SECOND ORDER METHODS IN TIME VARIANT RELIABILITY PROBLEMS

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ABSTRACT

Time variant structural reliability involving Gaussian random processes and time independent random variables is computed based on the so–called outcrossing approach. The required multi-dimensional integrations over probability spaces and time are performed by using the Laplace asymptotic method. Results are given for scalar and vectorial Gaussian processes whose parameters can be ergodic random sequences. Also a computation scheme for the integration over time–invariant random vectors, modeling e.g. resistance quantities, is presented. Numerical comparisons are made at some extreme examples. The proposed method is applied to the study of crack propagation and crack instability in a realistic offshore structure

INTRODUCTION

Structural reliability problems typically involve high–dimensional probability integrations. This high–dimensionality generally prohibits the application of direct integration methods. In the last two decades, several approximate numerical methods have been developed. Among them are the so–called first–order reliability method (FORM) and the second–order reliability method (SORM) and, more recently, various Monte Carlo integration methods based on importance sampling. Practical applications of these methods to time invariant reliability problems or to time variant reliability problems which could be reduced to simple probability integrals in marine engineering are numerous (for example, Ferro, 1984; Østergaard, 1988; Bjerager et al., 1988). However, much less effort has been spent in the development of general methods which also are capable to handle time variant reliability problems as they are usually present in marine engineering with time variant environmental loading and possibly deteriorating structural properties. Here, one generally needs to compute the probability that an adverse state is reached by the system for the first time in a given reference period. For this computational task virtually no closed form solution of practical interest is known. Therefore, the general approach to such reliability problems is of asymptotic nature. It rests on the construction of a Poisson process for the exits of the structural state function into the failure domain. The intensity parameter of this Poisson process is the outcrossing rate of the load effect process through the time variant limit state function. Given the outcrossing rate, the mean number of exits into the failure domain has to be determined by time integration. Whereas the calculation of the outcrossing rate in the general case is already a non–trivial task, a second difficulty usually arises when assuring the Poissonian nature (lack of memory) of the outcrossings under the presence of time invariant or at least non–ergodic basic variables.

Although various aspects of this approach have been recognized and formulated earlier in the context of scalar Gaussian process theory (see Cramer/Leadbetter, 1967; Bolotin, 1981 and Leadbetter et al., 1983), serious attempts to establish a sound theoretical basis and to design efficient numerical methods date back only several years. Important steps for a solution of the problem have, among others, been made by Veneziano et al. (1977), Ditlevsen (1983) and Breitung (1988) who derived in part closed form solutions for the outcrossing rate of stationary Gaussian vector processes. By using these in part asymptotic results, Fujita et al. (1987) designed an algorithm on the basis of SORM to handle the time invariant and non–ergodic variables consistently. It requires two nested SORM applications. First application to fatigue deteriorated systems can be found in Guers/Rackwitz (1987). Further, on the basis of the concepts in Breitung (1988), Plantec/Rackwitz (1989) derived solutions for the mean number of outcrossings of vector processes under non–stationary conditions by using asymptotic concepts for the time integrals over the crossing rates. When applying these more general results to fatigue reliability of marine structures, serious numerical difficulties are met, however. Furthermore, Schall et al. (1990) studied at simple examples the problem of the presence of time invariant basic variables together with ergodic sequences which, for example, are used to model the sea states. It was found that an accurate treatment of those variables is very important.

TIME VARIANT RELIABILITY METHODS

In the following, we consider the general task of estimating the probability $P_f(t)$ that a realization $s(\tau)$ of a random state vector $S(\tau)$ enters the failure domain $F = \{s(\tau) \mid g(s(\tau),\tau) \leq 0,\ 0 \leq \tau \leq t\}$ for the first time. $g(s(\tau),\tau) = 0$ is the limit state function. $S(\tau)$ may conveniently be separated into three components as

$$S(\tau)^T = (R^T, Q(\tau)^T, X(\tau)^T) \tag{1}$$

where R is a vector of random variables independent of time τ. $Q(\tau)$ is a slowly varying ergodic random vector sequence. It is assumed possible to divide $[0,t]$ into sub–intervals with $Q(\tau) = q$ on each sub–interval. $X(\tau)$ is a random process vector having fast fluctuations as compared to $Q(\tau)$ and having parameters depending on $Q(\tau)$. For example, a component of $Q(\tau)$ can be taken as the standard deviation of a component of $X(\tau)$ (see fig. 1).

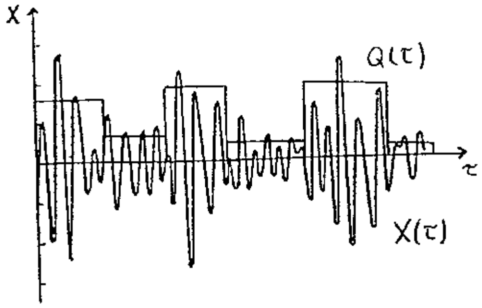

Figure 1: Variations of $Q(\tau)$ and $X(\tau)$ in time, $q = VAR[X(\tau)]$.

Consider first the case where only $X(\tau)$ (or equivalently $Q(\tau)$) is present. If it can be assumed that the stream of crossings of the vector $X(\tau)$ into the failure domain F is Poissonian, the failure probability $P_f(t)$ (see Leadbetter et al., 1983) can be estimated by

$$P_f(t) \approx 1 - (1 - P_f(0)) \exp(-E[N_X^+(t)]) \tag{2}$$

where $E[N_X^+(t)]$ is the expected number of crossings of $X(\tau)$ into the failure domain F in the considered time interval and $P_f(0)$ is the probability of failure at time zero.

In the case where both process variables $X(\tau)$ and time invariant random variables R are present, the Poissonian nature of outcrossings is lost. Eq. (2) furnishes only conditional probabilities $P_f(t|R{=}r)$. The total failure probability is obtained by integrating the conditional probability over all possible realizations of R.

$$P_f(t) \approx E_R[\, 1 - (1 - P_f(0)) \exp(- E[N_X^+(t|R)])] \tag{3}$$

In order to solve the multi–dimensional expectation operation over R, the failure probability is formulated as (Guers, 1987)

$$P_f(t) = P(T(R) - t \le 0) = P(g(R, U_T) \le 0) \tag{4}$$

where T(R) is the random lifetime with realizations t(r) as the solution to the equation

$$E[N_X^+(t(r)|r)] + \ln(-\Phi(u_T)) - \ln(1 - P_f(0)) = 0 \tag{5}$$

and u_T is a realization of an auxiliary standard normal variable. It is seen that eq. (4) is appropriate for FORM/SORM application and a computer program based on this formulation was successful in several practical applications (see Guers, 1987). However in the case of time varying limit state functions and/or non–stationary processes, the numerical solution of eq. (5) must be performed at least twice in each FORM/SORM iteration and the convergence of the algorithm becomes slow if not unreliable.

In the general case where all the different types of random variables R, $Q(\tau)$ and $X(\tau)$ are present, the failure probability $P_f(t)$ not only must be integrated up over the time in–variant variables R but an expectation operation must also be performed over the slowly varying variables $Q(\tau)$. In Schall et al. (1990) the following formula has been established

$$P_f(t) \approx E_R[1 - (1 - P_f(0)) \exp(- E_Q[E[N_X^+(t|R,Q)]])] \tag{6}$$

This asymptotic formula can easily be derived for stationary and ergodic processes $Q(\tau)$ and $X(\tau)$, respectively and time invariant limit state functions (see also Naess, 1984). For time variant limit state functions, it is only a usually conservative approximation.

Schall et al. (1990) concluded from their numerical studies that no further simplification is possible as one runs the risk of obtaining systematic errors up to several orders of magnitude. In the following, simple approximations for eq. (6) will be derived on the basis of asymptotic analysis.

BASIC FORMULATION AND NOTATIONS

For simplicity of presentation, we shall assume that $P_f(0) \simeq 0$. Eq. (6) can then be rewritten as

$$P_f(t) \simeq 1 - E_R[\exp(- E_Q[E[N_X^+(t|q,r)]])] \tag{7}$$

The first step consists in transforming the random vectors R and Q into vectors of non–correlated standard normal variable vectors U_r and U_q, by using the Rosenblatt transformation (see Hohenbichler/Rackwitz, 1981).

$$q = T_q(u_q) \tag{8 a}$$

$$r = T_r(u_r) \tag{8 b}$$

It is further convenient to introduce, for each realization of the variables R and Q, the standardized process $U_x(\tau)$ such that, for every time τ, $u_x(\tau)$ is a vector of non–correlated standardized normal random variables

$$x(\tau) = T_x(Q{=}q,\ R{=}r,\ u_x(\tau)) \tag{8 c}$$

For simplicity of notation, R, Q and $X(\tau)$ are also used for the transformed variables U_r, U_q and $U_x(\tau)$ in the following. Therefore, g(.) now denotes the state function formulated in the new standardized variables Q, R and $X(\tau)$, and ∂F denotes the corresponding failure surface. With these conventions, a method will first be described to estimate the expectation operations in the exponent of eq. (7).

From Cramer/Leadbetter (1967) and Plantec/Rackwitz (1989) the expected number of outcrossings $E[N_X^+(t|r,q)]$ can be obtained by using the relation between the expected number of crossings and the probabilities of $X(\tau)$ ending and starting in the failure domain.

$$E_Q[E[N_X^+(t|r,q)]] \approx \frac{1}{2} E_Q[E[N_X(t|r,q)]] - \frac{1}{2} E_Q[P(g(t,r,q,X(t)) \le 0) - P(g(0,r,q,X(0)) \le 0)] \tag{9}$$

In generalizing a formula given by Belyaev (1968), the expected number of crossings is (see Plantec/Rackwitz, 1989)

$$E_Q[E[N_X(t|q,r)]] = \int_{\mathbb{R}^{n_q}} \int_{\partial F} \int_{\dot{X}_n \in \mathbb{R}} \int_0^t |\dot{x}_n - \partial \dot{F}|\, f_{\dot{x}_n|x}(\dot{x}_n)\, f_x(x)\, f_q(q)\, d\tau\, d\dot{x}_n\, ds(x)\, dq \tag{10}$$

First, the calculation of $E[N_X(t|r,q)]$ is presented for a scalar Gaussian process $X(\tau)$. Then, asymptotic developments are exposed for the estimation of the whole integral (10) for a scalar process $X(\tau)$ and a vector–process $X(\tau)$. In both cases, the multidimensional integrals over τ, q, x and $\dot{x}$ are performed simultaneously by using only once Laplace's approximation. Finally, the calculation of the outer multidimensional expectation is discussed. For this problem, we will propose a formulation also based on asymptotic analysis although the asymptotic conditions will hardly be met in practical applications. It can however be shown by numerical studies that these concepts can also be applied with great numerical advantage in non–asymptotic cases.

EXPECTED NUMBER OF CROSSINGS OF NON–STATIONARY GAUSSIAN SCALAR PROCESSES FOR GIVEN r AND q

For a Gaussian scalar process $Z(\tau)$, the limit state function is

$$g(\tau,r,q,Z) = a(\tau,r,q) - Z(\tau) \tag{11}$$

where $a(\tau,q,r)$ is the original threshold function. The original process $Z(\tau)$ with mean value function $\mu(\tau)$ and standard deviation function $\sigma(\tau)$ and the threshold are standardized by the transformation T_x

$$X(\tau) = \frac{Z(\tau) - \mu(\tau)}{\sigma(\tau)} \tag{12}$$

$$b(\tau,r,q) = \frac{a(\tau,r,q) - \mu(\tau)}{\sigma(\tau)} \tag{13}$$

The limit state function then is

$$g(\tau,r,q,X) = b(\tau,r,q) - X(t) \tag{14}$$

The expected number of crossings of $b(\tau,r,q)$ by $X(\tau)$ in a given time interval $[0,t]$ conditional on $\mathbf{Q} = q$ and $\mathbf{R} = r$ is given as function of the mean crossing rate $\nu_X(\tau,r,q)$

$$E[N_X(t|r,q)] = \int_0^t \nu_X(\tau,r,q)\, d\tau \tag{15}$$

$$= \int_0^t \int_{\dot{x}\in\mathbb{R}} \frac{1}{\omega_0(\tau)} |\dot{x} - b_\tau(\tau,r,q)| \frac{1}{2\pi} \exp[-\tfrac{1}{2}\dot{x}^2/\omega_0^2(\tau)] \times \exp[-\tfrac{1}{2} b^2(\tau,r,q)]\, d\dot{x}\, d\tau \tag{16}$$

where ω_0 is the zero crossing frequency and $b_\tau(\tau,r,q)$ the first derivative of $b(\tau,r,q)$ with respect to τ. Using the well–known asymptotic arguments for integrals of this type for large $b(\tau,r,q)$ (see, for example, Bleistein/ Handelsmann, 1975; Breitung, 1988; Plantec/Rackwitz, 1989), the major contribution comes from the neighborhood of the point τ^* where $b^2(\tau,r,q)$ takes its minimum.

<u>If τ^* is an interior point of [0,t]</u>, the function $b^2(\tau,r,q)$ can be expanded to the second order as

$$b^2(\tau,r,q) = b^2(\tau^*,r,q) + 2\, b(\tau^*,r,q)\, b^*_{\tau\tau}\, \tau^2 + \dots \tag{17}$$

where $b^*_{\tau\tau}$ is the second order partial derivative of $b(\tau,r,q)$ in $\tau = \tau^*$. By introducing eq. (17) into eq. (16) it is seen that the result is similar to an expectation of the absolute value of a linear combination of Gaussian variates. This yields with $\omega_0 = \omega_0(\tau^*)$

$$E[N_X(t|r,q)] \approx \int_0^t \int_{\dot{x}\in\mathbb{R}} \frac{1}{\omega_0} |c^T \tilde{x}| \frac{1}{2\pi} \exp(-\tfrac{1}{2}(\tilde{x}^T S\, \tilde{x} - b^2(\tau^*,r,q)))\, d\dot{x}\, d\tau$$

$$= \int_0^t \int_{\dot{x}\in\mathbb{R}} \frac{1}{\omega_0} |c^T \tilde{x}|\; \varphi(b(\tau^*,r,q))\, (|\mathrm{Det}(S)|)^{-\frac{1}{2}} \frac{1}{\sqrt{2\pi}} (|\mathrm{Det}(S)|)^{\frac{1}{2}} \exp[-\tfrac{1}{2}\tilde{x}^T S\, \tilde{x}]\, d\dot{x}\, d\tau \tag{18}$$

where $\tilde{x} = (\dot{x},\tau)^T$, $S = \begin{bmatrix} 1/\omega_0^2 & 0 \\ 0 & 2b(\tau^*,r,q)b^*_{\tau\tau} \end{bmatrix}$, $c = (1, -2b^*_{\tau\tau})^T$

If the integration domain for τ is extended to the whole positive time axis which is consistent with the assumption that only a small neighborhood of τ^* contributes to the integral then one determines

$$E[N_X(t|r,q)] \approx \frac{2\; \varphi(b(\tau^*,r,q))}{\sqrt{2b(\tau^*,r,q)b^*_{\tau\tau}}} \left(\omega_0^2 + 2b^*_{\tau\tau}/b(\tau^*,r,q) \right)^{\frac{1}{2}} \tag{19}$$

for $b(\tau^*,r,q)$ sufficiently large.

<u>If τ^* is a boundary point of [0,t]</u>, the first order expansion of $b^2(\tau^*,r,q)$ yields

$$b^2(\tau,r,q) = b^2(\tau^*,r,q) + 2\, b(\tau^*,r,q)\, b^*_\tau\, \tau + \dots \tag{20}$$

where $b^*_\tau = b_\tau(\tau^*,r,q)$.

The integral for the expected number of crossings then is

$$E[N_X(t|r,q)] = \int_0^t \int_{\dot{x}\in\mathbb{R}} \frac{1}{\omega_0} |\dot{x} - b^*_\tau| \frac{1}{2\pi} \exp[-\tfrac{1}{2}\dot{x}^2/\omega_0^2] \exp[-\tfrac{1}{2} b^2(\tau,r,q)]\, d\dot{x}\, d\tau$$

$$\approx \int_0^t \int_{\dot{x}\in\mathbb{R}} \frac{1}{\omega_0} |\dot{x} - b^*_\tau|\; \varphi(b(\tau^*,r,q)) \exp[-\tfrac{1}{2}\dot{x}^2/\omega_0^2] \times \exp[-b(\tau^*,r,q)\, b^*_\tau\, \tau]\, d\dot{x}\, d\tau \tag{21}$$

Integrating first over $\dot{x}$ and extending the integration domain to all positive τ's yields

$$E[N_X(t|r,q)]) = \int_0^t \omega_0 [2\varphi(b^*_\tau/\omega_0) - b^*_\tau/\omega_0 + 2\, b^*_\tau\, \Phi(b^*_\tau/\omega_0)] \times \varphi(b(\tau^*,r,q)) \exp[-b(\tau^*,r,q)\, b^*_\tau \tau]\, d\tau$$

$$\approx \omega_0 [2\, \varphi(b^*_\tau/\omega_0) - b^*_\tau/\omega_0 + 2\, b^*_\tau/\omega_0\, \Phi(b^*_\tau/\omega_0)] \frac{\varphi(b(\tau^*,r,q))}{b(\tau^*,r,q) b^*_\tau} \tag{22}$$

where $b(\tau^*,r,q)$ is assumed sufficiently large.

Finally we consider the case where $b^2(\tau,r,q)$ takes its minimum on the boundary of the interval [0,t] and at the same point the first order partial derivatives of $b(\tau^*,r,q)$ vanishes. In this case the result is equal to half of the result for the interior point case which can be deduced from pure geometric reasoning.

EXPECTED NUMBER OF CROSSINGS OF NON–STATIONARY GAUSSIAN SCALAR PROCESSES FOR R=r

By introducing $z = \dot{x}/\sigma_{\dot{X}}$ into eq. (10), one obtains

$$E_Q[N(t|q,r)] = \int_{\mathbb{R}\times\mathbb{R}^{n_q}\times[0,t]} \frac{1}{(2\pi)^{1+n_q/2}} |k(\xi)| \exp(-\tfrac{1}{2} f(\xi))\, d\xi \tag{23}$$

with

$$\xi = (z, q, \tau)^T \tag{24 a}$$

$$k(\xi) = \sigma_{\dot{X}}\, z - b_\tau(\tau,q) \tag{24 b}$$

$$f(\xi) = z^2 + q^T q + b(\tau,q)^2 \tag{24 c}$$

Now it is necessary to first locate the point $\xi^* = (z^*, q^*, \tau^*)^T$ which minimizes $f(\xi)$ in [0,t]. Then expansions for functions f and k are used in the neighbourhood of ξ^*.

<u>If τ^* is an interior point of [0,t]</u>, these expansions are

$$f(\xi) = (b^{*2} + q^{*T} q^*) + \xi^T H\, \xi \tag{25 a}$$

$$k(\xi) = c^T \xi \tag{25 b}$$

where:
$$H = \left[\begin{array}{c|c|c} 1 & \dots 0 \dots & \\ \hline \vdots\; 0 & I_{n_q} + b^* \mathbf{B}^*_q & b^* b^*_{i\tau} \\ \hline 0 & b^* b^*_{i\tau} & b^* b^*_{\tau\tau} \end{array}\right] \tag{25 c}$$

and $\mathbf{B}^*_q$ denotes the Hessian matrix of b in the q–space at ξ^*,

$c^T = (\sigma_{\dot{X}}, b^*_{1\tau}, \ldots, b^*_{n_q\tau}, b^*_{\tau\tau})$ with: $b^*_{i\tau} = \frac{\partial^2 b}{\partial q_i \partial \tau}\Big|_{\xi^*}$ and $b^*_{\tau\tau} = \frac{\partial^2 b}{\partial \tau^2}\Big|_{\xi^*}$

Then using the usual asymptotic arguments, one obtains

$$E_Q[N(t|q,r)] \approx \sqrt{\frac{2}{\pi}} \exp(-\tfrac{1}{2}(b^{*2}+q^{*T}q^*)) \left[\frac{c^T H^{-1} c}{|\det(H)|}\right]^{1/2} \qquad (26)$$

If τ^* is a boundary point of [0,t], the expansions are

$$f(\xi) = (b^{*2}+q^{*T}q^*) + 2b^* b^*_\tau \tau + \eta^T K \eta \qquad (27\,a)$$

$$k(\xi) = b^*_\tau + \tilde{c}^T \eta \qquad (27\,b)$$

$$\text{with } \eta^T = (z,q),\ K = \left[\begin{array}{c|c} 1 & \ldots 0 \ldots \\ \hline 0 & I_{n_q} + b^* B_q \\ \vdots & \end{array}\right],\ \tilde{c}^T = (\sigma_{\dot{X}}, b^*_{1\tau}, \ldots, b^*_{n_q\tau}) \qquad (27\,c)$$

One finally determines

$$E_Q[N(t|q,r)] \approx$$

$$\frac{1}{(2\pi)^{\frac{1}{2}}} \frac{\exp(-\frac{1}{2}(b^{*2}+q^{*T}q^*))}{|b^* b^*_\tau|} \left[\frac{\tilde{c}^T K^{-1} \tilde{c}}{|\det(K)|}\right]^{1/2} (2\varphi(a) + a - 2a\Phi(a)) \qquad (28\,a)$$

where: $a = b^*_\tau / (\tilde{c}^T K^{-1} \tilde{c})^{1/2}$ (28 b)

EXPECTED NUMBER OF CROSSINGS OF NON-STATIONARY GAUSSIAN VECTOR PROCESSES FOR R=r

The vectorial case is only slightly more complex. In the case of a n_x–dimensional process $X(\tau)$, it is assumed that a twice differentiable parameterization of ∂F, $x_{nx} = b(\tau,q,x_1 \ldots x_{nx-1})$, exists such that for all $(\tau,q,x_1 \ldots x_{n_x-1}) \in \mathbb{R} \times \mathbb{R}^{nq} \times \mathbb{R}^{nx-1}$, there is

$$g(\tau,q,x_1 \ldots x_{n_x-1}, b(\tau,q,x_1 \ldots x_{n_x-1})) = 0 \qquad (29)$$

By this parameterization, the surface integral (10) can be transformed into a volume integral. One obtains

$$E_Q[N(t|q,r)] \approx \int_{\mathbb{R}^{n_x-1}} \int_{\mathbb{R}^{n_q}} \int_0^t \int_{\mathbb{R}} h(\xi)\, |k(\xi)|\, \exp(-\tfrac{1}{2} f(\xi))\, d\xi \qquad (30)$$

with $\quad \xi^T = (z,\tau,q^T,x_1 \ldots x_{n_x-1}) = (z,\tau,q^T,\tilde{x}^T)$ (31 a)

$$h(\xi) = \frac{1}{(2\pi)^{(n_x+n_q+1)/2}} \left[1 + \sum_{i=1}^{n_x-1} \left[\frac{g_i}{g_{n_x}}\right]^2\right]^{1/2} \qquad (31\,b)$$

$$k(\xi) = (\sigma z - \partial\dot{F} + m) \qquad (31\,c)$$

$$f(\xi) = z^2 + q^T q + \tilde{x}^T \tilde{x} + b(\tau,q,\tilde{x})^2 \qquad (31\,d)$$

$$z = (\dot{x}_n - m_{\dot{X}n}) / \sigma_{\dot{X}n} \qquad (31\,e)$$

and $\quad \sigma = \sigma_{\dot{X}n} = (n_x^T(\ddot{R} - \dot{R}^T \dot{R}) n_x)^{\frac{1}{2}}$ (31 f)

$$m = m_{\dot{X}n} = n_x^T \dot{R}^T x \qquad (31\,g)$$

$n_x^T = \frac{\nabla_x g}{\|\nabla_x g\|}$ denotes the normal vector to ∂F in the x–space.

After locating the critical point $\xi^{*T} = (z^*, \tau^*, q^{*T}, \tilde{x}^{*T})$ which minimizes $f(\xi)$, the functions f and k are expanded in the neighbourhood of ξ^*.

If τ^* is an interior point of [0,t], these expansions are

$$f(\xi) = \beta^2 + \xi^T H \xi \qquad (32\,a)$$

$$\beta^2 = f(\xi^*) = q^{*T} q^* + \tilde{x}^{*T} \tilde{x}^* + b(\tau^*, q^*, \tilde{x}^*)^2 \qquad (32\,b)$$

$$k(\xi) = c^T \xi \qquad (32\,c)$$

$$\text{where } H = \begin{bmatrix} 1 & \ldots & 0 & \ldots \\ \vdots & 0 & \ldots & 0 \ldots \\ 0 & 0 & I_{n_q+n_x-1} & \\ \vdots & \vdots & & \end{bmatrix} - \frac{b^*}{g_n(\xi^*)} \begin{bmatrix} 0 & \cdot & \cdot & 0 & \ldots \\ \vdots & & & & \\ 0 & M H_g^* M^T & & & \\ \vdots & & & & \end{bmatrix} \qquad (32\,d)$$

$$\text{with } M = \left[\begin{array}{c|c} I_{n_q+n_x} & \begin{matrix} b_\tau \\ b_q \\ b_{\tilde{x}} \end{matrix} \end{array}\right] \qquad (32\,e)$$

$$c^T = (\sigma^*, m^*_\tau - g^*_{\tau\tau}, m^*_q - g^*_{q\tau}, m^*_{\tilde{x}} - g^*_{\tilde{x}\tau}) = (\sigma^*, \nabla_\xi (m - g_\tau)^*) \qquad (32\,f)$$

$$b_q = -\left[\frac{\nabla_q g}{g_{n_x}}\right]_*,\ b_{\tilde{x}} = -\left[\frac{\nabla_{\tilde{x}} g}{g_{n_x}}\right]_* \text{ and } b_\tau = -\left[\frac{g_\tau}{g_{n_x}}\right]_* \qquad (32\,g)$$

H_g^* is the Hessian matrix of g in ξ^*.

The asymptotic arguments then yield

$$E_Q[N(t|q,r)] \approx \sqrt{\frac{2}{\pi}} \exp(-\tfrac{1}{2}\beta^2) \left[\frac{c^T H^{-1} c}{|\det(H)|}\right]^{1/2} J^* \qquad (33\,a)$$

$$\text{with: } \quad J^* = \left[1 + \sum_{i=1}^{n_x-1} \left[\frac{g_i}{g_{n_x}}\right]_*^2\right]^{1/2} \qquad (33\,b)$$

If τ^* is a boundary point of [0,t], the expansions are

$$f(\xi) = \beta^2 - 2b^* \left[\frac{g_\tau}{g_{n_x}}\right]_* + \eta^T K \eta \text{ with } \eta^T = (z,q,\tilde{x}) \qquad (34\,a)$$

$$k(\xi) = k_0(\tau) + \tilde{c}^T \eta \qquad (34\,b)$$

$$\text{where } \quad K = I_{n_q+n_x} - \frac{b^*}{g_n(\xi^*)} \cdot \begin{bmatrix} 0 & \ldots 0 \ldots \\ \vdots & \\ 0 & N H_g^* N^T \\ \vdots & \end{bmatrix} \qquad (34\,c)$$

$$\text{with } \quad N = \begin{bmatrix} 0 & \ldots 0 \ldots & 0 \\ \vdots & I_{n_q+n_x-1} & b_q \\ 0 & & b_{\tilde{x}} \end{bmatrix} \qquad (34\,d)$$

$$\text{and } \quad k_0(\tau) = m(\tau) - g_\tau(\tau) \qquad (34\,e)$$

$$\tilde{c}^T = (\sigma^*, m^*_q - g^*_{q\tau}, m^*_{\tilde{x}} - g^*_{\tilde{x}\tau}) \qquad (34\,f)$$

$$b_q = -\left[\frac{\nabla_q g}{g_{n_x}}\right]_* \text{ and } b_{\tilde{x}} = -\left[\frac{\nabla_{\tilde{x}} g}{g_{n_x}}\right]_* \qquad (34\,g)$$

H_g^* denotes the Hessian matrix of g in ξ^*.

The asymptotic arguments then lead to the approximation

$$E_Q[N(t|q,r)] \approx$$

$$\frac{1}{(2\pi)^{\frac{1}{2}}} \exp(-\tfrac{1}{2}\beta^2) \left(\frac{\tilde{c}^T K^{-1} \tilde{c}}{|\det(K)|}\right)^{1/2} (2\varphi(a) + a - 2a\Phi(a)) \frac{J^*}{|b^* b^*_\tau|} \qquad (35\,a)$$

$$\text{with } \quad a = \frac{k_0(\tau^*)}{(\tilde{c}^T K^{-1} \tilde{c})^{\frac{1}{2}}} \text{ and } J^* = \left[1 + \sum_{i=1}^{n_x-1} \left[\frac{g_i}{g_{n_x}}\right]_*^2\right]^{1/2} \qquad (35\,b)$$

EXPECTATION OVER THE NON–ERGODIC VARIABLES

We established in the three previous sections how to calculate the inner expectations in eq. (6) for a given realization of the non–ergodic variables $R = r$. The expectation over the non–ergodic variables can be written as

$$P_f(t) \simeq 1 - \int_{\mathbb{R}^{n_r}} \frac{1}{(2\pi)^{n_r/2}} \exp(-f_R^+(t,r))\, dr \qquad (36)$$

where $$f_R^+(t|r) = E_Q[E[N_X^+(t|r,q)]] + \frac{1}{2} r^T r \qquad (37)$$

The form of this integral is seen to be also suitable for applying the asymptotic results for an interior point. In this case, the point r^* where the function $f_R^+(t,r)$ takes its minimum with respect to r must be identified. The asymptotic solution is

$$P_f(t) \simeq 1 - \exp(-f_R^+(t,r^*))\,(|\det F^*|)^{-1/2} \qquad (38)$$

with F^* the Hessian matrix of $f_R^+(t,r)$ with respect to r taken in r^*.

NUMERICAL STUDY OF THE APPROXIMATION ACCURACY

In order to quantify the accuracy of the proposed approximation and to point out some simplifications in the numerical procedures some indicative numerical investigations are performed in the following. The approximation to the time integral for the mean outcrossing rate is considered first. The quality of the approximations in eqs. (19) and (22) depends on certain characteristics of the computational problem. The magnitude of $b(r,\tau^*)$ and thus the absolute value of the integral over $\nu(\tau)$ is expected to be most significant as the above results are only asymptotically exact $(b(r,\tau^*) \to \infty)$. In the non–asymptotic case the magnitude of the gradient or the curvature of the threshold function in the expansion point τ^* may have some influence and, finally, the length of the considered time interval may be important in some cases. In the following variations of these parameters are studied in the asymptotic approximations and exact results obtained by standard numerical integration of formula (6). A crack instability criterion of the form $\sigma(t) \geq K_{IC}(\pi a(t))^{-1/2}$ is used. The process $\sigma(t)$ is assumed stationary with zero mean and unit variance. The right hand side of the failure criterion is assumed deterministic depending only on the crack size a(t) which corresponds to the situation where $R = r$. Three situations for the behavior of the threshold function as presented in figure 2 are considered. In the first case the fracture toughness K_{IC} is constant and the crack size increases linearly with time. In the second and third case the fracture toughness K_{IC} is constant and the crack size increases nonlinearly in time. In the last case it is assumed that the crack tip initially is located in a part of the material where the fracture toughness has the value K_{IC1}. When it reaches a certain size it grows into a part of the material where the fracture toughnesses has a value $K_{IC2} \geq K_{IC1}$. This case simulates the situation where a crack with origin in a weld toe after some time grows out of the heat affected zone where the fracture toughness is reduced (see Shetty/Baker, 1990).

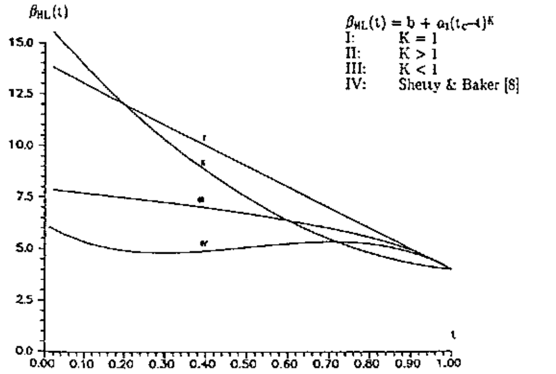

Figure 2

Figure 2 illustrates four different types of variation in the threshold function which are considered in the example.

For case I (see figure 2) an exact result for the expected number of outcrossings is available in Cramer/Leadbetter (1967). This result is plotted in figure 3 together with the results obtained by using the approximation in eq. (22) for different values of b and b_τ^* and with a fixed time ($\omega_0 = 2\pi N$, N = 1). Except for very small b_τ^* the approximations are very good for sufficiently large b. The small value of $b_\tau^* = 0.1$ corresponds to an almost stationary case and therefore some loss of accuracy must be expected.

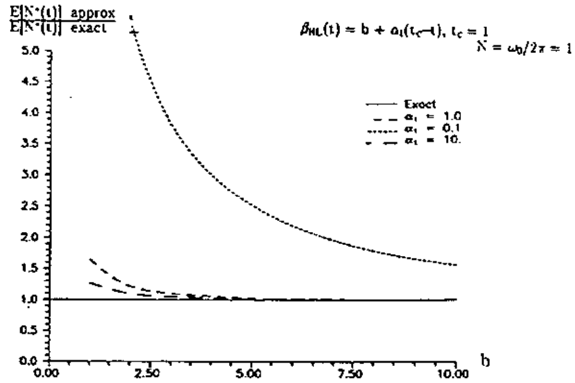

Figure 3: Ratio between the approximation and the exact solution for case I for a range of different values of b_τ^* and b (see figure 2).

In figure 4 case I is also considered but now with a fixed value for b_τ^* and different values of b and N. The approximations must also be viewed as excellent.

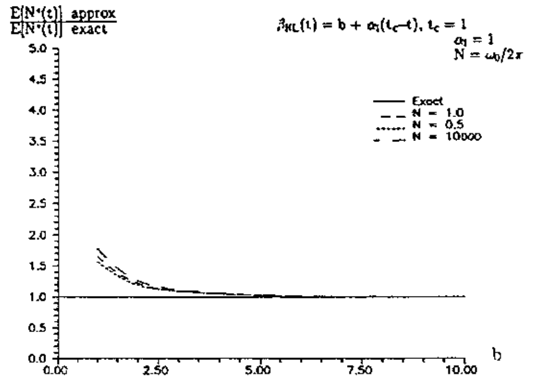

Figure 4: Ratio between the approximation and the exact solution for case I for constant b_τ^* and a range of different values of N and b (see figure 2).

In figure 5 and 6 the influence of the convexity properties of the threshold function is shown for the extreme case that the parameter t_c has a value of only 1.1. This means that the derivative of the threshold function is very small for case II and little variation in t_c would make the results for the interior point applicable. As expected the approximations for case III are slightly non–conservative but otherwise quite satisfactory.

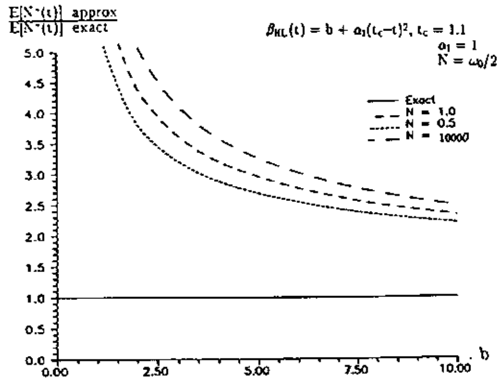

Figure 5: Ratio between the approximation and the exact solution for case II for constant b_7^* and a range of different values of N and b (see figure 2).

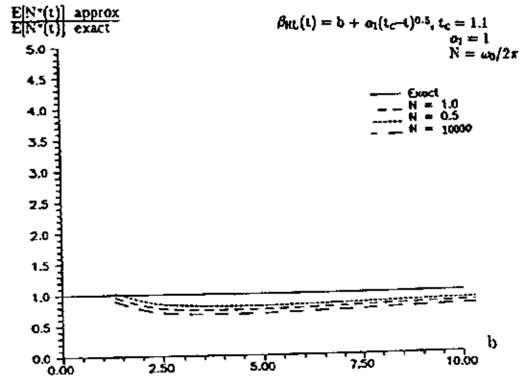

Figure 6: Ratio between the approximation and the exact solution for case III for constant b_7^* and a range of different values of N and b (see figure 2).

Next we study the quality of the approximations for the multidimensional expectation operations. Consider a R–vector with two components $R = (R_1, R_2)$ and a Q–vector with only one component Q. The failure criterion is taken as the first excursion of a stationary standardized Gaussian process over the threshold

$$b(R_1,R_2,Q) = \frac{1 + R_1 R_2}{Q} \tag{39}$$

All random variables are assumed to be normally distributed with parameters as shown below.

Variable	Distribution	Mean	S.D.
R_1	Normal	2.00	0.10
R_2	Normal	μ	0.10
Q	Normal	1.00	0.25

Numerical investigations are performed for $\mu \in [0.5;4.0]$. Table 1 gives a comparison between the probabilities of failure calculated according to (6) by using:

1 – numerical integration over R and Q,
2 – Laplace's approximation for R and Q at the optimal points $R = r^*$ and $Q = q^*$,
3 – Laplace's approximation for R and Q at the points $R = 0$ and $Q = q^*$,
4 – the same as 3, but only the diagonal terms of the Hessian matrix of $f_R^+(t,r)$ are determined

Mean of R_2: μ	1– Numerical integration	2– Expansion in $r = r^*$	3– Expansion in $r = 0$	4– Only diagonal terms of F^* considered
0.5	0.221	0.224	0.227	0.225
1.0	0.443×10^{-1}	0.449×10^{-1}	0.451×10^{-1}	0.452×10^{-1}
1.5	0.624×10^{-2}	0.630×10^{-2}	0.630×10^{-2}	0.629×10^{-2}
2.0	0.719×10^{-3}	0.720×10^{-3}	0.720×10^{-3}	0.725×10^{-3}
2.5	0.725×10^{-4}	0.716×10^{-4}	0.716×10^{-4}	0.716×10^{-4}
3.0	0.666×10^{-5}	0.642×10^{-5}	0.642×10^{-5}	0.635×10^{-5}
3.5	0.569×10^{-6}	0.582×10^{-6}	0.528×10^{-6}	0.530×10^{-6}
4.0	0.461×10^{-7}	0.404×10^{-7}	0.404×10^{-7}	0.403×10^{-7}
4.5	0.357×10^{-8}	0.291×10^{-8}	0.291×10^{-8}	0.292×10^{-8}
5.0	0.267×10^{-9}	0.199×10^{-9}	0.199×10^{-9}	0.199×10^{-9}

Table 1: Failure probabilities using different integration schemes

μ	r_1^*	r_2^*	$DET(F^*) - 1$ exact	$DET(F^*) - 1$ only diagonal terms
0.5	0.192×10^{-1}	0.760×10^{-1}	0.172×10^{-1}	0.216×10^{-1}
1.0	0.810×10^{-2}	0.161×10^{-1}	0.740×10^{-2}	0.723×10^{-2}
1.5	0.178×10^{-2}	0.238×10^{-2}	0.152×10^{-2}	0.153×10^{-2}
2.0	0.275×10^{-3}	0.275×10^{-3}	0.278×10^{-3}	0.248×10^{-3}
2.5	0.336×10^{-4}	0.269×10^{-4}	0.331×10^{-4}	0.331×10^{-4}
3.0	0.346×10^{-5}	0.231×10^{-5}	0.357×10^{-5}	0.377×10^{-5}
3.5	0.312×10^{-6}	0.178×10^{-6}	0.382×10^{-6}	0.378×10^{-6}
4.0	0.253×10^{-7}	0.127×10^{-7}	0.340×10^{-7}	0.341×10^{-7}
4.5	0.188×10^{-8}	0.838×10^{-9}	0.281×10^{-8}	0.280×10^{-8}
5.0	0.131×10^{-9}	0.528×10^{-10}	0.213×10^{-9}	0.213×10^{-9}

Table 2: Location of optimal point r^* and values of determinants

Table 2 gives the optimal point r^* corresponding to each value of μ. From table 2 it is seen that point r^* which minimizes the function $f_R^+(t,r)$ (see eq. 37) is always close to $r = 0$. Therefore, Laplace's approximation of the integral over R with $r = 0$ as the expansion point is usually satisfying. Moreover, in all cases, the Hessian matrix of $f_R^+(t,r)$ is of the form $F^* = Id + \epsilon$, where ϵ is a matrix with terms ϵ_{ij} such that $|\epsilon_{ij}| << 1$, $\forall i,j$. For illustration table 2 contains also the exact and simplified determinants.

DISCUSSION OF THE ASYMPTOTIC APPROXIMATIONS

The result obtained require some further discussion. Eqs. (19), (22), (26), (28 a–b), (33 a–b) and (35 a–b) can be shown to be asymptotically correct for increasing threshold $b(\tau,r,q)$ by the usual techniques of proof. Their accuracy has been demonstrated by some extreme examples in the last section. The same is not true for eq. (38). The application of these asymptotic arguments to justify eq. (38) is possible if we assume that the dispersion parameters of the R variables approach zero. The more is it surprising that quite accurate results are obtained as has been shown in the last section. Notwithstanding the generally high quality of Laplace's integral approximations, it is to be emphasized that the Poissonian nature of crossings is maintained throughout the developments. It is here where further improvements could be achieved.

AN APPLICATION TO AN OFF–SHORE INSTALLATION

Description of the Structure and Mechanical Model

Consider a simple tripod offshore structure (see fig. 7) consisting of one central column and three inclined legs made of steel which support a 3000 ton deck. The column and the legs are assumed to be clamped at the base. The water depth is 50 m and the topside of the platform is 20 m above the still water level. The following table summarizes the structural data:

Topside above still water level	20 m
Water depth	50 m
Diameter of the central column	3 m
Diameter of the legs	2 m
Thickness of all walls	100 mm
Total mass of the deck .	3000 tons
Marine growth thickness	100 mm
Young modulus of the steel	$2.1 \; 10^5$ MPa
Poisson modulus of steel	0.30

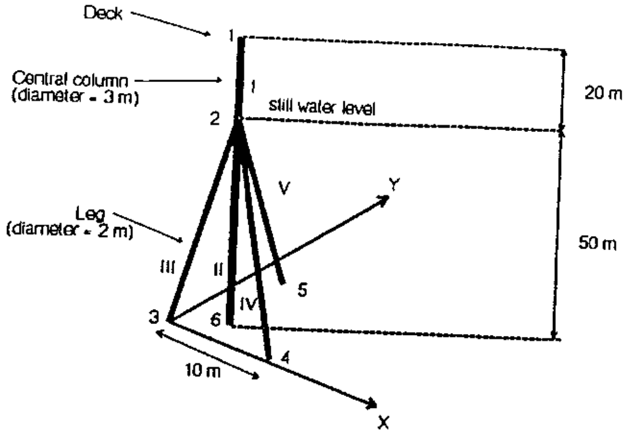

Figure 7: Tripod offshore structure from Karadeniz, 1989.

A finite element model using 5 beam–elements (one beam–element for each member) is applied for the modal analysis. In this model, the previously described structure has 12 degrees of freedom (6 corresponding to each of the nodes 1 and 2). The mass matrix and the flexibility matrix have been assembled and the eigenvectors and the corresponding eigenfrequencies have been found by using the finite–element code NASTRAN. Only the two (flexural) eigenmodes corresponding to the lowest frequency (2.11 Hz) are taken into consideration because the first and the second eigenfrequencies are well separated. The damping ratios for both lowest eigenmodes are 0.05.

The submerged structural members are loaded by wave forces which are calculated by means of the linearized Morison's equation. The inertia coefficient c_m and the drag coefficient c_d of this equation are 1.80 and 0.75 respectively. A modified spectral analysis is performed (Karadeniz, 1989). It leads to the stress transfer function $H_{s\eta}(\omega)$ such that in member j the spectrum of normal stresses is

$$S_{ss}(\omega)_j = H^*_{s\eta}(\omega)\, H_{s\eta}(\omega)\, S_{\eta\eta}(\omega) \tag{40}$$

where $S_{\eta\eta}(\omega)$ is the scalar spectrum of the wave elevation for an assumed sea–state. It is then possible to determine the spectral moments λ_i of the stresses from which some further quantities of interest are calculated, i.e.:

$\alpha = \frac{\lambda_2}{(\lambda_0\lambda_4)^{1/2}}$	bandwidth parameter of stress process	(41 a)
$\sigma_s = \sqrt{\lambda_0}$	standard deviation of the stress process	(41 b)
$\omega_0 = (\lambda_2/\lambda_0)^{\frac{1}{2}}$	mean–zero–crossing angular frequency	(41 c)
$\omega_m = (\lambda_4/\lambda_2)^{\frac{1}{2}}$	angular frequency of maxima	(41 d)

<u>Probabilistic Model for Sea–states</u>

For each short–term sea–state, the wave elevation η is assumed to be a stationary Gaussian process with zero–mean. Thus each sea–state is fully characterized by the spectral function of η. A Pierson–Moskowitz spectrum was chosen for this example

$$S_{\eta\eta}(\omega) = \frac{4\pi^3 H_s^2}{T_z^4\, \omega^5} \exp(-16\pi^3/(T_z\,\omega)^4) \tag{42}$$

Each sea–state is represented by the parameters H_s (significant wave height) and T_z (mean wave period). H_s and T_z are assumed to form two strongly cross–dependent Weibull–sequences with joint distribution function (see Houmb/Overvik, 1976)

$$F_{H_s,T_z}(x,y) = F_{T_z}(x|y)\, F_{H_s}(y) \tag{43}$$

where: $$F_i(x) = 1 - \exp\left[-\left[\frac{x-\epsilon_i}{u_i-\epsilon_i}\right]^{k_i}\right] \tag{44}$$

The parameters are:

$H_s = \text{Weibull}(u_s,k_s,\epsilon_s)$, $u_s = 2.02$ $k_s = 1.75$ $\epsilon_s = 0.46$
$T_z = \text{Weibull}(u_t,k_t,\epsilon_t)$

and for $H_s = h_s$:

$u_t = 6.05 \exp(0.07\, h_s)$, $k_t = 2.35 \exp(0.21\, h_s)$, $\epsilon_t = 0.00$.

We assume that for each sea–state the waves are unidirectional in the direction Ox (see fig. 7).

<u>Random Characteristics of the Structure</u>

The platform is assumed to have an initial crack of size a_0 on the member nº III, in the neighborhood of the joint nº 2. This crack propagates according to the Paris law with parameters C and m = 2. The initial crack size a_0 and the material constants C and K_{IC} are modeled by three independent non–ergodic random variables (see Table 3) and, hence: $R = (a_0, C, K_{IC})$.

For an assumed outcome of this vector R, the crack size a(t) is solution of the equation

$$\frac{da}{dN} = C\, Y^m\, E[\sigma_S^m]\, (\pi a)^{m/2} \tag{45}$$

where $a(0) = a_0$, Y is a geometry stress–concentration factor (which is assumed, for simplicity, to be independent of a) and σ_s is the standard deviation of the normal–stress process. For m = 2, eq. (45) provides

$$a(N) = a_0 \exp(\pi\, C\, Y^2\, E[\sigma_S^2]\, N) \tag{46}$$

where $N = t_s\, \omega_m/(2\pi)$ is the number of cycle maxima for the service–time t_s and a mean zero–crossing frequency ω_0 of the normal stress. $E[\sigma_S^2]$ is approximated by $E[\sigma_S^2] = 8\,\Gamma(2)\,\alpha\,\sigma_S^2$ (Madsen et al., 1986). For the considered joint it is reasonable to choose Y = 2.0, according to Kawamoto et al. (1985).

Variable	Distrib.	Mean	S.D.	units
(ergodic)				
H_s	Weibull	1.849	0.819	m
T_z	Weibull	$\mu_t = f_\mu(h_s)$	$\sigma_t = f_\sigma(h_s)$	s
(non–ergodic)				
a_0	Lognormal	0.003	0.004	m
C	Lognormal	$1.1\text{x}10^{-10}$	$2.9\text{x}10^{-11}$	
K_{IC}	Lognormal	100.00	26.00	$\text{MPa.m}^{1/2}$

Table 3: Parameters for ergodic and non–ergodic random variables

<u>The Failure Criterion</u>

Failure occurs when the stress intensity factor K for mode I (in the neighborhood of the crack) exceeds the corresponding fracture toughness K_{IC}

$$K_{IC} - K \leq 0 \tag{47}$$

By standardizing the normal–stress process S(t) into the process $s_n(t)$, one can express the failure criterion as the first passage of a standardized normal process over a time–varying threshold. This is precisely the required form for applying the previously explained concepts. The failure criterion is then rewritten as

$$s_n(t) \geq \frac{K_{IC}}{Y\ \sigma_s\ (\pi a)^{1/2}} \tag{48}$$

<u>General Method for Solving the Problem</u>

The application of the Laplace's approximation method for estimating both integrals over R and Q requires the utilization of an optimizer with respect to the vector **Q**. However, the spectral analysis for each outcome of the ergodic vector $Q = (H_s, T_z)$ would be extremely time–consuming. Therefore, interpolation response surfaces in the space of the ergodic variables were used. In the following, the main steps for the solution of the problem are summarized:

1 — Static and modal analysis (performed with the finite element code NASTRAN) leading to assembled mass– and stiffness–matrices, eigenfrequencies and eigenmodes of the structure.

2 — Spectral analysis for a set of out–comes of (H_s, T_z). The points of this set belong to a rectangular mesh of the plan (H_s, T_z). For each point of this set one calculates the spectral parameters α, σ_s, ω_0 and ω_m. The spectral analysis part was performed by using some modules of the code SAPOS (Karadeniz, 1989).

3 — The coefficients of bicubic spline interpolation polynomials were calculated for each of the four functions α, σ_s, ω_0 and ω_m.

4 — For an assumed service time t_s the probability of failure $P_f(t_s)$ of the structure was obtained by using the approximation of Laplace for calculating the integrals over time, **Q** and **R**.

Results: Variations of the Reliability Index β in Time

In figure 8, the reliability index $\beta(t) = -\Phi^{-1}(P_f(t))$ is shown for different times t.

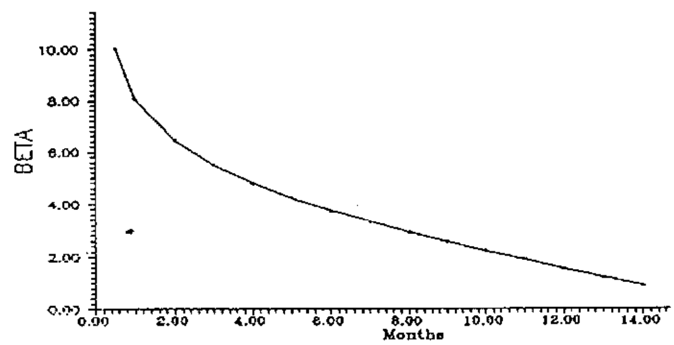

Figure 8: Reliability index β for various service times.

CONCLUSIONS

A new calculation scheme for approximations to time variant reliability in the presence of ergodic and non–ergodic basic variables is proposed. It can be shown to be far more reliable than earlier schemes from a numerical point of view. It is demonstrated by some extreme examples to be sufficiently accurate for most practical applications. The new approach has also been applied to the fatigue analysis of a realistic offshore structure.

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