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Fat brane, dark matter and localized kinetic terms in six dimensions

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Abstract Extra dimensions (ED) have been used as attempts to explain several phenomena in particle physics over the years. In this paper we investigate the role of an abelian gauge field as mediator of the interaction between dark matter (DM) and Standard Model (SM) particles, in a model with two flat and transverse ED compactified on the chiral square. DM is confined in a thin brane, localized at the origin of the chiral square, while the SM is localized in a finite width brane, lying in the opposite corner of the square. A brane-localized kinetic term is present in the DM brane, while in the fat brane it is not allowed. In this model the kinetic mixing is not required because we assume that the SM particles couple to the mediator through their B - Lcharges, while DM couples to it via a dark charge. Assuming a complex scalar field as DM candidate it is possible to obtain the observed DM relic abundance and avoid direct detection constraints for some parameter choices.

1 Introduction

Weakly interacting massive particles (WIMPs) have been the most well-known dark matter (DM) candidates [1] for decades, but the absence of any trace encourage us to look for different scenarios, both experimentally and theoretically. One promising way to chase DM would be if it interacts with the standard model (SM) particles through a new mediator. A relatively recent and very explored idea is the possible interaction between DM and SM via a new dark $U(1)_D$ gauge field, arising in turn from a kinetic mixing term between this new vector mediator (called dark photon, DP) and the hypercharge $U(1)_Y$ field [2–19].

Among other theoretical alternatives, extra dimensions (ED) have been considered over the decades as tools to address a wide range of issues in particle physics, such as the hierarchy [20–27] and flavor problems [28–30]. Mod-

els employing two ED, for example, may provide explanations for proton stability [31], origin of electroweak symmetry breaking [32-35], breaking of grand unified gauge groups [36–39] and the number of fermion generations [40– 45]. Many extensions of the SM appear by employing ED as well; indeed even the SM itself can be embedded in ED, whose fields propagate in the compact ED. In 4-D, the zero mode of each Kaluza-Klein (KK) tower of states is identified with the correspondent SM particle. These so-called Universal Extra Dimension (UED) models were build either with one [46] or two ED [47-50], for example, and current results from LHC [51,52] impose bounds on the UED compactification radius L for one $(L^{-1} > 1.4 - 1.5 \text{ TeV})$ [53–55] (for $\Lambda L \sim 5 - 35$, where Λ is the cutoff scale) or two ED $(L^{-1} > 900 \text{ GeV})$ [56]. In the context of ED, the DP model was embedded in a flat, single ED, along with DM candidates [57,58].

Much of the parameter space for the kinetic-mixing term has been excluded by several experiments and observations [11,59–75]. Its expected small value may be explained if one considers a single, flat ED and a thick brane [76], where the presence of a brane-localized-kinetic term (BLKT) spread inside the fat brane increase the suppression mechanism. BLKT appears as loop corrections associated with localized matter fields, giving rise to a massless spin-2 field [77] or massless spin-1 field [78]. The same mechanism also works for two ED [79-81], where the induced kinetic term is effectively 4-D, meaning that any expected extra scalar field, arising from the compactification of the ED has no contribution in 4-D. Thus for the graviton, for instance, the induced term on the brane describes a 4-D tensor gravity, rather than a 4-D tensor-scalar gravity. The role of BLKT has been investigated in several different scenarios [82–93], while the localization of matter or gauge fields in branes has been studied in other contexts, for thin [23,94–101] and thick branes [102,103].

In a recent paper [104], a model similar to the one presented in [76] was explored in 6-D, however it was shown

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that it is not possible to have a BLKT inside the fat brane, since the wave-functions do not satisfy the boundary conditions (BC) all along the boundary. Although it is expected to have BLKT in both thin and thick branes, one may investigate the case where the BLKT inside the fat brane is very small and can be neglected. This is the aim of this paper, where we show that it is possible to have a vector field in the bulk, which mediates the interaction between the SM particles localized in the fat brane and a DM candidate confined in a thin brane, without employing a kinetic-mixing term. The coupling with the SM, although not as suppressed as in [76], has a similar behavior. In this framework we can obtain the observed DM relic abundance for a range of parameter choices, as well as avoid DM direct detection constraints.

This paper is organized as follows. In Sect. 2 we present the vector mediator, a 6-D gauge field with BLKT on the chiral square. In Sect. 3 we analyze the resulting couplings with the SM and DM through the vector mediator. We examine the constraints on the SM interactions with the DM particle from both direct and indirect observations in Sect, 4, while Sect. 5 is reserved for conclusions.

2 Vector mediator in the bulk

We will consider two flat and transverse ED (x^4 and x^5) compactified on the chiral square. The chiral square is chosen in the UED model with two ED because it is the simplest compactification that leads to chiral quarks and leptons in 4-D [47]. The square has size πR , where *R* is the compactification radius of the ED, and the adjacent sides are identified (0, y) ~ (y, 0) and (πR , y) ~ (y, πR), with $y \in [0, \pi R]$. This means that the Lagrangians at those points have the same values for any field configuration: $\mathcal{L}(x^{\mu}, 0, y) = \mathcal{L}(x^{\mu}, y, 0)$ and $\mathcal{L}(x^{\mu}, \pi R, y) = \mathcal{L}(x^{\mu}, y, \pi R)$. A thin brane is localized at the origin (0, 0), where the DM candidate is confined, and the SM is contained within a fat brane, lying between ($\pi r, \pi r$) and ($\pi R, \pi R$), with a width $\pi (R - r) \equiv \pi L$, such that we assume $L \ll R$. The radius *r* represents the amount of the ED that is not part of the thick brane.

There is one abelian gauge field V^A , A = 0 - 3, 4, 5 in the bulk, interacting both with DM and SM. Since we are not assuming kinetic mixing, the vector field couples with DM and SM through the covariant derivative, which contains a term proportional to $\sim g_{6D}(B - L + Q_D)$, where g_{6D} is the 6-D dark gauge coupling. SM particles have $B - L \neq 0$ and $Q_D = 0$, while DM has B - L = 0 by assumption and, without loss of generality, $Q_D = 1$. We will use one of the four anomaly-free symmetries which do not need any additional SM fermion fields (beyond right-handed neutrinos, *i.e.*, the difference between baryon and lepton numbers (U_{B-L}) and the three differences between the lepton numbers $(U_{L_{\mu}-L_{e}}, U_{L_{e}-L_{\tau}}$ and $U_{L_{\mu}-L_{\tau}}$) [105–108], under which only baryons and/or leptons are charged. This is done in order to avoid dangerous couplings with the Higgs or gauge bosons, which in turn would spoil some of the well constrained electroweak predictions [55], such as the Z boson mass.

The action is similar to the one of UED model with two ED [47,48], given by

$$S = \int d^4x \int_0^{\pi R} dx^4 \int_0^{\pi R} dx^5 \left(-\frac{1}{4} V_{AB} V^{AB} + \mathscr{L}_{GF} + \mathscr{L}_{BLKT} \right), \tag{1}$$

where A is the 6-D index and the gauge fixing term has the following form to cancel the mixing between V_4 and V_5 with V_{μ} [48]

$$\mathscr{L}_{GF} = -\frac{1}{2} \Big[\partial_{\mu} V^{\mu} - (\partial_4 V_4 + \partial_5 V_5) \Big]^2, \qquad (2)$$

where we will work in the Feynman gauge. We will consider BLKT at the point (0, 0), where is localized the thin brane. Any BLKT on the fat brane should be very small [104] and will be neglected. Notice that KK parity is not preserved, although usually one invokes this Z_{KK}^2 symmetry in UED models in order for the lowest KK state to stable and to be the DM candidate, which is not needed in our case because the DM candidate is confined on the thin brane.

The BLKT at (0, 0) contributes with a term [77,80]

$$\mathscr{L}_{BLKT} = \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} - \frac{1}{2} (\partial_{\mu} V^{\mu})^2 \right] \cdot \delta_A R^2 \,\delta(x^4, x^5) \,,$$
(3)

where δ_A is positive constant.

Expanding the components of the 6-D gauge field in KK towers of states

$$V_{\mu}(x^{\nu}, x^{4}, x^{5}) = \sum_{j} \sum_{k} v_{0}^{(j,k)}(x^{4}, x^{5}) V_{\mu}^{(j,k)}(x^{\nu}), \quad (4)$$

$$V_4(x^{\nu}, x^4, x^5) = \sum_j \sum_k v_4^{(j,k)}(x^4, x^5) V_4^{(j,k)}(x^{\nu}), \qquad (5)$$

$$V_5(x^{\nu}, x^4, x^5) = \sum_j \sum_k v_5^{(j,k)}(x^4, x^5) V_5^{(j,k)}(x^{\nu}), \qquad (6)$$

leads to the solutions of the equations of motion for $v_4^{(j,k)}(x^4, x^5)$ and $v_5^{(j,k)}(x^4, x^5)$ [48]¹

$$v_4^{(j,k)}(x^4, x^5) = \frac{\sqrt{2}}{\pi R} \sin\left(\frac{jx^4 + kx^5}{R}\right),\tag{7}$$

$$v_5^{(j,k)}(x^4, x^5) = -\frac{\sqrt{2}}{\pi R} \sin\left(\frac{kx^4 - jx^5}{R}\right),\tag{8}$$

where *j* and *k* are integers. The physical masses of these scalar fields are $(M_{4,5}^{(j,k)})^2 = (j^2 + k^2)/R^2$, and notice that $V_4 = V_5 = 0$ for j = k = 0, from Eqs. (7) and (8). Notice

¹ In [48] the authors made the linear combinations $V_{\pm} = V_4 \pm i V_5$.

that because the scalar fields vanish at the thin brane, they do not interact with the DM (lying also in the thin brane).

The equation of motion for the wave-function $v_0^{(j,k)}(x^4, x^5)$ is

$$\left[\partial_4^2 + \partial_5^2 + M_{j,k}^2 + M_{j,k}^2 \delta_A R^2 \delta(x_4, x_5)\right] v_0^{(j,k)}(x^4, x^5) = 0,$$
(9)

where

$$M_{j,k}^2 = m_j^2 + m_k^2, (10)$$

whose solution yields [104]

$$v_{0}^{(j,k)}(x^{4}, x^{5}) = N_{j,k} \Big[\cos(m_{j}x^{4}) \cos(m_{k}x^{5}) \\ + \cos(m_{k}x^{4}) \cos(m_{j}x^{5}) \\ - \frac{\delta_{A}}{2} x_{j} x_{k} \Big(\sin(m_{j}x^{4}) \sin(m_{k}x^{5}) \\ + \sin(m_{k}x^{4}) \sin(m_{j}x^{5}) \Big) \Big],$$
(11)

where $m_j = x_j/R$ and $m_k = x_k/R$. The normalization constant $N_{j,k}$ is defined through

$$\int_{0}^{\pi R} dx^{4} \int_{0}^{\pi R} dx^{5} v_{i}^{(j,k)}(x^{4}, x^{5}) v_{i}^{(j',k')}(x^{4}, x^{5})$$
$$= \delta_{j,j'} \delta_{k,k'}, \qquad (12)$$

which results for $j \neq k$ in

$$N_{j,k}^{-2} = \frac{\pi^2 R^2}{2} \left\{ 1 + \frac{\delta_A}{4\pi^2} \cos^2(\pi x_j) \left[1 + \cos^2(\pi x_k) \right] + \frac{1}{4} \delta_A^2 x_j^2 x_k^2 - \frac{\delta_A}{2\pi} \left[x_k \cos^2(\pi x_j) \cot(\pi x_k) + x_j \cot(\pi x_j) \cos^2(\pi x_k) \right] - \frac{x_j x_k \sin(2\pi x_j) \sin(2\pi x_k)}{\pi^2 (x_j^2 - x_k^2)^2} + \frac{4x_k^2 \cos^2(\pi x_j) \csc^2(\pi x_k)}{\pi^2 (x_j^2 - x_k^2)^2} + \frac{4x_j^2 \csc^2(\pi x_j) \cos^2(\pi x_k)}{\pi^2 (x_j^2 - x_k^2)^2} + \frac{\sin(2\pi x_k)}{2\pi x_k} \right\},$$
 (13)

and for j = k in

$$N_{j=k}^{-2} = \pi^2 R^2 \left\{ 1 + \frac{\delta_A^2 x_j^4}{4\pi^2} + \frac{\delta_A^2 x_j^2}{16\pi^2} \sin^2 \left(2\pi x_j\right) - \frac{\delta_A^2 x_j^3}{4\pi} \sin \left(2\pi x_j\right) - \frac{\delta_A^2 x_j^3}{4\pi^2} \sin \left(2\pi x_j\right) + \frac{\sin^2 \left(2\pi x_j\right)}{4\pi^2 x_j^2} + \frac{\sin \left(2\pi x_j\right)}{\pi x_j} \right\}.$$
 (14)

At a first glance one might think that there are imaginary values for the normalization constants above, however the allowed values of x_j and x_k do not lead to complex numbers in Eqs. (13) and (14).

The 4-D gauge field is canonically normalized through the relations



Fig. 1 Solutions of the transcendental Eq. (16) for different values of δ_A

$$\int_{0}^{\pi R} dx^{4} \int_{0}^{\pi R} dx^{5} \left[1 + \delta_{A} R^{2} \delta(x^{4}, x^{5}) \right] v_{0}^{(j,k)} v_{0}^{(j',k')}$$

$$= Z_{(j,k)} \delta_{j,j'} \delta_{k,k'},$$

$$\int_{0}^{\pi R} dx^{4} \int_{0}^{\pi R} dx^{5} \left[\partial_{4} v_{0}^{(j,k)} \partial_{4} v_{0}^{(j',k')} + \partial_{5} v_{0}^{(j,k)} \partial_{5} v_{0}^{(j',k')} \right]$$

$$= Z_{(j,k)} M_{j,k}^{2} \delta_{j,j'} \delta_{k,k'},$$
(15)

where $Z_{(j,k)}$ is a normalization factor $Z_{(j,k)} = 1 + \delta_A R^2 v_0^{(j,k)}(0,0)$.

The transcendental equation that determines the roots x_j and x_k is found requiring the Dirichlet BC $v_0^{(j,k)}(\pi R, \pi R) = 0$, whose solutions depend only upon the parameter δ_A

$$\cot(\pi x_j)\cot(\pi x_k) = \frac{\delta_A}{2} x_j x_k \,. \tag{16}$$

The solutions of Eq. (16), given in [104], are reproduced in Fig. 1, for different values of δ_A . There are (2n+1) quantized masses for each curve *n*, where *n* is each one of the dashed lines. Each mode is described by the segments in the dashed lines, thus there is one mode for n = 0, a massive zero-mode $M_{0,0}$, while the second dashed line (n = 1) has three quantized masses $M_{0,1}$, $M_{1,0}$ and $M_{1,1}$, being the first two degenerate, etc. Notice that the masses $M_{j,k}$ and $M_{k,j}$ are degenerate. The whole continuous set of values (x_j, x_k) in each segment represent only one mass state, being narrow the range of each state [104].

3 Interactions

The couplings between the tower of (KK) mediators and DM is $g_{D,(j,k)} = g_D N_{j,k} / N_{0,0}$, where $g_D \equiv g_{6D} N_{0,0}$, g_{6D} is the 6-D dark gauge coupling and $N_{0,0}$ is the normalization of

the lowest KK state (j = k = 0). On the other hand, the interaction between the vector field and a generic zerothmode SM field ϕ , localized inside the fat brane, is given by the integral over the brane width

$$\int_{\pi r}^{\pi R} dx^4 \int_{\pi r}^{\pi R} dx^5 V^{\mu} J_{\mu} , \qquad (17)$$

where J_{μ} is the SM current. We are interested in the interaction with conventional SM particles, thus the zeroth-mode of SM field in the 6D UED model is $\phi/(\pi^2 L^2)$, where $(\pi L)^{-1}$ is the normalization constant. The 4-D gauge couplings between the SM fields and the KK mediators are defined as

$$g_{D,(j,k)}^{ED} \equiv g_{6D} \int_{\pi r}^{\pi R} dx^4 \int_{\pi r}^{\pi R} dx^5 \frac{v_0^{(j,k)}(x^4, x^5)}{\pi^2 L^2}$$
(18)

which in turn yields

$$g_{D,(j,k)}^{ED} = \frac{g_D N_{j,k} R^2}{N_{0,0} \pi^2 L^2} \left\{ \frac{2}{x_j x_k} \left[\sin\left(\pi x_j\right) - \sin\left(\frac{\pi r x_j}{R}\right) \right] \right. \\ \left. \times \left[\sin\left(\pi x_k\right) - \sin\left(\frac{\pi r x_k}{R}\right) \right] \right. \\ \left. - \delta_A \left[\cos\left(\pi x_j\right) - \cos\left(\frac{\pi r x_j}{R}\right) \right] \right. \\ \left. \times \left[\cos\left(\pi x_k\right) - \cos\left(\frac{\pi r x_k}{R}\right) \right] \right\},$$
(19)

for $j \neq k$, while for j = k the result is

$$g_{D,(j,j)}^{ED} = \frac{g_D N_{j,j} 2R^2}{N_{0,0} \pi^2 L^2 x_j^2} \bigg\{ \sin^2 \bigg(\frac{\pi x_j L}{2R} \bigg) \\ \times \bigg(\bigg(\delta_A x_j^2 + 2 \bigg) \cos \bigg(\frac{\pi x_j (r+R)}{R} \bigg) - \delta_A x_j^2 + 2 \bigg) \bigg\}.$$
(20)

From Eqs. (19) and specially (20) we see that in the limit of small roots x_j , since $L \ll R$, $g_{D,(j,j)}^{ED} \sim \frac{g_D N_{j,j}}{2N_{0,0}}$, while in 5-D the coupling is reduced by a factor proportional to L/R [76]. This behavior does not occur here because there is no significant BLKT in the fat brane.

For illustrative purposes, we will consider four specific benchmark models (BM), whose assumed set of values for the compactification radius R and the width of the fat brane L are shown in Table 1.

 Table 1 Illustrative sets of compactification radius R and fat brane width L

BM	Ι	II	III	IV
R^{-1}	1 GeV	1 GeV	100 MeV	100 MeV
L^{-1}	1 TeV	10 TeV	1 TeV	10 TeV

In Figs. 2, 3, 4 and 5 we plot the oscillatory behavior of the gauge coupling (19), for different values of δ_A and for the four BM in Table 1. The coupling decreases as x_j and x_k increases, and although it maintains the same pattern for different values of δ_A , the coupling is orders of magnitude smaller as δ_A is increased. The difference between the four BM is similar to the one presented in [76]: all BM show the same oscillatory pattern, but BM II (IV) reproduce the exact plot in II (III) after ten times more roots x_j , while decreasing the inverse of the compactification radius *R* makes the coupling smaller (compare BM II in Fig. 3 with III in Fig. 4).

4 Constraints from observations and experiments

We now consider the interactions between the DM candidate confined at the origin of the chiral square with the SM, mediated by the KK tower of states. We assume a complex scalar field as a DM candidate for simplicity, which is naturally stable via the Z_2 symmetry. The couplings between the KK mediators and the DM is $g_{D,(j,k)} \equiv g_D N_{j,k}/N_{0,0}$, while inside the fat brane the coupling is $g_{D,(j,k)}^{ED}$ as described in Eqs. (19) and (20). The mass and couplings of the DM particle are constrained by both direct and indirect experiments. In order for the DM not to annihilate into a pair of mediator particles (avoiding the s-wave annihilation excluded by Planck results [109]), DM should be lighter than the lightest mediator KK state. Thus the DM mass must be smaller than the lowest mediator mass $x_{0,0}/R$, whose root $x_{0,0}$ lies in the range ~ 0.4 to 0.5.

Assuming that the DM relic abundance is due to thermal freeze-out, the resulting final states from DM pair annihilation can be e^+e^- and $\mu^+\mu^-$, as well as three generations of nearly massless neutrinos, given the DM mass range of interest and only the final states that are charged under B-L. Note that the only accessible channels for BM II and IV are e^+e^- and neutrinos, because DM particle is lighter than muons. Considering the usual expansion of the thermally- averaged cross section (away from the resonance) in powers of the relative velocity of DM particles, v^2 , given by $\sigma v \approx a + bv^2$, we have the following coefficients for a vector mediator and a complex scalar DM: a = 0 and [110]

$$b_{f} = \frac{m_{DM}^{2}}{6\pi} \sqrt{1 - \frac{m_{f}^{2}}{m_{DM}^{2}}} \left(1 - \frac{m_{f}^{2}}{2m_{DM}^{2}}\right) \times \left(\sum_{n} \frac{g_{D,(j,k)}g_{D,(j,k)}^{ED}}{M_{j,k}^{2} - 4m_{DM}^{2}}\right)^{2}, \qquad (21)$$

where m_f is the mass of the final states. The (dominant) pwave DM annihilation is therefore not constrained by current observations [109, 111].



Fig. 2 Gauge KK couplings as a function of x_j with x_k fixed at around $x_k \sim 1.5$ (left) and $x_k \sim 10^3$ (right), for $g_D = 1$, $\delta_A = 1$ (first row), $\delta_A = 10$ (second row) and $\delta_A = 100$ (third row), for the BM I

The observed value of the DM relic density is obtained then through [110]

$$\Omega h^2 \simeq \frac{x_f \, 1.07 \times 10^9 \, \text{GeV}^{-1}}{g_*^{1/2} M_{Pl}(a+3b/x_f)} \,, \tag{22}$$

where M_{Pl} is the Planck mass and $x_f \equiv m_{DM}/T_f$ is the usual ratio between the DM mass and the temperature at the freeze-out, which can be taken to be $x_f = 20$. The effective number of degrees of freedom for the range of DM masses of

interest here (40–700 MeV) is $g_* \simeq 10.75$, since the temperature at the freeze-out is ~ 2–40 MeV. The value of the coupling g_D which gives the observed relic density ($\Omega h^2 = 0.12$ [109]) is calculated for the four BM with different values of $\delta_A = 0.1, 1, 10$. The lightest KK mediator mass is presented in Table 2, for these parameter choices. As we can see the mass of the lightest KK state increases as δ_A decreases.

Since DM masses here are relatively light and the corresponding recoil energies in direct detection is small, DM scat-



Fig. 3 Gauge KK couplings as a function of x_j with x_k fixed at around $x_k \sim 1.5$ (left) and $x_k \sim 10^4$ (right), for $g_D = 1$, $\delta_A = 1$ (first row), $\delta_A = 10$ (second row) and $\delta_A = 100$ (third row), for the BM II

tering off electrons provides greater sensitivity [61]. In our case the scattering cross section then becomes [110,112,113]

$$\sigma_e = \frac{\mu^2}{4\pi} \left(\sum_{j,k} \frac{g_{D,(j,k)} g_{D,(j,k)}^{ED}}{M_{j,k}^2} \right)^2, \qquad (23)$$

where a form factor of unity has been assumed and the reduced mass $\mu = m_e m_{DM}/(m_e + m_{DM}) \sim m_e$, since $m_{DM}^2 \gg m_e^2$. The resulting scattering cross section has

been constrained using the results from XENON10 [114], XENON100 [115], DarkSide-50 [116] and SENSEI [117]. Additionally, low energy accelerator experiments impose constraints on the $U(1)_{B-L}$ gauge field mass and coupling [108,118]. Although they are evaluated for a gauge field with kinetic mixing term, the constraints can be easily translated to the present model. The parameter space is very constrained for the range of masses presented in Table 2 (40–700 MeV). Only the BM II with $\delta_A = 10$ is not ruled out by direct



Fig. 4 Gauge KK couplings as a function of x_j with x_k fixed at around $x_k \sim 1.5$ (left) and $x_k \sim 10^4$ (right), for $g_D = 1$, $\delta_A = 1$ (first row), $\delta_A = 10$ (second row) and $\delta_A = 100$ (third row), for the BM III

detection experiments. Of course, these parameter choices are representative and other values of *R*, *L* and δ_A can give similar results.

In Fig. 6 we show the part of the parameter space that is allowed to explain the observed DM relic density.

5 Conclusions

In this paper we have investigated the role of an abelian gauge field as mediator of the interaction between a DM candidate and the SM, in a model with two ED compactified on the chiral square. DM is localized in one thin brane at the conical singularity (0, 0), while a fat brane is lying between $(\pi r, \pi r)$ and $(\pi R, \pi R)$. SM is confined in the fat brane and its fields propagate in the ED similarly to UED models, but the vector



Fig. 5 Gauge KK couplings as a function of x_j with x_k fixed at around $x_k \sim 1.5$ (left) and $x_k \sim 10^5$ (right), for $g_D = 1$, $\delta_A = 1$ (first row), $\delta_A = 10$ (second row) and $\delta_A = 100$ (third row), for the BM IV

mediator interacts with the visible sector only though B - L charged particles.

BLKT is present only in the thin brane because the BC do not allow them in the thick brane [104]. Notice that we did not need to introduce a kinetic-mixing term and the relative smallness of the coupling can be explained if a BLKT in the fat brane is very small and can be neglected. Due to BC the U(1) symmetry is broken without demanding any Higgs mechanism in the bulk, and the resulting roots that determine the masses of the KK states depend only upon

the BLKT parameter δ_A . The effective coupling between the mediator and the SM particles, due to the fat brane, has a similar behavior as in previous results [76], depending upon the 6-D compactification radius *R* and the SM brane thickness *L*, although it is not as suppressed as in the 5-D case. Considering a complex scalar field as a DM candidate, the DM relic abundance can be satisfied by some parameter choices, whose values also avoid direct detection constraints.

This model may lead to distinct signatures in the upcoming experiments and it resembles the 5-D case: the combination



Fig. 6 Allowed parameter space (BM II) that can satisfy the observed DM relic abundance (blue line). Notice that $m_{DM} < M_{0,0}$. The region in orange and brown are excluded by current direct detection experiments. As shown in Eq. (21) DM masses were assumed, for simplicity, to be away from the resonant region

Table 2 Lightest vector mediator mass $M_{0,0}$ for $\delta_A = 10^{-1}$, 1 or 10

BM	Ι	II	III	IV
$\delta_A = 10^{-1}$				
$M_{0,0}$ [MeV]	710	710	71	71
$\delta_A = 1$				
$M_{0,0}$ [MeV]	570	570	57	57
$\delta_A = 10$				
<i>M</i> _{0,0} [MeV]	420	420	42	42

of searches for KK vector mediators and UED particles in two ED, where for the latter the compactification radius Lis constrained through the missing energy from the cascade decay of SM KK particles. Moreover, the main final states of the lightest mediator decay are missing energy or charged leptons, while for the KK mediator modes, twice or more as heavy as DM, the resulting cascade decay gives a missing energy signature as well.

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