EXPERIMENTAL AND NUMERICAL ASSESSMENT OF AN ASYMPTOTICALLY CORRECT CHOKING MACH NUMBER EXPRESSION FOR ORGANIC VAPOR FLOWS THROUGH TURBINE CASCADES

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ABSTRACT

An analytical expression for predicting the choking Mach number is presented. The expression is derived employing an asymptotic method leading to a perturbation series with a dominant zeroth-order term. Hence, it is suggested that actual choking Mach numbers can be predicted accurately in all practical organic vapor turbine design applications, including the dense gas regime. The reliability of this expression in the dilute gas regime is assessed using an experimental investigation of the flow of an organic vapor through a representative transonic turbine cascade. The agreement between the predicted and measured choking Mach number is excellent for the considered test case. To assess the validity of the expression for dense gas flows or flows largely different from the dilute gas regime, a database of computational fluid dynamics (CFD) simulations are utilized. Two different turbine cascades employing various organic working fluids and thermodynamic operating conditions are analyzed regarding the computed choking Mach number behavior.

1 INTRODUCTION

In compressible flows through cascades, the choking Mach number prediction is of practical interest for turbine design. If sonic conditions are achieved at the cascade's throat cross-section, downstream influences are not felt upstream. Then, the cascade is said to be "choked," and the upstream Mach number is called the choking Mach number. Expressions for choking Mach numbers have been derived and are quoted for perfect gas flows, e.g., Lakshminarayana (1996) or Scholz (1965). The choking limit is relevant for the mass flux and hence for the operation and load behavior for the turbine (see, for instance, Lakshminarayana (1996) and Scholz (1965) or Dejc and Trojanovskij (1973)).
Relatively little is known regarding the choking Mach number in the case of non-perfect gas flows. For the design of axial flow turbines for organic Rankine cycle ORC applications, a better understanding of the choking behavior of real gas flows seems hence to be valuable. A severe challenge for any theoretical analysis of the choking behavior arises in the dense gas regime or close to the critical point or the vapor saturation line where substantial variations of the isentropic exponent $\gamma = c_p/c_v$ can occur, as discussed by Tosto et al. (2020). However, Scholz (1965) quoted that experience shows a relatively small impact of the isentropic exponent in the case of low or moderate choking Mach numbers (i.e., for about $Ma_{1,ch} \leq 0.3$). This important observation was supported by a recent asymptotic analysis of the choking Mach number expression and an experimental investigation (aus der Wiesche et al. (2021)) considering the dilute gas flow regime for vapors characterized by isentropic exponents close to unity.
The present work's objective is to further elaborate the asymptotic analysis method for the choking Mach number and assess its prediction using available experimental and numerical literature data. In this context, high-fidelity computational fluid dynamics (CFD) analyses covering the dense gas regime are of considerable value. This regime is difficult to investigate from an experimental perspective, but

many CFD studies were concerned with it. The recorded data sets of these studies were evaluated regarding the choking behavior. The outcome of this re-evaluation significantly increased the range of experiences and enabled a well-founded proof of the theoretical asymptotic expression.

2 THEORY

The flow through a turbine cascade with throat o and spacing s shown in Figure 1 is considered in the following. Assuming that the salient features are covered by a one-dimensional model considering isotropic flow, the exact mathematical expression for the choking Mach number

$$\frac{o}{s\,\sin\beta_1} = \frac{\rho_1 w_1}{\rho * a *} = Ma_{1,ch}\left(\frac{2+(\gamma_{pv}-1)Ma_{1,ch}^2}{\gamma_{pv}+1}\right)^{-\frac{\gamma_{pv}+1}{2(\gamma_{pv}-1)}} \tag{1}$$

results for a gas with isentropic exponent γ_{pv} (see, for instance, Scholz (1965)). The isentropic exponent

$$\gamma_{pv} = -\frac{c_p}{c_v}\frac{v}{p}\left(\frac{\partial p}{\partial v}\right)_T \tag{2}$$

reduces to the simple expression $\gamma = c_p/c_v$ in the case of a perfect gas, but for non-perfect gas dynamics, its general definition (2) has to be used (Traupel (1966)). In the case of a significant expansion pressure ratio or certain thermodynamic regions (e.g., close to the saturation lines or in the dense gas regime), substantial variations of the isentropic exponent value γ_{pv} can occur. In that case, it is necessary to use an effective or mean isentropic exponent $\gamma_{pv,m}$ representing the actual expansion flow. Its correct numerical value cannot be predicted easily, but in any case, it is possible to separate it into

$$\gamma_{pv} = 1 - x \tag{3}$$

with x as a function of pressure and temperature (and depending on the actual expansion path). Inserting (3) in equation (1) and expanding into a power series in respect to x yields

$$\frac{o}{s\,\sin\beta_1} = \frac{\sqrt{e}\;Ma_{1,ch}}{e^{Ma_{1,ch}^2/2}}\left(1 - \frac{Ma_{1,ch}^4 - 2\,Ma_{1,ch}^2 + 1}{8}x + \frac{3Ma_{1,ch}^8 - 28\,Ma_{1,ch}^6 + 42Ma_{1,ch}^4 - 12\,Ma_{1,ch}^2 - 5}{384}x^2 + \cdots\right) \tag{4}$$

Or

$$\frac{o}{s\,\sin\beta_1} = f_0(Ma_{1,ch}) + f_1(Ma_{1,ch})x + f_2(Ma_{1,ch})x^2 + \cdots \tag{5}$$

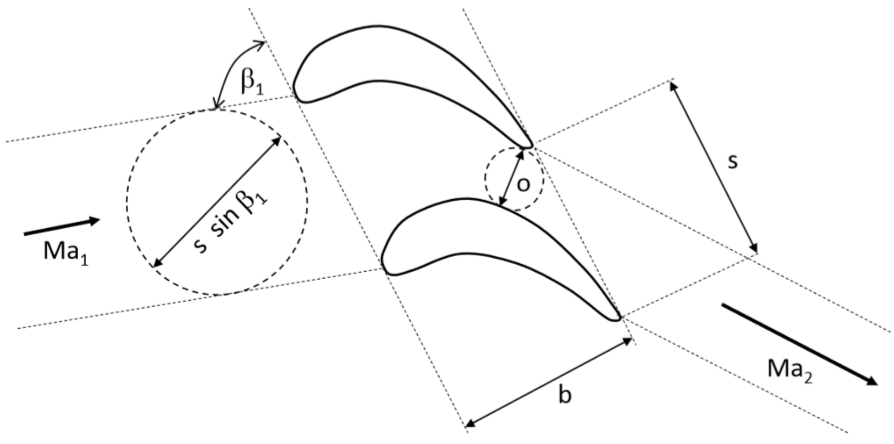

Figure 1: Flow through a turbine cascade and its nomenclature

Table 1: Values of $o/(s\ \sin\beta_1)$ for given choking Mach numbers $Ma_{1,ch}$ and isentropic exponents γ_{pv} calculated by different orders of the power series (5)

$Ma_{1,ch}$	γ_{pv}	f_0	$f_0 + f_1 x$	$f_0+f_1x+f_2x^2$	*exact*
0.2	0.5	0.3232	0.3046	0.3034	0.3033
	1.0	0.3232	0.3232	0.3232	0.3232
	1.5	0.3232	0.3418	0.3407	0.3408
	2.0	0.3232	0.3604	0.3559	0.3567
	2.5	0.3232	0.3791	0.3688	0.3712
	3.0	0.3232	0.3977	0.3795	0.3846
0.4	0.5	0.6088	0.5819	0.5796	0.5793
	1.0	0.6088	0.6088	0.6088	0.6088
	1.5	0.6088	0.6356	0.6333	0.6335
	2.0	0.6088	0.6625	0.6530	0.6547
	2.5	0.6088	0.6893	0.6681	0.6733
	3.0	0.6088	0.7162	0.6784	0.6897

The zeroth-order term f_0 in equation (5) is entirely independent of the gas properties, and it is a universal function of the choking Mach number $Ma_{1,ch}$. For small values of $x << 1$ (i.e., for a gas characterized by an isentropic exponent close to unity), the power series converges rapidly. Then, the zeroth-order term f_0 governs the choking Mach number leading to an asymptotically correct expression. However, by inserting other values $x \neq 0$, it is found that the above series (4) converges also well, see Table 1 and Figure 2. The impact of x remains relatively small. Hence, the choking Mach number through a turbine cascade can be reasonably approximated by the asymptotic expression f_0 provided that $Ma_{1,ch}$ is not too large.

In Table 1, some calculated values for $o/(s\ \sin\beta_1)$ are listed for given choking Mach numbers $Ma_{1,ch,}$ and isentropic exponents γ_{pv}. All values for $o/(s\ \sin\beta_1)$ are identical in the particular case of $\gamma_{pv} = 1$ because then the power series collapses to the zeroth-order term f_0. For more significant deviations of γ_{pv} from unity, the differences become noticeable between the term f_0 and higher-order series. However, a second-order series expansion provides results close to the exact expression (1), indicating the excellent convergence behavior of series (4).

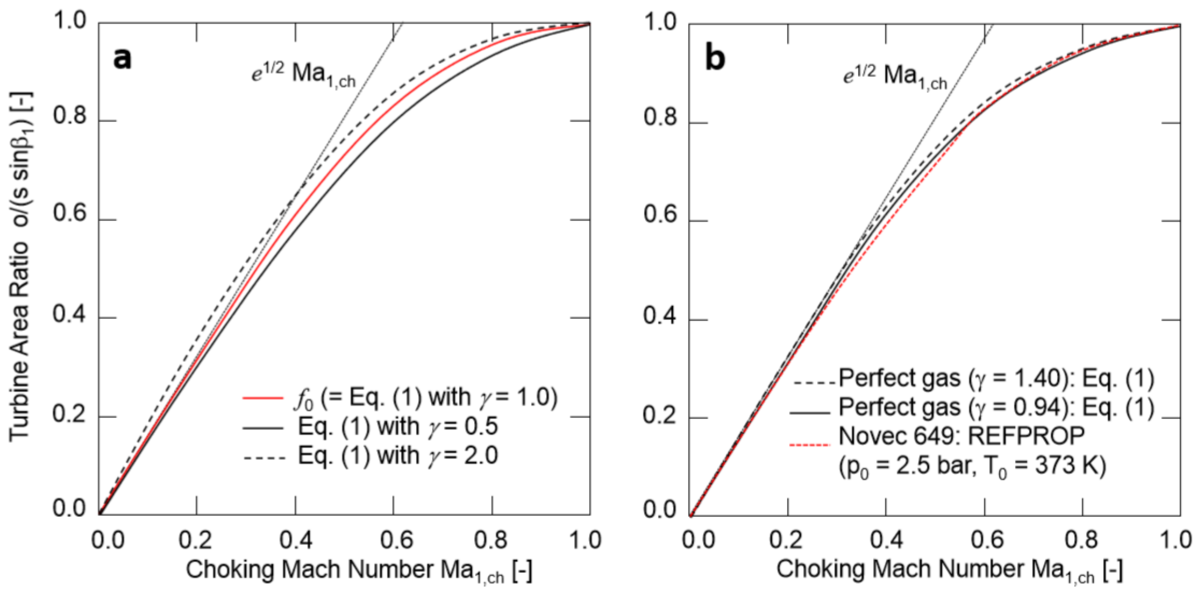

Figure 2: Choking Mach number relation in accordance with equation (1) for different isentropic exponents (**a**) and comparison with a calculation based on REFPROP data for Novec 649 (**b**)

In the case of a non-perfect gas flow through a turbine cascade, the appropriate value for the effective isentropic exponent $\gamma_{pv,m}$ is hard to determine, and a substantial scattering might occur in the dense gas regime due to uncertainties regarding the equation of state of the fluids. In that case, the maximum uncertainty of choking Mach number predictions is the deviation between the zeroth-order term f_0 and the exact value. For small or moderate choking Mach numbers (i.e., choking Mach numbers of order 0.4 or below), uncertainty remains small, and the term f_0 enables an accurate prediction of the choking Mach number. This is illustrated by means of Figure 2.a where the zeroth-order term f_0 (also identical to equation (1) with $\gamma = 1$) is compared with the calculations for $\gamma = 0.5$ and $\gamma = 2.0$, respectively. The zeroth-order term f_0 covers well the overall behavior. Even the simple linearization of f_0, $e^{1/2}Ma_{1,ch}$, describes the behavior reasonably well up to choking Mach numbers below 0.3. In Figure 2.b, the outcome of a REFPROP-based real-gas calculation is compared with equation (1) for an application in the dilute gas regime (for Novec 649 at stagnation condition 2.5 bar and 373 K).

3 TEST OF THE THEORY

3.1 Testing Approach and Overview

In the case of perfect gas flow, a good agreement between experimental data and the one-dimensional choking Mach number expression (1) is reported in the literature (e.g., Scholz (1965). In the case of small and moderate Mach numbers, the zeroth-order term f_0 describes the behavior well. Minor deviations between actual cascade data and the predictions by equation (1) can be attributed to boundary-layer thickness effects or to three-dimensional flow phenomena (Scholz (1965)) in the case of perfect gas flows.

To test the theory for non-perfect gas flows, it is useful to review available literature data and to deduce the choking Mach number. In the dilute gas flow regime, some experimental data for a representative transonic turbine cascade are available (see Section 3.2), but for the dense gas flow regime, corresponding experimental data are essentially missing in the open literature yet. For that case, the use of CFD data offers a way to test the theory (see section 3.3).

Among the numerous cascade flow simulations, two different turbine cascades were selected for the present study. Relevant cascade and flow configuration data are listed in Table 2, and Figure 4 illustrates the two cascades. The VKI-I (Kiock et al. (1986)) cascade was selected because reliable experimental results and literature data are well available. As the second independent configuration, the STCF 11 cascade (Fransson et al. (1999)) was selected because this cascade is characterized by a higher value of the area ratio parameter $o/(s\ \sin\beta_1)$, leading to a higher choking Mach number. Regarding the expansion series (4), higher choking Mach numbers are more critical since the asymptotic theory would be exact in the limit $Ma_{1,ch} \rightarrow 0$, but larger deviations can be expected for higher values of $o/(s\ \sin\beta_1)$. Please note that in the present study, the convention as shown in Figure 1 is used for the inflow angle β_1.

Table 2: Considered turbine cascade configurations

Cascade	Source (cascade & airfoil data)	Actual inflow angle β_1	$o/(s\ \sin\beta_1)$
VKI-I	Kiock et al. (1986)	60.00° (see Figure 1)	0.43564
STCF 11	Fransson et al. (1999)	52.85° (see Figure 1)	0.62982

3.2 Dilute Gas Flow Regime (Experimental Data for $\gamma \approx 0.95$)

The choking behavior of the so-called VKI-Sieverding cascade is well treated in literature (Kiock et al. (1986)), and the flow of Novec 649 through this representative transonic turbine cascade was recently experimentally investigated by aus der Wiesche et al. (2021). In that study, a closed-loop organic vapor wind tunnel driven by a centrifugal compressor and using a linear turbine cascade in the test section was employed. The mass flow rate through the linear cascade was controlled by a variable compressor running speed n. As working fluid, Novec 649 in the dilute gas flow regime at stagnation conditions T_{01} = 373 K and p_{01} = 2.5 bar was selected. At this stagnation condition, the isentropic exponent is about γ = 0.95. The inlet Mach number Ma_1 against normalized compressor running speed $n/(T_{01}/T_{ref})^{1/2}$ is shown in Figure 3 for Novec 649. In addition to the experimentally obtained data, the prediction of a one-dimensional calculation using the equation of state implemented in REFPROP 9.0 and assuming an isentropic flow through the turbine cascade is plotted in Figure 3. The deviations between the theoretical prediction and the measurements were within the experimental uncertainty level (of order 1 – 2 %). Based on the measurements, the choking Mach number $Ma_{1,\mathbf{ch}}$ was obtained as the limit value of the mass flow curve, see Figure 3. The experiment yielded a value of 0.278 $\pm$ 0.004, see Table 3. The fundamental term f_0, see equation (5), would predict a universal choking Mach number of $Ma_{1,ch} = 0.274$ for a cascade with $o/(s \sin\beta_1) = 0.43564$ as in the case of the considered VKI cascade configuration. All experimental and computed values for Novec 649 and dry air are close to that value. Using higher-order terms, see equation (5), the choking Mach number prediction would be 0.276. A calculation of the isentropic expansion using the equation of state of the database and thermodynamic program REFPROP 9.0 would give a value of $Ma_{1,ch} = 0.275$ for Novec 649.

In the case of air at usual wind tunnel conditions, equation (1) predicted the same value as equation (5), namely $Ma_{1,ch} = 0.263$, which is slightly smaller than the value for Novec 649 at the considered inlet conditions. The experimental data (Kiock et al. (1986)) obtained at four different European wind tunnels were in reasonable agreement with the theoretical values, but their significant internal scattering cannot be explained by their experimental uncertainties. However, that issue is out of the scope of the present analysis, and it cannot be resolved here.

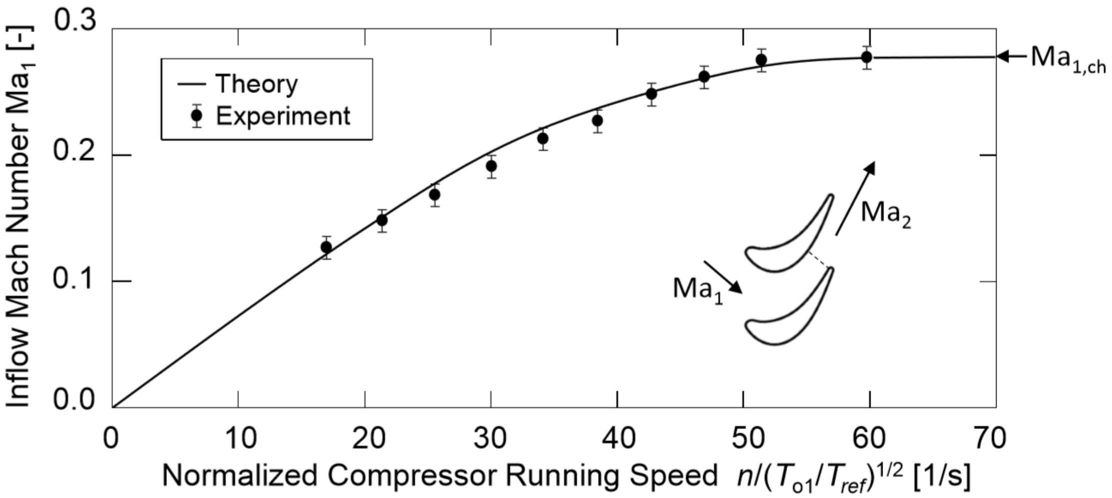

Figure 3: Inlet Mach number against normalized compressor running speed (VKI-I cascade with Novec 649 at 373 K and 2.5 bar stagnation condition)

Table 3: Experimental results for the choking Mach number for the VKI-I cascade (see Table 2)

Fluid	Flow regime (assumption)	Inlet isentropic exponent γ_{pv}	Method	Choking Mach number $Ma_{1,ch}$	Reference
n.a.	n.a.	n.a.	Eq. (5), only f_0,	0.274	-
Novec 649	Perfect gas	0.95	Eq. (1)	0.276	-
Novec 649	Perfect gas	0.95	Eq. (5), incl. f_2	0.276	-
Novec 649	Non-ideal gas	n.a.	REFPROP	0.275	Aus der Wiesche (2021)
Novec 649	Dilute gas flow	0.95	Experiment	0.278 $\pm$ 0.004	Aus der Wiesche (2021)
Air	Perfect gas	1.40	Eq. (1)	0.263	-
Air	Perfect gas	1.40	Eq. (5), incl. f_2	0.263	-
Air	Perfect gas	1.40	Experiment (RG)	0.282 $\pm$ 0.003	Kiock et al. (1986)
Air	Perfect gas	1.40	Experiment (BS)	0.260 $\pm$ 0.000	Kiock et al. (1986)
Air	Perfect gas	1.40	Experiment (GO)	0.282 $\pm$ 0.001	Kiock et al. (1986)
Air	Perfect gas	1.40	Experiment (OX)	0.252 $\pm$ 0.005	Kiock et al. (1986)

3.3 Computational Fluid Dynamics Analyses

The choking Mach number behavior for non-ideal and dense gas flows can be analyzed using available high-fidelity computation fluid dynamics simulations. For the present investigation, the numerical solution files of simulations published earlier (Cinnella et al. (2006) and Congedo et al. (2011)) for the VKI-I and the STFC 11 cascades were selected. As working fluids, Toluene was used for both cascades. In addition, PP10 was also used as a working fluid for the VKI-I cascade. Figure 4 shows examples for computed Mach number distributions for the cascades. Since the sonic line, $Ma = 1$, was reached at the throat region in the simulations, the inflow Mach numbers far upstream of the cascades represented the choking Mach numbers $Ma_{1,ch}$.

Details about the numerical methods used for the simulations can be found in Cinnella et al. (2006) and Congedo et al. (2011). The governing equations were solved numerically using a finite volume third-order accurate centered scheme. The scheme used a fourth-order accurate centered approximation of the fluxes, with the addition of a scalar numerical dissipation term. The comprehensive thermal equation of state of Martin and Hou (1955) was used to provide a realistic description of the non-ideal gas behavior.

For characterizing the inlet thermodynamic conditions, the values of the fundamental derivative Γ are also given in the following. The fundamental derivative Γ represents a measure of the rate of change of the sound speed a with density ρ in isentropic perturbations. In the case of a perfect gas, the relation $\Gamma = (\gamma +1)/2$ holds, leading to $\Gamma > 1$ since $\gamma = c_p/c_v > 1$. In the case of non-ideal gas behavior, the fundamental derivative depends on γ_{pv} and its derivative at constant entropy (see, for instance, Tosto et al. (2020)). If $\Gamma < 1$, the flow exhibits an uncommon sound speed variation in isentropic perturbations. The thermodynamic region characterized by negative values of Γ is called the inversion zone. It has been theoretically shown that, for $\Gamma < 0$, compression waves would smooth out, and rarefaction shocks could occur. The present CFD study covers a large range of Γ from negative up to high values. The thermodynamic state at the inlet and the numerical results for the inlet Mach number Ma_1 are listed in Table 4. In the CFD simulations, the inlet Mach number is also the choking Mach number if the choking limit was reached. The maximum Mach number $Ma_{2,max}$ is hence provided in Table 4 to enable a check the validity of the choking assumption. Using the values of the cascade flow area ratio $o/(s \sin\beta_1)$ for the configurations listed in Table 2, the choking Mach number was predicted using the universal term f_0 of equation (5). This predicted value does not depend on any thermodynamic property, and it can be easily calculated without any further thermodynamic computations. Finally, the relative deviations between the computed and the predicted inlet Mach numbers are listed in Table 4.

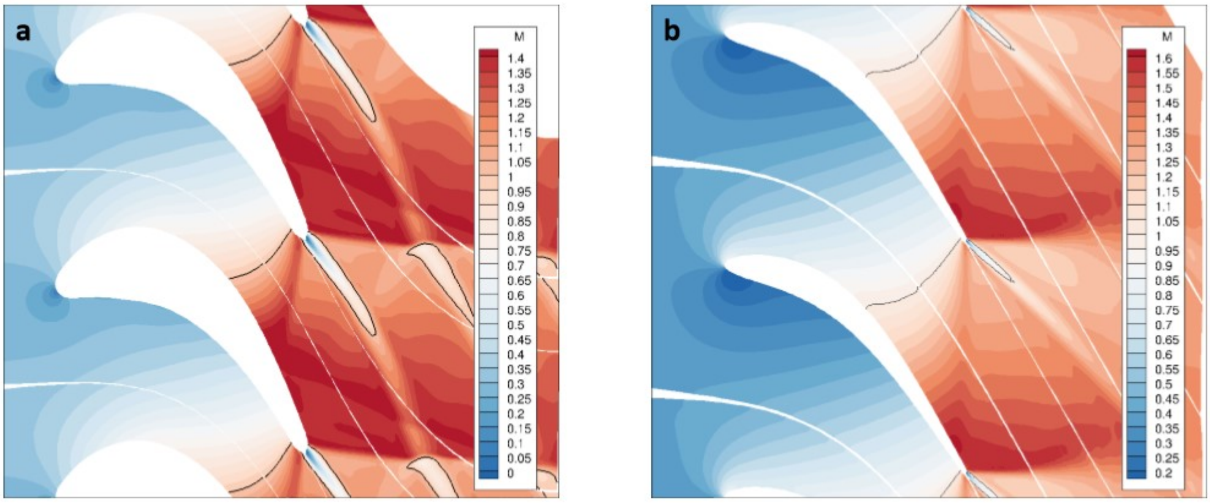

Figure 4: Computed Mach number distributions of flow of Toluene through the VKI-I (**a**) and the STCF 11 (**b**) cascades. The sonic line ($Ma = 1$) is shown as thin black line in the throat region.

Table 4: Numerical results for the choking Mach number for the VKI-I and STCF 11 cascades

Cascade	Fluid	ρ/ρ_{cr} at inlet	p/p_{cr} at inlet	Γ at inlet	Ma_1	$Ma_{2,max}$	Predicted by f_0	Deviation
VKI-I	Toluene	0.5160	0.9111	0.6779	0.2979	1.6030	0.275	+ 8.3 %
	Toluene	0.8130	1.0160	1.1350	0.2701	1.5770	0.275	- 1.8 %
	Toluene	0.9440	1.0500	1.0066	0.2612	1.5300	0.275	- 5.0 %
	Toluene	1.0890	1.0970	0.5346	0.2690	1.5020	0.275	- 2.2 %
	PP10	0.8258	1.0194	0.9756	0.3001	1.4260	0.275	+ 9.1 %
	PP10	0.9546	1.0531	1.6400	0.3001	1.4570	0.275	+ 9.1 %
	PP10	0.8855	0.9700	- 0.0689	0.2653	1.4390	0.275	- 3.5 %
STCF11	Toluene	0,4595	0.8468	0.6632	0.4551	1.7460	0.417	+ 9.1 %
	Toluene	0,7214	0.9561	0.9270	0.4608	1.5850	0.417	+ 10.5 %
	Toluene	0,8390	0.9869	0.8987	0.4643	1.5170	0.417	+ 11.3 %
	Toluene	0,7478	1.0274	0.7030	0.4550	1.4500	0.417	+ 9.1 %

The deviations between the numerically found choking Mach numbers and the predicted values using only the fundamental term f_0 remained below 10 % in the case of the VKI-I cascade. Even for an inlet condition in the inversion zone, $\Gamma = -0.0689$, the computed choking Mach number was close to the predicted value, and the deviation was only – 3.5 %.
In the case of the STCF 11 cascade, the computed choking Mach numbers Ma_1 were in a narrow band of about 0.46. That value is noticeably higher than the theoretically predicted value of order 0.417. That deviation could be explained by the sensitivity of the choking Mach number against the stagger angles. Using the nominal literature cascade data (with a stagger angle of 40.15°, Fransson et al. (1999)), the value $o/(s \sin\beta_1) = 0.60318$ results. This would lead to a predicted choking Mach number $Ma_{1,ch,f0} = 0.396$. That value is rather close to the experimentally observed value of $Ma_1 = 0.4$ in the case of the literature experiment with air as a working fluid (Fransson et al. (1999)). However, in the CFD study considered for the present assessment, the actual stagger angle was 44° instead of 40.15°, and hence a slightly higher value $o/(s \sin\beta_1) = 0.62982$ resulted. Using that value, a systematic deviation between the CFD results and the prediction using only the fundamental term f_0 of about 10 % results, see Table 4. This poorer agreement indicates that higher-order terms of the asymptotic expansion are required for an accurate prediction of the choking Mach number in the case of cascades characterized by higher values for the area ratio $o/(s \sin\beta_1)$. And indeed, the second-order expansion, including f_2, see equation (5), yielded a choking Mach number of $Ma_{1,\mathrm{ch}} = 0.450$ for Toluene with an inlet value of $\Gamma = 0.6632$, which is only 1.1 % away from the CFD result.

3.4 Discussion

The above results suggest that even the universal zeroth-order term, f_0 of equation (5), predicts the choking Mach number with reasonable accuracy. The observed deviations for non-ideal gas flows listed in Table 4 are of the same level as the scattering of the experimental data for air experiments conducted at different wind tunnels as reported by Kiock et al. (1986) and listed in Table 3. From a practical point of view, that accuracy level is hence acceptable for technical applications.
Following the idea of the asymptotic analysis approach, the choking Mach number might be predicted with higher accuracy utilizing a series including higher-order terms f_1 and f_2, see equation (5). This was, for instance, the case for the STCF 11 cascade. Then, it is necessary to calculate the isentropic exponent γ_{pv} for the actual expansion process to get the perturbation parameter x, see equation (3). That value can be calculated with good accuracy for quasi-perfect gas flows, but it might be relatively hard to get it in the case of non-classical gas flows or in expansion processes with substantial changes of the isentropic exponent. In such cases, it is recommended to use only the zeroth-order term and accept it as a reasonable approximation. It was, for example, not possible without further substantial numerical efforts to calculate the appropriate value for x in the case of the flow through the VKI-I cascade with an inlet value $\Gamma = -0.0689$.

From the viewpoint of turbomachinery design, the following two additional issues might be of some relevance, too. Firstly, the present method rests on the nominal cascade flow area ratio $o/(s \sin\beta_1)$. In reality, a small displacement thickness of the boundary layer flow reduces the effective throat area which affects the choking Mach number. This effect could be covered by reducing the geometric throat *o* by the displacement thicknesses of the flow, but some estimations regarding the boundary layer thickness are then required. Secondly, swirling flows, characterized by a non-negligible circumferential velocity component, are ubiquitous in internal flow devices like turbomachines. As mentioned by Tosto et al. (2020), it might be the case that swirling behavior could be substantially different than those exhibited by perfect gases. Such phenomena are not considered in the present linear cascade flow configuration, and they would require specific treatment. However, the linear cascade still represents the obvious starting point for any complex analyses.

4 CONCLUSIONS

It was demonstrated by experimental and numerical data that choking Mach numbers of linear turbine cascades can be predicted accurately, including the dense gas regime. A correlation for predicting the choking Mach number was derived employing an asymptotic method leading to a perturbation series with a dominant zeroth-order term. The observed deviations between the numerically found choking Mach numbers and the predictions of the zeroth-order term were of the same order as typical experimental uncertainty levels.

NOMENCLATURE

A	area	(m^2)	o	throat	(m)
b	axial chord	(m)	p	pressure	(Pa)
c	specific heat	(J/(kg K))	T	Temperature	(K)
f_i	i-th order function	(–)	s	spacing	(m)
Ma	Mach number	(–)	w	velocity	(m/s)
n	running speed	(1/s)	x	perturbation	(–)

Greek Symbols

β	angle	(°)	Γ	fundamental derivative	(–)
γ	isentropic exponent	(–)	ρ	density	(kg/m^3)

Subscripts

ch	choking	0	settling chamber or zeroth-order (f_0)
cr	critical point	1	inlet or first-order (f_1)
o	total	2	exit or second-order (f_2)

REFERENCES

Aus der Wiesche, S., Reinker, F., Wagner, R., Hake, L., Passmann, M., 2021, Critical and Choking Mach Numbers for Organic Vapor Flows Through Turbine Cascades, *Proceedings of ASME Turbo Expo 2021*, June 7-11, Virtual (paper-ID GT2021-59013)

Cinnella, P., Congedo, P. M., Laforgia, D., 2006, Transonic Flows of BZT Fluids Through Turbine Cascades, *Proceedings* DOI: 10.1007/3-540-31801-1_29

Congedo, P. M., Corre, C., Cinnella, P., 2011, Numerical investigation of dense-gas effects in turbomachinery, *Computer & Fluids*, vol. 49, pp. 290-301

Dejc, M. E., Trojanovskij, B. M., 1973, *Untersuchung und Berechnung axialer Turbinenstufen*, VEB Verlag Technik, Berlin

Fransson, T. H., Jöcker, M., Bölcs, A., Ott, P., 1999, Viscous and Inviscid Linear/Nonlinear Calculations Versus Quasi 3D Experimental Data for a new Aeroelastic Turbine Standard Configuration. *ASME Journal of Turbomachinery*, vol. 121, pp. 717-725

Kiock, R., Lehthaus, F., Baines, N. C., Sieverding, C. H., 1986, The Transonic Flow Through a Plane Turbine Cascade as Measured in Four European Wind Tunnels. *ASME Journal of Engineering for Gas Turbines and Power*, vol. 108, pp. 277-284

Lakshminarayana, B., 1996, *Fluid Dynamics and Heat Transfer of Turbomachinery*, John Wiley & Sons, New York

Martin, J. J., Hou, Y.-C., 1955, Development of an equation of state for gases, *AIChE Journal*, vol. 1(2), pp. 142-151

Scholz, N., 1965, *Aerodynamik der Schaufelgitter*, Verlag G. Braun, Karlsruhe (in German, English translation by A. Klein available as AGARDograph No. AG 220, 1970)

Tosto, F., Giuffre', A., Colonna, P., Pini, M., 2020, Non-Ideal Effects in Compressible Swirling Flows, *Proceedings 3rd International Seminar on Non-Ideal Compressible Fluid Dynamics for Propulsion and Power*, Delft, The Netherlands

Traupel, W., 1966, *Thermische Turbomaschinen – Band 1*, Springer, Berlin (in German, see especially Chapter 1)

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