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The Optimal Strategy for ε'/ε in the SM: 2019

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Abstract. Following the recent analysis done in collaboration with Jason Aebischer and Christoph Bobeth, I summarize the optimal, in our view, strategy for the present evaluation of the ratio ε'/ε in the Standard Model. In particular, I emphasize the importance of the correct matching of the long-distance and short-distance contributions to ε'/ε , which presently is only achieved by RBC-UKQCD lattice QCD collaboration and by the analytical Dual QCD approach. An important role play also the isospin-breaking and QED effects, which presently are best known from chiral perturbation theory, albeit still with a significant error. Finally, it is essential to include NNLO QCD corrections in order to reduce unphysical renormalization scheme and scale dependences present at the NLO level. Here μ_c in $m_c(\mu_c)$ in the case of QCD penguin contributions and μ_t in $m_t(\mu_t)$ in the case of electroweak penguin contributions play the most important roles. Presently the error on ε'/ε is dominated by the uncertainties in the QCD parameter $B_6^{(1/2)}$ and the isospin-breaking parameter $\hat{\Omega}_{\text{eff}}$. We present a table illustrating this.

1. Introduction

The present superstars in Kaon physics are in my view the ratio ε'/ε , describing the direct CP violation in $K_L \rightarrow \pi\pi$ decays relative to the indirect one, the $\Delta I = 1/2$ rule in $K \rightarrow \pi\pi$ decays, $K^0 - \bar{K}^0$ mixing with ε_K and ΔM_K and of course $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$. But there are several decays like $K_L \rightarrow \pi^0\ell^+\ell^-$, $K_{S,L} \rightarrow \mu^+\mu^-$ and $K_L \rightarrow \mu^+e^-$ that are important for the search for new physics (NP). They were presented in several talks at this conference.

I will concentrate here on ε'/ε which is rather difficult to evaluate. In order to see the reason for it, let us look at the effective Hamiltonian at the low energy scales. It has the following general structure:

$$\mathcal{H}_{\text{eff}} = \sum_i C_i \mathcal{O}_i^{\text{SM}} + \sum_j C_j^{\text{NP}} \mathcal{O}_j^{\text{NP}}, \quad C_i = C_i^{\text{SM}} + \Delta_i^{\text{NP}}, \quad (1)$$

where

- $\mathcal{O}_i^{\text{SM}}$ are local operators present in the Standard Model (SM) and $\mathcal{O}_j^{\text{NP}}$ are new operators having typically new Dirac structures, in particular scalar-scalar and tensor-tensor ones.
- C_i and C_j^{NP} are the Wilson coefficients of these operators. NP effects modify not only the Wilson coefficients of SM operators but also generate new operators with non-vanishing C_j^{NP} .

The amplitude for the transition $K \rightarrow \pi\pi$ can now be written as follows

$$\mathcal{A}(K \rightarrow \pi\pi) = \sum_i C_i \langle \pi\pi | \mathcal{O}_i^{\text{SM}} | K \rangle + \sum_j C_j^{\text{NP}} \langle \pi\pi | \mathcal{O}_j^{\text{NP}} | K \rangle. \quad (2)$$



The coefficients C_i and C_j^{NP} can be calculated in the renormalization group (RG) improved perturbation theory. The status of these calculations is by now very advanced, as reviewed in Ref. [1] and in the talk by Maria Cerdá-Sevilla at this conference. The complete NLO corrections have been calculated almost 30 years ago [2, 3, 4, 5, 6, 7]. The dominant NNLO QCD corrections to electroweak penguin (EWP) contributions have been presented in [8] and those to QCD penguins (QCDP) should be known soon [9, 10, 11]. We will see below that in the case of ε'/ε the NNLO QCD contributions play a significant role. On the whole, the status of present short distance (SD) contributions to ε'/ε is satisfactory.

The evaluation of the hadronic matrix elements is a different story. In $K \rightarrow \pi\pi$ decays, we have presently three approaches to our disposal:

- **Lattice QCD (LQCD)**. It is a sophisticated numerical method with very demanding calculations lasting many years. Yet, it is based on first principles of QCD and eventually in the case of $K \rightarrow \pi\pi$ decays and $K^0 - \bar{K}^0$ mixing it is expected to give the ultimate results for ε'/ε , $\Delta I = 1/2$ rule and $K^0 - \bar{K}^0$ mixing, both in the SM and beyond it. For $K \rightarrow \pi\pi$ only results for the SM operators are known and they are not yet satisfactory. The ones for BSM $K^0 - \bar{K}^0$ matrix elements are already known with respectable precision and interesting results have been obtained for long distance contributions to ΔM_K [12, 13].
- **Dual QCD (DQCD)** proposed already in the 1980s [14] and significantly improved in this decade [15]. This approach allows to obtain results for $K \rightarrow \pi\pi$ decays and $K^0 - \bar{K}^0$ mixing much faster than it is possible with the LQCD so that several relevant results have been obtained already in the 1980s and confirmed within uncertainties by LQCD in this decade. While not as accurate as the expected ultimate LQCD calculations, it allowed already to calculate hadronic matrix elements for all BSM operators entering $K \rightarrow \pi\pi$ decays [16] and $K^0 - \bar{K}^0$ mixing [17]. The latter paper allowed to get the insight into the QCD dynamics at low energy scales which is not possible using a purely numerical method like LQCD. For a recent review see Ref. [18]. More about it below.
- **Chiral Perturbation Theory (ChPT)** developed since 1978 [19, 20, 21, 22, 23, 24, 25] and discussed in several talks at this conference, in particular by Antonio Rodríguez Sánchez [26] and Toni Pich [27] in the context of ε'/ε . It is based on global symmetries of QCD with the QCD dynamics parametrized by low-energy constants L_i that enter the counter terms in meson loop calculations. L_i can be extracted from the data or calculated by LQCD but to this end the large N limit has to be taken. In the case of non-leptonic transitions this implies serious difficulties in matching long distance (LD) and short distance (SD) contributions in this framework. The point is that in the large N limit only factorizable contributions in hadronic matrix elements are present, whereas the dominant QCD dynamics in Wilson coefficients is given by non-factorizable contributions. This problem is absent in LQCD and DQCD as we will discuss below. Therefore, while the ChPT approach is very suitable for leptonic and semi-leptonic Kaon decays, it can only provide partial information on ε'/ε and $\Delta I = 1/2$ rule in the form of isospin breaking effects and final state interactions (FSI). Yet, in the case of isospin breaking contributions to ε'/ε the difficulties in the matching in question imply a rather significant error as we will see below.

This writing is arranged as follows. In Section 2 we will briefly describe the DQCD approach. In Section 3 we will present, in our view, the optimal strategy for the calculation of ε'/ε in the SM as of 2019, illustrating in particular the importance of NNLO QCD effects and isospin breaking corrections in the evaluation of this ratio. This presentation is fully based on the recent analysis of ε'/ε in collaboration with Jason Aebischer and Christoph Bobeth [28]. A brief outlook in Section 4 ends this presentation.

2. Grand View on the Dual QCD Approach

This analytic approach to $K \rightarrow \pi\pi$ decays and $K^0 - \bar{K}^0$ mixing in Refs. [14, 29, 15] is based on the ideas of 't Hooft and Witten who studied QCD with a large number N of colours. In this limit QCD is dual to a theory of weakly interacting mesons with the coupling $\mathcal{O}(1/N)$ and in particular in the strict large N limit it becomes a free theory of mesons, simplifying the calculations significantly. With non-interacting mesons the factorization of matrix elements of four-quark operators into matrix elements of quark currents and quark densities, used adhoc in the 1970s and early 1980s, is automatic and can be considered as a property of QCD in this limit [30]. But the factorization cannot be the whole story as the most important QCD effects related to asymptotic freedom are related to non-factorizable contributions generated by exchanges of gluons. In DQCD this role is played by meson loops that represent dominant non-factorizable contributions at the very low energy scales. Calculating these loops with a momentum cut-off Λ one finds then that the factorization in question does not take place at values of $\mu \geq 1$ GeV at which Wilson coefficients are calculated, but rather at very low momentum transfer between colour-singlet currents or densities.

Thus, even if in the large N limit the hadronic matrix elements factorize and can easily be calculated, in order to combine them with the Wilson coefficients, loops in the meson theory have to be calculated. In contrast to chiral perturbation theory, in DQCD a physical cut-off Λ is used in the integration over loop momenta. As discussed in detail in Refs. [14, 15] this allows to achieve a much better matching with short distance contributions than it is possible in ChPT, which uses dimensional regularization. The cut-off Λ is typically chosen around 0.7 GeV when only pseudoscalar mesons are exchanged in the loops [14] and can be increased up to 0.9 GeV when contributions from lowest-lying vector mesons are taken into account as done in Ref. [15]. These calculations are done in a momentum scheme, but as demonstrated in [15], they can be matched to the commonly used naive dimensional regularization (NDR) scheme. Once this is done it is justified to set $\Lambda \approx \mu$. We ask sceptical readers to study a detailed exposition of DQCD in Ref. [15], where also the differences from the usual ChPT calculations are emphasized.

The application of DQCD to weak decays consists in any NP model of the following steps:

Step 1: At Λ_{NP} one integrates out the heavy degrees of freedom and performs the RG evolution including Yukawa couplings and all gauge interactions present in the SM down to the electroweak scale. This evolution involves in addition to SM operators also beyond the SM (BSM) operators. This is the Standard Model effective field theory (SMEFT).

Step 2: At the electroweak scale W , Z , top quark and the Higgs are integrated out and the SMEFT is matched onto the effective field theory with only SM quarks except the top-quark, the photon and the gluons. Subsequently QCD and QED evolution is performed down to scales $\mathcal{O}(1 \text{ GeV})$.

Step 3: Around scales $\mathcal{O}(1 \text{ GeV})$ the matching to the theory of mesons is performed and the so-called *meson evolution* to the factorization scale is performed.

Step 4: The matrix elements of all operators are calculated in the large N limit, that is using factorization of matrix elements into products of currents or densities.

We do not claim that these are all QCD effects responsible for non-leptonic transitions, but these evolutions based entirely on non-factorizable QCD effects, both at short distance and long distance scales, appear to be the main bulk of QCD dynamics responsible for the $\Delta I = 1/2$ rule, ε'/ε and $K^0 - \bar{K}^0$ mixing. Past successes of this approach have been reviewed in Refs. [18, 31]. They are related in particular the non-perturbative parameter \hat{B}_K in $K^0 - \bar{K}^0$ mixing and $\Delta I = 1/2$ rule [15]. In fact DQCD allowed for the first time to identify already in 1986 the dominant mechanism behind this rule [14].

In 2018 a significant progress towards the general search for NP in ε'/ε with the help of DQCD has been made:

- The first to date calculations of the $K \rightarrow \pi\pi$ matrix elements of the chromo-magnetic dipole

operators [32] that are compatible with the LQCD results for $K \rightarrow \pi$ matrix elements of these operators obtained earlier in [33].

- The calculation of $K \rightarrow \pi\pi$ matrix elements of *all* four-quark BSM operators, including scalar and tensor operators, by DQCD [16].
- The derivation of a master formula for ε'/ε [34], which can be applied to any theory beyond the SM in which the Wilson coefficients of all contributing operators have been calculated at the electroweak scale. The relevant hadronic matrix elements of BSM operators used in this formula are from the DQCD, as lattice QCD did not calculate them yet, and the SM ones from LQCD.
- This allowed to perform the first to date model-independent anatomy of the ratio ε'/ε in the context of the $\Delta S = 1$ effective theory with operators invariant under QCD and QED and in the context of the SMEFT with the operators invariant under the full SM gauge group [35].
- Finally the insight from DQCD [17] into the values of BSM $K^0 - \bar{K}^0$ elements obtained by LQCD made sure that the meson evolution is hidden in lattice calculations.

The main messages from these papers are as follows:

- The inclusion of the meson evolution in the phenomenology of any non-leptonic transition like $K^0 - \bar{K}^0$ mixing and $K \rightarrow \pi\pi$ decays with ε'/ε and the $\Delta I = 1/2$ rule is mandatory!
- Meson evolution is hidden in LQCD results, but among analytic approaches only DQCD takes this important QCD dynamics into account. Whether meson evolution is present in the low energy constants L_i of ChPT is an interesting question, still to be answered.
- Most importantly, the meson evolution turns out to have the pattern of operator mixing, both for SM and BSM operators, to agree with the one found perturbatively at short distance scales. This allows for a satisfactory, even if approximate, matching between Wilson coefficients and hadronic matrix elements.

In summary DQCD turns out to be an efficient approximate method for obtaining results for non-leptonic decays, years and even decades, before useful results from numerically sophisticated and demanding lattice calculations could be obtained.

3. ε'/ε in the SM

3.1. Preliminaries

The situation of ε'/ε in the SM after the International Conference on Kaon Physics 2019 can be briefly summarized as follows:

- The analysis of ε'/ε by the RBC-UKQCD LQCD collaboration based on their 2015 results for $K \rightarrow \pi\pi$ matrix elements [36, 37], as well as the analyses performed in [38, 39] that are based on the same matrix elements but also include isospin breaking effects, found ε'/ε in the ballpark of $(1 - 2) \times 10^{-4}$. This is by one order of magnitude below the experimental world average from NA48 [40] and KTeV [41, 42] collaborations,

$$\boxed{(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}} \quad (3)$$

However, with an error in the ballpark of 5×10^{-4} obtained in these analyses, one can talk about an ε'/ε anomaly of at most 3σ . The RBC-UKQCD collaboration is expected to present soon new values of the $K \rightarrow \pi\pi$ hadronic matrix elements. Not only statistical errors have been significantly decreased, but also a better agreement with the experimental values of $\pi\pi$ -strong-interaction phases $\delta_{0,2}$ has been obtained [43, 44, 45]. Unfortunately, the inclusion of isospin-breaking and QED effects will still take more time.

- An independent analysis based on hadronic matrix elements from the DQCD approach [46, 47] gave a strong support to these values and moreover provided an *upper bound* on ε'/ε in the ballpark of 6×10^{-4} . However, this bound does not include the effects of final state interactions and it will be of interest to see how it will be modified when the latter are taken into account.
- A different view has been expressed in Refs. [26, 27, 48, 49], where, using ideas from ChPT, the authors found $\varepsilon'/\varepsilon = (14 \pm 5) \times 10^{-4}$ after the improved estimate of isospin-breaking corrections to ε'/ε . While in agreement with the measurement, the large uncertainty, that expresses the difficulties in matching long-distance and short-distance contributions in this framework, does not allow for any clear-cut conclusions. See also Refs. [47, 50] for a critical analysis of this approach as used in the context of ε'/ε .
- The preliminary result on NNLO QCD corrections to QCDP contributions [10, 11] demonstrates significant reduction of various scale uncertainties, foremost of μ_c , and indicates an additional, though modest, suppression of ε'/ε .

In contrast to the expected RBC-UKQCD result, the ChPT analysis includes isospin-breaking and QED corrections, but the known difficulties in matching long-distance and short-distance contributions in this approach imply a large uncertainty. In particular, the absence of the meson evolution in ChPT that suppresses ε'/ε within the DQCD approach [47, 50] is responsible for the poor matching and the large value of ε'/ε quoted above. The DQCD analysis [17] demonstrates on the example of BSM matrix elements in $K^0 - \bar{K}^0$ mixing that the effects of meson evolution are included in the present LQCD calculations. As shown in Ref. [17], neglecting this evolution in the case of $K^0 - \bar{K}^0$ mixing would miss the values of the relevant hadronic matrix by factors of 2–4, totally misrepresenting their values obtained by three LQCD collaborations [51, 52, 53, 54, 55]. Therefore, without the inclusion of these important QCD dynamics in the calculation of ε'/ε , the validity of the present ChPT result can be questioned.

Now all the analyses of ε'/ε until International Conference on Kaon Physics 2019, including the one in Ref. [49], used the known Wilson coefficients at the NLO level [2, 3, 4, 5, 6, 7] in the NDR scheme [56]. But already in Ref. [8] and recently in Refs. [35, 50] it has been pointed out that without NNLO QCD corrections to EWP contribution the results for ε'/ε are renormalization-scheme dependent and exhibit significant non-physical dependences on the scale μ_t at which the top-quark mass $m_t(\mu_t)$ is evaluated as well as on the matching scale μ_W .

Fortunately, all these uncertainties have been significantly reduced in the NNLO matching at the electroweak scale performed in Ref. [8] and it is of interest to look at them again in the context of new analyses with the goal to improve the present estimate of ε'/ε .

Now the LQCD calculations contain the meson evolution and have recently improved on FSI effects. On the other hand, DQCD has difficulties with the inclusion of the latter, while ChPT has difficulties in matching long distance and short distance contributions. Therefore, the optimal strategy for the evaluation of ε'/ε , as of 2019, appears to be as follows [28]:

Step 1: Use future RBC-UKQCD results for hadronic matrix elements of the dominant QCDP (Q_6) and EWP (Q_8) operators, represented by the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ respectively – with improved values of $\pi\pi$ -strong-interaction phases $\delta_{0,2}$ – but determine hadronic matrix elements of $(V - A) \otimes (V - A)$ operators from the experimental data on the real parts of the $K \rightarrow \pi\pi$ amplitudes, as done in Refs. [5, 38].

Step 2: Use the result for isospin-breaking and QED corrections from Ref. [49], which are compatible with the ones obtained already 30 years ago in Ref. [57].

Step 3: Use the NNLO QCD contributions to EWP in [8] in order to reduce the unphysical renormalization scheme and scale dependences in the EWP sector.

Step 4: Include NNLO QCD contributions to QCDP from [10, 11] in order to reduce left-over renormalization scale uncertainties.

	a^{QCDP}	$a_6^{(1/2)}$	a^{EWP}	$a_8^{(3/2)}$
$\mu_W = \mu_t = m_W$				
NLO	-4.19	17.68	-2.08	8.25
NNLO (EWP)	-4.19	17.68	-2.00	8.82
$\mu_W = m_W$ and $\mu_t = m_t$				
NLO	-4.18	17.63	-1.94	7.22
NNLO (EWP)	-4.18	17.63	-2.03	8.51

Table 1. Coefficients entering the semi-numerical formula of Eq. (4).

In view of the fact that meson evolution and the remaining three effects tend to suppress ε'/ε , the expectation based on the DQCD approach in Ref. [50] that $\varepsilon'/\varepsilon \approx (5 \pm 2) \times 10^{-4}$ in the SM is likely to be confirmed soon by LQCD.

The main goal of Ref. [28] was to illustrate the importance of isospin-breaking and QED corrections [49] and of the NNLO QCD contributions to EWP in Ref. [8], that were absent in the 2015 result of RBC-UKQCD and also in Refs. [38, 39].

Using the technology in Ref. [38] the analysis in Ref. [28] arrives at the formula ($a = 1.017$)

$$\frac{\varepsilon'}{\varepsilon} = \text{Im } \lambda_t \cdot \left[a(1 - \widehat{\Omega}_{\text{eff}}) \left(a^{\text{QCDP}} + a_6^{(1/2)} B_6^{(1/2)} \right) - a^{\text{EWP}} - a_8^{(3/2)} B_8^{(3/2)} \right], \quad (4)$$

with the numerical values of the coefficients given in Table 1 at NLO and NNLO from EWPs as discussed below. Explicit formulae for $a^{\text{QCDP}} = a_0^{(1/2)} - b a_{0,\text{EWP}}^{(1/2)}$, $a^{\text{EWP}} = a_0^{(3/2)} - a_{0,\text{EWP}}^{(1/2)}$, $a_6^{(1/2)}$ and $a_8^{(3/2)}$ in terms of Wilson coefficients and $\text{Re } A_{0,2}$ are given in Ref. [38], where we have introduced $a_{0,\text{EWP}}^{(1/2)}$ as the EWP contribution to $a_0^{(1/2)}$ and $b^{-1} = a(1 - \widehat{\Omega}_{\text{eff}})$. Other details are presented in Ref. [28]. $\lambda_t = V_{td} V_{ts}^*$ is the relevant CKM combination.

The $B_6^{(1/2)}$ and $B_8^{(3/2)}$ parameters, that enter the formula of Eq. (4), are defined as follows

$$\langle Q_6(\mu) \rangle_0 = -4h \left[\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2 (F_K - F_\pi) B_6^{(1/2)} = -0.473 h B_6^{(1/2)} \text{ GeV}^3, \quad (5)$$

$$\langle Q_8(\mu) \rangle_2 = \sqrt{2}h \left[\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2 F_\pi B_8^{(3/2)} = 0.862 h B_8^{(3/2)} \text{ GeV}^3. \quad (6)$$

In the large- N limit $B_6^{(1/2)} = B_8^{(3/2)} = 1$ [58, 57]. We have introduced the factor h in order to emphasize different normalizations of these matrix elements present in the literature. For instance RBC-UKQCD and the authors of Ref. [38] use $h = \sqrt{3/2}$, while $h = 1$ is used in Refs. [46, 47, 48, 49].

As an example we will first use the values [28]

$$B_6^{(1/2)}(m_c) = 0.80 \pm 0.08, \quad B_8^{(3/2)}(m_c) = 0.76 \pm 0.04, \quad (7)$$

to be compared with the 2015 values $B_6^{(1/2)}(m_c) = 0.57 \pm 0.19$ and $B_8^{(3/2)}(m_c) = 0.76 \pm 0.05$ from RBC-UKQCD [36, 37]. The increase of $B_6^{(1/2)}$ could be caused, as expected from ChPT, by an improved treatment of FSI in the update from the latter lattice collaboration.

3.2. Scale uncertainties at NLO

It should be emphasized that although the NLO QCD analyses of ε'/ε in Refs. [2, 3, 4, 5, 6, 7] reduced renormalization scheme dependence in the QCDP sector, the dependence of ε'/ε on the choice of μ_t in $m_t(\mu_t)$ remained. This dependence can only be removed through the NNLO QCD calculations, but in the QCDP sector it is already weak at the NLO level because of the weak dependence of the QCDP contributions on m_t . On the other hand, as pointed out already in Ref. [8], the EWP contributions at the NLO level suffer from a number of unphysical dependences.

- First of all there is the renormalization-scheme dependence with ε'/ε in the HV scheme, as used in Refs. [6, 7], generally smaller than in the NDR scheme used in Refs. [2, 3, 4, 5]. In what follows we will consider only the NDR scheme as this is the scheme used by the RBC-UKQCD collaboration and other analyses listed above.
- The dependence on μ_t , which is much larger than in the QCDP sector because the EWP contributions exhibit much stronger dependence on m_t as pointed out 30 years ago [59, 60]. Increasing μ_t makes the value of m_t smaller, decreasing the EWP contribution and thereby making ε'/ε larger. At NLO there is no QCD correction that could cancel this effect.
- The dependence on the choice of the matching scale μ_W . It turns out that with increasing μ_W in the EWP contribution, the value of ε'/ε decreases.

One should note that the scales μ_W and μ_t can be chosen to be equal or different from each other and they could be varied independently in the ranges illustrated in Figure 1, implying significant uncertainties in the NLO prediction for ε'/ε as demonstrated in Ref. [8]. In obtaining the values in Table 1 we provide the two settings from Ref. [8]: *i*) $\mu_W = \mu_t = m_W$ and *ii*) $\mu_W = m_W$ and $\mu_t = m_t$. For example *ii*) has been used in Ref. [38]. Other choices of these scales, like $\mu_t = 300$ GeV, would significantly change the NLO values of ε'/ε with significantly reduced change when NNLO corrections to EWPs are included.

We next evaluate ε'/ε for the values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ given in (7) and

- set $\mu_W = m_W$ and $\mu_t = m_t$ in the NLO formulae in the NDR scheme,
- set isospin breaking and QED corrections to zero $\widehat{\Omega}_{\text{eff}} = 0.0$,

as done by RBC-UKQCD. This results at NLO in

$$(\varepsilon'/\varepsilon)_{\text{NLO}, \widehat{\Omega}_{\text{eff}}=0.0} = (9.4 \pm 3.5) \times 10^{-4}. \quad (8)$$

The quoted error is a guess estimate based on the uncertainties in Eq. (7) and scale uncertainties as well as the omission of isospin-breaking effects. But as we will see soon its precise size is irrelevant for the point we want to make. The result in (8) is compatible with the experimental result of Eq. (3) with a tension of 1.7σ .

At first sight it would appear that this result confirms the claims in Refs. [48, 49, 27] because the result of Eq. (8) is quite consistent with ChPT estimate $\varepsilon'/\varepsilon = (14 \pm 5) \cdot 10^{-4}$. But such a conclusion would be false as we will illustrate now.

Indeed, as stated above at the NLO level, significant dependences on μ_W and μ_t are present and the impact of a non-vanishing $\widehat{\Omega}_{\text{eff}}$ is very significant. In order to exhibit these dependences we vary in Figure 1 the matching scale μ_W independently of the scale μ_t at which the top-quark mass $m_t(\mu_t)$ is evaluated and plot ε'/ε versus μ_t for the three values of $\mu_W = \{60, 80, 120\}$ GeV. We show these significant dependences both for $\widehat{\Omega}_{\text{eff}} = 0.0$ [green] and $\widehat{\Omega}_{\text{eff}} = 0.17$ [blue].

Fortunately all these uncertainties have been significantly reduced in the NNLO matching at the electroweak scale performed in Ref. [8]. In the NDR scheme, used in all recent analyses, these corrections enhance the EWP contribution implying a *negative* shift in ε'/ε as evident from Figure 1. Including NNLO QCD corrections in question and isospin breaking corrections

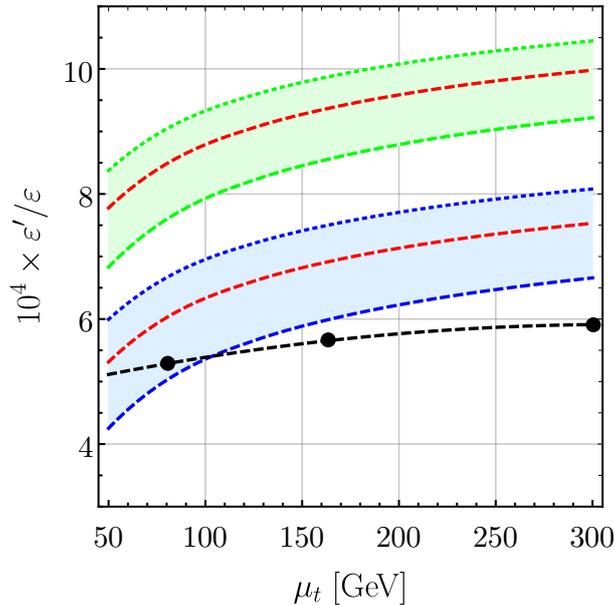


Figure 1. The dependence of ε'/ε for $B_6^{(1/2)} = 0.80$ on the scale μ_t of $m_t(\mu_t)$ for three values of the matching scale $\mu_W = \{60, 80, 120\}$ GeV [dotted, red, dashed] for $\widehat{\Omega}_{\text{eff}} = 0.0$ [green] and $\widehat{\Omega}_{\text{eff}} = 0.17$ [blue]. The black dots show the NNLO result for $\widehat{\Omega}_{\text{eff}} = 0.17$ at three scales μ_t from Ref. [8] with interpolation shown by the dashed line. We set $B_8^{(3/2)} = 0.76$. From Ref. [28].

from [49], $\widehat{\Omega}_{\text{eff}} = 0.17 \pm 0.09$, the result in Ref. (8) is changed with $\text{Im } \lambda_t = (1.43 \pm 0.04) \times 10^{-4}$ to [28]

$$\boxed{\varepsilon'/\varepsilon = (5.6 \pm 2.4) \times 10^{-4}} \quad (9)$$

Compared with the experimental value in Eq. (3), it signals an anomaly at the level of 3.3σ . But one should keep in mind that the central value in Eq. (9) will be shifted down by NNLO QCD corrections to QCDP by about 0.5×10^{-4} , as indicated in the preliminary plots in Refs. [10, 11] without modifying the error in (9). I am looking forward to the final results of these authors.

The largest remaining uncertainties in the evaluation of ε'/ε are present in the values of $\langle Q_6(m_c) \rangle_0$ (or $B_6^{(1/2)}$) and $\widehat{\Omega}_{\text{eff}}$. In Table 2 we give ε'/ε as a function of these two parameters for $B_8^{(3/2)} = 0.76$. This table should facilitate monitoring the values of ε'/ε in the SM when the LQCD calculations of hadronic matrix elements, including isospin-breaking corrections and QED effects, will improve with time. We observe a large sensitivity of ε'/ε to $B_6^{(1/2)}$, but for $B_6^{(1/2)} \geq 0.7$ also the dependence on $\widehat{\Omega}_{\text{eff}}$ is significant.

4. Summary and Outlook

Our analysis and in particular the comparison of the results in Eqs. (8) and (9), as well as Table 2, demonstrate the importance of NNLO QCD corrections and of isospin-breaking effects. Anticipating that the new RBC-UKQCD analysis will find $B_6^{(1/2)}(m_c) < 1.0$, as hinted by DQCD, the values of ε'/ε in the SM will be significantly below the data. Our example with $B_6^{(1/2)}(m_c)$ in the ballpark of 0.80 ± 0.08 illustrates a significant anomaly in ε'/ε of about 3.3σ .

However, even if ε'/ε anomaly hinted by DQCD, would be confirmed by new RBC-UKQCD results, it is very important to perform a number of the following steps:

$\widehat{\Omega}_{\text{eff}} / B_6^{(1/2)}$	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
NLO											
0.0 (A)	2.25	3.50	4.75	6.01	7.26	8.51	9.76	11.02	12.27	13.52	14.77
0.0 (B)	1.63	2.89	4.14	5.40	6.65	7.91	9.16	10.42	11.67	12.93	14.18
0.0 (C)	0.75	2.01	3.27	4.53	5.79	7.05	8.30	9.56	10.82	12.08	13.34
NNLO ($\mu_t = m_W$)											
0.0	0.02	1.28	2.54	3.80	5.06	6.32	7.58	8.83	10.09	11.35	12.61
0.10	-0.64	0.50	1.63	2.76	3.89	5.03	6.16	7.29	8.42	9.56	10.69
0.15	-0.97	0.10	1.17	2.24	3.31	4.38	5.45	6.52	7.59	8.66	9.73
0.20	-1.30	-0.29	0.71	1.72	2.73	3.74	4.74	5.75	6.76	7.76	8.77
0.25	-1.63	-0.69	0.26	1.20	2.15	3.09	4.03	4.98	5.92	6.87	7.81
0.30	-1.96	-1.08	-0.20	0.68	1.56	2.44	3.33	4.21	5.09	5.97	6.84

Table 2. The ratio $10^4 \times \varepsilon'/\varepsilon$ at NNLO for different values of the isospin corrections $\widehat{\Omega}_{\text{eff}}$ and the parameter $B_6^{(1/2)}(m_c)$ with more details in [28] and fixed value of $B_8^{(3/2)} = 0.76$ and $\text{Im } \lambda_t = 1.4 \times 10^{-4}$. In the first two rows we provide for comparison the NLO result for $\mu_t = 300 \text{ GeV}$ (A), $\mu_t = m_t$ (B) and $\mu_t = m_W$ (C), respectively. The results for $B_6^{(1/2)} \geq 1.0$ can be found in Ref. [28].

- Obtain satisfactory precision on $\langle Q_6(m_c) \rangle_0$ or $B_6^{(1/2)}$.
- Reduce the error on $\widehat{\Omega}_{\text{eff}}$. In particular isospin-breaking and QED effects should be taken into account in LQCD calculations.
- Even if the insight from DQCD allowed us to identify the dynamics (meson evolution) responsible for this anomaly, at least a second lattice QCD collaboration should calculate $K \rightarrow \pi\pi$ matrix elements and ε'/ε .
- Include the NNLO QCD corrections to the QCD penguin sector [10, 11] and the subleading NNLO QCD contributions to the electroweak penguin sector.
- Calculate BSM $K \rightarrow \pi\pi$ hadronic matrix elements of four-quark operators by lattice QCD. They are presently only known in the DQCD [16].

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