistance-demand problem*

As a first example from the structures area we take a rather simple case of a system where failure is defined if a random resistance or capacity is exceeded by a random demand. The demand is modelled as a one-dimensional, stationary marked Poissonian renewal process of disturbances (earthquakes, wind storms, explosions, etc.) with stationary renewal rate $\lambda$ and random, independent sizes of the disturbances $S_i$, $i=1,2,...$ The resistance is log-normally distributed with mean $p$ and a coefficient of variation $V_R$. The disturbances are also independently log-normally distributed with mean equal to unity and coefficient of variation $V_S$. A disturbance causes failure with probability:

$$P_f(p) = \Phi\left(-\frac{\ln\left\{p\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right\}}{\sqrt{\ln((1+V_R^2)(1+V_S^2))}}\right) \tag{60}$$

Thus, the failure rate is $\lambda P_f(\mathbf{p})$ and the Laplace transform of the renewal density is [Eq. (18)]:

$$h^*(\gamma, p) = \frac{\lambda P_f(p)}{\gamma + \lambda P_f(p)} \tag{61}$$

An appropriate objective function for maximization given systematic reconstruction then is:

$$\frac{Z(p)}{C_0} = \frac{b}{\gamma C_0} - \left(1 + \frac{C_1}{C_0}p^a\right) - \left(1 + \frac{C_1}{C_0}p^a + \frac{H_M}{C_0} + \frac{H_F}{C_0}\right)\frac{\lambda P_f(p)}{\gamma + \lambda P_f(p)} \tag{62}$$

The criterion (43) has the form:

$$\frac{\mathrm{d}}{\mathrm{d}p}(C_0 + C_1 p^a) \geq -K_F \frac{\mathrm{d}}{\mathrm{d}p}\lambda P_f(p) \tag{63}$$

Some more or less realistic, typical parameter assumptions are: $C_0=10^6$, $C_1=10^4$, $a=1.25$, $H_M=3\cdot C_0$, $V_R=0.2$, $V_S=0.3$ and $\lambda=1$[1/year]. The LQI-data is $e=e_r=77$, $g=25000$, $M=0.01$, $C_{F8}=0.15$, $w=0.125$, $N_F=1$ so that $H_F=1.9\cdot10^6$ and $K_F=2.6\cdot10^6$. Monetary values are in US$. Optimization is performed for the public and for the owner separately.

For the public $b=0.02\cdot C_0$ and $\gamma=0.0185$ are chosen. Optimization including the cost $H_F$ and the corresponding $b$ and $\gamma$ gives $p^*=4.199$, the corresponding failure rate is $1.9\cdot10^{-5}$. $Z_S(p^*)=0.015$ and $Z_S(p)$ is positive in the interval $p^*=[3.65,5.35]$. Solution of Eq. (46) yields a maximum admissible interest rate of $\gamma_{\max}=0.019$ at $p_\gamma=4.196$. For $\beta=\gamma=0.019$ instead, $p^*=4.19$ and $Z_S(p^*)\approx0$ (see Fig. 3). Criterion (43) is already fulfilled for $p_{\lim}=2.73$ corresponding to a failure rate of $1.8\cdot10^{-3}$ but $Z(p_{\lim})=-0.495$ being strongly negative.

The owner uses some typical values of $b=0.07\cdot C_0$ and $\gamma=0.05$ and does not include life saving cost. The calculations yield $p^*=3.76$, the corresponding failure rate is $7.1\cdot10^{-5}$. $Z_O(p^*)=0.342$



and $Z_O(p)$ is positive in the interval $p^*=[2.43,19.12]$. If the owner includes life saving cost maintaining his values for $b$ and $\gamma$ the optimal value is $p^*=3.92$, which is insignificantly larger. Comparing the construction cost from society's and owner's view point shows that society requires about 0.6% more investment cost. The cost $H_F$ become significant for buildings with comparatively low cost $H_M$ and/or low marginal cost $C_1$. This situation is typical for normal homes and smaller office buildings. The life saving cost dominate optimization whenever a large number of fatalities must be expected in the failure event. In this example one observes approximate inverse proportionality of the optimum failure rate and the expected number of fatalities.

*8.4. Example 4: definition of upper and lower bound of ALARP region*

In this example an attempt is made to define and reinterpret the bounds of the ALARP region mentioned in the introduction. As stated earlier the objective must be positive for each party. For the special but important case of Poissonian failure events with rate $\lambda(\mathbf{p})$ this gives the acceptability criterion for systematic reconstruction

$$\frac{\beta C_0}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)\frac{\lambda(\mathbf{p})}{\gamma} \geq 0$$

at the optimum $\mathbf{p}=\mathbf{p}^*$ or:

$$\lambda(\mathbf{p}^*) \leq \frac{\left(\beta\frac{C_0}{C(\mathbf{p}^*)} - \gamma\right)}{\left(1 + \frac{H_M+H_F}{C(\mathbf{p}^*)}\right)} \tag{64}$$

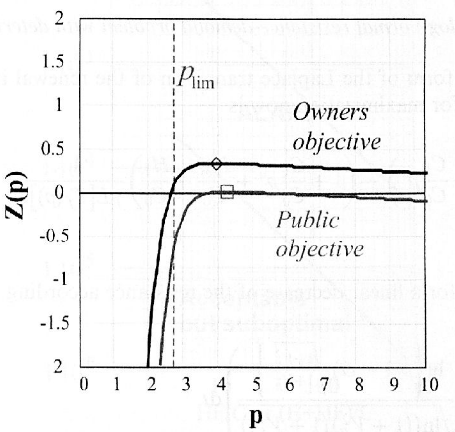


Fig. 3. Public's and owner's objective function for example 3.

The criterion says that the failure rate must be smaller than the difference of benefit (multiplied by $\frac{C_0}{C(\mathbf{p}^*)} \approx 1$) and interest rate divided by a factor which is between 1 and 10 or more under practical conditions. This suggests to construct a lower bound for the ALARP region. The upper bound may be constructed from Eq. (56). It is clear that at the minimum there must be $Z'(\mathbf{p})/C(\mathbf{p}) > 1$. With $K_F \approx H_F$ it follows that:

$$\lambda(\mathbf{p}_{\lim}) \leqslant \left(\frac{Z'(\mathbf{p}_{\lim})}{C(\mathbf{p}_{\lim})} - 1\right)\frac{1}{\frac{H_F}{C(\mathbf{p}_{\lim})}} \tag{65}$$

The values of $\mathbf{p}^*$ and $\mathbf{p}_{\lim}$ must be determined for each combination of $C(\mathbf{p})$ and $H_F$ or $H_M + H_F$, respectively.

For Fig. 4 the assumptions as in example 3 but $\beta = 0.045$ and $\gamma = 0.02$ are used. Both bounds show the well-known relationship between failure rates and losses. The usual slope of 45° in the log-log scale can be found for both curves in good approximation. But the curves depend on benefit and interest rates and on the specific stochastic problem at hand. Beyond a loss of $(H_M + H_F)/C(p^*) = 367$ the objective function for the lower curve is no more positive. Only a higher benefit rate would extend the admissible region to higher losses. The interpretation of the upper bound is as usual but $Z(p_{\lim})$ is negative throughout. The lower bound corresponds to the optimal solution. For given $\beta$ and $\gamma$ it would be possible to define a strip around the lower bound for which $Z(p)$ is positive and, thus, the area in which projects make socio-economic sense. Although the figure appears to be not unreasonable one is inclined to state that the factors influencing it rather provide an explanation for the differences of such diagrams in different application areas than can form a basis for general public acceptability.

### 8.5. Example 5: simple log-normal resistance-demand problem with deteriorating resistance

Here, the asymptotic form of the Laplace transform of the renewal intensity Eq. (15) is used. The objective function for maximization now is

$$\frac{Z(p)}{C_0} = \frac{b}{\gamma C_0} - \left(1 + \frac{C_1}{C_0}p^a\right) - \left(1 + \frac{C_1}{C_0}p^a + \frac{H_M}{C_0} + \frac{H_F}{C_0}\right)\frac{1}{\gamma E[T_f(p)]} \tag{66}$$

$E[T_f(p))$ is computed for a linear decrease of the resistance according to:

$$E[T_f(p)) = \int_0^{1/c} \Phi\left(\frac{\ln\left\{p(1-ct)\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right\}}{\sqrt{\ln\left((1+V_R^2)(1+V_S^2)\right)}}\right)\mathrm{d}t \tag{67}$$



The parameters are: $C_0 = 10^6$, $C_1 = 10^4$, $a = 1.25$, $H_M = 3\cdot10^6$, $H_F = 2.8\cdot10^6$, $b = 0.03\cdot C_0$, $\gamma = 0.02$ as well as $V_R = 0.2$, $V_S = 0.3$ and $c = 0.000001$. Optimization yields $p^* = 0.945$, corresponding to a failure rate of $\frac{1}{E[T_f(\mathrm{p})]} \approx 10^{-5}$ or a mean time between failures of $E[T(p)] \approx 10^5$. Criterion (43) has the form:

$$K_F\left|\frac{\mathrm{d}}{\mathrm{d}p}\left(\frac{1}{E[T_f(p)]}\right)\right| - \left|\frac{\mathrm{d}}{\mathrm{d}p}(C_0 + C_1 p^a)\right| \leqslant 0 \tag{68}$$

Its consideration results in $p_{\lim} = 0.504$ and a mean failure time of $E[T_f(p)) = 3782$.

### 8.6. Example 6: design acceleration in a seismic area

This example is based on a more detailed study in [30]. In a seismic zone Poissonian damage earthquakes occur with rate $\lambda = 2.9$ [1/year]. Only magnitudes between $m_l = 4.0$ and $m_u = 7.5$ are considered. They are distributed according to a Weibull distribution for maxima truncated at $m_l$.

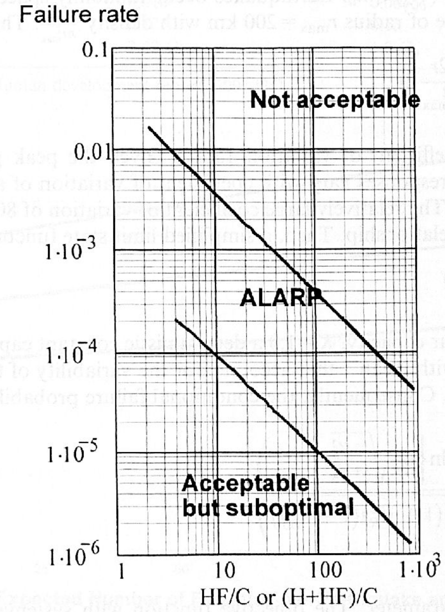


Fig. 4. ALARP region for example 3.

$$F_M(m) = \frac{\exp\left[-\left(\frac{m_u - m}{m_u - w}\right)^k\right] - \exp\left[-\left(\frac{m_u - m_l}{m_u - w}\right)\right]}{1 - \exp\left[-\left(\frac{m_u - m_l}{m_u - w}\right)^k\right]} \tag{69}$$

with $w = 4.35$ and $k = 8.11$. The attenuation law for the peak ground acceleration is according to Joyner and Boore [9]

$$a = h(m, r) = b_1 \exp(b_2 m)(r^2 + 7.3^2)^{-1/2} \exp(-b_3 r) = \exp(b_2 m) b(r) \tag{70}$$

where $b_1 = 0.0955g$, $b_2 = 0.573$, $b_3 = 0.00587$. Therefore, for given distance $r$

$$f_A(a, r) = \frac{k \frac{(m_u - h^{-1}(a,r))^{k-1}}{(m_u - w)^k} \exp\left[-\left(\frac{m_u - h^{-1}(a,r)}{m_u - w}\right)^k\right] \frac{\mathrm{d}h^{-1}(a,r)}{\mathrm{d}a}}{1 - \exp\left[-\left(\frac{m_u - m_l}{m_u - w}\right)^k\right]} \tag{71}$$

with $m_l \leqslant h^{-1}(a,r) = \frac{1}{b_2} \ln\left(\frac{a}{b(r)}\right) \leqslant m_u$. Earthquakes occur randomly spaced around the site under consideration with a circle of radius $r_{\max} = 200$ km with density $\frac{1}{\pi r_{\max}^2}$. This yields:

$$f_A(a) = \int_0^{r_{\max}} f_A(a, r) \frac{2r}{r_{\max}^2} \mathrm{d}r \tag{72}$$

with mean 0.075 and coefficient of variation 155%. Given the peak ground acceleration the ordinates of the spectral responses vary with coefficient of variation of about $V_S = 80\%$ and are log-normally distributed. The relatively large coefficient of variation of 80% also accounts for the error in the attenuation relationship. Then, a simplified limit state function is

$$g(\mathbf{X}) = R - KSA \leqslant 0 \tag{73}$$

$R$ is the log-normal shear capacity, $K = 1.0$ a deterministic constant capturing all system related properties, $S$ a quantity with mean 1 that accounts for the variability of the responses, and $A$ the peak ground acceleration. Consequently, the conditional failure probability is

$$P_f(p|a) = \Phi\left(-\frac{\ln\left\{\frac{p}{K \cdot a}\sqrt{\frac{1+V_S^2}{1+V_R^2}}\right\}}{\sqrt{\ln((1+V_R^2)(1+V_S^2))}}\right) \tag{74}$$

if $p = m_R$ is the design parameter. The objective function with systematic reconstruction after failure is (benefit term neglected)



$$\frac{Z(p)}{C_0} = \left(1 + \frac{C_1}{C_0} p^\delta\right) + E_A\left[\left(1 + \frac{C_2}{C_0} p^\delta a^\eta + \frac{H_O}{C_0} + \frac{0.11 a^\eta}{C_0}(H_M + H_F)\right) \frac{\lambda P_f(p|a)}{\gamma + \lambda P_f(p|a)}\right] \tag{75}$$

$C_2 p^\delta a^\eta$ are the rehabilitation cost depending on $a$ and $0.11 H_M a^\eta$ are the monetary damage cost also depending on $a$. Also, the cost for saving lives $0.11 H_F \eta_F^a$ depend on $a$. The indirect damage cost like loss of business are denoted by $H_O$. Furthermore, there is: $C_0 = 10^6$, $C_1 = 10^5$, $C_2 = 8000$, $\delta = 1.1$, $\eta = 1.25$, $H_M = 1.5 \cdot 10^5$, $V_R = 0.2$, $\gamma = 0.02$ and $H_F = ICAF\ N_F$ where the parameters are determined for three different socio-economic levels given in the Table 3 below [monetary values in US$ (1999)].

Table 3
Socio-economic levels for example 6[a]

| Socio-economic levels | High | Moderate | Low |
|---|---|---|---|
| $g$ | 23 500 | 6500 | 1500 |
| $e$ | 75 | 65 | 50 |
| $C_{F\delta}$ | 0.15 | 0.25 | 0.4 |
| $M$ | 0.010 | 0.0075 | 0.02 |
| $w$ | 0.125 | 0.15 | 0.18 |
| $H_O$ | 3.62 $C_0$ | 1.0 $C_0$ | 0.23 $C_0$ |

[a] From: united Nations. Human development report, 2001 [35].

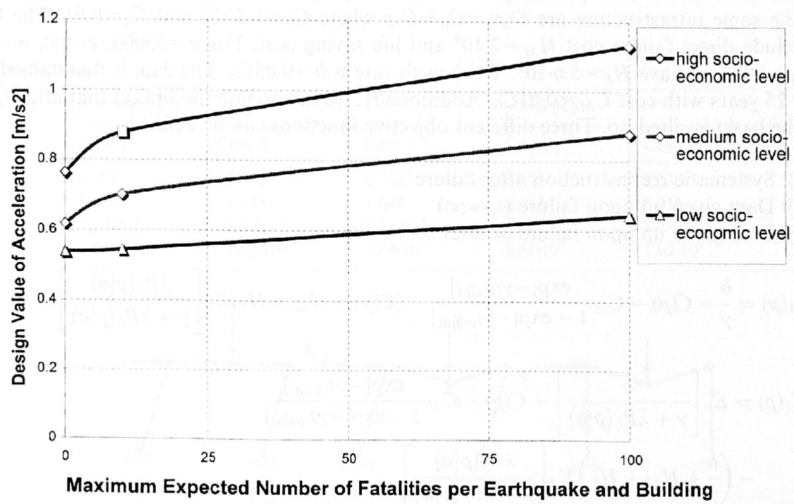


Fig. 5. Design acceleration versus maximum expected number of people in building for example 6.

The expression in brackets in Eq. (75) is valid only conditional on $a$. The expectation operation removes the condition. A subsequent FORM/SORM analysis can determine the so-called "design or most probable acceleration" $a^*$ associated with the optimum $p^*$ to be used in traditional design. In Fig. 5 some results of the analysis are shown.

Those design accelerations have mean return periods between 30 and 170 years. The corresponding design values for the random spectral enhancement are around 2 on the mean and, of course, need to be considered also as well as a small resistance reduction factor of about 1.1 (again on the mean). Structural failures will then occur with mean return periods between 74 and 2236 years, the lower values corresponding to the low socio-economic level. The calculations show that different social climates can have important influence on design rules. One can also apply criterion (43) alone. Let us assume that there exist about 10000 buildings in a town each with maximum possible occupation of $N_F=10$ per building. This corresponds to a population of 100 000. For the high socio-economic level one can determine a failure probability per building and earthquake of 0.084. From this one can predict a mean number of yearly fatalities of 1034. The corresponding objective function with $\beta=0.02$ and $\gamma=0.018$ at $p_{\lim}=0.374$ is strongly negative. If the optimum design is chosen the failure rate is $8\cdot10^{-4}$ and the mean number of yearly fatalities is 32. These results underline that optimal designs should be chosen leading to stronger structures providing the opportunity to benefit from their existence. Direct application of criterion (42) leads to economically unacceptable designs.

### 8.7. Example 7: earthquake design of a dam with different reconstruction policies

The same earthquake model is used to demonstrate the effect of different reconstruction policies and interest rates. A dam primarily for irrigation purposes is considered. The erection cost including some infrastructure are $C(p)=C_0+C_1p$ where $C_0=1.5\cdot10^9$ and $C_1=0.01$. The failure cost include direct failure cost $H_M=2\cdot10^9$ and life saving cost. For $g=35000$, $e=80$, $w=0.125$ and $N_F=2000$ we have $H_F=5.6\cdot10^9$. The benefit rate is $b=0.05C_0$. The dam is maintained every $t_{maint}=25$ years with cost $C_M=0.01C_0$. Additionally, we investigate the option that after $t_s=125$ years the basin is silted up. Three different objective functions can be defined:

$Z_1(p)$: Systematic reconstruction after failure
$Z_2(p)$: Dam given up upon failure ($t_s\to\infty$)
$Z_3(p)$: Dam given up upon failure or after service time $t_s$

$$Z_1(p)=\frac{b}{\gamma}-C(p)-C_M\frac{\exp[-\gamma t_{\mathrm{maint}}]}{1-\exp[-\gamma t_{\mathrm{maint}}]}-(C(p)+H_M+H_F)E_A\left[\frac{\lambda P_f(p|a)}{\gamma+\lambda P_f(p|a)}\right] \tag{76}$$

$$Z_2(p)=E_A\left[\frac{b}{\gamma+\lambda P_f(p|a)}\right]-C(p)-C_M\frac{\exp[-\gamma t_{\mathrm{maint}}]}{1-\exp[-\gamma t_{\mathrm{maint}}]}$$
$$-\left(\frac{b}{\gamma}+H_M+H_F\right)E_A\left[\frac{\lambda P_f(p|a)}{\gamma+\lambda P_f(p|a)}\right] \tag{77}$$



$$Z_3(p)=E_A\left[\frac{b}{\gamma+\lambda P_f(p|a)}(1-\exp[-(\gamma+\lambda P_f(p|a)t_s])\right]$$
$$-C(p)-C_M\sum_{k=1}^{t_s/t_{\mathrm{maint}}}\exp[-\gamma k t_{\mathrm{maint}}] \tag{78}$$
$$(H_M+H_F)E_A\left[\frac{\lambda P_f(p|a)}{\gamma+\lambda P_f(p|a)}(1-\exp[-(\gamma+\lambda P_f(p|a)t_s)])\right]$$

The third objective has been determined directly from Eq. (10). The discount functions for cases 1 and 2 are exact whereas it is slightly conservative for case 3. Table 4 shows some results for two interest rates. From Table 4 one concludes that the different reconstruction strategies have very little influence for both interest rates. Here, of course, an interest rate of $\gamma=0.045$ is preferable. Criterion (43) demands for a maximum failure rate of $8.8\cdot10^{-4}$ at only $p^*=3.238$ but the objective function is strongly negative.

### 8.8. Example 8: optimal portal frame

This example illustrates the case of multiple failure modes with vector parameter and multiple failure modes. A rigid–plastic frame as shown in Fig. 6 is considered. The three state functions for the three failure modes considered here are given by:

$$\begin{aligned}M_1&=X_1+X_2+X_4+X_5-X_6h\leqslant0\\M_2&=X_1+2X_3+2X_4+X_5-X_6h-X_7h\leqslant0\\M_3&=X_2+2X_3+X_4-X_7h\leqslant0\end{aligned} \tag{79}$$

Table 4
Results for example 7

| | $\gamma=0.018$ | | | $\gamma=0.045$ | | |
|---|---|---|---|---|---|---|
| | Case 1 | Case 2 | Case 3 | Case 1 | Case 2 | Case 3 |
| $p^*$ | 9.107 | 10.481 | 9.744 | 7.475 | 8.036 | 7.770 |
| $Z(p^*)/C_0$ | 1.647 | 1.643 | 1.641 | 0.012 | 0.011 | 0 |
| $\lambda P_f(p^*)$ | $7.1\cdot10^{-5}$ | $4.0\cdot10^{-5}$ | $5.3\cdot10^{-5}$ | $1.6\cdot10^{-4}$ | $1.2\cdot10^{-4}$ | $1.3\cdot10^{-4}$ |
| $C(p^*)$ | $1.64\cdot10^9$ | $1.68\cdot10^9$ | $1.64\cdot10^9$ | $1.61\cdot10^9$ | $1.62\cdot10^9$ | $1.62\cdot10^9$ |

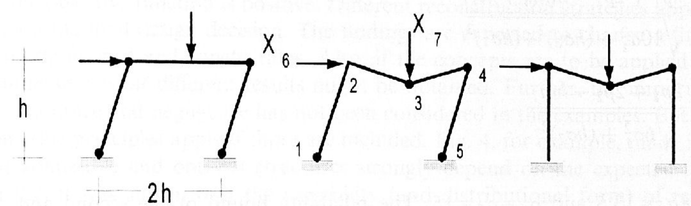


Fig. 6. Failure modes of portal frame.

The constant is $h=5[m]$ and the stochastic model is:

| Variable | Distr.-Type | $m_i$ | $\sigma_i$ |
|---|---|---|---|
| $X_1$, $X_2$,$X_4$, $X_5$, | Normal | $p_1$ [kNm] | 13.5 [kNm] |
| $X_3$ | Normal | $p_2$ [kNm] | 13.5 [kNm] |
| $X_6$ | Normal | 50 [kN] | 15 [kN] |
| $X_7$ | Normal | 40 [kN] | 15 [kN] |

The plastic hinges 1 and 2 as well as 4 and 5 form in the columns, hinge 3 in the beam. The cost function is $3p_1+2p_2$, the damage cost are $H_M=10^5$ and the cost for saving lives are $H_F=2.4\cdot 10^6$. In this case the failure probabilities are analytic but we retain the Kuhn-Tucker conditions formally [14]. We require additionally $P_{f,k}\leqslant 10^{-6}$, $k=1,2,3$ derived from serviceability considerations, for example. Then, for so-called separable systems (independent failure modes) and systematic reconstruction using Eq. (48)

$$\text{Maximize:}\quad Z(\mathbf{p})=B^*-C(\mathbf{p})-(C(\mathbf{p})+H_M+H_F)\cdot\frac{\sum_{k=1}^{3}P_{f,k}(\mathbf{p})}{1-\sum_{k=1}^{3}P_{f,k}(\mathbf{p})}$$

$$\begin{aligned}\text{Subject to:}\quad & g_k(\mathbf{u}_k,\mathbf{p})=0;\ k=1,2,3\\ & u_{i,k}\|\nabla_{\mathbf{u}}g_k(\mathbf{u}_k,\mathbf{p})\|+\nabla_{\mathbf{u}}g_k(\mathbf{u}_k,\mathbf{p})_i\|\mathbf{u}_k\|=0;\\ & \quad i=1,\dots,n_k-1;\ k=1,2,3\\ & P_{f,k}(\mathbf{p})\leqslant 10^{-6};\ k=1,2,3\\ & \mathbf{p}\geqslant 0;\end{aligned}\tag{80}$$

where the $\mathbf{u}_k$ are independent standard normal vectors with dimension $n_k$ each. Note that the last gradient condition has to be ommitted for each failure mode. Otherwise the Kuhn–Tucker condition is overdetermined. The reliability indices are

$$\begin{aligned}\beta_1(\mathbf{p})&=\frac{4p_1-hm_6}{\sqrt{4\sigma_1^2+(h\sigma_6)^2}}\\ \beta_2(\mathbf{p})&=\frac{4p_1+2p_2-h(m_6+m_7)}{\sqrt{10\sigma_1^2+(h\sigma_6)^2+(h\sigma_7)^2}}\\ \beta_3(\mathbf{p})&=\frac{2p_1+2p_2-hm_7}{\sqrt{6\sigma_1^2+(h\sigma_7)^2}}\end{aligned}\tag{81}$$

with $\sigma_1=..=\sigma_5=13.5$ and $\sigma_6=\sigma_7=15$. The reliability bound of the second and third failure mode are active. The design parameter is $\mathbf{p}^*=(190.1,41.1)$ at total cost of 657.4. The expected failure cost has value 5.0 and is expectedly small. It is noted that the assumption of separability is only slightly conservative as can be shown by reiteration.



The same example is also used to demonstrate the time-variant stationary case. Now, the loads are independent, intermittent rectangular wave processes with jump rate $\lambda_{6,7}=1000$ and "on"-probabilities $q_6=0.048$, $q_7=0.434$ and $q_{67}=0.043$, respectively. Then it is

$$\sum_{k=1}^{3}\nu_k^+(\mathbf{p})=q_6\lambda_6\Phi(-\beta_1(\mathbf{p}))+q_7\lambda_7\Phi(-\beta_3(\mathbf{p}))+q_{67}(\lambda_6+\lambda_7)\Phi_2(\beta_2(\mathbf{p}),-\beta_2(\mathbf{p});\rho)\tag{82}$$

with reliability indices as in Eq. (81) and:

$$\rho=1-\frac{(h\sigma_6)^2}{4\sigma_1^2+(h\sigma_6)^2+(h\sigma_7)^2}$$

For $\gamma=0.05$ simple optimization of

$$Z(\mathbf{p})=C(\mathbf{p})+(C(\mathbf{p})+H_M+H_F)\frac{1}{\gamma}\sum_{k=1}^{3}\nu_k^+(\mathbf{p})\tag{83}$$

without any reliability restriction results in the design parameter vector $\mathbf{p}^*=(220.1,129.0)$ at a failure rate of $1.2\cdot 10^{-7}$ and total cost 933.7. A reliability restriction would not be active for $\nu_k^+(\mathbf{p})\leqslant 10^{-6}$, $k=1,2,3$ nor is criterion (43) active.

### 8.9. Discussion

The examples have demonstrated that an optimal solution with life saving cost included can be found. Usually, it requires only little more expenditures than a solution without life saving cost. Only if the socio-economic level is high and a large number of fatalities must be expected in case of failure the differences with and without life saving cost can become larger. Criteria of the type of Eq. (43) are almost always fulfilled at the optimum as predicted from Eqs. (54) and (55). Application of criterion (43) alone, in general, results in significantly larger yearly failure probabilities than optimization but installations just fulfilling the criterion are suboptimal. However, a structure must still be considered as acceptable if the LQI-criterion (43) is satisfied and, of course, provided that the objective function is positive. Different reconstruction strategies appear to have little influence on the final design decision. The findings are expected to change a little for systematically varying interest and benefit rates. Also, if the concepts are to be applied to existing technical facilities somewhat different results might be obtained. Further, the important role of human error, ignorance and negligence has not been considered in the examples. But there is no doubt that the same principles apply if those are included. Fig. 4, for example, made it clear that the designs of admissible and optimal structures strongly depend on the expected failure consequences. In [22] it was shown that the variability (and distributional form) of resisting and acting quantities also plays an important role as well as the cost efficiency of the design parameters. These observations might, at least in part, explain the non-uniformity of safety levels inherent in traditional design rules mentioned in the introduction.

## 9. Conclusions

Optimization techniques are essential ingredients of reliability-oriented optimal designs of technical facilities. Suitable objective functions are presented based on a renewal model. A special one-level optimization is proposed for general cost-benefit analysis. Structural facilities can involve risks to human life and limb and this invokes the question "how safe is safe enough". An attempt has been made to give at least a partial answer based on optimization and on the recently proposed Life Quality Index (LQI). This defines the necessary and affordable investments into life saving via safety regulations in building codes, safety related measures in risky technological projects and precautions against natural hazards. In particular, specific values for the life saving cost to be inserted into the objective function are proposed. The life saving cost are between less than 0.5 and up to 3 million US$ per person depending on the socio-economic level of the society considered. Also, a differential criterion is derived to determine the maximum acceptable failure rate. A key question of optimization in the name of the public, whose constitutional imperative is to safe lives, is how to set benefit and interest rates. It is proposed to use as social benefit rate for public interventions a number close to the long term growth rate of the GDP which is around 2% for developed countries. The corresponding interest rate then must be slightly smaller. Further, it is concluded that optimal structures almost always are also acceptable structures but just acceptable structures are almost always suboptimal.

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